SEARCH OF GOOD ROTATION PATTERNS THROUGH EXPONENTIAL TYPE REGRESSION ESTIMATOR IN SUCCESSIVE SAMPLING OVER TWO OCCASIONS

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Abstract. The problem of estimating the finite population mean on the second (current) occasion in successive sampling over two occasions has been discussed. Utilizing all the readily available information from first and second occasions, an efficient estimation procedure through exponential type regression estimator has been developed. It has been shown that the proposed procedure is more efficient than the one recently reported by Singh and Homa [17]. Optimum replacement policy relevant to the suggested estimation procedure has been advocated. In support of the present study a numerical illustration is given.

Keywords: exponential type regression estimator, successive sampling, variance, optimum replacement policy, efficiency.

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1. Introduction

There are many useful situations where survey needs to be repeated many times. The purpose of a repeated survey is allowing one or more items to be monitored over time. If situations that are required to be monitored are concerned with very large group of individuals (population or universe), such situations are difficult, time taking and costly affairs. To meet these requirements, rotation (successive) sampling provides a strong tool for generating the reliable estimates at different occasions. In successive sampling the use of information collected on an earlier occasion in improving the current estimates is well recognized, for instance, see Jessen [1], Patterson [2], Eckler [3], Rao and Graham [4], Das [5], Singh et al. [6], among others. Sen [7] applied this theory successfully in designing the strategies

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for estimating the population mean on the current occasion using information on two auxiliary variables readily available on variables.

Sen [8], [9] extended his work for multiple auxiliary variables. Singh et al.[10] used auxiliary information on current occasion and suggested estimators of the current population mean in two-occasion successive (rotation) sampling. Feng and Zou [11] and Biradar and Singh [12] used the auxiliary information on both the occasions for estimating the current population mean in the successive sampling. Singh [13], Singh and Vishwakarma [14], [15], [16], Singh and Homa [17], Singh and Pal [18], [19] have used the auxiliary information on both the occasions and envisaged several estimators for the estimating the population mean on current (second) occasion in two-occasion successive (rotation) sampling.

In this paper, we have made an effort to propose an exponential type regression estimator for estimating the current (second) population mean in two-occasion successive (rotation) sampling. The behavior of the suggested estimators has been examined through numerical illustration and suitable recommendations are made.

2. The suggested estimator

Let $U = (U_1, U_2, ..., U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study be denoted by $x(y)$ on the first (second) occasions respectively. We assume that information on an auxiliary variable z (stable over occasion), whose population mean $\bar{Z}$ is known, is available on both the occasions. It is assumed that the auxiliary variable z is correlated to x and y on the first and second occasions respectively. A simple random sample S_n of size n is drawn without replacement on the first occasion and a random sub sample S_m of the size $m\ (= n\lambda)$ units from S_n is retained (matched) for its use on the current (second) occasion. A fresh unmatched sample S_u of size $u = (n-m) = n\theta$ units is drawn on the current (second) occasion from the whole population by simple random sample (without replacement) procedure so that the sample size on the current (second) occasion remains. n, λ and θ $(\lambda + \theta = 1)$ are the fractions of the matched and fresh samples, respectively, on the current occasion. The values of λ and θ should be selected optimally. For the simplicity, we assume that the population size N is large enough so that the finite population correction (fpc) terms are ignored.

In what follows we shall use the following notations throughout this paper.

- $\bar{X}$, $\bar{Y}$, $\bar{Z}$: The population means of the variables x, y and z respectively.
- $\bar{x_n}$, $\bar{x_m}$, $\bar{y_u}$, $\bar{y_m}$, $\bar{z_m}$, $\bar{z_n}$, $\bar{z_u}$: The sample means of the respective variables based on the sample sizes indicated in subscripts.
- ρ_{yx}, ρ_{yz}, ρ_{xz}: The correlation coefficients between the variables shown in subscripts.
- S_x^2, S_y^2, S_z^2: The population mean square of x, y, z respectively.
- $\beta_{yx\cdot z}$: The population partial regression coefficient of y on x for fixed z.

- $R_x = \frac{\bar{Y}}{\bar{X}}$, $R_z = \frac{\bar{Y}}{\bar{Z}}$.
- $\beta_{yz \cdot x}$: The population partial regression coefficient of y on z for fixed x.
- S_{yx}, S_{yz}, S_{zx}: The population covariance between the variables shown in the subscripts.
- β_{yx}: The population regression coefficient of y on x.
- β_{yz}: The population regression coefficient of y on z.
- β_{zx}: The population regression coefficient of z on x.
- β_{xz}: The population regression coefficient of x on z.
- $\beta^{(m)}_{yx \cdot z}$: The sample partial regression coefficient of y on x for fixed z based on the sample size m.
- $\beta^{(m)}_{yz \cdot x}$: The sample partial regression coefficient of y on z for fixed x based on sample size m,
- $\beta^{(n)}_{yz}$: The sample regression coefficient of y on z based on sample size n.
- C_x, C_y, C_z: The population coefficient of variation of the variables given in the subscript.

To estimate the population mean $\bar{Y}$ on the current (second) occasion, two classes of estimators can be formed. First, based on sample of size $(u = n\theta)$drawn afresh on the second occasion and second based on the matched sample of side $(m = n\lambda)$ common to both the occasions.

2.1. A class of estimators based on the sample drawn afresh on the second occasion

Let $(\bar{y}_u, \bar{z}_u)$, be the unbiased estimators of population means $(\bar{Y}, \bar{Z})$ respectively based on u units. We consider a class of estimators of the population mean$\bar{Y}$, defined by

$$d_u = \bar{y}_u \exp\left[\frac{\alpha_0 \ (\bar{Z} - \bar{z}_u)}{(\bar{Z} + \bar{z}_u)}\right], \tag{2.1}$$

where α_0 is a constant to be determined such that variance of d_u is least. For $\alpha_0 = 1$, it reduces to Singh and Homa [17] estimator.

