CHARACTERIZATIONS OF ORDERED $\mathcal{AG}$-GROUPOIDS IN TERMS OF SOFT SETS

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Abstract. We introduced the concepts of soft intersection left (right, two-sided) ideals, (generalized) bi-ideals, interior ideals, quasi-ideals in ordered $\mathcal{AG}$-groupoids and obtained significant characterizations of intra-regular ordered $\mathcal{AG}$-groupoid via soft intersection left (right, two-sided) ideals, (generalized) bi-ideals, interior ideals, quasi-ideals of ordered $\mathcal{AG}$-groupoids.

Keywords: soft set, soft intersection left (right, two-sided, interior, quasi-) ideal, soft intersection (generalized) bi-ideal, interior ideal, intra-regular ordered $\mathcal{AG}$-groupoid.

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Introduction

Molodtsov in 1999 [1], introduced the fundamental concept of soft set which provides a natural frame work for generalization of several basic notions of algebra such as groups [2], [3], semirings [4], rings [5], BCK/BCI-algebras [6], [7], [8], BL-algebras [9], near-rings [10], [28], and soft substructures and union soft substructures [12], [13], and some other structures such as [14], [15], [16], [17].

Many related concepts with soft sets, especially soft sets operations, have also undergone tremendous studies. Maji et al. [18], presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [19], introduced several operations on soft sets and Sezgin and Atagun [20], and Ali et al. [21], studied on soft sets as well.

Nowadays the theory of soft sets and its applications has been rapidly growing. These applications are used in various fields such as computer science and in soft decision making as in the following studies [24], [25], [26], [27], [28].

The concept of Abel-Grassmann's Groupoids ($\mathcal{AG}$-groupoids) [32], was first given by M.A. Kazim and M. Naseeruddin in 1972.

An $\mathcal{AG}$-groupoid is a groupoid having the left invertive law

$$(ab)c = (cb)a, \text{ for all } a,\ b,\ c, \in S. \tag{1}$$

An $\mathcal{AG}$-groupoid is a non-associative and non-commutative algebraic structure mid way between a groupoid and a commutative semigroup, nevertheless, it posses many interesting properties which we usually find in associative and commutative algebraic structures. The left identity in an $\mathcal{AG}$-groupoid if exists is unique [33].

In an $\mathcal{AG}$-groupoid, the medial law [32], holds

$$(ab)(cd) = (ac)(bd), \text{ for all } a,\ b,\ c,\ d \in S. \tag{2}$$

In an $\mathcal{AG}$-groupoid S with left identity, the paramedial law [33] holds

$$(ab)(cd) = (dc)(ba), \text{ for all } a, b, c, d \in S. \tag{3}$$

If an $\mathcal{AG}$-groupoid contains a left identity, then by using medial law, the following law [33] holds

$$a(bc) = b(ac), \text{ for all } a,\ b,\ c \in S. \tag{4}$$

The purpose of this paper, we have a new approach to the ordered $\mathcal{AG}$-groupoid theory via soft set theory with the concept of soft intersection $\mathcal{AG}$-groupoids and soft intersection $\mathcal{AG}$-ideals. The paper is organized as follows: First we remind some basic definitions about ordered $\mathcal{AG}$-groupoids, soft sets, soft intersection product and soft characteristic function, soft intersection ordered $\mathcal{AG}$-groupoid left (right, two-sided) ideal, (generalized) bi-ideals, interior ideal, quasi-ideal and soft semiprime ideals in ordered $\mathcal{AG}$-groupoids. Also we have shown that all these ideals coincide in intra-regular ordered $\mathcal{AG}$-groupoids with left identity. In the following sections intra-regular ordered $\mathcal{AG}$-groupoids are characterized by the properties of these soft intersection ideals.

Preliminary notes

In this section, we recall some basic notions and results on ordered $\mathcal{AG}$-groupoids.

Definition 1 [35] An ordered $\mathcal{AG}$-groupoid S (po-$\mathcal{AG}$-groupoid) is a structure $(S, \cdot, \leq)$ in which the following conditions hold:

(i) $(S, \cdot)$ is an $\mathcal{AG}$-groupoid.

(ii) $(S, \leq)$ is a poset. (Reflexive, anti symmetric and transitive)

(iii) $a \leq b \Longrightarrow ax \leq bx$ and $xa \leq xb$, $\forall\ a, b, x \in S$.

Example 1 Let $S = \{a, b, c\}$ be an $\mathcal{AG}$-groupoid with the following multiplication table.

$\cdot$	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

Then, S is an ordered $\mathcal{AG}$-groupoid with the following two orders:

$$\leq := \{(a,a),(b,b),(c,c),(c,a),(c,b)\}$$

$$\leq := \{(a,a),(b,b),(c,c),(a,c),(a,b)\}$$

Let $(S,\cdot,\leq)$ be an ordered $\mathcal{AG}$-groupoid. A fuzzy subset or a fuzzy set f of a non-empty set S is an arbitrary mapping $f : S \to [0,1]$, where $[0,1]$ is the unit segment of real line. A fuzzy subset f is a class of objects with a grades of membership having the form $f = \{(s,\ f(s))\ /s \in S\}$.

Let f and g be any two fuzzy subsets of $(S,\cdot,\leq)$. Then, the product of f and g is defined as:

$$(f \circ g)(x) = \begin{cases} \bigvee\limits_{x \leq yz} \{f(y) \wedge g(z)\} & \text{if } \exists\ y, z \in S \text{ such that } x \leq yz, \\ 0 & \text{otherwise.} \end{cases}$$

The order relation $\subseteq$ between any two fuzzy subsets f and g of $(S,\cdot,\leq)$ is defined as:

$$f \subseteq g \text{ if and only if } f(a) \leq g(a)\ \forall\ a \in S.$$

We define the symbols $f \wedge g$ and $f \vee g$, which will means the following fuzzy subsets of an ordered $\mathcal{AG}$-groupoid $(S,\cdot,\leq)$

$$(f \wedge g)(a) = \min\{f(a), g(a)\} \text{ for all } a \in S.$$
$$(f \vee g)(a) = \max\{f(a), g(a)\} \text{ for all } a \in S.$$

Let A be a non-empty subset of an ordered $\mathcal{AG}$-groupoid $(S,\cdot,\leq)$, then

$$(A] = \{t \in S \mid t \leq a, \text{ for some } a \in A\}.$$

For $A = \{a\}$, we usually written as $(a]$.

