CERTAIN TRANSFORMATIONS OF BASIC AND POLY-BASIC HYPERGEOMETRIC SERIES

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Abstract. In the present work, certain new transformations of basic and poly-basic hypergeometric series have been established with the help of Bailey lemma.

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1. Introduction

In 1944, Bailey [5] established a powerful series identity which was later known as Bailey's lemma. The Bailey's lemma states that, if

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \tag{1.1}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{n+r} \tag{1.2}$$

then, under the suitable convergence conditions and if change in the order of summations is allowable

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.3}$$

where α_r, δ_r, u_r and v_r are functions of r, such that γ_n exists. The proof of the lemma is trivial.

The Bailey lemma has been a powerful tool in proving Rogers-Ramanujan type of identities and also a verity of transformations of basic and Poly-basic hypergeometric series. Slater [15], [16] used Bailey's lemma systematically to produced long list of 130 identities of Roger-Ramanujan type. The theory of basic hypergeometric series has been extensively studied and developed during last decades due to its various application in mathematics as well as in other disciplines. For a complete account of the theory and its some of the applications one is referred to the notes of Gasper and Rahman [12], Exton [9], Agarwal [2], Fine[10], Andrew [3], [4] Ernst [8] and Tariboon et al [17].

In the present paper, we have made an attempt to establish some interesting transformations and summations of basic and polybasic hypergeometric series by making use of Bailey lemma. Some applications have also been discussed.

In what follows, the following notations and definitions [12] have been used.

For $|q| < 1$,

$$(a;q)_n = \begin{cases} a(1-a)(1-aq)(1-aq^2)...(1-aq^{n-1}); & n \in N \\ 1; & n = 0 \end{cases}$$

or, equivalently,

$$(a;q)_n = \prod_{j=0}^{\infty} \frac{(1-aq^j)}{(1-aq^{n+j})} \equiv \frac{(a;q)_\infty}{(aq^n;q)_\infty},$$

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n),$$

where a is real or complex.

A Basic Hypergeometric Series is defined as

$${}_r\Phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{array}\right] = \sum_{n=0}^{\infty} \frac{(a_1;q)_n\ (a_2;q)_n...(a_r;q)_n}{(q;q)_n(b_1;q)_n\ (b_2;q)_n....(b_s;q)_n}[(-1)^n q^{\frac{n(n-1)}{2}}]^{1+s-r} z^n$$

For $0 < |q| < 1$, the series converges absolutely for all z if $r \leq s$ and for $|z| < 1$ if $r = s+1$.

This series also converges absolutely if $|q| > 1$ and $|z| < |b_1 b_2 ... b_s| / |a_1 a_2 ... a_r|$.

We define the Poly-Basic Hypergeometric Series as

$$\Phi\left[\begin{array}{lllll} a_1, a_2, ...a_r & : c_{1,1}, ...c_{1,r_1} & ; ...; c_{m,1}; ...c_{m,r_m} & ; q, q_1 q_2 ... q_m, z \\ b_1, b_2, ...b_s & : d_{1,1}, ...d_{1,s_1} & ; ...; d_{m,1}; ...d_{m,s_m} & \end{array}\right]$$

$$= \sum_{t=0}^{n-1} \frac{(a_1, a_2....a_r;q)_t}{(q, b_1, b_2....b_s;q)_t} z^t \prod_{j=1}^{m} \frac{(c_{j,1}....c_{j,r_j};q_j)_t}{(d_{j,1}....d_{j,s_j};q_j)_t}$$

which converges for max $(|q|, |q_1|, ..., |q_m|) < 1$.

We shall also require the following known results in our work

$$(1.4) \qquad {}_2\Phi_1\left[\begin{array}{cccc} a, & y; & q; & q \\ & ayq & & \end{array}\right]_n = \frac{(aq, yq;q)_n}{(q, ayq;q)_n} \qquad ([1], \text{App.II}(8))$$

$$(1.5) \quad {}_4\Phi_3\left[\begin{array}{ccccc} \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & e; & q;\ 1/e \\ & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/e & \end{array}\right]_n = \frac{(\alpha q, eq;q)_n}{(q, \alpha q/e;q)_n e^n} \quad ([1], \text{App.II}(8))$$

$$ {}_6\Phi_5\left[\begin{matrix} \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & \beta, & \gamma, & \delta; & q; \; q \\ & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/\beta, & \alpha q/\gamma, & \alpha q/\delta & \end{matrix}\right]_n = \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_n}{(q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_n} \quad \text{([1], App.II(25))} \tag{1.6} $$

