ON L-FUZZY 2-ABSORBING IDEALS

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Abstract. Let L be a complete lattice. In this paper we introduce various definitions of L-fuzzy 2-absorbing ideals of a commutative ring R and give some basic results concerning these classes of ideals.

Keywords: L-fuzzy 2-absorbing ideal, L-fuzzy strongly 2-absorbing ideal, L-fuzzy weakly completely 2-absorbing ideal, L-fuzzy K-2-absorbing ideal, L-fuzzy prime ideal.
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1. Introduction

Zadeh in [14] introduced the notion of a fuzzy subset μ of a non-empty set X as a function from X to $[0;1]$. Goguen in [2] generalized the notion of a fuzzy subset of X to that of an L-fuzzy subset, namely a function from X to a lattice L.

Later Rosenfeld considered the fuzzification of algebraic structures [12]. Liu [5], introduced and examined the notion of a fuzzy ideal of a ring. Since then several authors have obtained interesting results on L-fuzzy ideals of R and L-fuzzy modules. See [8] for a comprehensive survey of the literature on these developments.

L-fuzzy prime ideals play an important role in fuzzy commutative ring theory. Let R be a commutative ring. Of course for a non-constant fuzzy ideal $\xi : R \to L$, ξ is called an L-fuzzy prime ideal of R if for any L-fuzzy points $x_r, y_s \in F(R)$, $x_r y_s \in \xi$ implies that either $x_r \in \xi$ or $y_s \in \xi$. A particular attention was paid to the fuzzy prime ideals and prime fuzzy ideals (see for example [3], [6], [9], [10], [11], [13], [15], [16] and the papers cited there). In this paper, we introduce some generalizations of L-fuzzy prime ideals, namely *L-fuzzy 2-absorbing ideals.*

2. Preliminaries

Throughout this paper R is a commutative ring with a nonzero identity and L stands for a complete lattice with least element 0 and greatest element 1. Given a nonempty set X, an L-fuzzy subset μ is a function from X to L. We denote by $F(X)$ the set of all L-fuzzy subsets of X. For $\mu, \nu \in F(X)$ we say $\mu \subseteq \nu$ if and only if $\mu(x) \leq \nu(x)$, for all $x \in X$. Also, $\mu \subset \nu$ if and only if $\mu \subseteq \nu$ and $\mu \neq \nu$.

Let $\mu \in F(X)$ and $t \in L$. Then the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called the t-level subset of X with respect to μ. By an L-fuzzy point x_r of X, $x \in X$; $r \in L\backslash\{0\}$, we mean $x_r \in F(X)$ defined by

$$x_r(y) = \begin{cases} r, & \text{if y=x;} \\ 0, & \text{otherwise.} \end{cases}$$

If x_r is an L-fuzzy point of X and $x_r \subseteq \mu \in F(X)$, we write $x_r \in \mu$. For $A \subseteq X$ the characteristic function of A, $\chi_A \in F(X)$, is defined by

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{otherwise.} \end{cases}$$

We recall two following basic definitions given in [8].

Definition 2.1 Let $\xi \in F(R)$. Then ξ is called an L-fuzzy ideal of R if for all $x; y \in R$,

(i) $\xi(x - y) \geq \xi(x) \wedge \xi(y)$,

(ii) $\xi(xy) \geq \xi(x) \vee \xi(y)$.

(iii) $\xi(0) = 1$.

We denote by $I(R)$ the set of all L-fuzzy ideals of R.

The following are two basic operations which will be used to define L-fuzzy prime ideals and their generalizations.

Definition 2.2 Let $\xi, \mu \in F(R)$. Define the composition $\xi o \mu$ and the product $\xi\mu$ (both are L-fuzzy subsets of R) respectively as follows: For all $w \in R$,

$$\begin{aligned} (\xi o \mu)(w) &= \sup\{\xi(r) \wedge \mu(s) \mid r, s \in R,\ w = rs\}, \\ (\xi\mu)(w) &= \sup\left\{ \inf_{i=1}^{n}\{\xi(r_i) \wedge \mu(s_i)\} \mid r_i, s_i \in R,\ n \in \mathbb{N},\ w = \sum_{i=1}^{n} r_i s_i \right\}, \end{aligned}$$

where as usual the supremum of an empty set is taken to be 0.