For further information about $\mathcal{AG}$-groupoids ordered $\mathcal{AG}$-groupoids and fuzzy $\mathcal{AG}$-groupoids, we invite the reader to [35], [36], [37], [38]. A semilattice is a structure where "$\cdot$" is an infix binary operation, called semilattice operation, such that "$\cdot$" is commutative, associative and idempotent. From now on, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and A, B are the subsets of E.

Definition 2 ([23, 1]) A soft set f_A over U is defined by

$$f_A : E \longrightarrow P(U) \text{ such that } f_A(x) = \phi \text{ if } x \notin A.$$

Here, f_A is also called approximate function. A soft set over U can be represented by the set of ordered pair

$$f_A = \{(x, f_A) : x \in E,\ f_A(x) \in P(U)\}.$$

Clearly, we see that a soft set is a parametrized family of subsets of set U. Note that throughout this paper $S(U)$ will denote the set of all soft sets over U.

Definition 3 [23] Let f_A and $f_B \in S(U)$. Then, f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$ if and only if $f_A \leq f_B\ \forall\ x \in E$.

Definition 4 [23] Let f_A and $f_B \in S(U)$. Then, the union of f_A and f_B, denoted by $f_A \cup f_B$, is defined as $f_A \cup f_B = f_{A\cup B}$, where $f_{A\cup B}(x) = f_A(x) \vee f_B(x)\ \forall\ x \in E$.

Definition 5 [23] Let f_A and $f_B \in S(U)$. Then, the intersection of f_A and f_B, denoted by $f_A \cap f_B$, is defined as $f_A \cap f_B = f_{A\cap B}$, where $f_{A\cap B}(x) = f_A(x) \wedge f_B(x)\ \forall\ x \in E$.

Definition 6 Let f_A and $f_B \in S(U)$. Then, the product of f_A and f_B, denoted by $f_A \Theta f_B$, is defined as $f_A \Theta f_B = f_{A\Theta B}$, where $f_{A\Theta B}(x,y) = f_A(x) \wedge f_B(y)\ \forall\ (x,y) \in E\times E$.

Definition 7 Let S be an ordered $\mathcal{AG}$-groupoid and f_A and f_B be soft sets over the common universe set U. Then, the soft intersection product $f_A \circ f_B$ is defined by

$$(f_A \circ f_B)(x) = \begin{cases} \bigvee\limits_{x\leq yz} \{f_A(y) \wedge f_B(z)\} & \text{if } \exists\ y,z \in S \text{ such that } x \leq yz, \\ \phi & \text{otherwise,} \end{cases}$$

for all $x \in S$.

Theorem 1 [39] *Let f_A, f_B, $f_C \in S(U)$. Then,*

(i) $(f_A \circ f_B) \circ f_C = f_A \circ (f_B \circ f_C)$.

(ii) $f_A \circ (f_B \cup f_C) = (f_A \circ f_B) \cup (f_A \circ f_C)$ *and* $(f_A \cup f_B) \circ f_C = (f_A \circ f_C) \cup (f_B \circ f_C)$.

(iii) $f_A \circ (f_B \cap f_C) = (f_A \circ f_B) \cap (f_A \circ f_C)$ *and* $(f_A \cap f_B) \circ f_C = (f_A \circ f_C) \cap (f_B \circ f_C)$.

(iv) *if* $f_A \subseteq f_B$, *then* $f_A \circ f_C \subseteq f_B \circ f_C$ *and* $f_C \circ f_A \subseteq f_C \circ f_B$.

(v) *If* $t_A, t_B \in S(U)$ *such that* $t_A \subseteq f_A$ *and* $t_B \subseteq f_B$, *then* $t_A \circ t_B \subseteq f_A \circ f_B$.

Proposition 1 [39] *Let S be an ordered $\mathcal{AG}$-groupoid. Then, the medial law holds in $S(S)$.*

Theorem 2 [39] *Let S be an ordered $\mathcal{AG}$-groupoid with left identity and $f_A, f_B, f_C \in S(S)$. Then, the following properties hold in $S(S)$:*

(i) $f_A \circ (f_B \circ f_C) = f_B \circ (f_A \circ f_C)$.

(ii) $(f_A \circ f_B) \circ (f_C \circ f_D) = (f_D \circ f_C) \circ (f_B \circ f_A)$.

Definition 8 [39] Let X be a subst of S. We denote by $\mathcal{S}_X$, the soft characteristic function of X and define as

$$\mathcal{S}_X(x) = \begin{cases} U, & \text{if } x \in X, \\ \phi, & \text{if } x \notin X. \end{cases}$$

Lemma 1 [35] *In an ordered $\mathcal{AG}$-groupoid S, the following are true.*

(i) $A \subseteq (A]\,, \forall\ A \subseteq S$.

(ii) $A \subseteq B \subseteq G \Longrightarrow (A] \subseteq (B]\,,\ \forall\ A, B \subseteq S$.

(iii) $(A]\,(B] \subseteq (AB]\,,\ \forall\ A, B \subseteq S$.

(iv) $(A] = ((A]]\,,\ \forall\ A \subseteq S$.

(vi) $((A]\,(B]] = (AB]\,,\ \forall\ A, B \subseteq S$.