where $\alpha = \beta\gamma\delta$

$$ \sum_{i=0}^{n} \frac{(1-\alpha p^i q^i)(\alpha;p)_i(\beta;q)_i\beta^{-i}}{(1-\alpha)(q;q)_i(\alpha p/\beta;p)_i} = \frac{(\alpha p;p)_n(\beta q;q)_n\beta^{-n}}{(q;q)_n(\alpha p/\beta;p)_n} \quad \text{([12],App.(II.34))} \tag{1.7} $$

$$ \sum_{i=0}^{n} \frac{(1-\alpha p^i q^i)(1-\beta p^i q^{-i})(\alpha;p)_i(\beta;p)_i(\gamma;q)_i(\alpha/\beta\gamma;q)_i q^i}{(1-\alpha)(1-\beta)(q;q)_i(\alpha q/\beta;q)_i(\alpha p/\gamma;p)_i(\beta\gamma p;p)_i} = \frac{(\alpha p, \beta p; p)_n(\gamma q;q)_n(\alpha q/\beta\gamma;q)_n}{(q, \alpha q/\beta;q)_n(\alpha p/\gamma;p)_n(\beta\gamma p;p)_n} \quad \text{([12] , App.(II.35))} \tag{1.8} $$

$$ \begin{aligned} &\sum_{r=0}^{n} \frac{(1-\alpha\delta p^r q^r)(1-\beta p^r/\delta q^r)(\alpha,\beta;p)_r(\gamma,\alpha\delta^2/\beta\gamma;q)_r}{(1-\alpha\delta)(1-\beta/\delta)(\delta q,\alpha\delta q/\beta;q)_r(\alpha\delta p/\gamma,\beta\gamma p/\delta;p)_r} q^r \\ &\qquad \frac{(1-\alpha)(1-\beta)(1-\gamma)(1-\alpha\delta^2/\beta\gamma)}{\delta(1-\alpha\delta)(1-\beta/\delta)(1-\gamma/\delta)(1-\alpha\delta/\beta\gamma)} \\ &\times\left(\frac{(\alpha p,\beta p;p)_n(\gamma q,\alpha\delta^2 q/\beta\gamma;q)_n}{(\delta q,\alpha\delta q/\beta;q)_n(\alpha\delta p/\gamma,\beta\gamma p/\delta;p)_n} - \frac{(\gamma/\alpha\delta,\delta/\beta\gamma;p)_1(1/\delta,\beta/\alpha\delta;q)_1}{(1/\gamma,\beta\gamma/\alpha\delta^2;q)_1(1/\alpha,1/\beta;p)_1}\right) \end{aligned} \tag{1.9} $$

([12],App.(II.36) for $m = 0$)

2. Main results

The following results have been established in this section.

$$ \begin{aligned} &{}_4\Phi_3\left[\begin{matrix} a, & y, & \alpha q, & eq; & q; \; q/e \\ & q, & ayq, & \alpha q/e & \end{matrix}\right] \\ &\quad + {}_6\Phi_5\left[\begin{matrix} aq, & yq, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & e; & q; \; 1/e \\ & q, & ayq, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/e & \end{matrix}\right] \\ &\quad = {}_6\Phi_5\left[\begin{matrix} a, & y, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & e; & q; \; q/e \\ & q, & ayq, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/e & \end{matrix}\right] \end{aligned} \tag{2.1} $$

where $|q/e| < 1$ and $|1/e| < 1$. Choose $a = 0$ in (2.1), we have

$$ \begin{aligned} &{}_3\Phi_2\left[\begin{matrix} y, & \alpha q, & eq; & q; q/e \\ & q, & \alpha q/e & \end{matrix}\right] \\ &\quad + {}_5\Phi_4\left[\begin{matrix} yq, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & e; & q, 1/e \\ & q, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/e & \end{matrix}\right] \\ &\quad = {}_5\Phi_4\left[\begin{matrix} y, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & e; & q; \; q/e \\ & q, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/e & \end{matrix}\right] \end{aligned} \tag{2.2} $$

