OPTIMAL INVESTMENT DECISION
USING NEURAL REDUCT FUZZY SET OF IVFS

M. Muthumeenakshi
School of Social Sciences and Languages
VIT University
Vellore – 632 014, Tamilnadu
India
e-mail: muthumeenakshi@live.com

P. Muralikrishna
PG and Research Department of Mathematics
Muthurangam Government Arts College(Autonomous)
Vellore – 632002, Tamilnadu
India
e-mail: pmkrishna@rocketmail.com

Abstract. Taking optimal investment decision is very challenging in the current scenario. Traditionally, the investors were followed fundamental analysis or technical analysis in the investment decision. But, today the market and technology are growing very faster than the investors’ ideas. Every new method is coming for the investment analysis. All the decisions are future oriented which relies uncertainty. It is inevitable to apply a tool which is linked with the uncertainty. Hence, in this study, ‘IVFS’ technique has been used which measures the uncertainty. Finding out the exact membership in Interval Valued Fuzzy is a crucial part in the investment analysis. To reduce this problem, here the notion of neural reduct fuzzy set of interval valued fuzzy set has been applied. In a clear sense, this paper finds the membership using interval value and converge into one single point that act as member for the fuzzy set.

Keywords: interval valued fuzzy set, soft set, ivfs-set, neural reduct fuzzy set, investment decision.

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1. Introduction

2. Interval valued fuzzy soft set and interval valued fuzzy soft decision set

In this section, the basic definitions of interval valued fuzzy set and soft set have been presented. Definition of interval valued fuzzy soft set (IVFS-set) and interval valued fuzzy decision set of an IVFS-set are also used to construct a decision method using neural reduct fuzzy set of Interval valued fuzzy set.

**Definition 2.1** [6] An Interval-Valued Fuzzy Set \( \tilde{\mu} \) on a universe \( U \) is a mapping such that \( \tilde{\mu}: U \rightarrow D([0,1]), \) where \( D([0,1]) \) is the set of all closed sub intervals of \([0,1] \). The set of all interval-valued fuzzy sets on \( U \) is denoted by \( \mathcal{I}(U) \). Also, \( \forall x \in U, \tilde{\mu}_x(x) = [\mu^L_x(x), \mu^U_x(x)] \) is called the degree of membership of an element \( x \) to \( \tilde{\mu} \) where \( \mu^L_x(x) \) and \( \mu^U_x(x) \) are the lower and upper degrees of membership of \( x \) to \( \tilde{\mu} \) respectively such that \( 0 \leq \mu^L_x(x) \leq \mu^U_x(x) \leq 1 \).

**Example 2.2** Let \( U = \{u_1, u_2, u_3, u_4\} \) be the Universe. Then \( \tilde{\mu}(U) \) defined following is an Interval-Valued Fuzzy Set.

\[
\begin{align*}
\tilde{\mu}(u_1) &= [0.4, 0.7] \\
\tilde{\mu}(u_2) &= [0.3, 0.5] \\
\tilde{\mu}(u_3) &= [0.1, 0.6] \\
\tilde{\mu}(u_4) &= [0.2, 0.8]
\end{align*}
\]

**Definition 2.3** [3] An Neural Reduct fuzzy set \( \mu_N(U) \) of Interval-Valued Fuzzy Set \( \tilde{\mu}(U) \) is defined by \( \mu_N(U) = \left\{ \frac{\mu^L_x(x) + \mu^U_x(x)}{2}, \quad x \in U \right\} \).

**Example 2.4** For the Universe defined in Example 2.2, the Neural Reduct fuzzy set \( \mu_N(U) \) is defined as follows: \( \mu_N(U) = \{\langle u_1, 0.55 \rangle, \langle u_2, 0.4 \rangle, \langle u_3, 0.35 \rangle, \langle u_4, 0.5 \rangle\} \).

**Definition 2.5** [2] Let \( U \) refers to an initial universe, \( E \) is a set of all parameters, \( P(U) \) is the power set of \( U \), and \( A \subseteq E \). A Soft set \( F_A \) over \( U \) is a set defined by a function \( f_A \) representing a mapping \( f_A: E \rightarrow P(U) \) such that \( f_A = \emptyset \) if \( x \notin A \). Thus a soft set over \( U \) can be represented by the set of ordered pairs \( F_A = \{(x, f_Ax) : x \in E, f_Ax \in P(U) \} \).

**Example 2.6** Consider the Universe \( U = \{u_1, u_2, u_3, u_4\} \) defined in Example 2.2 Suppose \( E = \{x_1, x_2, x_3, x_4, x_5\} \) is the set of parameters. Then the following is a Soft set \( F_A \) over \( U \) for the subset \( A = \{x_1, x_2, x_3\} \) of \( E \): \( F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\}), (x_3, \{u_1, u_2, u_3\})\} \).

