CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES IN ŠOSTAK'S SENSE

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Abstract. In this paper, we introduce various types of fuzzy connectedness in intuitionistic fuzzy topological spaces in view of Šostak's sense. The interrelationship between different notions of intuitionistic fuzzy connectedness are investigate. Also, we inspect some interrelations between these types of intuitionistic fuzzy connectedness together with the preservation properties under intuitionistic fuzzy continuous maps.

Keywords: intuitionistic fuzzy topology; intuitionistic fuzzy ($c_i^{\alpha,\beta}$, $c_S^{\alpha,\beta}$,$c_M^{\alpha,\beta}$, $O^{\alpha,\beta}$,$O_q^{\alpha,\beta}$)-connectedness; (α,β)-intuitionistic fuzzy super connectedness.

2010 Mathematics Subject Classification: 54A40.

1. Introduction and preliminaries

Zadeh[34] introduced the fundamental concept of a fuzzy set. Later Chang [7] defined fuzzy topological spaces. Šostak [29] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and Chang's fuzzy topology. The fuzzy topology in Šostak's sense were rediscovered by Chattopadhyay et al. [8]. In the same year, Ramadan [22] gave a similar definition of a fuzzy topology under the name "smooth topology".

On the other hand, Atanassove and his colleagues [2]–[6] introduced the fundamental concept of an intuitionistic fuzzy set. Çoker [12], [14] used this type of generalized fuzzy set to define "intuitionistic fuzzy topological spaces". Also,

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Çoker and Demirci [13] introduced the basic definition and properties of "intuitionistic fuzzy topological spaces in Šostak's sense" which is a generalized form of "fuzzy topological spaces" developed by Šostak [29], [30]. In this sense many works have been launched [15], [17]–[20], [25], [32].

Connectedness of fuzzy sets is an important subject in fuzzy topology, it won the attention of many researchers [1], [9], [16], [21], [24], [26]–[28], [33].

In this paper, many different notions of connectedness of fuzzy sets are extended to intuitionistic fuzzy topological spaces in Šostak's sense and the interrelationship between them are studied. Also, we inspect some interrelations between these types of intuitionistic fuzzy connectedness together with the preservation properties under intuitionistic fuzzy continuous maps.

Definition 1.1. ([2]) Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly, IFS) A is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\},$$

where the map $\mu_A : X \longrightarrow I$ and $\gamma_A : X \longrightarrow I$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS having the form

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}.$$

For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ for the IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$. For a given nonempty set X, let us denote the family of all IFSs in X by the symbol ζ^X.

Definition 1.2. ([2],[14]) Let X be a nonempty set, and the IFSs A and B in X be in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Furthermore, let $\{A_i : i \in J\}$ be an arbitrary family of IFSs in X. Then,

(i) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$, for all $x \in X$;

(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(iii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;

(iv) $A - B = A \cap \overline{B}$;

(v) $\cap A_i = \{\langle x, \bigwedge \mu_{A_i}(x), \bigvee \gamma_{A_i}(x)\rangle : x \in X\}$;

(vi) $\cup A_i = \{\langle x, \bigvee \mu_{A_i}(x), \bigwedge \gamma_{A_i}(x)\rangle : x \in X\}$;

(vii) $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 1.3. ([11]) Let a and b be two real numbers in [0,1] satisfying the inequality $a + b \leq 1$. Then the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair.

Let $\langle a_1, b_1\rangle, \langle a_2, b_2\rangle$ be two intuitionistic fuzzy pairs. Then we define

(i) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle \Leftrightarrow a_1 \leq a_2$ and $b_1 \geq b_2$;

(ii) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$;

(iii) If $\{\langle a_i, b_i\rangle : i \in J\}$ is a family of intuitionistic fuzzy pairs, then $\bigvee\langle a_i, b_i\rangle = \langle\bigvee a_i, \bigwedge b_i\rangle$ and $\bigwedge\langle a_i, b_i\rangle = \langle\bigwedge a_i, \bigvee b_i\rangle$;

(iv) The complement of an intuitionistic fuzzy pair $\langle a, b\rangle$ is the intuitionistic fuzzy pair defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$;

(v) $1^\sim = \langle 1, 0\rangle$ and $0^\sim = \langle 0, 1\rangle$.

Definition 1.4. ([14]) Let X and Y be two nonempty sets and $f : X \to Y$ be a map.

(i) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(ii) If $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f , denoted by $f(A)$ is the IFS in Y defined by
$f(A) = \{\langle y, f(\mu_A)(y), \underline{f}(\gamma_A)(y)\rangle : y \in Y\}$, where $\underline{f}(\gamma_A) = 1 - f(1 - \gamma_A)$.

Definition 1.5. ([10]) Let $A, B \in \zeta^X$. Then, A and B are said to be quasi-coincident, denoted by AqB iff there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$, otherwise $A \not{q} B$.

Theorem 1.6. ([10],[31]) *Let $A, B \in \zeta^X$. Then,*

(i) $A \not{q} \overline{B}$ *iff* $A \subseteq B$,

(ii) AqB *iff* $A \not\subseteq \overline{B}$,

(iii) *if* $A \cap B = 0_\sim$, *then* $A \subseteq \overline{B}$,

(iv) *if* $A \not\subseteq \overline{B}$, *then* $A \cap B \neq 0_\sim$.

Definition 1.7. ([11]) An IFS ξ on the set ζ^X is called an intuitionistic fuzzy family (IFF for short) on X. In symbols, denote such an IFF in form $\xi = \langle\mu_\xi, \gamma_\xi\rangle$.

Let ξ be an IFF on X. Then the complemented IFF of ξ on X is defined by $\xi^* = \langle\mu_{\xi^*}, \gamma_{\xi^*}\rangle$, where $\mu_{\xi^*}(A) = \mu_\xi(\overline{A})$ and $\gamma_{\xi^*}(A) = \gamma_\xi(\overline{A})$, for each $A \in \zeta^X$.

If τ is an IFF on X, then for any $A \in \zeta^X$, construct the intuitionistic fuzzy pair $\langle\mu_\tau(A), \gamma_\tau(A)\rangle$ and use the symbol $\tau(A) = \langle\mu_\tau(A), \gamma_\tau(A)\rangle$.

Definition 1.8. ([13]) An intuitionistic fuzzy topology in Šostak's sense (IFT for short) on a nonempty set X is an IFF τ on X satisfying the following axioms:

(T_1) $\tau(0_\sim) = \tau(1_\sim) = 1^\sim$;

(T_2) $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$, for any $A, B \in \zeta^X$;

(T_3) $\tau(\cup A_i) \geq \bigwedge \tau(A_i)$ for any $\{A_i : i \in J\} \subseteq \zeta^X$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space in Šostak's sence (IFTS for short). For any $A \in \zeta^X$, the number $\mu_\tau(A)$ is called the openness degree of A, while $\gamma_\tau(A)$ is called the nonopenness degree of A.

Definition 1.9. ([13]) Let (X, τ) be an IFTS on X. Then, the IFF τ^* of complemented IFSs on X is defined by: $\tau^*(A) = \tau(\overline{A})$. The number $\mu_{\tau^*}(A) = \mu_\tau(\overline{A})$ is called the closedness degree of A, while $\gamma_{\tau^*}(A) = \gamma_\tau(\overline{A})$ is called the nonclosedness degree of A.

Theorem 1.10. ([13]) *The IFF τ^* on X satisfies the following properties:*

(C_1) $\tau^*(0_\sim) = \tau^*(1_\sim) = 1^\sim$;

(C_2) $\tau^*(A \cup B) \geq \tau^*(A) \wedge \tau^*(B)$, *for any* $A, B \in \zeta^X$;

(C_3) $\tau^*(\cap A_i) \geq \bigwedge \tau^*(A_i)$, *for any* $\{A_i : i \in J\} \subseteq \zeta^X$.

Definition 1.11. ([13]) Let (X, τ) be an IFTS and A be an IFS in X. Then the fuzzy closure and fuzzy interior of A are defined by

$$\begin{aligned} cl_{\alpha,\beta}(A) &= \cap\{K \in \zeta^X : A \subseteq K, \tau^*(K) \geq \langle \alpha, \beta \rangle\} \\ int_{\alpha,\beta}(A) &= \cup\{G \in \zeta^X : G \subseteq A, \tau(G) \geq \langle \alpha, \beta \rangle\}, \end{aligned}$$

where $\alpha \in I_0 = (0, 1], \beta \in I_1 = [0, 1)$ with $\alpha + \beta \leq 1$.