Theorem 3 *Let S be an ordered $\mathcal{AG}$-groupoid and X, Y be non-empty subsets of X. Then, the following properties hold:*

(i) *If $X \subseteq Y$, then $\mathcal{S}_X \subseteq \mathcal{S}_Y$.*

(ii) $\mathcal{S}_X \cap \mathcal{S}_Y = \mathcal{S}_{X \cap Y}$, $\mathcal{S}_X \cup \mathcal{S}_Y = \mathcal{S}_{XUY}$.

(iii) $\mathcal{S}_X \circ \mathcal{S}_Y = \mathcal{S}_{(XY]}$.

Proof. (i) It is obvious.

(ii) Let $x \in \mathcal{S}$. If $x \in X \cap Y$, then $x \in X$ and $x \in Y$. Thus, we have

$$(\mathcal{S}_X \cap \mathcal{S}_Y)(x) = \mathcal{S}_X(x) \cap \mathcal{S}_Y(x) = U = \mathcal{S}_{X \cap Y}(x).$$

If $x \notin X \cap Y$, then $x \notin X$ or $x \notin Y$. Hence, we have

$$(\mathcal{S}_X \cap \mathcal{S}_Y)(x) = \mathcal{S}_X(x) \cap \mathcal{S}_Y(x) = \phi = \mathcal{S}_{X \cap Y}(x).$$

Therefore, $\mathcal{S}_X \cap \mathcal{S}_Y = \mathcal{S}_{X \cap Y}$.

Similarly, we can show that $\mathcal{S}_X \cup \mathcal{S}_Y = \mathcal{S}_{XUY}$.

(iii) For any $x \in \mathcal{S}$. Let $x \in XY$. Then, there exist $y \in X$ and $z \in Y$ such that $x \leq yz$. Thus, we have

$$(\mathcal{S}_X \circ \mathcal{S}_Y)(x) = \bigvee_{x \leq yz} \{\mathcal{S}_X(y) \cap \mathcal{S}_Y(z)\} \supseteq \mathcal{S}_X(y) \cap \mathcal{S}_Y(z) = U,$$

and so $(\mathcal{S}_X \circ \mathcal{S}_Y)(x) = U$. Since $x \in XY$, we get $\mathcal{S}_{(XY]}(x) = U$. Suppose $x \notin XY$. Then, $x \nleq yz$ for any $y \in X$ and $z \in Y$. If $x \leq yz$ for some $y, z \in \mathcal{S}$, then $y \notin X$ or $z \notin Y$. Hence

$$(\mathcal{S}_X \circ \mathcal{S}_Y)(x) = \bigvee_{x \leq yz} \{\mathcal{S}_X(y) \cap \mathcal{S}_Y(z)\} = \phi = \mathcal{S}_{(XY]}(x).$$

If $x \nleq yz$ for all $x, y \in \mathcal{S}$, then

$$(\mathcal{S}_X \circ \mathcal{S}_Y)(x) = \phi = \mathcal{S}_{(XY]}(x).$$

In any case, we have $\mathcal{S}_X \circ \mathcal{S}_Y = \mathcal{S}_{(XY]}$. ∎

Definition 9 Let S be an ordered $\mathcal{AG}$-groupoid and f_A be a soft set over U. Then, f_A is called soft intersection ordered $\mathcal{AG}$-groupoid of S, if

$$f_A(xy) \geq f_A(x) \wedge f_A(y) \text{ for all } x, y \in S.$$

Definition 10 Let S be an ordered $\mathcal{AG}$-groupoid and f_A be a soft set over U. Then, f_A is called soft intersection left $(right)$ ideal of S,if

(i) $x \leq y \Rightarrow f_A(x) \geq f_A(y) \ \forall \ x, y \in S.$

(ii) $f_A(xy) \geq f_A(y) \ (f_A(xy) \geq f_A(x)) \ \forall \ x, y \in S.$

A soft set over U is called a soft intersection two-sided ideal (soft intersection ideal) of S, if it is both soft intersection left and soft intersection right ideal of S over U.

Definition 11 A soft intersection ordered $\mathcal{AG}$-groupoid f_A over U is called a soft intersection bi-ideal of S over U if

(i) $x \leq y \Rightarrow f_A(x) \geq f_A(y) \ \forall \ x, y \in S$.

(ii) $f_A((xy)z) \geq f_A(x) \wedge f_A(z) \ \forall \ x, y, z \in S$.

Definition 12 A soft set f_A over U is called a soft intersection interior ideal of S over U, if

(i) $x \leq y \Rightarrow f_A(x) \geq f_A(y) \ \forall \ x, y \in S$.

(ii) $f_A((xy)z) \geq f_A(y) \ \forall \ x, y, z \in S$.

Definition 13 A soft set f_A over U is called a soft intersection generalized bi-ideal of S over U, if

(i) $x \leq y \Rightarrow f_A(x) \geq f_A(y) \ \forall \ x, y \in S$.

(ii) $f_A((xy)z) \geq f_A(x) \wedge f_A(z) \ \forall \ x, y, z \in S$.

Now, throughout this paper, soft intersection ordered $\mathcal{AG}$-groupoid soft intersection left (right, two-sided, interior, quasi-, generalized bi-) ideal are abbreviated by SI-ordered $\mathcal{AG}$-groupoid, SI-right (left, two-sided, interior, quasi-, generalized bi-) ideal respectively.

It is easy to show that $f_A(x) = U \ \forall \ x \in S$, then f_A is an SI-ordered $\mathcal{AG}$-groupoid (right ideal, left ideal, interior ideal, quasi-ideal, generalized bi-ideal) of S over U. We denote such type of SI-ordered $\mathcal{AG}$-groupoid (right ideal, left ideal, interior ideal, quasi-ideal, generalized bi-ideal) by $\widetilde{\mathcal{S}}$ [39].

Definition 14 A soft set f_A over U is called soft intersection quasi-ideal of S over U if

$$f_A(f_A \circ \widetilde{\mathcal{S}}) \wedge (\widetilde{\mathcal{S}} \circ f_A) \leq f_A$$

Lemma 2 [39] *Let f_A be any SI-ordered $\mathcal{AG}$-groupoid over U. Then, we have the following:*

(i) $\widetilde{\mathcal{S}} \circ \widetilde{\mathcal{S}} \subseteq \widetilde{\mathcal{S}}$.