To obtain the variance of d_u, we write

$$\bar{y}_u = \bar{Y}(1 + e_{yu}), \ \bar{z}_u = \bar{Z}(1 + e_{zu})$$

such that

$$E(e_{yu}) = E(e_{zu}) = 0$$

and

$$\begin{cases} E(e^2_{yu}) &= (C^2_y/u), \\ E(e^2_{zu}) &= (C^2_z/u), \\ E(e^2_{yu}e^2_{zu}) &= (1/u)\rho_{yz}C_yC_z. \end{cases} \tag{2.2}$$

Expressing (2.1) in terms of e_{yu} and e_{zu} we have

$$\begin{aligned} d_u &= \bar{Y}(1+e_{yu})exp[(-\alpha_0 e_{zu})/(2+e_{zu})] \\ &= \bar{Y}(1+e_{yu})exp[(-\alpha_0 e_{zu}/2)(1+(e_{zu}/2))^{-1}] \\ &\cong \bar{Y}[(1+e_{yu})-(\alpha_0 e_{zu}/2)], \end{aligned}$$

or

$$(d_u-\bar{Y})=\bar{Y}[e_{yu}-(\alpha_0\ e_{zu}/2)]. \quad (2.3)$$

Squaring both sides of (2.3) we have

$$(d_u-\bar{Y})^2=\bar{Y}^2(e_{yu}^2+(\alpha_0^2 e_{zu}^2/4)-\alpha_0 e_{yu}e_{zu}). \quad (2.4)$$

Taking expectation of both sides of (2.4) we get the variance of d_u to the first degree of approximation (ignoring (fpc) term) as

$$Var(d_u)=(1/u)[S_y^2+(\alpha_0/4)R_z S_z^2(\alpha_0-4(\beta_{yz}/R_z))], \quad (2.5)$$

which is minimized for

$$\alpha_0=2(\beta_{yz}/R_z)=\alpha_{0(opt)}. \quad (2.6)$$

Substitution of $\alpha_{0(opt)}$ in place of α_0 in (2.1) d_u yields the asymptotically optimum estimator (AOE) for $\bar{Y}$ as

$$d_u^{(0)}=\bar{y}_u \exp\left[\frac{2\beta_{yz}\ (\bar{Z}-\bar{z}_u)}{R_z(\bar{Z}+\bar{z}_u)}\right]. \quad (2.7)$$

Putting $\alpha_{0(opt)}$ in (2.5) we get the minimum variance of d_u or the variance of the AOE $d_u^{(0)}$ as

$$Min.Var(d_u)=Var(d_u^{(0)})=(S_y^2/u)(1-\rho_{yz}^2) \quad (2.8)$$

It is to be mentioned that the $AOEd_u^{(0)}$ in (2.7) depends on the unknown parameters (β_{yz},R_z) which lacks the practical utility of $AOE\ d_u^{(0)}$. However, the values of parameters (β_{yz}, R_z) can be guessed quite accurately either through past data or experience gathered in due course of time. For discussion on this subject the reader is referred to Reddy [20], Srivenkataramana and Tracy [21], and Singh and Ruiz-Espezo [22] among others. If the guessed values of the parameters (β_{yz}, R_z) are not available, then it is worth advisable to replace (β_{yz}, R_z) by their consistent estimators $[(b_{yz}^{(u)}, \hat{R}_z^{(u)})=(\bar{y}_u/\bar{Z})$, ($\bar{Z}$ is the known population mean of the variable z)] respectively in (2.7). Thus, we get an estimator of the population mean $\bar{Y}$ based on estimated optimum value $\hat{\alpha}_{0(opt)}=b_{yz}^{(u)}\bar{Z}/\bar{y}_u$ as

$$\hat{d}_u^{(0)}=\bar{y}_u \exp\left[\frac{2b_{yz}^{(u)}\bar{Z}(\bar{Z}-\bar{z}_u)}{\bar{y}_u(\bar{Z}+\bar{z}_u)}\right]. \quad (2.9)$$

Following [Sukhatme et al. ([23], p. 238-239)], it can be shown that

$$E(b_{yz}^{(u)}) = \beta_{yz} + \left(\frac{1}{u}\right)\left(\frac{N-n}{N-2}\right)\beta_{yz}\left(\frac{\mu_{003}}{\mu_{002}} - \frac{\mu_{012}}{\mu_{011}}\right),$$

or

$$E(b_{yz}^{(u)}) = \beta_{yz} + \left(\frac{1}{u}\right)\left(\frac{N}{N-2}\right)\left(1-\frac{u}{N}\right)\beta_{yz}\left(\frac{\mu_{003}}{\mu_{002}} - \frac{\mu_{012}}{\mu_{011}}\right),$$

where $\mu_{rst} = E[(x_i - \bar{X})^r(y_i - \bar{Y})^s(z_i - \bar{Z})^t]$, (r, s, t) being non-negative integers. For large N,$(N/(N-2)) \cong 1$ and $(u/N) \cong 0$, we have

$$E(b_{yz}^{(u)}) = \beta_{yz} + \left(\frac{\beta_{yz}}{u}\right)\left(\frac{\mu_{003}}{\mu_{002}} - \frac{\mu_{012}}{\mu_{011}}\right). \tag{2.10}$$

which can be written as

$$E(b_{yz}^{(u)} = \beta_{yz} + O(u^{-1}).$$

Also from (2.10) we have

$$E\left(\frac{b_{yz}^{(u)} - \beta_{yz}}{\beta_{yz}}\right) = O(u^{-1}) \Longrightarrow E(\delta b_{yz}^{(u)}) = O(u^{-1}).$$