Theorem 1.12. ([13]) *The closure and interior operators satisfy the following properties:*

(i) $A \subseteq cl_{\alpha,\beta}(A)$;

(ii) $int_{\alpha,\beta}(A) \subseteq A$;

(iii) $cl_{\alpha,\beta}(cl_{\alpha,\beta}(A)) = cl_{\alpha,\beta}(A)$;

(iv) $int_{\alpha,\beta}(int_{\alpha,\beta}(A)) = int_{\alpha,\beta}(A)$;

(v) $cl_{\alpha,\beta}(A \cup B) = cl_{\alpha,\beta}(A) \cup cl_{\alpha,\beta}(B)$;

(vi) $int_{\alpha,\beta}(A \cap B) = int_{\alpha,\beta}(A) \cap int_{\alpha,\beta}(B)$;

(vii) $\overline{cl_{\alpha,\beta}(A)} = int_{\alpha,\beta}(\overline{A})$;

(viii) $\overline{int_{\alpha,\beta}(A)} = cl_{\alpha,\beta}(\overline{A})$.

Definition 1.13. ([13]) Let (X, τ_1) and (Y, τ_2) be two IFTSs and $f : X \to Y$ be a map. Then, f is said to be intuitionistic fuzzy continuous iff

$$\tau_1(f^{-1}(B)) \geq \tau_2(B),$$

for each $B \in \zeta^Y$.

Theorem 1.14. ([13]) *The following properties are equivalent:*

(i) $f : (X, \tau_1) \to (Y, \tau_2)$ *is an intuitionistic fuzzy continuous.*

(ii) $\tau_1^*(f^{-1}(B)) \geq \tau_2^*(B)$, *for each* $B \in \zeta^Y$.

Definition 1.15. ([23]) Let A be an IFS in an IFTS (X, τ). For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, A is called:

(i) an (α, β)-intuitionistic fuzzy regular open (briefly, (α, β)-ifro) set of X, if $int_{\alpha,\beta}(cl_{\alpha,\beta}A) = A$,

(ii) an (α, β)-intuitionistic fuzzy regular closed (briefly, (α, β)-ifrc) set of X, if $cl_{\alpha,\beta}(int_{\alpha,\beta}A) = A$.

Theorem 1.16. ([23]) *Let A be an IFS in an IFTS (X, τ). Then, for $\alpha \in I_0$, $\beta \in I_1$ with $\alpha + \beta \leq 1$.*

(i) *If A is an (α, β)-ifro (resp. (α, β)-ifrc) set then,* $\tau(A) \geq \langle \alpha, \beta \rangle$ *(resp.* $\tau^*(A) \geq \langle \alpha, \beta \rangle$*).*

(ii) *A is an (α, β)-ifro set iff $\overline{A}$ is an (α, β)-ifrc set.*

2. Different notions of connectedness of intuitionistic fuzzy sets

Definition 2.1. Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$,

(i) If there exist the IFSs $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha, \beta \rangle$, $\tau(V) \geq \langle \alpha, \beta \rangle$ satisfying the following properties, then N is called an intuitionistic fuzzy $c_i^{\alpha,\beta}$-disconnected(briefly, IF$c_i^{\alpha,\beta}$-disconnected), $(i = 1, 2, 3, 4)$:
IF$c_1^{\alpha,\beta}$: $N \subseteq U \cup V$, $U \cap V \subseteq \overline{N}$, $N \cap U \neq 0_\sim$, $N \cap V \neq 0_\sim$.
IF$c_2^{\alpha,\beta}$: $N \subseteq U \cup V$, $N \cap U \cap V = 0_\sim$, $N \cap U \neq 0_\sim$, $N \cap V \neq 0_\sim$.
IF$c_3^{\alpha,\beta}$: $N \subseteq U \cup V$, $U \cap V \subseteq \overline{N}$, $U \nsubseteq \overline{N}$, $V \nsubseteq \overline{N}$.
IF$c_4^{\alpha,\beta}$: $N \subseteq U \cup V$, $N \cap U \cap V = 0_\sim$, $U \nsubseteq \overline{N}$, $V \nsubseteq \overline{N}$.

(ii) N is said to be intuitionistic fuzzy $c_i^{\alpha,\beta}$-connected(briefly, IF$c_i^{\alpha,\beta}$-connected) if N is not an IF$c_i^{\alpha,\beta}$-disconnected, $(i = 1, 2, 3, 4)$.

Remark 2.2. From Definition 2.1, we have the following implication between IF$c_i^{\alpha,\beta}$-connectedness $(i = 1, 2, 3, 4)$.

$$\begin{array}{ccc} \text{IF}c_1^{\alpha,\beta}\text{-connectedness} & \Longrightarrow & \text{IF}c_2^{\alpha,\beta}\text{-connectedness} \\ \Downarrow & & \Downarrow \\ \text{IF}c_3^{\alpha,\beta}\text{-connectedness} & \Longrightarrow & \text{IF}c_4^{\alpha,\beta}\text{-connectedness} \end{array}$$

But the reciprocal implication are not true in general as shown by the following examples.

Example 2.3. Let $X = \{a, b, c\}$ and $N_1, N_2, G_i \in \zeta^X$ $(i = 1, 2, 3, 4)$ defined as follows:

$$N_1 = \langle x, (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.2}), (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.4})\rangle$$

$$N_2 = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3})\rangle$$

$$G_1 = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$$

$$G_3 = \langle x, (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}), (\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$$

$$G_4 = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.6, 0.3\rangle, & \text{if } A = G_i (i = 1, 2, 3, 4) \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.4, \beta = 0.5$. Then, N_1 is both an IF$c_2^{\alpha,\beta}$-connected and IF$c_3^{\alpha,\beta}$-connected but not an IF$c_1^{\alpha,\beta}$-connected (i.e., N_1 is an IF$c_2^{\alpha,\beta}$-connected since, for every $G_i \in \zeta^X$ with $\tau(G_i) \geq \langle \alpha, \beta\rangle$, $i = 1, 2, 3, 4$, and satisfies $N_1 \subseteq G_1 \cup G_2$, $N_1 \subseteq G_1 \cup G_3$, $N_1 \subseteq G_1 \cup G_4$, $N_1 \subseteq G_3 \cup G_2$, $N_1 \subseteq G_3 \cup G_4$, we have $N_1 \cap G_1 \cap G_2 \neq 0_{\sim}$, $N_1 \cap G_1 \cap G_3 \neq 0_{\sim}$, $N_1 \cap G_1 \cap G_4 \neq 0_{\sim}$, $N_1 \cap G_3 \cap G_2 \neq 0_{\sim}$, $N_1 \cap G_3 \cap G_4 \neq 0_{\sim}$. Similarly, N_1 is an IF$c_3^{\alpha,\beta}$-connected. N_1 is not an IF$c_1^{\alpha,\beta}$-connected since, there exist $G_1, G_2 \in \zeta^X$ with $\tau(G_i) \geq \langle \alpha, \beta\rangle$, $i = 1, 2$ and satisfies $N_1 \subseteq G_1 \cup G_2$, $G_1 \cap G_2 \subseteq \overline{N_1}$, $N_1 \cap G_1 \neq 0_{\sim}$ and $N_1 \cap G_2 \neq 0_{\sim}$). By the same technique we have, N_2 is an IF$c_4^{\alpha,\beta}$-connected but not an IF$c_3^{\alpha,\beta}$-connected.

Example 2.4. Let $X = \{a, b, c\}$ and $N, G_i \in \zeta^X$ $(i = 1, 2, 3, 4)$ defined as follows:

$$N = \langle x, (\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.0}), (\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{1.0})\rangle$$

$$G_1 = \langle x, (\frac{a}{0.0}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{1.0}, \frac{b}{0.5}, \frac{c}{0.4})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.6}, \frac{b}{0.0}, \frac{c}{0.2}), (\frac{a}{0.3}, \frac{b}{1.0}, \frac{c}{0.4})\rangle$$

$$G_3 = \langle x, (\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4})\rangle$$

$$G_4 = \langle x, (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{0.2}), (\frac{a}{1.0}, \frac{b}{1.0}, \frac{c}{0.4})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.5, 0.2\rangle, & \text{if } A \in \{G_1, G_1\} \\ \langle 0.7, 0.2\rangle, & \text{if } A \in \{G_3, G_4\} \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.3, \beta = 0.6$. Then, N is an IF$c_4^{\alpha,\beta}$-connected but not an IF$c_2^{\alpha,\beta}$-connected.