(ii) $f_A \circ \widetilde{\mathcal{S}} \subseteq \widetilde{\mathcal{S}}$ *and* $\widetilde{\mathcal{S}} \circ f_A \subseteq \widetilde{\mathcal{S}}$.

(iii) $f_A \cup \widetilde{\mathcal{S}} = \widetilde{\mathcal{S}}$ *and* $f_A \cap \widetilde{\mathcal{S}} = \widetilde{\mathcal{S}}$.

Theorem 4 [39] *Let X be a nonempty subset of an ordered $\mathcal{AG}$-groupoid S. Then, X is left (right, two-sided, bi-, interior, quasi-, generalized bi-) ideal of S if and only if S_X is an SI-left (left, right, two-sided, bi-, interior, quasi-, generalized bi-) ideal of S.*

Proposition 2 [39] *Let f_A be a soft set over U. Then,*

(i) *f_A is an SI-ordered $\mathcal{AG}$-groupoid over U if $f_A \circ f_A \subseteq f_A$*

(ii) *f_A is an SI-left (right) ideal of S over U if and only if $\widetilde{\mathcal{S}} \circ f_A \subseteq f_A(f_A \circ \widetilde{\mathcal{S}}) \subseteq f_A$.*

(iii) *f_A is an SI-bi-ideal of S over U if and only if $f_A \circ f_A \subseteq f_A$ and $f_A \circ \widetilde{\mathcal{S}} \circ f_A \subseteq f_A$.*

(iv) *f_A is an SI-interior ideal of S over U if and only if* $\widetilde{\mathcal{S}} \circ f_A \circ \widetilde{\mathcal{S}} \subseteq f_A$.

(v) *f_A is an SI-generalized bi-ideal of S over U if and only if* $f_A \circ \widetilde{\mathcal{S}} \circ f_A \subseteq f_A$.

Theorem 5 [39] *Every SI-left (right, two-sided) ideal of an ordered $\mathcal{AG}$-groupoid S over U is an SI-bi-ideal of S.*

Proposition 3 [39] *For an ordered $\mathcal{AG}$-groupoid S, the following conditions are equivalent:*

(i) *Every SI-ideal of an ordered $\mathcal{AG}$-groupoid S over U is an SI-interior ideal of S over U.*

(ii) *Every SI-quasi-ideal of an ordered $\mathcal{AG}$-groupoid S is an SI-ordered $\mathcal{AG}$-groupoid of S.*

(iii) *Every one sided SI-ideal of S is an SI-quasi-ideal of S.*

(iv) *Every SI-quasi-ideal of S is an SI-bi-ideal of S.*

Characterizations of intra-regular ordered AG-groupoids

In this section, we characterize an intra-regular ordered $\mathcal{AG}$-groupoids in terms of SI-ideals. An ordered $\mathcal{AG}$-groupoid S is called intra-regular if for every $a \in S$ there exist elements $x, y \in S$ such that

$$a \leq (xa^2)y$$

Example 2 Let us consider an $\mathcal{AG}$-groupoid S with left identity e defined in the following multiplication table.

$\cdot$	a	b	c	d	e
a	e	a	b	c	d
b	d	e	a	b	c
c	c	d	e	a	b
d	b	c	e	d	a
e	a	b	c	d	e

Then, S is an ordered $\mathcal{AG}$-groupoid with the following order:

$$\leq := \{(a,a),(a,b),(c,c),(a,c),(d,d),(a,e),(e,e),(b,b)\}.$$

Note that S is an intra-regular ordered $\mathcal{AG}$-groupoid. Indeed for every $a \in S$ there exist elements $x, y \in S$ such that $a \leq (xa^2)y$.

Proposition 4 *Let S be an intra-regular ordered $\mathcal{AG}$-groupoid and f_A be a soft set of S. Then, f_A is an SI-right ideal if and only if it is an SI-left ideal.*

Proof. Suppose that f_A is an SI-right ideal of S. Since S is intra-regular, so for each $a \in S$ there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. So, by using (1),

$$f_A(ab) \geq f_A((xa^2)y)b) = f_A((by)(xa^2)) \geq f_A(by) \geq f_A(b)$$

Thus, f_A is an SI-left ideal of S.

Conversely, suppose that f_A is an SI-left ideal of S. Then, by using (1), we have

$$f_A(ab) \geq f_A((xa^2)y)b) = f_A((by)(xa^2)) \geq f_A(xa^2) \geq f_A(a^2) \geq f_A(a).$$

Thus, f_A is an SI-right ideal of S. ■

Proposition 5 *Every SI-two-sided ideal of an intra-regular ordered $\mathcal{AG}$-groupoid with left identity is idempotent.*

Proof. Let f_A be an SI-two sided ideal of S. Then,

$$f_A \circ f_A \subseteq f_A \circ \widetilde{\mathcal{S}} \subseteq f_A.$$

Since S is intra-regular, so for each $a \in S$ there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. So,

$$a \leq (ax^2)y = (x(aa))y = (y(xa))a.$$

Thus, we have

$$\begin{aligned}(f_A \circ f_A)(a) &= \bigvee_{a \leq (y(xa))a} \{f_A(y(xa)) \wedge f_A(a)\} \\ &\geq f_A(y(xa)) \wedge f_A(a) \\ &\geq f_A(a) \wedge f_A(a) \\ &= f_A(a).\end{aligned}$$

Hence, $f_A \circ f_A = f_A$. ■

Corollary 1 *Every SI-left ideal of an ordered $\mathcal{AG}$-groupoid S with left identity is idempotent.*

Proposition 6 *Let S be an intra-regular ordered $\mathcal{AG}$-groupoid with left identity. Then,*

$$f_A = (\widetilde{\mathcal{S}} \circ f_A)^2 \text{ for all } SI\text{-left ideals } f_A \text{ of } S.$$