**Definition 2.7** [4] Interval Valued Fuzzy Soft [IVFS] set. Let \( U \) refers to an initial universe, \( E \) is a set of all parameters, \( P(U) \) is the power set of \( U \), and \( X \) be a fuzzy set over \( E \). An IVFS-set on \( U \) is defined by the pair \( F_X = \{\tilde{\mu}_X(x)/x, f_Xx) : x \in E, f_Xx \in P(U), \tilde{\mu}_X(x) \in D([0,1])\} \), where the function \( f_X : E \rightarrow P(U) \) is called the approximate function such that \( f_X(x) = \emptyset \) if \( \tilde{\mu}_X(x) = [0,0] \), and \( \tilde{\mu}_X : E \rightarrow D([0,1]) \) is called the membership function of IVFS-set \( F_X \). The value of \( \tilde{\mu}_X(x) \) is the degree of importance of the parameter \( x \), and depends on the decision maker's requirements. The sets of all IVFS-sets over \( U \) will be denoted by IVFS(\( U \)).
Example 2.8 Consider the Universe \( U = \{u_1, u_2, u_3, u_4\} \) and the set of parameters \( E = \{x_1, x_2, x_3\} \). Then \( F_X = \{(0.3, 0.6)/x_1, \{u_1, u_2\}\}, \{(0.5, 0.8)/x_2, \{u_2, u_3\}\}, \{(0.1, 0.4)/x_3, \{u_1, u_2, u_3\}\} \) is an Interval Valued Fuzzy Soft set.

Definition 2.9 Interval Valued Fuzzy Soft Decision (IVFSD) set. Let \( F_X \in IVFS(U) \). Then, an interval valued fuzzy soft decision (IVFSD) set of \( F_X \) denoted by \( F_X^D \) and is defined by \( F_X^D = \{\tilde{\mu}_{F_X^D}(u) : u \in U\} \), which is an interval valued fuzzy set over \( U \) and its membership function \( \tilde{\mu}_{F_X^D} \) is defined by
\[
\tilde{\mu}_{F_X^D} : U \rightarrow D([0, 1]) \quad \text{and} \quad \tilde{\mu}_{F_X^D}(u) = \left[ \frac{1}{|E|} \sum_{x \in E} \mu^L_X(x), \frac{1}{|E|} \sum_{x \in E} \mu^U_X(x) \right].
\]
The following section gives an illustration for the IVFSD set.

3. Analysis of optimal investment decision

In this section, an algorithm has been introduced using Neural Reduct Fuzzy set of IVFSD-set to choose the optimal investment avenue from the alternatives.

1. Design a Soft set \( F_A \) over \( U \).
2. Design a IVFS-set \( F_X \) over \( U \).
3. Construct the IVFSD-set \( F_X^D \).
4. Obtain Neural Reduct Fuzzy set of \( F_X^D \).
5. Choose the maximum membership value from Neural Reduct Fuzzy set of \( F_X^D \).

The study has been carried out by considering 50 investors in a basket of Middle Income Group having the annual income of Rs.3 Lacs to Rs.5 Lacs. A structured questionnaire has been circulated by asking their preference towards the selected investments along with corresponding constraints. Four major varieties of investment avenues namely Fixed Deposits in Bank, Gold, Real estate and Shares have been identified. In symbolic form, it is noted as \( I_1, I_2, I_3 \) and \( I_4 \). The criteria for the preferences of investments are indicated as Return, Growth, Risk less security, Tax benefit. These are denoted as \( C_1, C_2, C_3 \) and \( C_4 \) respectively.

Step 1. To apply IVFS Decision Set in Investment decision, a universal set \( U = \{I_1, I_2, I_3, I_4\} \) has been framed and the parameters used for analyzing the investment characteristics by the investors are considered as the set of Constraints \( C = \{C_1, C_2, C_3, C_4\} \). Based on the responses, the II (FD) having the characteristic of Return, Risk less security and Tax benefit, the I2 (Gold) is having the characteristic of Risk less security and Growth, I3(Real Estate) is having the characteristic of Growth and I4 (Shares) is having the characteristic of Return and Growth. Hence the Soft set \( F_A \) is designed as
\[
F_A = \{((C_1), (I_3, I_4)); ((C_2), (I_2, I_3, I_4)); ((C_3), (I_1, I_2)); ((C_4), (I_1))\}.
\]

Step 2. The points are given by the investors from 0 to 1 based on their preferences towards the selected investment avenues. From these points the lowest point and the highest point are taken as lower membership and upper membership grades for the IVF set respectively. By following the above procedure, the IVFS set has been framed.
The IVFS-set $F_X$ is designed as
\[
\left\{ \begin{array}{l}
(([0.4, 0.6]/C_1), (I_3, I_4)) ; (([0.28, 0.72]/C_2), (I_1, I_3, I_4)) ; \\
(([0.36, 0.64]/C_3), (I_1, I_2)) ; (([0.44, 0.56]/C_4), (I_4))
\end{array}\right. \}
\]

Step 3. The IVFSD-set $F^D_X$ has been attained using Definition 2.5.
\[
F^D_X = \{ [0.30, 0.45]/I_1, [0.23, 0.34]/I_2, [0.07, 0.18]/I_3, [0.17, 0.33]/I_4 \}
\]

Step 4. Neural Reduct Fuzzy set of the IVFSD-set $F^D_X$ of given by
\[
\left\{ 0.375/I_1, 0.285/I_2, 0.125/I_3, 0.250/I_4 \right\}
\]

Step 5. It is clear that the maximum membership value 0.375 occurs on $I_1$, i.e, Fixed Deposit is decided as the optimal investment.

4. Conclusion
The responses by the respondents are favor to the Fixed Deposit. It is a Risk less security having a reasonable return with tax benefit which impress the middle income group. Having all these factors, the FD is preferable investment with the selected criteria. Hence it is concluded that FD is the optimal investment decision for the middle income group. One can apply the notion of IVFSD in medical diagnosis, neural networks, Managerial and Engineering decision making situation to make the decision optimally.

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References


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