where $|q/e| < 1$ and $|1/e| < 1$

$$
\begin{aligned}
&{}_6\Phi_5\left[\begin{matrix} a, & y, & \alpha q, & \beta q, & \gamma q, & \delta q; & q; & q \\ & q, & ayq, & \alpha q/\beta, & \alpha q/\gamma, & \alpha q/\delta & & \end{matrix}\right] \\
&\quad + {}_8\Phi_7\left[\begin{matrix} aq, & yq, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & \beta, & \gamma, & \delta; & q; & q \\ & q, & ayq, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/\beta, & \alpha q/\gamma, & \alpha q/\delta & & \end{matrix}\right] \\
&\quad = \frac{(aq, yq, \alpha q, \beta q, \gamma q, \delta q; q)_\infty}{(q, qyq, q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_\infty} \\
&\quad + {}_8\Phi_7\left[\begin{matrix} a, & y, & \alpha, & q\sqrt{\alpha}, & -q\sqrt{\alpha}, & \beta, & \gamma, & \delta; & q; & q^2 \\ & q, & ayq, & \sqrt{\alpha}, & -\sqrt{\alpha}, & \alpha q/\beta, & \alpha q/\gamma, & \alpha q/\delta & & \end{matrix}\right]
\end{aligned} \tag{2.3}
$$

$$
\begin{aligned}
&\Phi\left[\begin{matrix} a, & y, & \beta q: & \alpha p; & q, & p; & q/\beta \\ q, & ayq: & \alpha p/\beta & & & & \end{matrix}\right] \\
&\quad + \Phi\left[\begin{matrix} aq, & yq, & \beta: & \alpha: & \alpha pq; & q, & p, & pq; & 1/\beta \\ q, & ayq: & \alpha p/\beta: & \alpha & & & & & \end{matrix}\right] \\
&\quad = \Phi\left[\begin{matrix} a, & y, & \beta: & \alpha: & \alpha pq; & q, & p, & pq; & q/\beta \\ q, & ayq: & \alpha p/\beta: & \alpha & & & & & \end{matrix}\right]
\end{aligned} \tag{2.4}
$$

Taking $p = q$ in (2.4), we get

$$
\begin{aligned}
&{}_4\Phi_3\left[\begin{matrix} a, & y, & \beta q, & \alpha q; & q; & q/\beta \\ & q, & ayq, & \alpha q/\beta & & \end{matrix}\right] \\
&\quad + \Phi\left[\begin{matrix} aq, & yq, & \beta, & \alpha: & \alpha q^2; & q, & q^2; & 1/\beta \\ q, & ayq, & \alpha q/\beta: & \alpha & & & & \end{matrix}\right] \\
&\quad = \Phi\left[\begin{matrix} a, & y, & \beta, & \alpha: & \alpha q^2; & q, & q^2; & q/\beta \\ q, & ayq, & \alpha q/\beta: & \alpha & & & & \end{matrix}\right]
\end{aligned} \tag{2.5}
$$

$$
\begin{aligned}
&\Phi\left[\begin{matrix} aq, & yq, \ \gamma, \ \alpha/\beta\gamma: & \alpha, & \beta: & \alpha pq: & \beta p/q; & q, \ p, \ pq, \ p/q; \ q \\ q, & ayq, \ \alpha q/\beta: & \alpha p/\gamma, & \beta\gamma p: & \alpha: & \beta & \end{matrix}\right] \\
&\quad + \Phi\left[\begin{matrix} a, & y, & \gamma q, & \alpha q/\beta\gamma: & \alpha p, & \beta p; & q, & p; & q \\ q, & ayq, & \alpha q/\beta: & \alpha p/\gamma, & \beta\gamma p & & & & \end{matrix}\right] \\
&\quad = \frac{(aq, yq, \gamma q, \alpha q/\beta\gamma; q)_\infty (\alpha p, \beta p; p)_\infty}{(q, q, ayq, \alpha q/\beta; q)_\infty (\alpha p/\gamma, \beta\gamma p; p)_\infty} \\
&\quad + \Phi\left[\begin{matrix} a, \ y, \ \gamma \ \alpha/\beta\gamma: & \alpha \ , \beta: & \alpha pq: & \beta p/q; & q \ , p \ , pq, \ p/q; \ q^2 \\ q, \ ayq, \ \alpha q/\beta: & \alpha p/\gamma, \ \beta\gamma p: & \alpha: & \beta & \end{matrix}\right]
\end{aligned} \tag{2.6}
$$

$$\frac{(1-\alpha)(1-\beta)(1-\gamma)(1-\alpha\delta^2/\beta\gamma)}{\delta(1-\alpha\delta)(1-\beta/\delta)(1-\gamma/\delta)(1-\alpha\delta/\beta\gamma)}$$

$$\left(\Phi\left[\begin{matrix} a, & y, & \gamma q, & \alpha\delta^2 q/\beta\gamma : & \alpha p, & \beta p; & q, & p; & q \\ ayq, & \delta q, & \alpha\delta q/\beta : & \alpha\delta p/\gamma, & \beta\gamma p/\delta & & & & \end{matrix}\right]\right.$$