Notice that $\xi o \mu$ is the case $n = 1$ in the definition of $\xi\mu$. Thus $\xi o \mu \subseteq \xi\mu$.

Definition 2.3 ([10]) Let $A \in F(R)$. Then the L-fuzzy ideal of R generated by A, denoted by $\langle A \rangle$, is defined to be the intersection of all L-fuzzy ideals of R containing A.

Lemma 2.4 *Let R be a commutative ring with identity and let x_r and y_s be two L-fuzzy points of R. Then*

(i) $x_r y_s = (xy)_{inf\{r,s\}}$.

(ii) $\langle x_r\rangle\langle y_s\rangle = \langle x_r y_s\rangle$.

Proof. [10]. ■

Definition 2.5 ([6]) For a non-constant $\xi \in I(R)$, ξ is called an L-fuzzy prime ideal of R if for any L-fuzzy ideals $\mu, \nu \in I(R)$, $\mu\nu \subseteq \xi$ implies that either $\mu \subseteq \xi$ or $\nu \subseteq \xi$.

Definition 2.6 ([11]) For a non-constant $\xi \in I(R)$, ξ is called an L-fuzzy completely prime ideal of R if for any L-fuzzy points $x_r, y_s \in F(R)$, $x_r y_s \in \xi$ implies that either $x_r \in \xi$ or $y_s \in \xi$.

Definition 2.7 ([11]) A fuzzy ideal ξ of R is said to be a fuzzy weakly completely prime ideal if ξ is a non-constant function and for all $x, y \in R$, $\xi(xy) = \max\{\xi(x), \xi(y)\}$.

Definition 2.8 ([4]) Let ξ be a non-constant L-fuzzy ideal of R. ξ is said to be an L-fuzzy K-prime ideal if $\xi(xy) = \xi(0)$ implies either $\xi(x) = \xi(0)$ or $\xi(y) = \xi(0)$ for any $x, y \in R$.

These different definitions of prime are not independent. In fact, in the commutative case, Definition 2.5 is equivalent to Definition 2.6.

Moreover, Definition 2.6 $\Rightarrow$ Definition 2.7 $\Rightarrow$ Definition 2.8 ([3]).

Definition 2.9 ([1]) Let R be a commutative ring with identity. A proper ideal I of R is said to be 2-absorbing provided that whenever $a, b, c \in R$ with $abc \in I$ then either $ab \in I$ or $ac \in I$ or $bc \in I$. R is called a 2-absorbing ring if and only if its zero ideal is 2-absorbing.

3. L-fuzzy weakly completely 2-absorbing ideals and L-fuzzy K-2-absorbing ideals

In this section we first give two definitions for fuzzy 2-absorbing ideals of R, and then prove some fundamental properties between these classes of ideals and fuzzy quotient rings.

Definition 3.1 Let η be an L-fuzzy ideal of R.

- η is called an L-fuzzy weakly completely 2-absorbing ideal of R if for all $x, y, z \in R$, $\eta(xyz) = \eta(xy)$ or $\eta(xyz) = \eta(xz)$ or $\eta(xyz) = \eta(yz)$.
- η is called an L-fuzzy K-2-absorbing ideal of R if $\eta(xyz) = \eta(0)$ implies that $\eta(xy) = \eta(0)$ or $\eta(xz) = \eta(0)$ or $\eta(yz) = \eta(0)$.

Let η be a non-constant L-fuzzy ideal of R. It is easy to see that η is an L-fuzzy weakly completely 2-absorbing ideal of R if and only if, for every $x, y, z \in R$, $\eta(xyz) = max\{\eta(xy), \eta(xz), \eta(yz)\}$. Note also that every L-fuzzy weakly completely 2-absorbing ideal is L-fuzzy K-2-absorbing. But the following example shows that the converse is not necessarily true.