Definition 2.5. Let X be a nonempty set and $A, B \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, A and B are said to be

(i) (α, β)-intuitionistic fuzzy weakly separated (briefly,(α, β)-IFWS) if there exist IFSs $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha, \beta\rangle$, $\tau(V) \geq \langle \alpha, \beta\rangle$ such that $A \subseteq U$, $B \subseteq V$, $A \not{q} V$, $B \not{q} U$.

(ii) (α, β)-intuitionistic fuzzy q-separated (briefly, (α, β)-IFqS) if $Cl_{\alpha,\beta}(A) \cap B = 0_{\sim}$ and $A \cap Cl_{\alpha,\beta}(B) = 0_{\sim}$.

Theorem 2.6. *Let (X, τ) be an IFTS and $A, B \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, A and B are (α, β)-IFWS iff $cl_{\alpha,\beta}A \subseteq \overline{B}$ and $cl_{\alpha,\beta}B \subseteq \overline{A}$.*

Proof. Suppose that A, B are (α, β)-IFWS. Then, there exist $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha, \beta\rangle$ such that $A \subseteq U$, $B \subseteq V$, $A \not{q} V$, $B \not{q} U$. By Theorem 1.6, $A \subseteq \overline{V}$, since $\tau^*(\overline{V}) = \tau(V) \geq \langle \alpha, \beta\rangle$ then, $cl_{\alpha,\beta}A \subseteq \overline{V} \subseteq \overline{B}$. Similarly, $cl_{\alpha,\beta}B \subseteq \overline{A}$.

Conversely, suppose that $cl_{\alpha,\beta}A \subseteq \overline{B}$ and $cl_{\alpha,\beta}B \subseteq \overline{A}$. Then, $B \subseteq \overline{cl_{\alpha,\beta}A} = V$, $A \subseteq \overline{cl_{\alpha,\beta}B} = U$ which implies that, $\tau(U) = \tau(\overline{cl_{\alpha,\beta}B}) = \tau^*(cl_{\alpha,\beta}B) \geq \langle \alpha, \beta\rangle$ and $\tau(V) = \tau(\overline{cl_{\alpha,\beta}A}) = \tau^*(cl_{\alpha,\beta}A) \geq \langle \alpha, \beta\rangle$. Also, $A \subseteq cl_{\alpha,\beta}A = \overline{V}$ and $B \subseteq cl_{\alpha,\beta}B = \overline{U}$, which implies that $A \not{q} V$, $B \not{q} U$. Hence, A, B are (α, β)-IFWS.

Definition 2.7. Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$,

(i) N is called an intuitionistic fuzzy $c_s^{\alpha,\beta}$-disconnected (briefly, IF$c_s^{\alpha,\beta}$-disconnected) if there exist an (α, β)-IFWS sets $A, B \in \zeta^X$ such that $A \cup B = N$ and $A \neq 0_{\sim}$, $B \neq 0_{\sim}$.

(ii) N is called an intuitionistic fuzzy $c_M^{\alpha,\beta}$-disconnected (briefly, IF$c_M^{\alpha,\beta}$-disconnected) if there exist (α, β)-IFqS sets $A, B \in \zeta^X$ such that $A \cup B = N$ and $A \neq 0_{\sim}$, $B \neq 0_{\sim}$.

(iii) N called an IF$c_s^{\alpha,\beta}$-connected if N is not an IF$c_s^{\alpha,\beta}$-disconnected.

(iv) N called an IF$c_M^{\alpha,\beta}$-connected if N is not an IF$c_M^{\alpha,\beta}$-disconnected.

Theorem 2.8. *Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an IF$c_s^{\alpha,\beta}$-connected then, N is an IF$c_M^{\alpha,\beta}$-connected.*

Proof. Suppose for a contradiction that N is an IF$c_M^{\alpha,\beta}$-disconnected. Then, there exist $A, B \in \zeta^X$ such that $A \cup B = N$, $(cl_{\alpha,\beta}A) \cap B = 0_{\sim}$, $(cl_{\alpha,\beta}B) \cap A = 0_{\sim}$, $A \neq 0_{\sim}$, $B \neq 0_{\sim}$. By Theorem 1.6, we have $cl_{\alpha,\beta}A \subseteq \overline{B}$, $cl_{\alpha,\beta}B \subseteq \overline{A}$. Then by Theorem 2.6, N is an IF$c_s^{\alpha,\beta}$-disconnected which is a contradiction. Hence, N is an IF$c_M^{\alpha,\beta}$-connected.

Theorem 2.9. *Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an IF$c_1^{\alpha,\beta}$-connected then, N is an IF$c_s^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$c_s^{\alpha,\beta}$-disconnected. Then, there exist $A, B \in \zeta^X$ such that, $A \cup B = N$, $cl_{\alpha,\beta}A \subseteq \overline{B}$, $cl_{\alpha,\beta}B \subseteq \overline{A}$, $A \neq 0_\sim$, $B \neq 0_\sim$. By Theorem 2.6, there exist $U, V \in \zeta^X$ with, $\tau(U) \geq \langle\alpha, \beta\rangle$, $\tau(V) \geq \langle\alpha, \beta\rangle$ such that, $A \subseteq U$, $B \subseteq V$, $A \not{q} V$, $B \not{q} U$. Then, $N = A \cup B \subseteq U \cup V$. Also, $N \cap U \neq 0_\sim$. For, if $N \cap U = 0_\sim$, then $N \cap A = 0_\sim$, so that $A = 0_\sim$(since, $A \subseteq N$) which a contradicts that $A \neq 0_\sim$. Similarly, $N \cap V \neq 0_\sim$. Also, $U \cap V \subseteq \overline{N}$. For, if $U \cap V \not\subseteq \overline{N}$ then, $(U \cap V)qN$ which implies that $(U \cap V)qA$ or $(U \cap V)qB$. Then $(UqA$ and $VqA)$ or $(UqB$ and $VqB)$, a contradiction with $A \ /qV$ and $B \ /qU$. Thus, N is an IF$c_1^{\alpha,\beta}$-disconnected, which is a contradiction. Hence, N is an IF$c_s^{\alpha,\beta}$-connected.

Theorem 2.10. *Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an IF$c_s^{\alpha,\beta}$-connected then, N is an IF$c_2^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$c_2^{\alpha,\beta}$-disconnected. Then, there exist $U, V \in \zeta^X$ with, $\tau(U) \geq \langle\alpha, \beta\rangle$, $\tau(V) \geq \langle\alpha, \beta\rangle$ such that, $N \subseteq U \cup V$, $N \cap U \cap V = 0_\sim$, $N \cap U \neq 0_\sim$, $N \cap V \neq 0_\sim$. put $A = N \cap U \subseteq U$ and $B = N \cap V \subseteq V$. Then, $A \cup B = (N \cap U) \cup (N \cap V) = N \cap (U \cup V) = N$. Now, if AqV then, there exists $x \in X$ such that $\mu_A(x) > \gamma_V(x)$ or $\gamma_A(x) < \mu_V(x)$. First if $\mu_A(x) > \gamma_V(x)$ then, $\mu_A(x) > 0$. Since $N = A \vee B$ and $A \leq U$ then, $\mu_N(x) > 0$ and $\mu_U(x) > 0$. $\mu_V(x) \neq 0_\sim$ (since, if $\mu_V(x) = 0_\sim$, then $\gamma_V(x) = 1$, a contradiction with $\mu_A(x) > \gamma_V(x)$). Thus, $(\mu_N(x) \wedge \mu_U(x) \wedge \mu_V(x)) > 0$. So, $N \cap U \cap V \neq 0_\sim$, a contradiction with $N \cap U \cap V = 0_\sim$. Thus $A \not{q} V$. Second: if $\gamma_A(x) < \mu_V(x)$ then, $\mu_V(x) > 0$. $\mu_N(x) > 0$ (for, if $\mu_N(x) = 0$ then, $N = A \cup B$ implies that, $\mu_A(x) = \mu_B(x) = 0$, so $\gamma_A(x) = 1$ a contradicts with $\gamma_A(x) < \mu_V(x)$). Since, $N \subseteq U \cup V$, $\mu_V(x) > 0$, $\mu_N(x) > 0$ then, $(\mu_N \wedge \mu_U \wedge \mu_V)(x) \neq 0$ and so, $N \cap U \cap V \neq 0_\sim$, a contradiction, then $A \not{q} V$. Similarly, $B \not{q} U$. Then, N is an IF$c_s^{\alpha,\beta}$-disconnected, a contradiction. Thus, N is an IF$c_2^{\alpha,\beta}$-connected.