$$\left. -\frac{(aq,yq;q)_\infty}{(q,ayq;q)_\infty}\times\frac{(1-\gamma/\alpha\delta)(1-\delta/\beta\gamma)(1-1/\delta)(1-\beta/\alpha\delta)}{(1-1/\gamma)(1-\beta\gamma/\alpha\delta^2)(1-1/\alpha)(1-1/\beta)}\right)$$

$$(2.7)\qquad \begin{aligned} &= \frac{(aq,yq,\gamma q,\alpha\delta^2 q/\gamma;q)_\infty(\alpha p,\beta p;p)_\infty}{(q,ayq,\delta q,\alpha\delta q/\beta;q)_\infty(\alpha\delta p/\gamma,\beta\gamma p/\delta;p)_\infty}\frac{(1-\alpha)(1-\beta)(1-\gamma)}{\delta(1-\alpha\delta)(1-\beta/\delta)} \\ &\times\frac{(1-\alpha\delta^2/\beta\gamma)}{(1-\gamma/\delta)(1-\alpha\delta/\beta\gamma)} - \frac{(aq,yq;q)_\infty}{(q,ayq;q)_\infty}\frac{(1-\gamma/\alpha\delta)(1-\delta/\beta\gamma)}{(1-1/\alpha)(1-1/\beta)} \\ &\times\frac{(1-1/\delta)(1-\beta/\alpha\delta)}{(1-1/\gamma)(1-\beta\gamma/\alpha\delta^2)}\frac{(1-\alpha)(1-\beta)(1-\gamma)(1-\alpha\delta^2/\beta\gamma)}{\delta(1-\alpha\delta)(1-\beta/\delta)(1-\gamma/\delta)(1-\alpha\delta/\beta\gamma)} \\ &-\Phi\left[\begin{matrix} aq,yq,\gamma,\alpha\delta^2/\beta\gamma:\alpha,\beta:\alpha\delta pq:\beta p/\delta q; & q, & p, & pq, & p/q; & q \\ ayq,\ \delta q,\ \alpha\delta q/\beta:\alpha\delta p/\gamma,\ \beta\gamma p/\delta:\alpha\delta:\beta/\delta & & & & & \end{matrix}\right] \\ &+\Phi\left[\begin{matrix} a,\ y,\ \gamma,\ \alpha\delta^2/\beta\gamma:\alpha,\ \beta:\alpha\delta pq:\beta p/\delta q; & q,\ p,\ pq,\ p/q;\ q^2 \\ ayq,\ \delta q,\ \alpha\delta q/\beta:\alpha\delta p/\gamma,\ \beta\gamma p/\delta:\alpha\delta:\beta/\delta & \end{matrix}\right] \end{aligned}$$

Proof of 2.1. Let us choose $u_r = v_r = 1$ and $\delta_r = \frac{(a,y;q)_r}{(q,ayq;q)_r}q^r$ in (1.1) and (1.2), we get

$$(2.8)\qquad \begin{aligned} \beta_n &= \sum_{r=0}^{n}\alpha_r \\ \gamma_n &= \sum_{r=n}^{\infty}\delta_r \end{aligned}$$

and γ_r can be written as,

$$\gamma_r = \sum_{r=0}^{\infty}\delta_r - \sum_{r=0}^{n}\delta_r + \delta_n$$

By choosing $\delta_r = \frac{(a,y;q)_n}{(q,ayq;q)_n}q^n$ in the above relation, gives

$$\gamma_r = \sum_{r=0}^{\infty}\frac{(a,y;q)_r}{(q,ayq;q)_r}q^r - \sum_{r=0}^{n}\frac{(a,y;q)_r}{(q,ayq;q)_r}q^r + \frac{(a,y;q)_n}{(q,ayq;q)_n}q^n$$

By using (1.4), we have,

$$(2.9)\qquad \gamma_r = \frac{(aq,yq;q)_\infty}{(q,ayq;q)_\infty} - \frac{(aq,yq;q)_n}{(q,ayq;q)_n} + \frac{(a,y;q)_n}{(q,ayq;q)_n}q^n$$

Substituting (2.8) and (2.9) in (1.3), we obtain the following relation

$$\sum_{n=0}^{\infty}\alpha_n\left[\frac{(aq,yq;q)_\infty}{(q,ayq;q)_\infty}-\frac{(aq,yq;q)_n}{(q,ayq;q)_n}+\frac{(a,y;q)_n}{(q,ayq;q)_n}q^n\right] = \sum_{n=0}^{\infty}\beta_n\frac{(a,y;q)_n}{(q,ayq;q)_n}q^n \tag{2.10}$$

By choosing $\alpha_r = \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/e; q)_r e^r}$ in (2.8), we get

$$\beta_n = \sum_{r=0}^{n}\frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/e; q)_r e^r}$$

By using (1.5), we get

$$\beta_n = \frac{(\alpha q, eq; q)_n}{(q, \alpha q/e; q)_n e^n}$$

Now, put α_n and β_n in (2.10). We have (2.1).