Proof. Let S be an intra-regular ordered $\mathcal{AG}$-groupoid with left identity and f_A be any SI-left ideal of S. Then, $\widetilde{\mathcal{S}} \circ f_A \subseteq f_A$ and since $\widetilde{\mathcal{S}} \circ f_A$ is an SI-left ideal of S, it is idempotent. Thus,

$$(\widetilde{\mathcal{S}} \circ f_A)^2 = \widetilde{\mathcal{S}} \circ f_A \subseteq f_A$$

Moreover,

$$f_A = f_A \circ f_A \subseteq \widetilde{\mathcal{S}} \circ f_A = (\widetilde{\mathcal{S}} \circ f_A)^2.$$

This implies that, $f_A = (\widetilde{\mathcal{S}} \circ f_A)^2$. ■

Theorem 6 *The following conditions are equivalent for an ordered $\mathcal{AG}$-groupoid S with left identity:*

(i) *f_A is an SI-ideal of S.*

(ii) *f_A is an SI-bi-ideal of S.*

Proof. (i) $\Rightarrow$ (ii) : Follows from Theorem 5.

(ii) $\Rightarrow$ (i) : Let f_A be an SI-bi-ideal of S. Since S is intra-regular, so for each $a, b \in S$ there exist $u, v \in S$ such that $a \leq (xa^2)y$ and $b \leq (ub^2)v$. So,

$$\begin{aligned} f_A(ab) &\geq f_A(((xa^2)y)b) = f_A((by)(xa^2)) = f_A((a^2x)(yb)) \\ &= f_A(((yb)x)a^2) = f_A(((yb)x)(aa)) \\ &= f_A((aa)(x(yb))) = f_A(((x(yb))a)a) \\ &= f_A(((x(yb)((xa^2)y))a) = f_A(((xa^2)((x(yb))y))a) \\ &= f_A(((y(x(yb)))(a^2x))a) = f_A((a^2((y(x(yb)))x))a) \\ &= f_A(((aa)((y(x(yb)))x))a) = f_A(((x(y(x(yb))))(aa))a) \\ &= f_A((a((x(y(x(yb))))a))a) \geq f_A(a) \wedge f_A(a) = f_A(a) \end{aligned}$$

and

$$\begin{aligned} f_A(ab) &\geq f_A(a((ub^2)v)) = f_A((ub^2)(av)) = f_A((va)(b^2u)) \\ &= f_A(b^2((va)u)) = f_A((bb)((va)u)) \\ &= f_A((((va)u)b)b) = f_A([((va)u)((ub^2)v)]b) \\ &= f_A([(ub^2)(((va)u)v)]b) = f_A([(v(va)u))(b^2u)]b) \\ &= f_A([b^2((v((va)u))u)]b) = f_A(((bb)((v((va)u))u))b) \\ &= f_A(((u(v((va)u)))(bb))b) = f_A((b((u(v((va)u)))b))b) \\ &\geq f_A(b) \wedge f_A(b) = f_A(b). \end{aligned}$$

Hence, f_A is an SI-ideal of S. ■

Corollary 2 *Let S be an ordered $\mathcal{AG}$-groupoid with left identity. Then, an SI-right ideal of S is an SI-bi-ideal of S.*

Proposition 7 *For a soft set f_A of an ordered $\mathcal{AG}$-groupoid with left identity, the following conditions are equivalent:*

(i) *f_A is an SI-ideal of S.*

(ii) *f_A is an SI-interior ideal of S.*

Proof. (i) $\Rightarrow$ (ii) : It is easy.

(ii) $\Rightarrow$ (i) : Suppose that (ii) holds. Let S be an intra-regular ordered $\mathcal{AG}$-groupoid with left identity. Then, for any $a, b \in S$ there exist $x, y, u, v \in S$ such that $a \leq (xa^2)y$ and $b \leq (ub^2)v$. Since f_A is an SI-interior ideal of S, we have

$$\begin{aligned} f_A\,(ab) &\geq f_A\,(((xa^2)y)b) = f_A\,(((by)(xa^2)) = f_A\,((by)(x(aa))) \\ &= f_A\,((by)(a(xa))) = f_A\,((ba)(y(xa))) \geq f_A\,(a) \end{aligned}$$

and

$$\begin{aligned} f_A\,(ab) &\geq (a((ub^2)v)) = f_A\,((ub^2)(av)) = f_A\,((b(ub))(av)) \\ &= f_A\,((va)((ub)b)) = f_A\,((ub)((va)b)) \geq f_A\,(b). \end{aligned}$$

Hence, f_A is an SI-ideal of S. ■

Proposition 8 *Let f_A be a soft set of an intra-regular ordered $\mathcal{AG}$-groupoid S with left identity. Then, the following conditions are equivalent:*

(i) f_A *is an SI-bi-ideal of S.*

(ii) f_A *is an SI-generalized bi-ideal of S.*

Proof. (i) $\Rightarrow$ (ii) : It is easy.

(ii) $\Rightarrow$ (i) : Suppose that (ii) holds. Let a be any element of an intra-regular ordered $\mathcal{AG}$-groupoid S. Then, there exist $x, y \in S$ such that $a \leq (xa^2)y$. Thus, we have

$$\begin{aligned} f_A(ab) &\geq f_A(((xa^2)y)b) = f_A(((xa^2)(ey)b) = f_A(((ye)(a^2x))b) \\ &= f_A((a^2((ye)x))b) = f_A(((aa)((ye)x))b) = f_A(((x(ye)(aa))b) \\ &= f_A((a((ye))a))b) \geq f_A(a) \wedge f_A(b). \end{aligned}$$

Hence, f_A is an SI-bi-ideal of S. ∎

Proposition 9 *Let S be an intra-regular ordered $\mathcal{AG}$-groupoid S with left identity and f_A be a soft set of S. Then, the following conditions are equivalent:*

(i) f_A *is an SI-ideal of S.*

(ii) f_A *is an SI-quasi-ideal of S.*

Proof. (i) $\Rightarrow$ (ii) : It is easy.