Theorem 2.11. *Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an IF$c_s^{\alpha,\beta}$-connected then, N is an IF$c_3^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$c_3^{\alpha,\beta}$-disconnected. Then, there exist $U, V \in \zeta^X$ with, $\tau(U) \geq \langle\alpha, \beta\rangle$, $\tau(V) \geq \langle\alpha, \beta\rangle$ such that, $N \subseteq U \cup V$, $U \cap V \subseteq \overline{N}$, $U \not\subseteq \overline{N}$, $V \not\subseteq \overline{N}$. Put $A = N \cap U \subseteq U$ and $B = N \cap V \subseteq V$. Then, $A \cup B = N$.
Let $R = \langle x, \mu_R(x), \gamma_R(x)\rangle$ and $S = \langle x, \mu_S(x), \gamma_S(x)\rangle$. Where,

$$\mu_R(x) = \begin{cases} \mu_A(x), & \text{if } \mu_U(x) \geq \mu_V(x) \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_R(x) = \begin{cases} \gamma_A(x), & \text{if } \gamma_U(x) \leq \gamma_V(x) \\ 1, & \text{otherwise} \end{cases}$$

$$\mu_S(x) = \begin{cases} \mu_B(x), & \text{if } \mu_U(x) < \mu_V(x) \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_S(x) = \begin{cases} \gamma_B(x), & \text{if } \gamma_U(x) > \gamma_V(x) \\ 1, & \text{otherwise} \end{cases}.$$

Then, $N = R \cup S$. Now, $R \neq 0_\sim$ (since, if $R = 0_\sim$, then $U \subset V$ which implies that, $U = U \cap V \subseteq \overline{N}$, a contradiction). Similarly, $S \neq 0_\sim$. Also, $R \subseteq A \subseteq U$ and $S \subseteq B \subseteq V$. Now, $R \not{q} V$. For, if RqV then, there exists, $x \in X$ such that, $\mu_R(x) > \gamma_V(x)$ or $\gamma_R(x) < \mu_V(x)$. First, if $\mu_R(x) > \gamma_V(x)$ then, $\mu_R(x) > 0$ which implies that, $\mu_U(x) \geq \mu_V(x)$. Since $N = R \cup S$ then, $\mu_N(x) \geq \mu_R(x) > \gamma_V(x)$. Since, $U \cap V \subseteq \overline{N}$ then, $\mu_N(x) \leq \gamma_U(x) \vee \gamma_V(x)$ but, $\mu_N(x) > \gamma_V(x)$ this implies that $\gamma_U(x) > \gamma_V(x)$ then, $\gamma_R(x) = 1$ this implies that $\mu_R(x) = 0$, a contradiction. Then, $R \not{q} V$. Second, if $\gamma_R(x) < \mu_V(x)$ then , $\gamma_R(x) < 1$ which implies that $\gamma_U(x) \leq \gamma_V(x)$. Since $N = R \cup S$ then, $\gamma_N(x) = (\gamma_R \wedge \gamma_S)(x)$ implies that $\gamma_N(x) \leq \gamma_R(x)$ then, $\gamma_N(x) < \mu_V(x)$. Since, $U \cap V \subseteq \overline{N}$ then, $\gamma_N(x) \geq (\mu_U \wedge \mu_V)(x)$ but, $\gamma_N(x) < \mu_V(x)$ this implies that $\mu_V(x) > \mu_U(x)$ then, $\mu_R(x) = 0$ implies $\gamma_R(x) = 1$ a contradiction. Then, $R \not{q} V$. Similarly, $S \not{q} U$. Then N is an IF$c_s^{\alpha,\beta}$-disconnected, which is a contradiction. Thus, N is an IF$c_3^{\alpha,\beta}$-connected.

Theorem 2.12. *Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if N is an IF$c_3^{\alpha,\beta}$-connected then, N is an IF$c_M^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$c_M^{\alpha,\beta}$-disconnected. Then, there exist $A, B \in \zeta^X$ such that, $A \cup B = N$, $(cl_{\alpha,\beta}A) \cap B = 0_\sim$, $(cl_{\alpha,\beta}B) \cap A = 0_\sim$, $A \neq 0_\sim$, $B \neq 0_\sim$. Let $U = \overline{cl_{\alpha,\beta}A}$ and $V = \overline{cl_{\alpha,\beta}B}$. Then, $\tau(U) = \tau^*(\overline{U}) = \tau^*(cl_{\alpha,\beta}A) \geq \langle\alpha, \beta\rangle$ and $\tau(V) = \tau^*(\overline{V}) = \tau^*(cl_{\alpha,\beta}B) \geq \langle\alpha, \beta\rangle$.

Now, $U \cap V = \overline{cl_{\alpha,\beta}A} \cap \overline{cl_{\alpha,\beta}B} = \overline{cl_{\alpha,\beta}A \cup cl_{\alpha,\beta}B} = \overline{cl_{\alpha,\beta}(A \cup B)} = \overline{cl_{\alpha,\beta}N} \subseteq \overline{N}$. Also, $U \cup V = \overline{cl_{\alpha,\beta}A} \cup \overline{cl_{\alpha,\beta}B} = \overline{cl_{\alpha,\beta}A \cap cl_{\alpha,\beta}B} = \overline{cl_{\alpha,\beta}(A \cap B)} = \overline{cl_{\alpha,\beta}0_\sim} = \overline{0_\sim} = 1_\sim$. Then, $N \subseteq U \cup V$. Also, $U \not\subseteq \overline{N}$. For if $U \subseteq \overline{N}$ then, $N \subseteq \overline{U} = cl_{\alpha,\beta}A$ this implies that, $cl_{\alpha,\beta}A \supseteq A \cup B$ implies $cl_{\alpha,\beta}A \supseteq B$ implies $B \cap cl_{\alpha,\beta}A \neq 0_\sim$, a contradiction. Similarly, $V \not\subseteq \overline{N}$. Therefore, N is an IF$c_3^{\alpha,\beta}$-disconnected which is a contradiction, then N is an IF$c_M^{\alpha,\beta}$-connected.

Definition 2.13. Let X be a nonempty set and $A, B \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, A and B are said to be

(i) (α, β)-intuitionistic fuzzy separated (briefly, (α, β)-IFS), if there exist IFSs $U, V \in \zeta^X$ with $\tau(U) \geq \langle\alpha, \beta\rangle$ and $\tau(V) \geq \langle\alpha, \beta\rangle$ such that $A \subseteq U$, $B \subseteq V$, $U \cap B = 0_\sim$ and $A \cap V = 0_\sim$.

(ii) (α, β)-intuitionistic fuzzy strongly separated (briefly, (α, β)-IFSS) if there exist IFSs $U, V \in \zeta^X$ with $\tau(U) \geq \langle\alpha, \beta\rangle$ and $\tau(V) \geq \langle\alpha, \beta\rangle$ such that $A \subseteq U$, $B \subseteq V$, $U \cap B = 0_\sim$, $A \cap V = 0_\sim$, UqA and VqB.

Definition 2.14. Let (X, τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$,

(i) N is called an intuitionistic fuzzy $O^{\alpha,\beta}$-disconnected (briefly, IF$O^{\alpha,\beta}$-disconnected) if there exist an (α, β)-IFS sets $A, B \in \zeta^X$ such that $A \cup B = N$, $A \neq 0_\sim$ and $B \neq 0_\sim$.

(ii) N is called an intuitionistic fuzzy $O_q^{\alpha,\beta}$-disconnected (briefly,IF$O_q^{\alpha,\beta}$-disconnected) if there exist an (α, β)-IFSS sets $A, B \in \zeta^X$ such that $A \cup B = N$, $A \neq 0_\sim$ and $B \neq 0_\sim$.

(iii) N is called an IF$O^{\alpha,\beta}$-connected if N is not an IF$O^{\alpha,\beta}$-disconnected.

(iv)) N is called an IF$O_q^{\alpha,\beta}$-connected if N is not an IF$O_q^{\alpha,\beta}$-disconnected.