Now, expressing (2.9) in terms of e_{zu}, e_{yu} and $\delta b_{yz}^{(u)}$ we have

$$\begin{aligned}
\hat{d}_u^{(0)} &= \bar{Y}(1+e_{yu})exp\left[\frac{(-2\beta_{yz}(1+\delta\ b_{yz}^{(u)})\bar{Z}e_{zu})}{(\bar{Y}(1+e_{yz})(2+e_{zu}))}\right] \\
&= \bar{Y}(1+e_{yu})exp[(-\beta_{yz}\bar{Z}(1+\delta\ b_{yz}^{(u)})e_{zu})/(\bar{Y}(1+e_{yu})(1+(e_{zu}/2))^{-1}] \\
&= \bar{Y}(1+e_{yu})[1-\{(-\beta_{yz}\bar{Z}(1+\delta\ b_{yz}^{(u)})e_{zu})/(\bar{Y}(1+e_{yu}))(1+(e_{zu}/2))^{-1}\}+\cdots] \\
&= \bar{Y}(1+e_{yu}) - \beta_{yz}\bar{Z}e_{zu},
\end{aligned}$$

or

$$(\hat{d}_u^{(0)} - \bar{Y}) = (\bar{Y}e_{yu} - \beta_{yz}\bar{Z}e_{zu}). \tag{2.11}$$

Squaring both sides of (2.11) we have

$$(\hat{d}_u^{(0)} - \bar{Y})^2 = (\bar{Y}^2e_{yu}^2 + \beta_{yz}^2\bar{Z}^2e_{zu}^2 - 2\beta_{yz}\bar{Y}\bar{Z}e_{yu}e_{zu}). \tag{2.12}$$

Taking expectation of both sides of (2.12), we get the variance of $\hat{d}_u^{(0)}$ to the first degree of approximation (ignoring fpc terms) as

$$Var(\hat{d}_u^{(0)}) = (1/u)(S_y^2 + \beta_{yz}^2S_z^2 - 2\beta_{yz}^2S_z^2) = (1/u)S_y^2(1-\rho_{yz}^2), \tag{2.13}$$

which equals to the minimum variance of d_u given by (2.8). Thus, the estimator $\hat{d}_u^{(0)}$ based on estimated optimum value can be used in practice.

2.2. A class of estimators based on the matched sample common to both occasions

We define the following class of estimators for population mean $\bar{Y}$ as

$$d_m = \bar{y}_m \exp\left[\frac{\alpha_1(\bar{x}_n - \bar{x}_m)}{(\bar{x}_n + \bar{x}_m)} + \frac{\alpha_2(\bar{z}_n - \bar{z}_m)}{(\bar{z}_n + \bar{z}_m)} + \frac{\alpha_3(\bar{Z} - \bar{z}_n)}{(\bar{Z} + \bar{z}_n)}\right] \tag{2.14}$$

where $\alpha_i's$ $(i = 1, 2, 3)$ are suitably chosen scalars.

To obtain the variance of d_m we write $\bar{y}_m = \bar{Y}(1 + e_{ym})$, $\bar{x}_m = \bar{X}(1 + e_{xm})$, $\bar{z}_m = \bar{Z}(1 + e_{zm})$, $\bar{x}_n = \bar{X}(1 + e_{xn})$ and $\bar{z}_n = \bar{Z}(1 + e_{zn})$, such that

$$E(e_{ym}) = E(e_{xm}) = E(e_{xn}) = E(e_{zm}) = E(e_{zn}) = 0$$

and, ignoring the finite population correction (fpc) terms, we have

$$\begin{aligned}
&E(e_{ym}^2) = m^{-1}C_y^2,\ E(e_{xm}^2) = m^{-1}C_x^2,\ E(e_{zm}^2) = m^{-1}C_z^2\\
&E(e_{xn}^2) = n^{-1}C_x^2,\ E(e_{zn}^2) = n^{-1}C_z^2,\\
&E(e_{ym}e_{xm}) = m^{-1}\rho_{yx}C_yC_x,\ E(e_{ym}e_{xn}) = n^{-1}\rho_{yx}C_yC_x,\ E(e_{ym}e_{zm}) = m^{-1}\rho_{yz}C_yC_z,\\
&E(e_{xm}e_{xn}) = n^{-1}C_x^2,\ E(e_{xm}e_{zm}) = m^{-1}\rho_{xz}C_xC_z,\ E(e_{xm}e_{zn}) = n^{-1}\rho_{xz}C_xC_z,\\
&E(e_{xn}e_{zm}) = n^{-1}\rho_{xz}C_xC_z,\ E(e_{xn}e_{zn}) = n^{-1}\rho_{xz}C_xC_z \text{ and } E(e_{zm}e_{zn}) = n^{-1}C_z^2,
\end{aligned}$$

Expressing (2.14) in terms ofe_{ym}, e_{xm}, e_{zm}, e_{xn} and e_{zn} we have

$$\begin{aligned}
d_m &= \bar{Y}(1 + e_{ym})\exp[-(\alpha_1/2)(e_{xm} - e_{xn})\{1 + ((e_{xm} + e_{xn})/2)\}^{-1}\\
&\quad -(\alpha_2/2)(e_{zm} - e_{zn})\{1 + ((e_{zm} + e_{zn})/2)\}^{-1} - (\alpha_3)(e_{zn}/2)\{1 + (e_{zn}/2)\}^{-1}]\\
&\cong \bar{Y}[1 + e_{ym} - (\alpha_1/2)(e_{xm} - e_{xn}) - (\alpha_2/2)(e_{zm} - e_{zn}) - (\alpha_3/2)(e_{zn})]
\end{aligned}$$

or

$$(d_m - \bar{Y}) = \bar{Y}[e_{ym} - (\alpha_1/2)(e_{xm} - e_{xn}) - (\alpha_2/2)(e_{zm} - e_{zn}) - (\alpha_3/2)e_{zn}]. \tag{2.15}$$