(ii) $\Rightarrow$ (i) : Suppose that (ii) holds. Since S is intra-regular so for any element $a \in S$ there exist $x, y \in S$ such that $a \leq (xa^2)y$. Thus, we have

$$\begin{aligned} a &\leq (xa^2)y = (xa^2)(ey) = (xe)(a^2y) = a^2((xe)y) \\ &= ((aa)((xe)y) = (ya)((xe)a) = (y(xe))(aa) = a((y((xe))a). \end{aligned}$$

Also,

$$\widetilde{\mathcal{S}} \circ f_A = (\widetilde{\mathcal{S}} \circ \widetilde{\mathcal{S}}) \circ f_A = (f_A \circ \widetilde{\mathcal{S}}) \circ \widetilde{\mathcal{S}}.$$

Therefore,

$$\begin{aligned} (\widetilde{\mathcal{S}} \circ f_A)(a) = ((f_A \circ \widetilde{\mathcal{S}}) \circ \widetilde{\mathcal{S}})(a) &= \bigvee_{a \leq a((y((xe))a)} \{(f_A \circ \widetilde{\mathcal{S}})(a) \wedge \widetilde{\mathcal{S}}((y((xe))a)\} \\ &\geq (f_A \circ \widetilde{\mathcal{S}})(a). \end{aligned}$$

Therefore,

$$f_A \circ \widetilde{\mathcal{S}} \subseteq (f_A \circ \widetilde{\mathcal{S}}) \cap (\widetilde{\mathcal{S}} \circ f_A) \subseteq f_A$$

Thus, f_A is an SI-right ideal of S. Now by proposition 4, f_A is an SI-left ideal of S. Hence f_A is an SI-ideal of S. ∎

Theorem 7 *For a soft set f_A of an intra-regular ordered $\mathcal{AG}$-groupoid S with left identity, the following conditions are equivalent:*

(i) f_A *is an SI-right ideal of S.*

(ii) f_A *is an SI-left ideal of S.*

(iii) *f_A is an SI-ideal of S.*

(iv) *f_A is an SI-bi-ideal of S.*

(v) *f_A is an SI-generalized bi-ideal of S.*

(vi) *f_A is an SI-interior ideal of S.*

(vii) *f_A is an SI-quasi-ideal of S.*

Definition 15 A soft set f_A over U is called soft semiprime* if

$$f_A\,(a) \geq f_A\,(a^2) \text{ for all } a \in S.$$

Theorem 8 *For a non-empty set A of an intra-regular ordered $\mathcal{AG}$-groupoid S, the following conditions are equivalent:*

(i) *A is semiprime.*

(ii) *The soft characteristic function $\mathcal{S}_A$ of A is soft semiprime*.*

Proof. Suppose that (i) holds. Let a be any arbitrary element of S. We need to show that $\mathcal{S}_A(a) \geq \mathcal{S}_A(a^2)$ for all $a \in S$. Now if $a^2 \in A$, then since A is semiprime, so $a \in A$. Thus,

$$\mathcal{S}_A(a) = U = \mathcal{S}_A(a^2).$$

If $a^2 \notin A$, then

$$\mathcal{S}_A(a) \geq \varphi = \mathcal{S}_A(a^2).$$

In both cases we see that, $\mathcal{S}_A(a) \geq \mathcal{S}_A(a^2)$ for all $a \in S$. Thus $\mathcal{S}_A$ is soft semiprime*. Hence, $(i) \Rightarrow (ii)$.

Conversely, suppose that (ii) holds. Let $a^2 \in A$. Since $\mathcal{S}_A$ is soft semiprime*, we have

$$\mathcal{S}_A(a) \geq \mathcal{S}_A(a^2) = U.$$

This implies that, $\mathcal{S}_A(a) = U$ and $a \in A$. Thus, A is semiprime. Hence, $(ii) \Rightarrow (i)$. ∎

Theorem 9 *For any SI-ordered $\mathcal{AG}$-groupoid f_A, the following conditions are equivalent:*

(i) *f_A is soft semiprime*.*

(ii) *$f_A(a) = f_A(a^2)$ for all $a \in S$.*

Proof. (ii) $\Rightarrow$ (i) : It is easy.

(i) $\Rightarrow$ (ii) : Let $a \in S$ be any element of S. Since f_A is an SI-ordered $\mathcal{AG}$-groupoid, we have

$$f_A(a) \geq f_A(a^2) = f_A(aa) \geq f_A(a) \wedge f_A(a) = f_A(a)$$

So, $f_A(a^2) = f_A(a)$. Hence,$(i) \Rightarrow (ii)$. ∎

Proposition 10 *In an intra-regular ordered $\mathcal{AG}$-groupoid S, every SI-interior ideal of S is a soft semiprime*.*

Proof. Let f_A be any SI-interior ideal of S and $a \in S$. Since S is intra-regular, there exist elements $x, y \in S$ such that $a \leq (xa^2)y$. Thus, we have

$$f_A(ab) \geq f_A((xa^2)y) \geq f_A(a^2).$$

Hence, f_A is soft semiprime*. ■

Theorem 10 *Let S be an ordered $\mathcal{AG}$-groupoid with left identity. Then, the following conditions are equivalent:*

(i) *S is intra-regular.*

(ii) *$L \cap R \subseteq (LR]$ for every left ideal L and for every right ideal R of S and R is semiprime.*

(iii) *$f_A \cap f_B \subseteq f_A \circ f_B$ for every SI-left ideal f_A and for every SI-right ideal f_B of S and SI-right ideal f_B is soft semiprime*.*