Theorem 2.15. *Let (X,τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, N is an IF$c_2^{\alpha,\beta}$-connected iff N is an IF$O^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$O^{\alpha,\beta}$-disconnected. Then, there exist $A, B \in \zeta^X$ such that, A, B are (α,β)-IFS, $N = A \cup B$, $A \neq 0_\sim$, $B \neq 0_\sim$. Since A, B are (α,β)-IFS, there exist $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha,\beta \rangle$ and $\tau(V) \geq \langle \alpha,\beta \rangle$ such that, $A \subseteq U$, $B \subseteq V$, $U \cap B = 0_\sim$, $V \cap A = 0_\sim$. $N = A \cup B \subseteq U \cup V$. Now, $N \cap U \cap V = (A \cup B) \cap (U \cap V) = (A \cap U \cap V) \cup (B \cap U \cap V) = 0_\sim$. Also, $N \cap U = (A \cup B) \cap U = (A \cap U) \cup (B \cap U) = A \cup 0_\sim = A \neq 0_\sim$. Similarly, $N \cap V \neq 0_\sim$. Then, N is an IF$c_2^{\alpha,\beta}$-disconnected, which is a contradiction. Hence, N is an IF$O^{\alpha,\beta}$-connected.

Conversely, suppose that N is an IF$c_2^{\alpha,\beta}$-disconnected. Then, there exist $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha,\beta \rangle$ and $\tau(V) \geq \langle \alpha,\beta \rangle$ such that, $N \subseteq U \cup V$, $N \cap U \cap V = 0_\sim$, $N \cap U \neq 0_\sim$, $N \cap V \neq 0_\sim$. Let $A = N \cap U \subseteq U$ and $B = N \cap V \subseteq V$. Then, $A \cup B = (N \cap U) \cup (N \cap V) = N \cap (U \cup V) = N$. Also, $U \cap B = U \cap N \cap V = 0_\sim$. Similarly, $V \cap A = 0_\sim$. So, N is an IF$O^{\alpha,\beta}$-disconnected which is a contradiction. Then, N is an IF$c_2^{\alpha,\beta}$-connected.

Theorem 2.16. *Let (X,τ) be an IFTS and $N \in \zeta^X$. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, if N is an IF$c_4^{\alpha,\beta}$-connected then, N is an IF$O_q^{\alpha,\beta}$-connected.*

Proof. Suppose that N is an IF$O_q^{\alpha,\beta}$-disconnected.Then, there exist $A, B \in \zeta^X$ such that, A, B are (α,β)-IFSS, $N = A \cup B$. Since A, B are (α,β)-IFSS, there exist $U, V \in \zeta^X$ with $\tau(U) \geq \langle \alpha,\beta \rangle$ and $\tau(V) \geq \langle \alpha,\beta \rangle$ such that, $A \subseteq U$, $B \subseteq V$, $U \cap B = 0_\sim$, $V \cap A = 0_\sim$, UqA, VqB. $N = A \cup B \subseteq U \cup V$. Now, $N \cap U \cap V = (A \cup B) \cap U \cap V = (A \cap U \cap V) \cup (B \cap U \cap V) = 0_\sim$. Also, since UqA and $A \subseteq N$, there exists $x \in X$ such that $\mu_U(x) > \gamma_A(x) \geq \gamma_N(x)$ or $\gamma_U(x) < \mu_A(x) \leq \mu_N(x)$ this implies that, $U \not\subseteq \overline{N}$. Similarly, $V \not\subseteq \overline{N}$. Therefore, N is an IF$c_4^{\alpha,\beta}$-disconnected which is a contradiction. Then, N is an IF$O_q^{\alpha,\beta}$-connected.

Remark 2.17. From Remark 2.2 and Theorems 2.8-2.12,2.15,2.16 we can build the following diagram

IF$c_1^{\alpha,\beta}$-disconnectedness
$\Downarrow$
IF$c_s^{\alpha,\beta}$-disconnectedness
$\swarrow \quad \searrow$
IF$c_3^{\alpha,\beta}$-disconnectedness IF$c_2^{\alpha,\beta}$-disconnectedness =
$\Downarrow$ $\searrow$ $\swarrow$ IF$O^{\alpha,\beta}$-disconnectedness
IF$c_M^{\alpha,\beta}$-disconnectedness IF$c_4^{\alpha,\beta}$-disconnectedness
$\Downarrow$
IF$o_q^{\alpha,\beta}$-disconnectedness

Examples 2.3, 2.4 and the next examples show that the reverse implications in Remark 2.17 are not true in general.

Example 2.18. Let $X = \{a, b\}$ and $N, G_i \in \zeta^X$ $(i = 1, 2)$ be defined as follows:

$$N = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$$

$$G_1 = \langle x, (\frac{a}{0.7}, \frac{b}{0.8}), (\frac{a}{0.3}, \frac{b}{0.1}) \rangle$$

$$G_2 = \langle x, (\frac{a}{0.2}, \frac{b}{0.3}), (\frac{a}{0.4}, \frac{b}{0.5}) \rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.5, 0.3 \rangle, & \text{if } A = G_1 \\ \langle 0.7, 0.2 \rangle, & \text{if } A = G_2 \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.4$, $\beta = 0.5$. Then, N is an IF$c_s^{\alpha,\beta}$-connected but not an IF$c_1^{\alpha,\beta}$-connected.

Example 2.19. Let $X = \{a, b\}$ and $N, G_i \in \zeta^X$ $(i = 1, 2, 3, 4, 5, 6)$ defined as follows:

$$N = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle$$

$$G_1 = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$$

$$G_2 = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}) \rangle$$

$$G_3 = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \rangle$$

$$G_4 = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}) \rangle$$

$$G_5 = \langle x, (\frac{a}{0.0}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.6}) \rangle$$

$$G_6 = \langle x, (\frac{a}{0.3}, \frac{b}{0.0}), (\frac{a}{0.4}, \frac{b}{0.5}) \rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.4, 0.2 \rangle, & \text{if } A \in \{G_1, G_2\} \\ \langle 0.5, 0.4 \rangle, & \text{if } A \in \{G_3, G_4\} \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.4, \beta = 0.5$. Then, N is both IF$c_2^{\alpha,\beta}$-connected and IF$c_3^{\alpha,\beta}$-connected but not IF$c_s^{\alpha,\beta}$-connected.

Example 2.20. Let $X = \{a, b, c\}$ and $N, G_i \in \zeta^X$ $(i = 1, 2, 3)$ defined as follows:

$$N = \langle x, (\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.4})\rangle$$

$$G_1 = \langle x, (\frac{a}{0.6}, \frac{b}{0.0}, \frac{c}{0.0}), (\frac{a}{0.3}, \frac{b}{1.0}, \frac{c}{1.0})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.0}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{1.0}, \frac{b}{0.2}, \frac{c}{0.3})\rangle$$

$$G_3 = \langle x, (\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.3})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ < 0.3, 0.5 >, & \text{if } A \in \{G_1, G_2\} \\ < 0.7, 0.2 >, & \text{if } A = G_3 \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.1, \beta = 0.8$. Then, N is an IF$O_q^{\alpha,\beta}$-connected but not an IF$c_4^{\alpha,\beta}$-connected.

Example 2.21. Let $X = \{a, b, c\}$ and $N, G_i \in \zeta^X$ $(i = 1, 2, 3, 4)$ defined as follows:

$$N = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.2})\rangle$$

$$G_1 = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.1}), (\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.2})\rangle$$

$$G_3 = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.2})\rangle$$

$$G_4 = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.1}), (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_{\sim}, 1_{\sim}\} \\ \langle 0.3, 0.6\rangle, & \text{if } A \in \{G_1, G_2\} \\ \langle 0.5, 0.4\rangle, & \text{if } A \in \{G_3, G_4\} \\ 0^{\sim}, & \text{otherwise.} \end{cases}$$

Let $\alpha = 0.2, \beta = 0.7$. Then, N is an IF$C_M^{\alpha,\beta}$-connected but not an IF$c_3^{\alpha,\beta}$-connected.

3. Intuitionistic fuzzy $c_5^{\alpha,\beta}$-connectedness

Definition 3.1. Let (X, τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$,

(i) X is called an intuitionistic fuzzy $c_5^{\alpha,\beta}$-disconnected (briefly, IF$c_5^{\alpha,\beta}$-disconnected) if there exist an IFS $A \in \zeta^X$ such that $\tau(A) \geq \langle \alpha, \beta \rangle$, $\tau^*(A) \geq \langle \alpha, \beta \rangle$, $A \neq 0_\sim$ and $A \neq 1_\sim$.