Squaring both sides of (15), we have

$$\begin{aligned}
(d_m - \bar{Y})^2 = \bar{Y}^2[&e_{ym}^2 + (\alpha_1^2/4)(e_{xm} - e_{xn})^2 + (\alpha_2^2/4)(e_{zm} - e_{zn})^2\\
&+ (\alpha_3^2/4)(e_{zn}^2) - \alpha_1(e_{ym}e_{xm} - e_{ym}e_{xn})\\
&- \alpha_2(e_{ym}e_{zm} - e_{ym}e_{zn}) - \alpha_3(e_{ym}e_{zn})\\
&+ (\alpha_1\alpha_2/2)(e_{xm}e_{zm} - e_{xn}e_{zm} - e_{xm}e_{zn} + e_{xn}e_{zn})\\
&+ (\alpha_1\alpha_3/2)(e_{xm}e_{zn} - e_{xn}e_{zn}) + (\alpha_2\alpha_3/2)(e_{zm}e_{zn} - e_{zn}^2)].
\end{aligned} \tag{2.16}$$

Taking expectation of both sides (2.16), we get the variance of d_m to the first degree of approximation (ignoring fpc terms) as

$$
\begin{aligned}
Var(d_m) &= [(1/m)S_y^2 + ((1/m) - (1/n))(1/4)\{R_x^2 S_x^2 \alpha_1^2 + R_z^2 S_z^2 \alpha_2^2 \\
&\quad - 4\alpha_1 R_x \rho_{yx} S_y S_x - 4\alpha_2 R_z \rho_{yz} S_y S_z + 2\alpha_1 \alpha_2 R_x R_z \rho_{xz} S_x S_z\} \\
&\quad + (1/4n)(R_z^2 S_z^2 \alpha_3^2 - 4\alpha_3 R_z S_y S_z \rho_{yz})] \\
&= [(1/m)S_y^2 + ((1/m) - (1/n))(1/4)\{R_x^2 S_x^2 \alpha_1(\alpha_1 - (4\beta_{yx}/R_x)) \\
&\quad + R_z^2 S_z^2 \alpha_2(\alpha_2 - (4\beta_{yz}/R_z)) + 2\alpha_1\alpha_2 R_x R_z \rho_{xz} S_x S_z\} \\
&\quad + (R_z^2 S_z^2 \alpha_3/4n)(\alpha_3 - (4\beta_{yz}/R_z))].
\end{aligned} \tag{2.17}
$$

Differentiating (2.17) with respect to $\alpha_1, \alpha_2, \alpha_3$ and equating them to zero, we get

$$
\begin{cases}
\{\alpha_1 + \alpha_2 R_z(\beta_{zx}/R_x)\} = 2(\beta_{yx}/R_x), \\
\{\alpha_1 R_x(\beta_{zx}/R_z) + \alpha_2\} = 2(\beta_{yz}/R_z), \\
\alpha_3 = 2(\beta_{yz}/R_z).
\end{cases} \tag{2.18}
$$

Solving (2.18) we get the optimum values of α_1,α_2 and α_3 respectively, as

$$
\begin{cases}
\alpha_{1(opt)} = 2\{(\beta_{yx} - \beta_{yz}\beta_{zx})/R_x(1 - \beta_{xz}\beta_{zx})\} = 2(\beta_{yx.z}/R_x), \\
\alpha_{2(opt)} = 2\{(\beta_{yz} - \beta_{yx}\beta_{xz})/R_x(1 - \beta_{xz}\beta_{zx})\} = 2(\beta_{yz.x}/R_z), \\
\alpha_{3(opt)} = 2(\beta_{yz}/R_z).
\end{cases} \tag{2.19}
$$

Substitution of the optimum values $\alpha_{i(opt)}$'s $(i = 1, 2, 3)$ in (2.14) we get the asymptotically optimum estimator (AOE) in the class of estimators d_m as

$$
d_m^{(0)} = \bar{y}_m \exp\left[\frac{2\left(\frac{\beta_{yx.z}}{R_x}\right)(\bar{x}_n - \bar{x}_m)}{(\bar{x}_n + \bar{x}_m)} + \frac{2\left(\frac{\beta_{yz.x}}{R_z}\right)(\bar{z}_n - \bar{z}_m}{(\bar{z}_n + \bar{z}_m)} + \frac{2\left(\frac{\beta_{yz}}{R_z}\right)(\bar{Z} - \bar{z}_n)}{(\bar{Z} + \bar{z}_n)}\right] \tag{2.20}
$$

Substituting the optimum values $\alpha_{i(opt)}$ $(i{=}1, 2, 3)$ in the place of α_i's $(i{=}1, 2, 3)$ in (2.17) we get the minimum variance of d_m or the variance of the AOE $d_m^{(0)}$ as

$$
Min.Var(d_m) = (S_y^2/m)[1 - \rho_{yz}^2 - \theta(R_{y.xz}^2 - \rho_{yz}^2)] = Var(d_m^{(0)}), \tag{2.21}
$$

where $R_{y.xz}^2 = (\rho_{yx}^2 + \rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{zx})/(1 - \rho_{xz}^2)$. It is observed from (2.20) that the AOE depends on the unknown parameters $(\beta_{yz}, \beta_{yx.z}, \beta_{yz.x}, R_x, R_z)$ which limits the practical utility of the AOE $d_m^{(0)}$. However, one can get the guessed values of the parameters$(\beta_{yz}, \beta_{yx.z}, \beta_{yz.x}, R_x, R_z)$ quite accurately either through the past data or experience gathered in due course of time. For detailed discussion on this issue the reader is referred to Murthy [24], Reddy [20], and Srivenkataramana and Tracy [21], among others. Thus, the guessed values of the parameters $(\beta_{yz}, \beta_{yx.z}, \beta_{yz.x}, R_x, R_z)$ can be obtained in practice and used in the AOE for its use in practice. Alternately, if the guessed values of the parameters $(\beta_{yz}, \beta_{yx.z}, \beta_{yz.x}, R_x, R_z)$ cannot be made available, then it is worth advisable to replace them by their consistent estimators $(b_{yz}^{(n)}, b_{yx.z}^{(m)}, b_{yz.x}^{(m)},\ \hat{R}_x = (\bar{y}_m/\bar{x}_n),$