Proof. (i) $\Rightarrow$ (iii) : Let f_A and f_B be any SI-left ideal and SI-right ideal of S, respectively and a be any element of S. Since S is intra-regular, there exist $x, y \in S$ such that $a \leq (xa^2)y$. Thus,

$$\begin{aligned} a &\leq (xa^2)y = (a(xa))y = (y(xa))a = (y(xa))(ea) \\ &= (ye)((xa)a) = (xa)((ye)a) = (xa)((ye)a) = (xa)((ae)y). \end{aligned}$$

Thus, we have

$$\begin{aligned} (f_A \circ f_B)(a) &= \bigvee_{a \leq (xa)((ae)y)} \{f_A(xa) \wedge f_B((ae)y)\} \\ &\geq f_A(a) \wedge f_B \\ &= (f_A \cap f_B)(a). \end{aligned}$$

Thus, $f_A \circ f_B \supseteq f_A \cap f_B$, and

$$f_B(a) \geq f_B((xa^2)y) = f_B((xa^2)(ey)) = f_B((ye)(a^2x)) = f_B(a^2((ye)x))) \geq f_B(a^2).$$

Hence, f_B is soft semiprime*.

(iii) $\Rightarrow$ (ii) : Let L and R be any left and right ideal of S. Then $\mathcal{S}_L$ and $\mathcal{S}_R$ are SI-left and SI-right ideal of S, respectively. Let $a \in L \cap R$. Then, $a \in L$ and $a \in R$, so by assumption, we have

$$U = \mathcal{S}_{L \cap R}(a) = (\mathcal{S}_L \cap \mathcal{S}_R)(a) \supseteq (\mathcal{S}_L \circ \mathcal{S}_R)(a) = \mathcal{S}_{(LR]}(a).$$

Thus, $L \cap R \subseteq (LR]$. By our assumption, the soft characteristic function $\mathcal{S}_R$ is soft semiprime*, and so, by Theorem 8, R is semiprime.

(ii) $\Rightarrow$ (i) : Clearly, $(Sa]$ and $(a^2S]$ are left and right ideals of S such that $a \in (Sa]$ and $a^2 \in (a^2S]$. Since by assumption, $(a^2S]$ is semiprime, therefore, $a \in (a^2S]$. Now by using Lemma 1 and $(1), (2), (3)$ and (4), we have

$$\begin{aligned} a &\in (a^2S] \cap (Sa] = ((a^2S](Sa]] \subseteq ((a^2S)(Sa)] = ((aS)(Sa^2)] \\ &= (((Sa^2)S)a] = (((Sa^2)(eS))a] \subseteq (((Sa^2)(SS))a] \\ &= (((SS)(a^2S))a] = ((a^2((SS)S))a] \subseteq ((a^2S)S] \\ &= ((SS)(aa)] = ((aa)(SS)] \subseteq ((aa)S] = ((Sa)a] \\ &\subseteq ((Sa)(a^2S)] = (((a^2S)a)S] = (((aS)a^2)S] \subseteq ((Sa^2)S]. \end{aligned}$$

This shows that there exist $x, y \in S$, such that $a \leq (xa^2)y$. Hence, S is intra-regular. ■

Theorem 11 *Let S be an ordered $\mathcal{AG}$-groupoid with left identity. Then, the following conditions are equivalent:*

(i) *S is intra-regular.*

(ii) *$R \cap L = (RL]$ for every left ideal L and for every right ideal R of S and R is semiprime.*

Theorem 12 *The following conditions are equivalent for an ordered $\mathcal{AG}$-groupoid S with left identity.*

(i) *S is intra-regular.*

(ii) *$f_A \cap f_B \subseteq f_A \circ f_B$, for every SI-right ideal f_A and for every SI-left ideal f_B of S and f_A is soft semiprime*.*

Proof. (i) $\Longrightarrow$ (ii) : Let f_A be any SI-right ideal and f_A be any SI-left ideal of S with left identity. Then, we have

$$f_A \circ f_B \subseteq f_A \cap f_B.$$

Since S is right-regular, for each $a \in S$ there exist $x, y \in G$ such that $a \leq (xa^2)y$. Thus,

$$a \leq (xa^2)y = (x(aa))y = (a(xa))y = (y(xa))a = ((ey)(xa))a = ((ax)(ye))a.$$

Hence,

$$\begin{aligned}(f_A \circ f_B)(a) &= \bigvee_{a \leq ((ax)(ye))a} \{f_A((ax)(ye)) \wedge f_B(a)\} \\ &\geq \{f_A((ax)(ye)) \wedge f_B(a)\} \\ &\geq f_A(a) \wedge f_B(a) \\ &= (f_A \cap f_B)(a).\end{aligned}$$

Thus, $f_A \cap f_B \subseteq f_A \circ f_B$.

Moreover,

$$f_A(a) \geq f_A((xa^2)y) = f_A((xa^2)(ey)) = f_A((ye)(a^2x)) = f_A(a^2((ye)x)) \geq f_A(a^2).$$

Hence, f_A is a soft semiprime.

(ii) $\Longrightarrow$ (i) : Let R be any right ideal and L be any left ideal of S. Then, $\mathcal{S}_L$ and $\mathcal{S}_R$ are SI-left ideal and SI-right ideal of S.Let $a \in R \cap L$. Then, $a \in R$ and $a \in L$. So, by assumption, we have

$$U = \mathcal{S}_{R\cap L}(a) = (\mathcal{S}_R \cap \mathcal{S}_L)(a) = (\mathcal{S}_R \circ \mathcal{S}_L)(a) = (\mathcal{S}_{(Rl]}(a).$$

Thus, $R \cap L \subseteq (RL]$ and clearly $(RL] \subseteq R \cap L$. Thus $(RL] = R \cap L$. Hence, S is intra-regular by Theorem 11. Moreover, by assumption, the soft characteristic function $\mathcal{S}_R$ is soft semiprime*, and so R is semiprime by Theorem 8. This complete the proof. ■

Corollary 3 *For an ordered $\mathcal{AG}$-groupoid S with left identity, $f_A \cap f_B = f_A \circ f_B$ for every SI-Ideals f_A and f_B of S.*