(ii) X is called an (α, β)-intuitionistic fuzzy disconnected (briefly, (α, β) IF-disconnected) if there exist an IFSs $A, B \in \zeta^X$ with $\tau(A) \geq \langle \alpha, \beta \rangle$, $\tau(B) \geq \langle \alpha, \beta \rangle$, such that $A \cup B = 1_\sim$, $A \cap B = 0_\sim$, $A \neq 0_\sim$ and $B \neq 0_\sim$.

(iii) X called an IF$c_5^{\alpha,\beta}$-connected if X is not an IF$c_5^{\alpha,\beta}$-disconnected.

(iv) X called an (α, β)IF-connected if X is not an (α, β)IF-disconnected.

Theorem 3.2. *Let (X, τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if (X, τ) is an IF$c_5^{\alpha,\beta}$-connected then, (X, τ) is an (α, β)IF-connected.*

Proof. Suppose that (X, τ) is an (α, β)IF-disconnected.Then, there exist $A, B \in \zeta^X$ with, $\tau(A) \geq \langle \alpha, \beta \rangle$, $\tau(B) \geq \langle \alpha, \beta \rangle$ such that, $A \cup B = 1_\sim$, $A \cap B = 0_\sim$, $A \neq 0_\sim$, $B \neq 0_\sim$. This implies that, $\mu_A \vee \mu_B = 1_X$, $\gamma_A \wedge \gamma_B = 0_X$, $\mu_A \wedge \mu_B = 0_X$, $\gamma_A \vee \gamma_B = 1_X$. Let $C = \{x \in X : \mu_A(x) > 0\}$ and $D = \{x \in X : \mu_A(x) = 0\}$.

If $x \in C$ then, $\mu_A(x) > 0 \Rightarrow \mu_B(x) = 0 \Rightarrow \mu_A(x) = 1 \Rightarrow \gamma_A(x) = 0 \Rightarrow \gamma_B(x) = 1$.

If $x \in D$ then, $\mu_A(x) = 0 \Rightarrow \gamma_A(x) = 1 \Rightarrow \gamma_B(x) = 0 \Rightarrow \mu_B(x) = 1$. Then, $\mu_A = \gamma_B$ and $\gamma_A = \mu_B$; in other words, $B = \overline{A}$ then, $\tau^*(A) = \tau(\overline{A}) = \tau(B) \geq \langle \alpha, \beta \rangle$. and since $B \neq 0_\sim$, $A \neq 1_\sim$. Thus, (X, τ) is an IF$c_5^{\alpha,\beta}$-disconnected which is a contradiction. Hence, (X, τ) is an (α, β)IF-connected.

Theorem 3.3. *Let (X, τ_1), (Y, τ_2) be two IFTSs and $f : (X, \tau_1) \to (Y, \tau_2)$ be an intuitionistic fuzzy continuous and surjective map. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if (X, τ_1) is an (α, β)IF-connected then so is (Y, τ_2).*

Proof. Suppose that (Y, τ_2) is an (α, β)IF-disconnected. Then, there exist $U, V \in \zeta^Y$ with, $\tau_2(U) \geq \langle \alpha, \beta \rangle$, $\tau_2(V) \geq \langle \alpha, \beta \rangle$ such that, $U \cup V = 1_\sim$, $U \cap V = 0_\sim$, $U \neq 0_\sim$ and $V \neq 0_\sim$. Since f is an intuitionistic fuzzy continuous then,

$\tau_1(f^{-1}(U)) \geq \tau_2(U) \geq \langle \alpha, \beta \rangle$ and $\tau_1(f^{-1}(V)) \geq \tau_2(V) \geq \langle \alpha, \beta \rangle$. Let $A = f^{-1}(U)$, $B = f^{-1}(V)$, then $\tau_1(A) \geq \langle \alpha, \beta \rangle$ and $\tau_1(B) \geq \langle \alpha, \beta \rangle$. Since f is surjective and $U \neq 0_\sim$ then, $A = f^{-1}(U) \neq 0_\sim$ (For, if $f^{-1}(U) = 0_\sim$ then, $U = f(f^{-1}(U)) = f(0_\sim) = 0_\sim$ a contradiction). Similarly, $B = f^{-1}(V) \neq 0_\sim$. Now, $A \cup B = f^{-1}(U) \cup f^{-1}(V) = f^{-1}(U \cup V) = f^{-1}(1_\sim) = 1_\sim$. Similarly, $A \cup B = 0_\sim$. Thus, (X, τ_1) is an (α, β)IF-disconnected which is a contradiction. Hence, (Y, τ_2) is an (α, β)IF-connected.

Theorem 3.4. *Let (X, τ_1), (Y, τ_2) be two IFTSs and $f : (X, \tau_1) \to (Y, \tau_2)$ be an intuitionistic fuzzy continuous and surjective map. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if (X, τ_1) is an IF $c_5^{\alpha,\beta}$-connected then so is (Y, τ_2).*

Proof. It is similar to Theorem 3.3.

Theorem 3.5. *Let (X, τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, (X, τ) is an IF $c_5^{\alpha,\beta}$-connected iff there is no exist IFSs $A, B \in \zeta^X$ with $\tau(A) \geq \langle \alpha, \beta \rangle$, $\tau(B) \geq \langle \alpha, \beta \rangle$ such that $A = \overline{B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$.*

Proof. Assume that there exist $A, B \in \zeta^X$ with $\tau(A) \geq \langle\alpha,\beta\rangle$, $\tau(B) \geq \langle\alpha,\beta\rangle$ such that $A = \overline{B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$. Now, $\tau^*(B) = \tau(\overline{B}) = \tau(A) \geq \langle\alpha,\beta\rangle$ and $A \neq 0_\sim$ implies that $B \neq 1_\sim$. Then, (X,τ) is an IF $c_5^{\alpha,\beta}$-disconnected which is a contradiction. Conversely, assume that (X,τ) is an IF $c_5^{\alpha,\beta}$-disconnected. Then, there exists an IFS $A \in \zeta^X$ such that $\tau(A) \geq \langle\alpha,\beta\rangle$, $\tau^*(A) \geq \langle\alpha,\beta\rangle$, $A \neq 0_\sim$, $A \neq 1_\sim$. Now, take $B = \overline{A}$ then, $\tau(B) = \tau^*(\overline{B}) = \tau(A) \geq \langle\alpha,\beta\rangle$ and since $A \neq 1_\sim$ then $B \neq 0_\sim$, which is a contradiction. Hence, (X,τ) is an IF $c_5^{\alpha,\beta}$-connected.

Theorem 3.6. *Let (X,τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, (X,τ) is an IF $c_5^{\alpha,\beta}$-connected iff there is no exist IFSs $A, B \in \zeta^Y$ such that $B = \overline{A}$, $B = \overline{cl_{\alpha,\beta}A}$, $A = \overline{cl_{\alpha,\beta}B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$.*

Proof. Assume that there exist $A, B \in \zeta^Y$ such that $B = \overline{A}$, $B = \overline{cl_{\alpha,\beta}A}$, $A = \overline{cl_{\alpha,\beta}B}$, $A \neq 0_\sim$, $B \neq 0_\sim$. Then, $\tau(A) = \tau(\overline{cl_{\alpha,\beta}B}) = \tau^*(cl_{\alpha,\beta}B) \geq \langle\alpha,\beta\rangle$ and $\tau(\overline{A}) = \tau(B) = \tau(\overline{cl_{\alpha,\beta}A}) = \tau^*(cl_{\alpha,\beta}A) \geq \langle\alpha,\beta\rangle$. Then, (X,τ) is an IF $c_5^{\alpha,\beta}$-disconnected, a contradiction. Conversely, suppose that, (X,τ) is an IF $c_5^{\alpha,\beta}$-disconnected. Then there exists an IFS $A \in \zeta^Y$ such that $\tau(A) \geq \langle\alpha,\beta\rangle$, $\tau^*(A) \geq \langle\alpha,\beta\rangle$, $A \neq 0_\sim$ and $A \neq 1_\sim$. Let $B = \overline{A}$. Then, $B \neq 1_\sim$, $B \neq 0_\sim$ and since $\tau^*(A) \geq \langle\alpha,\beta\rangle$ then, $A = cl_{\alpha,\beta}A$, this implies that $B = \overline{cl_{\alpha,\beta}A}$. Since $\tau^*(B) = \tau^*(\overline{A}) = \tau(A) \geq \langle\alpha,\beta\rangle$, then $A = \overline{B} = \overline{cl_{\alpha,\beta}B}$, a contradiction.