$\hat{R}_z = (\bar{y}_m/\bar{Z}))$ as respectively in (2.20). Thus, we define an estimator of the population $\bar{Y}$ (based on the estimated optimum values) as

$$\hat{d}_m^{(0)} = \bar{y}_m \exp\left[\left\{2b_{yx.z}^{(m)} \frac{\bar{x}_n(\bar{x}_n - \bar{x}_m)}{\bar{y}_m(\bar{x}_n + \bar{x}_m)}\right\} + \left\{2b_{yz.x}^{(m)} \frac{\bar{Z}(\bar{z}_n - \bar{z}_m)}{(\bar{z}_n + \bar{z}_m)}\right\} + \left\{2b_{yz}^{(n)} \frac{\bar{Z}(\bar{Z} - \bar{z}_n)}{\bar{y}_n(\bar{z}_n + \bar{Z})}\right\}\right], \tag{2.22}$$

To obtain the variance of (2.22) we further, write

$$b_{yz}^{(n)} = \beta_{yz}(1 + \delta b_{yz}^{(n)}),\ b_{yx.z}^{(m)} = \beta_{yx.z}(1 + \delta b_{yx.z}^{(m)}),\ b_{yz.x}^{(m)} = \beta_{yz.x}(1 + \delta b_{yz.x}^{(m)})$$

such that

$$E(b_{yz}^{(n)}) = \beta_{yz} + O(n^{-1}) \Longrightarrow E(\delta b_{yz}^{(n)}) = O(n^{-1}),$$
$$E(b_{yx.z}^{(m)}) = \beta_{yz} + O(m^{-1}) \Longrightarrow E(\delta b_{yx.z}^{(m)}) = O(m^{-1}),$$
$$E(b_{yz.x}^{(n)}) = \beta_{yz} + O(m^{-1}) \Longrightarrow E(\delta b_{yz.x}^{(m)}) = O(m^{-1}).$$

Expressing (2.22) in terms of $e_{ym}, e_{xm}, e_{zm}, e_{xn}, e_{zn}, \delta b_{yz}^{(n)}, \delta b_{yx.z}^{(m)}$ and $\delta b_{yz.x}^{(m)}$, we have

$$\hat{d}_m^{(0)} = \bar{Y}(1+e_{ym}) \exp\left[\left\{\frac{-(\beta_{yx.z}\bar{X}(1+\delta b_{yx.z}^{(m)})(1+e_{xn}))(e_{xm}-e_{xn})}{\bar{Y}(1+e_{ym})}\right\}\left\{1+\frac{(e_{xm}+e_{xn})}{2}\right\}^{-1} - \left\{\frac{(\beta_{yz.x}\bar{Z}(1 + \delta b_{yz.x}^{(m)}))(e_{zm} - e_{zn})}{\bar{Y}(1 + e_{ym})}\right\}\left\{1 + \frac{(e_{zm} + e_{zn})}{2}\right\}^{-1} - \left\{\frac{(\beta_{yz}(1 + \delta b_{yz}^{(n)})\bar{Z}e_{zn})}{\bar{Y}(1 + e_{ym})}\right\}(1 + e_{zn})^{-1}\right]$$

which may be approximately written as

$$\hat{d}_m^{(0)} \cong [\bar{Y}(1 + e_{ym}) - \beta_{yx.z}\bar{Z}(e_{xm} - e_{xn}) - \beta_{yz.x}\bar{Z}(e_{zm} - e_{zn}) - \beta_{yz}\bar{Z}e_{zn}],$$

or

$$(\hat{d}_m^{(0)} - \bar{Y}) \cong [\bar{Y}e_{ym} - \beta_{yx.z}\bar{Z}(e_{xm} - e_{xn}) - \beta_{yz.x}\bar{Z}(e_{zm} - e_{zn}) - \beta_{yz}\bar{Z}e_{zn}]. \tag{2.23}$$

Squaring both sides of (2.23) and then taking the expectation, we get the variance $\hat{d}_m^{(0)}$ to the first degree of approximation (ignoring fpc terms) as

$$Var(\hat{d}_m^{(0)}) = (S_y^2/m)[1 - \rho_{yz}^2 - \theta(R_{y.xz}^2 - \rho_{yz}^2)], \tag{2.24}$$

which equals to the minimum variance of d_m given by (2.21). Thus, the estimator $\hat{d}_m^{(0)}$ in (2.22) is more useful in practice as it does not depend on the unknown constant.

2.3. Combined estimator of the population mean on second (current) occasion

Now, for estimating the population mean $\bar{Y}$ on second (current) occasion, we define the following estimator of $\bar{Y}$ as

$$d = [(1-\phi)\hat{d}_u^{(0)} + \phi\hat{d}_m^{(0)}], \tag{2.25}$$

where ϕ is a constant to be determined under certain criterion, and $\hat{d}_u^{(0)}$ and $\hat{d}_m^{(0)}$ are respectively defined in (2.9) and (2.22). The variance of d is given by

$$Var(d) = [(1-\phi)^2 Var(\hat{d}_u^{(0)}) + \phi^2 Var(\hat{d}_m^{(0)}) + 2Cav(\hat{d}_u^{(0)}, \hat{d}_m^{(0)})].$$

Under the supposition that population size N is large enough so that finite population correction (fpc) terms, are ignored [i.e., $(n/N) \cong 0$, $(m/N) \cong 0$, $(u/N) \cong 0$], the covariance term will be negligible,i.e., $Cav(\hat{d}_u^{(0)}, \hat{d}_m^{(0)}) \cong 0$. We note that to the first order of approximation, $Var(d) \cong MSE(d)$, where $MSE(\bullet)$ stands for mean square error $(\bullet)$.