Proposition 11 *Let f_A and f_B be SI-left (right, two-sided) ideal of an ordered $\mathcal{AG}$-groupoid S with left identity. Then, $f_A \circ f_B$ is an SI-left (right, two-sided) ideal of S over U.*

Theorem 13 *Let S be an intra–regular ordered $\mathcal{AG}$-groupoid with left identity. Then, the set of all SI-ideals of S forms a samilattice structure with identity $\widetilde{\mathcal{S}}$.*

Proof. Let I_S be the set of all SI-ideals of an ordered $\mathcal{AG}$-groupoid S and f_A, f_B and $f_C \in I_S$. Clearly I_S is closed by Proposition 11. Moreover, by Proposition 5, we have $f_A = (f_A)^2$ and by Corollary 3, $f_A \circ f_B = f_A \cap f_B$, where f_A and f_B are $SI-$ideals. Obviously, $f_A \circ f_B = f_B \circ f_A$. Moreover by using (1),

$$(f_A \circ f_B) \circ f_C = f_C \circ (f_B \circ f_A).$$

Also by using (1) and Proposition 11,

$$f_A \circ \widetilde{\mathcal{S}} = (f_A \circ f_A) \circ \widetilde{\mathcal{S}} = (\widetilde{\mathcal{S}} \circ f_A) \circ f_A = f_A \circ f_A = f_A.$$

This complete the proof. ■

Theorem 14 *Let S be an ordered $\mathcal{AG}$-groupoid with left identity. Then, the following conditions are equivalent:*

(i) *S is intra-regular.*

(ii) *$f_A \cap f_B \subseteq f_A \circ f_B$ for every SI-right ideal f_A and every SI-left ideal f_B of S and SI-right ideal f_A is soft semiprime*.*

(iii) *$f_A \cap f_B \subseteq f_A \circ f_B$ for every SI-right ideal f_A and every SI-bi- ideal (SI-quasi-ideal) f_B of S and SI-right ideal f_A is soft semiprime*.*

Proof. It is obvious that (i) ⇒ (iii) and (iii) ⇒ (ii).
(ii) ⇒ (i) follows from Theorem 12. ■

Theorem 15 *Let S be an ordered $\mathcal{AG}$-groupoid with left identity. Then, the following conditions are equivalent:*

(i) *S is intra-regular.*

(ii) *$f_A \cap f_B \subseteq (f_A \circ f_B) \circ f_A$ for every SI-right ideal f_A and every SI-left ideal f_B of S and SI-right ideal f_A is soft semiprime*.*

(iii) *$f_A \cap f_B \subseteq (f_A \circ f_B) \circ f_A$ for every SI-right ideal f_A and every SI-bi- ideal f_B of S and SI-right ideal f_A is soft semiprime*.*

Proof. (i) ⇒ (iii) : Suppose S is intra-regular. Let f_A and f_B be any SI-right and SI-bi-ideal of S, respectively. Since S is intra-regular, for each $a \in S$, there exist $x, y \in S$ such that $a \leq (xa^2)y$. Thus,

$$\begin{aligned} a &\leq (xa^2)y = (a(xa))y = (y(xa))a = (y(x(xa^2)y))a \\ &= (y((xa^2)(xy)))a = ((xa^2)(y(xy)))a \\ &= ((x(aa))(y(xy)))a = ((a(xa))(y(xy)))a \\ &= (((y(xy)(xa))a)a = (((ax((xy)y))a)a. \end{aligned}$$

Thus, we have

$$\begin{aligned}((f_A \circ f_B) \circ f_A(a)) &= \bigvee_{a \leq (((ax((xy)y))a)a} \{f_A((ax((xy)y)) \wedge f_B(a) \wedge f_A(a)\} \\ &\geq f_A((ax((xy)y)) \wedge f_B(a) \wedge f_A(a) \\ &\geq f_A(a) \wedge f_B(a) \wedge f_A(a) \\ &= (f_A \cap f_B)(a).\end{aligned}$$

Thus, $f_A \cap f_B \subseteq (f_A \circ f_B) \circ f_A$.

Moreover,

$$f_A(a) \geq f_A((xa^2)y) = f_A((xa^2)(ey)) = f_A((ye)(a^2x)) = f_A(a^2((ye)x)) \geq f_A(a^2).$$

Hence, f_A is a soft semiprime*.

(iii) $\Rightarrow$ (ii) : It is obvious.

(ii) $\Rightarrow$ (i) : Suppose f_A and f_B be any SI-right and SI-left ideal of S, respectively. Then, by our assumption, we have

$$f_B \cap f_A = f_A \cap f_B \subseteq (f_A \circ f_B) \circ f_A \subseteq (\widetilde{\mathcal{S}} \circ f_B) \circ f_A \subseteq f_B \circ f_A.$$

Hence, by Theorem 10, S is intra-regular. ■

Conclusions

Order theory is a branch of Mathematics which investigates our intuitive notion of order using binary relations. It provides a formal framework for describing statements such as "this is less than that" or "this precedes that". The study of an algebraic structure using the order theory plays a prominent role in Mathematics with wide ranging applications in many disciplines such as control engineering, computer arithmetics, coding theory, sequential machines and formal languages.

In this paper, we first remind the concept of soft intersection left (right, two-sided) ideals, generalized bi-ideals, interior ideals quasi-ideals of an ordered $\mathcal{AG}$-groupoid. We then show that all these ideals coincide in intra-regular ordered $\mathcal{AG}$-groupoid with left identity. Moreover, we characterize intra-regular ordered $\mathcal{AG}$-groupoid by the properties of soft intersection ideals. We are of the opinion that some results in this paper have already constituted a foundation for further investigation related with further development of ordered $\mathcal{AG}$-groupoids. In the following study of soft intersection ordered $\mathcal{AG}$-groupoids, applying soft intersection ordered $\mathcal{AG}$-groupoid to some applied fields, such as decision making, data analysis and forecasting and so on are worth to be considered.

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