Example 3.2 Let $R = Z$, the ring of integers. Define the fuzzy ideal η of $\mathbb{Z}$ by

$$\eta(x) = \begin{cases} 1, & \text{if x=0;} \\ \frac{1}{2}, & \text{if } x \in 8\mathbb{Z} - \{0\}; \\ \frac{1}{3}, & \text{if } x \in \mathbb{Z} - 8\mathbb{Z}. \end{cases} \tag{3.1}$$

Then η is a fuzzy K-2-absorbing ideal of $\mathbb{Z}$. But since

$$\eta(40) = \frac{1}{2} > \frac{1}{3} = \max\{\eta(20), \eta(4), \eta(20)\},$$

η is not a fuzzy weakly completely 2-absorbing ideal.

Proposition 3.3

(1) *Every L-fuzzy weakly completely prime ideal of R is L-fuzzy weakly completely 2-absorbing.*

(2) *Every L-fuzzy K-prime ideal of R is L-fuzzy K-2-absorbing.*

Proof. (1) Let μ be an L-fuzzy weakly completely prime ideal of R. Then, for every $x, y, z \in R$, $\mu(xyz) = \mu(x)$ or $\mu(xyz) = \mu(y)$ or $\mu(xyz) = \mu(z)$. Assume that $\mu(xyz) = \mu(x)$. Then from $\mu(xyz) \geq \mu(xy) \geq \mu(x)$ we get $\mu(xyz) = \mu(xy)$. In a similar way we can show that if $\mu(xyz) = \mu(y)$ or $\mu(xyz) = \mu(z)$, then $\mu(xyz) = \mu(yz)$ or $\mu(xyz) = \mu(xz)$. Hence μ is L-fuzzy weakly completely 2-absorbing.

(2) The proof is similar to that of (1). ■

Theorem 3.4 *Let η be an L-fuzzy ideal of R. The following statements are equivalent:*

(i) *η is an L-fuzzy weakly completely 2-absorbing ideal of R.*

(ii) *For every $t \in L$, the t-level subset η_t of η is a 2-absorbing ideal of R.*

Proof. (i)⇒(ii) Assume that η is L-fuzzy weakly completely 2-absorbing and let $x, y, z \in R$ be such that $xyz \in \eta_t$ for some $t \in L$. Then $\max\{\eta(xy), \eta(xz), \eta(yz)\} = \eta(xyz) \geq t$. Hence $\eta(xy) \geq t$ or $\eta(xz) \geq t$ or $\eta(yz) \geq t$, that is $xy \in \eta_t$ or $xz \in \eta_t$ or $yz \in \eta_t$. Therefore that is η_t is 2-absorbing in R.

(ii)⇒(i) Assume that η_t is a 2-absorbing ideal of R for every $t \in L$. For $x, y, z \in R$ set $\eta(xyz) = t$. Then $xyz \in \eta_t$ and η_t 2-absorbing gives $xy \in \eta_t$ or $xz \in \eta_t$ or $yz \in \eta_t$. Thus $\eta(xy) \geq t$ or $\eta(xy) \geq t$ or $\eta(yz) \geq t$, that is

$\max\{\eta(xy), \eta(xz), \eta(yz)\} \geq t = \eta(xyz)$. Also since η is a is an L-fuzzy ideal of R, we have $\eta(xyz) \geq \max\{\eta(xy), \eta(xz), \eta(yz)\}$.

Hence $\eta(xyz) = \max\{\eta(xy), \eta(xz), \eta(yz)\}$, that is η is an L-fuzzy weakly completely 2-absorbing ideal. ∎

Here, we recall the definition of an L-fuzzy quotient ring of R induced by an L-fuzzy ideal of R. Let X be a set and μ an L-fuzzy relation on X. Then μ is called a fuzzy equivalence relation if (i) $\mu(x, x) = 1$ for all $x \in X$; (ii) $\mu(x, y) = \mu(y, x)$ for all $x, y \in X$; and (iii) for all $x, y \in X$, $\mu(x, y) \geq \sup_{z\in X} \min\{\mu(x, z), \mu(z, y)\}$. If μ is an L-fuzzy equivalence relation on X, then, for each $a \in X$, we shall denote $\mu[a](x) = \mu(a, x)$ for every $x \in X$. We shall say that $\mu[a]$ is the fuzzy class corresponding to a. In this case the set $X/\mu = \{\mu[a] : a \in X\}$ is called the fuzzy quotient set. Now let ν be an L-fuzzy ideal of R. It is proved in [4] that the relation μ on R defined by $\mu(x, y) = \nu(x-y)$ is a fuzzy equivalence relation. Note that, for $x, y \in R$, $\mu[x] = \mu[y]$ if and only if $\nu(x - y) = 1$. Now define summation and multiplication of L-fuzzy classes as follows:

$$\mu[x] + \mu[y] = \mu[x + y] \quad \text{and} \quad \mu[x]\mu[y] = \mu[xy].$$

Then R/μ is an L-fuzzy ring with this operations. We will write R/ν for R/μ and call it L-fuzzy quotient ring of R induced by the L-fuzzy ideal ν.

Theorem 3.5 *Let η be a non-constant L-fuzzy ideal of R. Then η is an L-fuzzy K-2-absorbing ideal of R if and only if R/η is a 2-absorbing ring.*

Proof. Assume first that η is an L-fuzzy K-2-absorbing ideal of R and let $\mu[x], \mu[y], \mu[z] \in R/\eta$ be such that $\mu[x]\mu[y]\mu[z] = \mu[0]$. Since $\mu[x]\mu[y]\mu[z] = \mu[xyz]$, we have $\eta(xyz) = \eta(xyz-0) = 1 = \eta(0)$. As η is considered to be L-fuzzy K-2-absorbing, $\eta(xy) = \eta(0) = 1$ or $\eta(xz) = \eta(0) = 1$ or $\eta(yz) = \eta(0) = 1$. This implies that $\mu[x]\mu[y] = \mu[0]$ or $\mu[x]\mu[z] = \mu[0]$ or $\mu[y]\mu[z] = \mu[0]$, that is R/η is a 2-absorbing ring.

Conversely, assume that R/η is a 2-absorbing ring and let $\eta(xyz) = \eta(0) = 1$ for $x, y, z \in R$. Then we have $\mu[x]\mu[y]\mu[z] = \mu[xyz] = \mu[0]$. Since R/η is 2-absorbing, $\mu[x]\mu[y] = \mu[0]$ or $\mu[x]\mu[z] = \mu[0]$ or $\mu[y]\mu[z] = \mu[0]$. Hence $\eta(xy) = \eta(0)$ or $\eta(xz) = \eta(0)$ or $\eta(yz) = \eta(0)$, that is η is L-fuzzy K-2-absorbing. ∎

Corollary 3.6 *If η is an L-fuzzy weakly completely 2-absorbing ideal of R, then R/η is a 2-absorbing ring.*

4. L-fuzzy 2-absorbing ideals and L-fuzzy strongly 2-absorbing ideals

Definition 4.1 Let η be an L-fuzzy ideal of R. η is called an L-fuzzy 2-absorbing ideal of R if for any L-fuzzy points $x_r, y_s, z_t \in F(R)$ $(x, y, z \in R$ and $r, s, t \in L)$, $x_r y_s z_t \in \eta$ implies that either $x_r y_s \in \eta$ or $x_r z_t \in \eta$ or $y_s z_t \in \eta$.

Lemma 4.2 *Every L-fuzzy prime ideal of R is L-fuzzy 2-absorbing.*

Proof. The proof is straightforward. ∎

Proposition 4.3 *The intersection of every pair of distinct L-fuzzy prime ideals of R is an L-fuzzy 2-absorbing ideal of R.*

Proof. Let η and μ be two distinct L-fuzzy prime ideals of R. Assume that $x_r, y_s, z_t \in F(R)$ are L-fuzzy points such that $x_r y_s z_t \in \eta \cap \mu$ but $x_r y_s \notin \eta \cap \mu$ and $x_r z_t \notin \eta \cap \mu$. Then we have the following cases:

Case 1. $x_r y_s \notin \eta$ and $x_r z_t \notin \eta$. As η is an L-fuzzy prime ideal of R we will have $z_t \in \eta$. Therefore $x_r z_t \in \eta$ which is a contradiction.

Case 2. $x_r y_s \notin \eta$ and $x_r z_t \notin \mu$. In this case from $x_r y_s z_t \in \eta \cap \mu$ we get $z_t \in \eta$ and $y_s \in \mu$. Hence $y_s z_t \in \eta \cap \mu$.