Proof of 2.3. Let $\alpha_r = \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_r}q^r$ in (2.8). We get

$$\beta_r = \sum_{r=0}^{n}\frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_r}q^r$$

By employing (1.6), we have

$$\beta_n = \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_n}{(q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_n}$$

By substituting α_n and β_n in (2.10), we get (2.3).

Proof of 2.4. By taking $\alpha_r = \frac{(\alpha pq; pq)_r(\alpha; p)_r(\beta; q)_r\beta^{-r}}{(\alpha; pq)_r(\alpha p/\beta; p)_r(q; q)_r}$ in (2.8), we have

$$\beta_n = \sum_{r=0}^{n}\frac{(\alpha pq; pq)_r(\alpha; p)_r(\beta; q)_r}{(\alpha; pq)_r(\alpha p/\beta; p)_r(q; q)_r}\beta^{-r}$$

By using (1.7), we obtain

$$\beta_n = \frac{(\alpha p; p)_n(\beta q; q)_n}{(q; q)_n(\alpha p/\beta; p)_n}\beta^{-n}$$

Now, put α_n and β_n in (2.10). We obtain (2.4).

Proof of 2.6. Let us choose

$$\alpha_r = \frac{(\alpha pq; pq)_r(\beta p/q; p/q)_r(\alpha, \beta; p)_r(\gamma, \alpha/\beta\gamma; q)_r q^r}{(\alpha; pq)_r(\beta; p/q)_r(q, \alpha q/\beta; q)_r(\alpha p/\gamma, \beta\gamma p; p)_r}$$

in (2.8) and we have

$$\beta_n=\sum_{r=0}^{n}\frac{(\alpha pq;pq)_r(\beta p/q;p/q)_r(\alpha,\beta;p)_r(\gamma,\alpha/\beta\gamma;q)_r}{(\alpha;pq)_r(\beta;p/q)_r(q,\alpha q/\beta;q)_r(\alpha p/\gamma,\beta\gamma p;p)_r}q^r$$

By employing (1.8), we get

$$\beta_n=\frac{(\alpha p,\beta p;p)_n(\gamma q,\alpha q/\beta\gamma;q)_n}{(q,\alpha q/\beta;q)_n(\alpha p/\gamma,\beta\gamma p;p)_n}$$

By substituting α_n and β_n in (2.10), we get (2.6).

Proof of 2.7. Let us choose

$$\alpha_r=\frac{(\alpha\delta pq;pq)_r(\beta p/\delta q;p/q)_r(\alpha,\beta;p)_r(\gamma,\alpha\delta^2/\beta\gamma;q)_r q^r}{(\alpha\delta;pq)_r(\beta/\delta;p/q)_r(\alpha\delta p/\gamma,\beta\gamma p/\delta;p)_r(\delta q,\alpha\delta q/\beta;q)_r}$$

in (2.8) and, by employing (1.9), we have

$$\beta_n=\frac{(1-\alpha)(1-\beta)(1-\gamma)(1-\alpha\delta^2/\beta\gamma)}{\delta(1-\alpha\delta)(1-\beta/\delta)(1-\gamma/\delta)(1-\alpha\delta/\beta\gamma)}$$
$$\times\left(\frac{(\alpha p,\beta p;p)_n(\gamma q,\alpha\delta^2 q/\beta\gamma;q)_n}{(\delta q,\alpha\delta q/\beta;q)_n(\alpha\delta p/\gamma,\beta\gamma p/\delta;p)_n}-\frac{(\gamma/\alpha\delta,\delta/\beta\gamma;p)_1(1/\delta,\beta/\alpha\delta;q)_1}{(1/\gamma,\beta\gamma/\alpha\delta^2;q)_1(1/\alpha,1/\beta;p)_1}\right)$$

By substituting α_n and β_n in (2.10), we get (2.7).

Concluding remark

In the above section, we have demonstrated the power of Bailey lemma as a tool for discovering new transformations of basic hypergeometric series from the known summations and transformations. Some of the transformations in the previous section generalize the known transformation formulae. The study may be continued to further develop the theory of summation and transformation of basic hypergeometric series

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