Now, using expressions in (2.13) and (2.24), we get the variance of d to the first degree of approximation (ignoring fpc terms) as

$$\begin{aligned} Var(d) = {} & (S_y^2/n\theta(1-\theta))[(1-\theta)(1-\rho_{yz}^2)+\phi^2\{(1-\rho_{yz}^2)-\theta^2(R_{y.xz}^2-\rho_{yz}^2)\} \\ & - 2\phi(1-\theta)(1-\rho_{yz}^2)], \end{aligned} \tag{2.26}$$

which is minimum when

$$\phi = \frac{(1-\theta)(1-\rho_{yz}^2)}{\{(1-\rho_{yz}^2)-\theta^2(R_{y.xz}^2-\rho_{yz}^2)\}} = \phi_(opt), (say). \tag{2.27}$$

Thus, the resulting minimum variance of d is given by

$$Min.Var(d) = \left(\frac{S_y^2}{n}\right)\frac{[(1-\rho_{yz}^2)-\theta(R_{y.xz}^2-\rho_{yz}^2)]}{[(1-\rho_{yz}^2)-\theta^2(R_{y.xz}^2-\rho_{yz}^2)]}. \tag{2.28}$$

Singh and Homa [17] have proposed an estimator of the population mean $\bar{Y}$ as

$$d_1 = [(1-\phi_1)d_{1u} + \phi_1 d_{1m}], \tag{2.29}$$

where ϕ_1 is unknown constant to be determined so as to make the estimator d_1 is more precise,

$$d_{1u} = \bar{y}_u exp\left[\frac{(\bar{Z}-\bar{Z}_u)}{(\bar{Z}+\bar{Z}_u)}\right] \tag{2.30}$$

is an estimator of the population mean $\bar{Y}$ based on the fresh sample of size $u = (n\theta)$ drawn on the second occasion, for instance, see Bahl and Tuteja [25];

$$d_{1m} = \bar{y}_m \exp\left[\frac{(\bar{Z}-\bar{Z}_m)}{(\bar{Z}+\bar{Z}_m)}\right] + b_{yx}^{(m)}\left[\bar{x}_n \exp\frac{(\bar{Z}-\bar{Z}_n)}{(\bar{Z}+\bar{Z}_n)} - \bar{x}_m \exp\frac{(\bar{Z}-\bar{Z}_m)}{(\bar{Z}+\bar{Z}_m)}\right], \tag{2.31}$$

is an estimator based on the sample of size $m(=n(1-\theta))$ common to both the occasions. Singh and Homa [17] have derived the mean squared error / variance of d_1 up to first order of approximation under the supposition $(C_x \cong C_y \cong C_z)$.

Putting $\alpha_0 = 1$ and$(C_x \cong C_y \cong C_z)$ in (2.5), we get the variance of the estimator d_{1u} due to Singh and Homa [17] to the first degree of approximation (ignoring fpc term) as

$$Var(d_{1u}) = (S_y^2/u)\{(5/4) - \rho_{yz}\}. \tag{2.32}$$

When $(C_x \cong C_y \cong C_z)$, the correct variance expression of the estimator d_{1m} to the first degree of approximation (ignoring fpc terms) is given by

$$\begin{aligned} Var(d_{1m}) &= S_y^2[(1/m)\{(5/4) - \rho_{yz}\} \\ &-\{(1/m) - (1/n)\}\{\rho_{yx}^2((3/4) + \rho_{xz}) - \rho_{yx}(\rho_{yz} + \rho_{xz} - (1/2))\}]. \end{aligned} \tag{2.33}$$

From (2.24) and (2.33) we have

$$\begin{aligned} Var(d_{1m}) - Var(\hat{d}_m^{(0)}) &= (S_y^2/m)[(1-\theta)((1/2) - \rho_{yz})^2 \\ &+\theta\{[\rho_{yx}((1/2)-\rho_{xz})-((1/2)-\rho_{yz})]^2+\rho_{xz}^2(\rho_{yz}-\rho_{xz}\rho_{yx})^2/(1-\rho_{xz}^2)\}], \end{aligned} \tag{2.34}$$

which is positive. Thus, the proposed estimator$\hat{d}_m^{(0)}$ is more efficient than the estimator d_{1m} due to Singh and Homa [17]. So the use of $\hat{d}_m^{(0)}$ would be more profitable than the one considered by Singh and Homa [17].

It is to be noted that the variance expression of d_{1m} in (2.33) is correct while the variance expression of the estimatord_{1m} due to Singh and Homa [17] is incorrect. The variance of d_{1m} to the first degree of approximation (ignoring fpc terms) due to Singh and Homa [17] is given by

$$\begin{aligned} Var(d_{1m}) &= S_y^2[(1/m)\{-\rho_{yx}^2((3/4) + \rho_{xz}) + \rho_{yx}(\rho_{yz} + \rho_{xz}-(1/2))-\rho_{yz}\} \\ &+ (1/n)\rho_{yx}\{((-3/4)\rho_{yx}) + (\rho_{xz}\rho_{yx}-\rho_{xz}+(1/2))\}]. \end{aligned} \tag{2.35}$$

[See Singh and Homa ([17] eq. (13), p.150), for large population size N, i.e., for large N, i.e., $(n/N) \cong 0$, $(m/N) \cong 0$, $(u/N) \cong 0$, $(1/N) \cong 0$,]