Case 3. $x_r y_s \notin \mu$ and $x_r z_t \notin \eta$. By a similar argument as in the Case 2 we may show that $y_s z_t \in \eta \cap \mu$.

Case 4. $x_r y_s \notin \mu$ and $x_r z_t \notin \mu$. A similar argument as in the Case 1 leads us to a contradiction.

Therefore $\eta \cap \mu$ is L-fuzzy 2-absorbing. ∎

Theorem 4.4 *Let η be an L-fuzzy 2-absorbing ideal of R. Then, for every $t \in L$ with $\eta_t \neq R$, η_t is a 2-absorbing ideal of R.*

Proof. Assume that $x, y, z \in R$ are such that $xyz \in \eta_t$. Then $\eta(xyz) \geq t$. Then we have $x_t y_t z_t = (xyz)_t \in \eta$. since η is an L-fuzzy 2-absorbing ideal of R, we get $(xy)_t = x_t y_t \in \eta$ or $(xz)_t = x_t z_t \in \eta$ or $(yz)_t = y_t z_t \in \eta$. If $a_t \in \eta$ for some $a \in R$, then $\eta(a) \geq t$. So $a \in \eta_t$. Therefore $xy \in \eta_t$ or $xz \in \eta_t$ or $yz \in \eta_t$. Hence η_t is a 2-absorbing ideal of R. ∎

Corollary 4.5 *If η is an L-fuzzy 2-absorbing ideal of R, then $\eta_* = \{x \in R \mid \eta(x) = \eta(0)\}$ is a 2-absorbing ideal of R.*

Proof. Since η is a non-constant L-fuzzy ideal of R, $\eta_* \neq R$. Now the result follows from Theorem 4.4. ∎

Definition 4.6 Let $\alpha \in L \backslash \{1\}$. Then α is called a 2-absorbing element of L if $r \wedge s \wedge t \leq \alpha$ implies that $r \wedge s \leq \alpha$ or $r \wedge t \leq \alpha$ or $s \wedge t \leq \alpha$ for all $r, s, t \in L$.

Proposition 4.7 *Let η be an L-fuzzy 2-absorbing ideal of R. Then $\alpha = \eta(1)$ is a 2-absorbing element of L.*

Proof. Assume that $x, y, z \in L$ with $x \wedge y \wedge z \leq \alpha$. Consider the three L-fuzzy points $1_x, 1_y, 1_z$ of R. Then we have $1_x 1_y 1_z = 1_{x \wedge y \wedge z} \in \eta$. Since η is 2-absorbing we get $1_{x \wedge y} = 1_x 1_y \in \eta$ or $1_{x \wedge z} = 1_x 1_z \in \eta$ or $1_{y \wedge z} = 1_y 1_z \in \eta$. Hence $x \wedge y \leq \eta(1) = \alpha$ or $x \wedge z \leq \eta(1) = \alpha$ or $y \wedge z \leq \eta(1) = \alpha$, that is α is 2-absorbing in L. ∎

Theorem 4.8 *Let A be an 2-absorbing ideal of R and α a 2-absorbing element of L. If η is the L-fuzzy subset of R defined by*

$$\eta(x) = \begin{cases} 1, & \text{if } x \in A; \\ \alpha, & \text{otherwise.} \end{cases} \tag{4.1}$$

for all $x \in R$, then η is an L-fuzzy 2-absorbing ideal of R.

Proof. (a) Since A is a 2-absorbing ideal of R, $A \neq R$. Therefore η is a non-constant L-fuzzy ideal of R. Suppose that $x_r, y_s, z_t \in F(R)$ are L-fuzzy points such that $x_r y_s z_t \in \eta$ but $x_r y_s \notin \eta$ and $x_r z_t \notin \eta$ and $y_s z_t \notin \eta$. In this case $\eta(xy) = \alpha$ and so $xy \notin A$. Similarly, $xz \notin A$ and $yz \notin A$. As A is assumed to be 2-absorbing, we get $xyz \notin A$. So that $\eta(xyz) = \alpha$. Also from $(xyz)_{r\wedge s\wedge t} = x_r y_s z_t \in \eta$ we have $r \wedge s \wedge t \leq \eta(xyz) = \alpha$. Hence $r \wedge s \leq \alpha$ or $r \wedge t \leq \alpha$ or $s \wedge t \leq \alpha$, since α is a 2-absorbing element, which is a contradiction. Therefore $x_r y_s \in \eta$ or $x_r z_t \in \eta$ or $y_s z_t \in \eta$, that is η is 2-absorbing. ∎