Definition 3.7. Let (X,τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, X is called an (α,β)-intuitionistic fuzzy strong connected(briefly, (α,β)IF-strong connected) if there exist IFSs $A, B \in \zeta^X$ with $\tau^*(A) \geq \langle\alpha,\beta\rangle$, $\tau^*(B) \geq \langle\alpha,\beta\rangle$, such that $\mu_A + \mu_B \leq 1$, $\gamma_A + \gamma_B \geq 1$, $A \neq 0_\sim$ and $B \neq 0_\sim$.

Remark 3.8. The notions of $c_5^{\alpha,\beta}$-connectedness and (α,β)IF-strong connectedness are independent as indicated by the following examples.

Example 3.9. Let $X = \{a,b,c\}$ and $G_i \in \zeta^X$ $(i = 1,2,3,4)$ defined as follows:

$$G_1 = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.2})\rangle$$

$$G_3 = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3})\rangle$$

$$G_4 = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.2})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^\sim, & \text{if } , A \in \{0_\sim, 1_\sim\} \\ \langle 0.4, 0.5\rangle, & \text{if } A \in \{G_1, G_2\} \\ \langle 0.5, 0.5\rangle, & \text{if } A \in \{G_3, G_4\} \\ 0^\sim, & \text{otherwise} \end{cases}$$

Let $\alpha = 0.3, \beta = 0.5$. Then, X is an (α,β)-strong connected but not an IF$c_5^{\alpha,\beta}$-connected.

Example 3.10. Let $X = \{a, b, c\}$ and $G_1, G_2 \in \zeta^X$ defined as follows:

$$G_1 = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.7}), (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.2})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.7})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^{\sim}, & \text{if } A \in \{0_\sim, 1_\sim\} \\ \langle 0.4, 0.3\rangle, & \text{if } A = G_1 \\ \langle 0.6, 0.2\rangle, & \text{if } A = G_2 \\ 0^{\sim}, & \text{otherwise} \end{cases}$$

Let $\alpha = 0.2, \beta = 0.5$. Then, X is an IF$c_5^{\alpha,\beta}$-connected but not an (α, β)-strong connected.

Theorem 3.11. *Let (X, τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, (X, τ) is an (α, β)-IF strong connected iff there is no exist IFSs $A, B \in \zeta^Y$ with $\tau(A) \geq \langle\alpha, \beta\rangle$, $\tau(B) \geq \langle\alpha, \beta\rangle$ such that $\mu_A + \mu_B \geq 1$, $\gamma_A + \gamma_B \leq 1$, $A \neq 1_\sim$ and $B \neq 1_\sim$.*

Proof. Let $A, B \in \zeta^X$ with $\tau(A) \geq \langle\alpha, \beta\rangle$, $\tau(B) \geq \langle\alpha, \beta\rangle$ such that $\mu_A + \mu_B \geq 1$, $\gamma_A + \gamma_B \leq 1$, $A \neq 1_\sim$ and $B \neq 1_\sim$. If we take $C = \overline{A}$ and $D = \overline{B}$, then $\tau^*(C) = \tau^*(\overline{A}) = \tau(A) \geq \langle\alpha, \beta\rangle$, $\tau^*(D) = \tau^*(\overline{B}) = \tau(B) \geq \langle\alpha, \beta\rangle$, $C \neq 0_\sim$ and $D \neq 0_\sim$. Moreover, $\mu_C + \mu_D = \gamma_A + \gamma_B \leq 1$, $\gamma_C + \gamma_D = \mu_A + \mu_B \geq 1$, a contradiction.

The converse of the proof is obtained by using a similar technique.

Theorem 3.12. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be an intuitionistic fuzzy continuous and surjective map from an IFTS (X, τ_1) to another IFTS (Y, τ_2). For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, if (X, τ_1) is an (α, β)IF-strong connected then so is (Y, τ_2).*

Proof. Suppose that (Y, τ_2) is not an (α, β)IF-strong connected. Then, there exist $C, D \in \zeta^Y$ with, $\tau_2^*(C) \geq \langle\alpha, \beta\rangle$, $\tau_2^*(D) \geq \langle\alpha, \beta\rangle$ such that, $\mu_C + \mu_D \leq 1$, $\gamma_C + \gamma_D \geq 1$, $C \neq 0_\sim$, $D \neq 0_\sim$. By using Theorem 1.14, we have, $\tau_1^*(f^{-1}(C)) \geq \tau_2^*(C) \geq \langle\alpha, \beta\rangle$ and $\tau_1^*(f^{-1}(D)) \geq \tau_2^*(D) \geq \langle\alpha, \beta\rangle$. Also, $\mu_{f^{-1}(C)} + \mu_{f^{-1}(D)} = f^{-1}(\mu_C) + f^{-1}(\mu_D) = \mu_C \circ f + \mu_D \circ f \leq 1_\sim$(since, $\mu_C + \mu_D \leq 1_\sim$). Similarly, $\gamma_{f^{-1}(C)} + \gamma_{f^{-1}(D)} = f^{-1}(\gamma_C) + f^{-1}(\gamma_D) = \gamma_C \circ f + \gamma_D \circ f \geq 1_\sim$(since, $\gamma_C + \gamma_D \geq 1_\sim$). Moreover, $f^{-1}(C) \neq 0_\sim$ (For, if $f^{-1}(C) = 0_\sim$ then, $C = f(f^{-1}(C)) = f(0_\sim) = 0_\sim$, a contradiction). Similarly, $f^{-1}(C) \neq 0_\sim$. This is a contradiction, thus (Y, τ_2) is (α, β)IF-strong connected.

4. (α, β)-intuitionistic fuzzy super connectedness

Definition 4.1. Let (X, τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$,

(i) X is called an (α, β)-intuitionistic fuzzy super disconnected(briefly, (α, β)IF-super disconnected) if there exist an (α, β)-ifro set A in X such that, $A \neq 0_\sim$ and $A \neq 1_\sim$.

(ii) X called an (α,β)IF-super connected if X is not an (α,β)IFS-disconnected.

Theorem 4.2. *Let (X,τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, the following statements are equivalent:*

(i) *X is an (α,β)IF-super connected.*

(ii) *For each $A \in \zeta^X$, $A \neq 0_\sim$ such that $\tau(A) \geq \langle\alpha,\beta\rangle$ we have $cl_{\alpha,\beta}A = 1_\sim$.*

(iii) *For each $A \in \zeta^X$, $A \neq 1_\sim$ such that $\tau^*(A) \geq \langle\alpha,\beta\rangle$ we have $int_{\alpha,\beta}A = 0_\sim$.*

(iv) *There is no exist IFSs $A, B \in \zeta^X$ with, $\tau(A) \geq \langle\alpha,\beta\rangle$ and $\tau(B) \geq \langle\alpha,\beta\rangle$ such that $A \subseteq \overline{B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$.*

(v) *There is no exist IFSs $A, B \in \zeta^X$ with, $\tau(A) \geq \langle\alpha,\beta\rangle$ and $\tau(B) \geq \langle\alpha,\beta\rangle$ such that $B = \overline{cl_{\alpha,\beta}A}$, $A = \overline{cl_{\alpha,\beta}B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$.*

(vi) *There is no exist IFSs $A, B \in \zeta^X$ with, $\tau^*(A) \geq \langle\alpha,\beta\rangle$ and $\tau^*(B) \geq \langle\alpha,\beta\rangle$ such that $B = \overline{int_{\alpha,\beta}A}$, $A = \overline{int_{\alpha,\beta}B}$, $A \neq 1_\sim$ and $B \neq 1_\sim$.*

Proof. (i)⇒(ii) Assume that there exist $A \in \zeta^X$, $A \neq 0_\sim$ with $\tau(A) \geq \langle\alpha,\beta\rangle$ such that $cl_{\alpha,\beta}A \neq 1_\sim$. Then, $B = int_{\alpha,\beta}(cl_{\alpha,\beta}A) \neq 1_\sim$ is an (α,β)-ifro set in X and $0_\sim \neq A \subseteq int_{\alpha,\beta}(cl_{\alpha,\beta}A) = B$, which is a contradiction. Then, $cl_{\alpha,\beta}A = 1_\sim$.