Using expressions (2.32) and (2.33), the minimum variance of Singh and Homa [17] estimator d_1 is given by

$$Min.Var(d_1) = \frac{S_y^2\delta_0(\delta_0 - \theta A)}{n(\delta_0 - \theta^2 A)}, \tag{2.36}$$

for the optimum value of ϕ_1:

$$\phi_{1(opt)} = \frac{\delta_0(1-\theta)}{(\delta_0 - \theta^2 A)}, \tag{2.37}$$

where $\delta_0 = ((5/4) - \rho_{yz})$ and $A = [\rho_{yx}^2((3/4) + \rho_{xz}) - \rho_{yx}(\rho_{yz} + \rho_{xz} - (1/2))]$. From (2.28) and (2.36) we have

$$\begin{aligned} &Min.Var(d_1) - Min.Var(d) \\ &= \frac{S_y^2[\delta_0(1-\rho_{yz}^2)A^* + A^*\{(1-\rho_{yz}^2)-\theta(R_{y.xz}^2-\rho_{yz}^2)\}((1/2)-\rho_{yz})^2]}{n(\delta_0-\theta^2 A)[(1-\rho_{yz}^2)-\theta^2(R_{y.xz}^2-\rho_{yz}^2)]} \geq 0, \end{aligned} \tag{2.38}$$

where

$$A^* = [(1-\theta)((1/2)-\rho_{yz})^2 + \theta\{[\rho_{yx}((1/2)-\rho_{xz}) - ((1/2)-\rho_{yz})]^2 + (\rho_{xz}^2(\rho_{yz}-\rho_{yx}\rho_{xz})^2/(1-\rho_{xz}^2))\}]$$

and

$$A_1^* = (\delta_0 - \theta A).$$

It follows from (2.38) that the proposed estimator d is more efficient than the Singh and Homa [17] estimator d_1.

Minimizing (2.36) with respect to θ we obtain the optimum value of θ as

$$\theta_{opt} = \frac{\delta_0 \pm \sqrt{\delta_0(\delta_0 - A)}}{A}. \tag{2.39}$$

The real values of θ_{opt} exist; if $(\delta_0 - A) \geq 0$. For any combination of correlation coefficients $(\rho_{yx}, \rho_{yz}, \rho_{zx})$ that satisfies the condition of real solution, two real values of θ_{opt} are possible. Hence, while selecting the value of θ_{opt}, it should be remembered that $0 \leq \theta_{opt} \leq 1$, all other values of are inadmissible. If both the solutions of θ_{opt} are admissible, the lowest one is the best choice as it will reduce the cost of survey. Putting the admissible value of θ_{opt}, say θ_0^*, from (2.39) in (2.36), we get the resulting value of $Min.Var(d_1)$ as

$$Min.Var(d_1)_{opt} = \frac{S_y^2\delta_0(\delta_0 - \theta_0^* A)}{n(\delta_0 - \theta_0^{*2} A)}. \tag{2.40}$$

3. Optimum replacement policy

To obtain the optimum value of θ (fraction of a sample to be taken fresh at the second occasion) so that the population mean $\bar{Y}$ may be estimated with maximum precision. Minimization of minimum variance of d in (2.28) with respect to θ yields the optimum value of θ as

$$\hat{\theta} = \frac{1 \pm \sqrt{(1-\rho_{yx.z}^2)}}{\rho_{yx.z}^2} = \theta_{(opt)}(say), \tag{3.1}$$

where

$$\rho_{yx.z} = (\rho_{yx} - \rho_{yz}\rho_{xz})/\sqrt{(1-\rho_{yx}^2)}\sqrt{(1-\rho_{yx}^2)}, \tag{3.2}$$

Since $(1-\rho_{yx.z}^2) \geq 0$, for any combination ofρ_{yx}, ρ_{yz} and ρ_{xz} two real values of $\hat{\theta}$ are possible, hence, to select a value of $\hat{\theta}$, it should be remembered that $0 \leq \hat{\theta} \leq 1$. All other values of $\hat{\theta}$ are inadmissible. In case if both the solutions of $\hat{\theta}$ are admissible, we select the minimum of these two as θ_0, where

$$\theta_0 = \frac{\left\{1 - \sqrt{(1-\rho_{yx.z}^2)}\right\}}{\rho_{yx.z}^2} \tag{3.3}$$

Putting θ_0 in (2.28) we get the resulting value of $Min.Var(d)$ as

$$Min.Var(d)_{opt} = \frac{S_y^2(1-\rho_{yx}^2)\left[1-\sqrt{(1-\rho_{yx.z}^2)}\right]}{2n}. \tag{3.4}$$

4. Efficiency comparisons

The percent relative efficiency of the proposed estimator d with respect to (i)$\bar{y}_n$, when there is no matching, (ii) $\bar{y}_\psi = (1-\psi)+\psi\bar{y}_{lm}$, when no auxiliary information is used at any occasion, where $[\bar{y}_{lm} = \bar{y}_m + b_{yx}^{(m)}(\bar{x}_n - \bar{x}_m)]$; ψ being suitably chosen scalar and (iii) Singh and Homa's [17] estimator d_1; have been obtained for different choices of ρ_{xz}, ρ_{yz} and ρ_{yx}.

To the first degree of approximation and for large population size N, i.e., $N \longrightarrow \infty$, i.e., $[(n/N) \cong 0,\ (m/N) \cong 0, (u/N) \cong 0,\ (1/N) \cong 0]$, the minimum variance of $\bar{y}_\psi$ is given by

$$Min.Var(\bar{y}_\psi) = \frac{S_y^2(1-\theta\rho_{yx}^2)}{n(1-\theta^2\rho_{yx}^2)}. \tag{4.1}$$

for the optimal value of ψ:

$$\psi_{opt} = (1-\theta)/(1-\theta^2\rho_{yx}^2). \tag{4.2}$$

Minimization of (4.1) with respect to θ yields the optimal value of θ as

$$\theta_{opt} = \left[1+\sqrt{(1-\rho_{yz}^2)}\right]^{-1}. \tag{4.3}$$

Thus, the resulting value of $Min.Var(\bar{y}_\psi$ is given by

$$Min.Var(\bar{y}_\psi)_{opt} = \frac{S_y^2}{2n}\left[1+\sqrt{(1-\rho_{yz}^2)}\ \right] \tag{4.4}$$

[see Cochran ([26], p. 346)].