Example 4.9 By Lemma 4.2, every L-fuzzy prime ideal of R is L-fuzzy 2-absorbing, but the converse does not necessarily hold. For example, consider the case where R=$\mathbb{Z}$. Let p and q be a pair of distinct prime numbers, and set A=$pq\mathbb{Z}$. It is not difficult to show that A is a 2-absorbing ideal of $\mathbb{Z}$. Now define $\eta : \mathbb{Z} \to [0, 1]$ by

$$\eta(x) = \begin{cases} 1, & \text{if } pq|x; \\ 0, & \text{otherwise.} \end{cases}$$

Then η is a fuzzy 2-absorbing ideal of R by Theorem 4.8. Moreover $\eta_0 = A$ is a 2-absorbing ideal of $\mathbb{Z}$ that is not a prime ideal. Hence η is not a fuzzy prime ideal of R.

Definition 4.10 Let η be an L-fuzzy ideal of R. η is said to be an L-fuzzy strongly 2-absorbing ideal of R provided that it is non-constant and whenever $\lambda, \mu, \nu \in I(R)$ with $\lambda\mu\nu \subseteq \eta$, then $\lambda\mu \subseteq \eta$ or $\lambda\nu \subseteq \eta$ or $\mu\nu \subseteq \eta$.

Theorem 4.11

(1) *Every L-fuzzy prime ideal of R is L-fuzzy strongly 2-absorbing.*

(2) *Every L-fuzzy strongly 2-absorbing ideal of R is L-fuzzy 2-absorbing.*

Proof. (1) Straightforward.

(2) Suppose that η is an L-fuzzy strongly 2-absorbing ideal of R. Assume that $x_r, y_s, z_t \in \eta$ for some L-fuzzy points x_r, y_s, z_t of R. Then, by Lemma 2.4, we have $\langle x_r\rangle\langle y_s\rangle\langle z_t\rangle = \langle x_r y_s z_t\rangle \subseteq \eta$. Since η is L-fuzzy strongly 2-absorbing, we have $\langle x_r y_s\rangle = \langle x_r\rangle\langle y_s\rangle \subseteq \eta$ or $\langle x_r z_t\rangle = \langle x_r\rangle\langle z_t\rangle \subseteq \eta$ or $\langle y_s z_t\rangle = \langle y_s\rangle\langle z_t\rangle \subseteq \eta$. Therefore $x_r y_s \in \eta$ or $x_r z_t \in \eta$ or $y_s z_t \in \eta$, that is η is an L-fuzzy 2-absorbing ideal of R. ∎

Let A be an ideal of the commutative ring R. It is proved in [1] that A is a 2-absorbing ideal of R if and only if whenever I, J, K are ideals of R with $IJK \subseteq A$, then $IJ \subseteq A$ or $IK \subseteq A$ or $JK \subseteq A$. It is also well known that a nonconstant ideal $\xi \in I(R)$ is an L-fuzzy prime ideal of R if and only if for any two L-fuzzy ideals μ and ν of R, $\mu\nu \subseteq \xi$ implies that either $\mu \subseteq \eta$ or $\nu \subseteq \eta$. Let I be a nonconstant L-fuzzy ideal of R. We proved in Theorem 4.11 that every L-fuzzy strongly 2-absorbing ideal of R is L-fuzzy 2-absorbing, but I was unable to prove or disprove the converse.

Question 1. Is every L-fuzzy 2-absorbing ideal of R of the form 4.1, where A is a 2-absorbing ideal of R and α is a 2-absorbing element of L?

Question 2. Is every L-fuzzy 2-absorbing ideal of R L-fuzzy strongly 2-absorbing?

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