(ii)⇒(iii) Let $A \neq 1_\sim$ be an IFS in X such that $\tau^*(A) \geq \langle\alpha,\beta\rangle$. Then, $\overline{A} \neq 0_\sim$ and $\tau(\overline{A}) = \tau^*(A) \geq \langle\alpha,\beta\rangle$. By (ii) we have, $cl_{\alpha,\beta}(\overline{A}) = 1_\sim$ implies that $\overline{cl_{\alpha,\beta}(\overline{A})} = 0_\sim$ and by Theorem 1.12, we have $int_{\alpha,\beta}A = 0_\sim$.

(iii)⇒(iv) Let $A, B \in \zeta^X$ with, $\tau(A) \geq \langle\alpha,\beta\rangle$ and $\tau(B) \geq \langle\alpha,\beta\rangle$ such that $A \subseteq \overline{B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$. Then, $\overline{B} \neq 1_\sim$ and $\tau^*(\overline{B}) = \tau(B) \geq \langle\alpha,\beta\rangle$. By (iii) we have $int_{\alpha,\beta}\overline{B} \neq 0_\sim$, and since $A \leq \overline{B}$, then $0_\sim \neq A = int_{\alpha,\beta}A \subseteq int_{\alpha,\beta}\overline{B} = 0_\sim$ which is a contradiction.

(iv)⇒(i) Assume for a contradiction that X is an (α,β)IF-super disconnected. Then, there exists an (α,β)-ifro set A in X such that $A \neq 0_\sim$ and $A \neq 1_\sim$. By Theorem 1.16, $\tau(A) \geq \langle\alpha,\beta\rangle$. If we take $B = \overline{cl_{\alpha,\beta}A}$, then $\tau(B) \geq \langle\alpha,\beta\rangle$ and $B \neq 0_\sim$ (For, if $B = 0_\sim \Rightarrow \overline{cl_{\alpha,\beta}A} = 0_\sim \Rightarrow cl_{\alpha,\beta}A = 1_\sim \Rightarrow A = int_{\alpha,\beta}(cl_{\alpha,\beta}A) = 1_\sim$ which is a contradiction with the fact $A \neq 0_\sim$). We also , have $A \leq \overline{B}$ and this is a contradiction too.

(i)⇒(v) Suppose that there exist IFSs $A, B \in \zeta^X$ with, $\tau(A) \geq \langle\alpha,\beta\rangle$ and $\tau(B) \geq \langle\alpha,\beta\rangle$ such that $B = \overline{cl_{\alpha,\beta}A}$, $A = \overline{cl_{\alpha,\beta}B}$, $A \neq 0_\sim$ and $B \neq 0_\sim$. Then, $int_{\alpha,\beta}(cl_{\alpha,\beta}A) = int_{\alpha,\beta}\overline{B} = \overline{cl_{\alpha,\beta}B} = A$ and $A \neq 0_\sim$, $A \neq 1_\sim$ (For, if $A = 1_\sim$, then $1_\sim = \overline{cl_{\alpha,\beta}B}$ implies $0_\sim = cl_{\alpha,\beta}B$ implies $B = 0_\sim$). A contradiction with X is an (α,β)IF-super connected.

(v)⇒ (i) Suppose that X is an (α,β)IF-super disconnected. Then, there is an (α,β)-ifro set A in X such that, $A \neq 0_\sim$, $A \neq 1_\sim$. Now, take $B = \overline{cl_{\alpha,\beta}A}$. Then, $\tau(B) \geq \langle\alpha,\beta\rangle$, $B \neq 0_\sim$ and $\overline{cl_{\alpha,\beta}B} = \overline{cl_{\alpha,\beta}(\overline{cl_{\alpha,\beta}A})} = \overline{\overline{int_{\alpha,\beta}(cl_{\alpha,\beta}A)}} = int_{\alpha,\beta}(cl_{\alpha,\beta}A) = A$ which is a contradiction.

(v)⇒(vi) Let A, B IFSs in X with $\tau^*(A) \geq \langle\alpha,\beta\rangle$ and $\tau^*(B) \geq \langle\alpha,\beta\rangle$ such that $B = \overline{int_{\alpha,\beta}A}$, $A = \overline{int_{\alpha,\beta}B}$, $A \neq 1_\sim$ and $B \neq 1_\sim$. Take $C = \overline{A}$ and $D = \overline{B}$.

Then, $C \neq 0_\sim$, $D \neq 0_\sim$, $\tau(C) = \tau(\overline{A}) = \tau^*(A) \geq \langle\alpha,\beta\rangle$, $\tau(D) = \tau(\overline{B}) = \tau^*(B) \geq \langle\alpha,\beta\rangle$ and $\overline{cl_{\alpha,\beta}C} = \overline{cl_{\alpha,\beta}\overline{A}} = \overline{\overline{int_{\alpha,\beta}A}} = int_{\alpha,\beta}A = \overline{B} = D$. Similarly, $\overline{cl_{\alpha,\beta}D} = C$. This is a contradiction.

(vi)⇒(v) It is similarly to that (v)⇒(vi).

Theorem 4.3. *Let (X,τ) be an IFTS. For $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, if X is an (α,β)IF-super connected then, X is an IF$c_5^{\alpha,\beta}$-connected.*

Proof. It is clear.

The converse of Theorem 4.3, is not true in general as shows in the following example:

Example 4.4. Let $X = \{a,b,c,d\}$ and $G_i \in \zeta^X$ $(i = 1,2,3,4)$ defined as follows:

$$G_1 = \langle x, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{0.0}), (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{1.0})\rangle$$

$$G_2 = \langle x, (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{1.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{0.0})\rangle$$

$$G_3 = \langle x, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{1.0}), (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{0.0})\rangle$$

$$G_4 = \langle x, (\frac{a}{0.0}, \frac{b}{0.0}, \frac{c}{0.0}, \frac{d}{0.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}, \frac{d}{1.0})\rangle$$

Let $\tau : \zeta^X \longrightarrow I \times I$ defined as follows:

$$\tau(A) = \begin{cases} 1^\sim, & \text{if }, A \in \{0_\sim, 1_\sim\} \\ \langle 0.5, 0.4\rangle, & \text{if } A \in \{G_1, G_2\} \\ \langle 0.7, 0.3\rangle, & \text{if } A \in \{G_3, G_4\} \\ 0^\sim, & \text{otherwise} \end{cases}$$

Let $\alpha = 0.4, \beta = 0.6$. Then, X is an IF$c_5^{\alpha,\beta}$-connected but not an (α,β)IF-super connected.

Theorem 4.5. *Let (X,τ_1), (Y,τ_2) be two IFTSs and $f : (X,\tau_1) \to (Y,\tau_2)$ be a surjective intuitionistic fuzzy continuous map. Then, for $\alpha \in I_0, \beta \in I_1$ with $\alpha+\beta \leq 1$, if X is an (α,β)IF-super connected, then so is Y.*

Assume that Y is an (α,β)IF-super disconnected. By Theorem 4.4(iv), there exist IFSs $C, D \in \zeta^Y$ with $\tau_2(C) \geq \langle\alpha,\beta\rangle$ and $\tau_2(D) \geq \langle\alpha,\beta\rangle$ such that $C \subseteq \overline{D}$, $C \neq 0_\sim$ and $D \neq 0_\sim$. Since f is intuitionistic fuzzy continuous, $\tau_1(f^{-1}(C)) \geq \tau_2(C) \geq \langle\alpha,\beta\rangle$ and $\tau_1(f^{-1}(D)) \geq \tau_2(D) \geq \langle\alpha,\beta\rangle$. $C \subseteq \overline{D}$ implies that $f^{-1}(C) \subseteq f^{-1}(\overline{D}) = \overline{f^{-1}(D)}$. Also, $f^{-1}(C) \neq 0_\sim$ and $f^{-1}(D) \neq 0_\sim$. By Theorem 4.4(iv), X is an (α,β)IF-super disconnected, a contradiction.

5. Conclusions

We defined and studied several types of fuzzy connectedness in intuitionistic fuzzy topological spaces in view of Šostak's sense. The relationships between different kinds of intuitionistic fuzzy connectedness were investigated. We Built a diagram to sum up the interrelationships between these types of intuitionistic fuzzy connectedness and illustrated that the converses are not true in general by giving several examples.

Acknowledgements. The authors are highly grateful to the referees for their valuable comments and suggestions.

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Accepted: 26.10.2015