The variance of $\bar{y}_n$ for large population size N, i.e., $[N \longrightarrow \infty$, i.e., $(n/N) \cong 0]$ is given by $Var(\bar{y}_n = (S_y^2/n)$.

The percent relative efficiencies (PREs) of the proposed estimator d with respect to $\bar{y}_n$, $\bar{y}_\psi$ and Singh and Homa's [17] estimator d_1 have been computed by using the formulae:

$$E_1 = \frac{2}{(1-\rho_{yz}^2)\left[1+\sqrt{(1-\rho_{yx.z}^2)}\ \right]} \times 100, \tag{4.5}$$

$$E_2 = \frac{\left[1+\sqrt{(1-\rho_{yz}^2)}\ \right]}{(1-\rho_{yz}^2)\left[1+\sqrt{(1-\rho_{yx.z}^2)}\ \right]} \times 100, \tag{4.6}$$

$$E_3 = \frac{2\delta_0(\delta_0-\theta_0^* A)}{(\delta_0-\theta_0^{*2}A)(1-\rho_{yz}^2)\left[1+\sqrt{(1-\rho_{yx.z}^2)}\ \right]} \times 100. \tag{4.7}$$

for different combinations of $(\rho_{xz}, \rho_{yz}, \rho_{yx})$ and findings are shown in Table 1. We have also computed the values of θ_0^* and θ_0 by using the formula (2.38) and (3.3) for the same combination of $(\rho_{xz},\ \rho_{yz},\ \rho_{yx})$ for which E_1, E_2, E_3 have been computed and the results are given in Table 1.

Table 1: Optimum values (θ_0, θ_0^*) and $PREs$ of the proposed estimator with respect to $\bar{y}_n, \bar{y}_\psi$ and Singh and Homa [17] estimator d_1

ρ_{xz}	ρ_{yz}	ρ_{yx}	0.3	0.4	0.5	0.6	0.7	0.8
0.9	0.65	θ_0	0.662	0.547	0.509	0.500	0.516	0.568
		θ_0^*	0.469	0.471	0.479	0.493	0.516	0.556
		E_1	229.405	189.297	176.109	173.249	178.718	196.691
		E_2	224.122	181.395	164.312	155.924	153.174	157.353
		E_3	146.608	120.535	110.406	105.486	103.840	106.213
0.8	0.75	θ_0	0.604	0.537	0.508	0.000	0.508	0.537
		θ_0^*	0.462	0.463	0.470	0.483	0.506	0.545
		E_1	276.277	245.284	232.320	228.571	232.320	245.284
		E_2	269.914	235.045	216.757	205.714	199.114	196.227
		E_3	149.350	132.411	123.661	118.262	114.719	112.479
0.7	0.85	θ_0	0.617	0.539	0.508	0.500	0.510	0.544
		θ_0^*	0.453	0.452	0.457	0.470	0.493	0.531
		E_1	444.736	388.493	366.296	360.376	367.666	392.019
		E_2	434.494	372.276	341.759	324.339	315.116	313.615
		E_3	196.469	171.903	160.134	153.300	149.285	147.648
0.6	0.90	θ_0	0.580	0.522	0.502	0.504	0.530	0.600
		θ_0^*	0.449	0.447	0.452	0.465	0.487	0.525
		E_1	610.053	549.428	528.059	530.270	557.383	631.683
		E_2	596.003	526.493	492.685	477.243	477.717	505.346
		E_3	237.947	214.914	204.234	199.644	200.309	210.472

It is observed from Table 1 that:

- for fixed values of (ρ_{xz}, ρ_{yz}), no trends for θ_0, θ_0^* and E_i $'s$ $(i = 1, 2, 3)$ are observed as the value of ρ_{yx} increases,
- for fixed values of (ρ_{yx}, ρ_{yz}), the values ofE_i's $(i = 1, 2, 3)$ increase while the value of θ_0^* decreases and no trend is observed forθ_0 as the value of ρ_{xz} decreases. Similar patterns are observed for fixed value of (ρ_{xz}, ρ_{yx}), and increasing values of ρ_{yz}.

It is further observed from Table 1 that the values of E_i's $(i = 1, 2, 3)$ are greater than 100 which follow that the proposed estimator d is more efficient than the usual unbiased estimator $\bar{y}_n$, $\bar{y}_\psi$ and the Singh and Homa [17] estimator d_1 with appreciable gain in efficiency. Thus, we recommend our proposed estimator d for its use in practice.

5. Conclusion

It is well established fact that in successive sampling, it is advantageous to utilize the entire information gathered in the previous investigation. The information on an auxiliary variable is commonly available for all the units of a finite population in survey sampling. In this paper, an exponential type regression estimator has been proposed using information on the auxiliary variable in successive (rotation) sampling over two occasions. We have studied the properties of the proposed estimator and compared with (i) $\bar{y}_n$, when there is no matching, (ii) $\bar{y}_\psi$, when no auxiliary information is used at any occasion, and (iii) the Singh and Homa's [17] estimatord_1 .It has been shown theoretically and empirically that the proposed estimator is better than $\bar{y}_n$, $\bar{y}_\psi$ and Singh and Homa's [17] estimator d_1 . Empirically it has been shown that the use of auxiliary information is highly rewarding in terms of the suggested estimator. From the values of Table 1 it is concluded that if information on moderately / highly positively correlated auxiliary variable is available in successive sampling, it is appropriate to use the envisaged estimator$d$.

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