SEMIGROUP IDEALS AND PERMUTING 3-GENERALIZED DERIVATIONS IN PRIME NEAR RINGS

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Abstract. The purpose of the present paper is to prove some commutativity theorems in the setting of a semigroup ideal of a 3-prime near ring admitting a permuting generalized 3-derivation, thereby extending some known results of derivations, biderivations and 3-derivations.

Keywords: 3-prime near ring, semigroup ideal, permuting 3-derivation, permuting 3-generalized derivation.

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1. Introduction

Throughout the paper, N denotes a zero-symmetric left near ring with multiplicative centre Z; and for any pair of elements $x, y \in N$, $[x, y]$ denotes the commutator $xy - yx$ while the symbol (x, y) denotes the additive commutator $x + y - x - y$. A near ring N is called zero-symmetric if $0x = 0$, for all $x \in N$ (recall that left distributivity yields that $x0 = 0$). The near ring N is said to be 3-prime if $xNy = \{0\}$ for $x, y \in N$ implies that $x = 0$ or $y = 0$. A near ring N is called $n-$torsion-free, where n is a positive integer, if $(N, +)$ has no element of order n. For $x \in N$, $C(x) = \{a \in N \mid ax = xa\}$ denotes the centralizer of x in N. A nonempty subset U of N is called a semigroup right (resp. semigroup left) ideal if $UN \subseteq U$ (resp. $NU \subseteq U$) and if U is both a semigroup right ideal and a semigroup left ideal, it is called a semigroup ideal.

A mapping $\Delta : N \times N \times N \longrightarrow N$ is said to be permuting if $\Delta(x, y, z) = \Delta(y, x, z) = \Delta(z, y, x) = \Delta(x, z, y) = \Delta(y, z, x) = \Delta(z, x, y)$ for all $x, y, z \in N$. A mapping $\delta : N \longrightarrow N$ defined by $\delta(x) = \Delta(x, x, x)$ is called trace of Δ, where $\Delta : N \times N \times N \longrightarrow N$ is a permuting mapping. It is obvious that, if $\Delta : N \times N \times N \longrightarrow N$ is a permuting mapping which is also 3-additive (i.e., additive in all arguments), then the trace δ of Δ satisfies the relation

$$\delta(x + y) = \delta(x) + 2\Delta(x, x, y) + \Delta(x, y, y) + \Delta(x, x, y) + 2\Delta(x, y, y) + \delta(y)$$

for all $x, y \in N$.

A permuting 3-additive mapping $\Delta : N \times N \times N \longrightarrow N$ is called a permuting 3-derivation if $\Delta(xw, y, z) = \Delta(x, y, z)w + x\Delta(w, y, z)$ is fulfilled for all $x, y, z, w \in N$.

Motivated by the notion of permuting 3-derivations in near rings [10], Park and Jung [9] defined permuting 3-generalized derivations in near rings. Let $\Delta : N \times N \times N \longrightarrow N$ be a permuting 3-derivation of N. A permuting 3-additive mapping $F : N \times N \times N \longrightarrow N$ is said to be a permuting 3-right (resp. left) generalized derivation of N associated with Δ if $F(xw, y, z) = F(x, y, z)w + x\Delta(w, y, z)$ (resp. $F(xw, y, z) = \Delta(x, y, z)w + xF(w, y, z)$) for all $x, y, z, w \in N$. Also, F is said to be a permuting 3-generalized derivation of N associated with Δ if it is both a permuting 3-right and a permuting 3-left generalized derivation associated with Δ. In case of rings and near rings derivations and biderivations have received significant attention in recent years (see [1], [2], [3], [4], [12], [13], [14]). The purpose of the present paper is to prove some commutativity theorems in the setting of a semigroup ideal of a near ring admitting a permuting 3-generalized derivation, thereby extending some known results on derivations and biderivations.

Example 1.1. Let S be a commutative near-ring and let

$$N = \left\{ \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} \mid a, b \in S \right\}.$$

Then N is a near-ring under matrix addition and matrix multiplication. Define the maps $\Delta, F : N \times N \times N \longrightarrow N$ by

$$\left(\begin{pmatrix} 0 & a_1 \\ 0 & b_1 \end{pmatrix}, \begin{pmatrix} 0 & a_2 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} 0 & a_3 \\ 0 & b_3 \end{pmatrix} \right) \longmapsto \begin{pmatrix} 0 & a_1a_2a_3 \\ 0 & 0 \end{pmatrix} \text{ and}$$

$$\left(\begin{pmatrix} 0 & a_1 \\ 0 & b_1 \end{pmatrix}, \begin{pmatrix} 0 & a_2 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} 0 & a_3 \\ 0 & b_3 \end{pmatrix} \right) \longmapsto \begin{pmatrix} 0 & 0 \\ 0 & b_1b_2b_3 \end{pmatrix} \text{ respectively.}$$

It can be easily verified that Δ is a 3-derivation of N and F is a permuting 3-right generalized derivation of N with associated 3-derivation Δ.

Example 1.2 Let S be a commutative near-ring and let

$$N = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in S \right\}.$$

Then N is a near-ring under matrix addition and matrix multiplication. Define the maps $\Delta, F : N \times N \times N \longrightarrow N$ by

$$\left(\begin{pmatrix} a_1 & b_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} a_3 & b_3 \\ 0 & 0 \end{pmatrix} \right) \longmapsto \begin{pmatrix} 0 & b_1 b_2 b_3 \\ 0 & 0 \end{pmatrix} \text{ and}$$
$$\left(\begin{pmatrix} a_1 & b_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} a_3 & b_3 \\ 0 & 0 \end{pmatrix} \right) \longmapsto \begin{pmatrix} a_1 a_2 a_3 & 0 \\ 0 & 0 \end{pmatrix} \text{ respectively.}$$

It can be verified that Δ is a 3-derivation of N and F is a 3-left generalized derivation of N with associated 3-derivation Δ.

Example 1.3 Let S be a commutative near-ring and let

$$N = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \mid a, b, c \in S \right\}.$$

Then N is a near-ring under matrix addition and matrix multiplication. Define maps $\Delta, F : N \times N \times N \longrightarrow N$ by

$$\left(\begin{pmatrix} a_1 & 0 \\ b_1 & c_1 \end{pmatrix}, \begin{pmatrix} a_2 & 0 \\ b_2 & c_2 \end{pmatrix}, \begin{pmatrix} a_3 & 0 \\ b_3 & c_3 \end{pmatrix} \right) \longmapsto \begin{pmatrix} 0 & 0 \\ b_1 b_2 b_3 & 0 \end{pmatrix} \text{ and}$$
$$\left(\begin{pmatrix} a_1 & 0 \\ b_1 & c_1 \end{pmatrix}, \begin{pmatrix} a_2 & 0 \\ b_2 & c_2 \end{pmatrix}, \begin{pmatrix} a_3 & 0 \\ b_3 & c_3 \end{pmatrix} \right) \longmapsto \begin{pmatrix} 0 & 0 \\ b_1 b_2 b_3 & 0 \end{pmatrix} \text{ respectively.}$$

It can be verified that Δ is a 3-derivation of N and F is a permuting 3-right generalized derivation and a permuting 3-left generalized derivation of N with associated 3-derivation Δ and hence F is a permuting 3-generalized derivation of N.

2. Preliminary results

We begin with the following Lemmas. Lemmas 2.7, 2.9 and 2.10 are proved using same techniques with some variations as those of Lemmas 7, 8 and 10 in [8].

Lemma 2.1 [2, Lemma 1.2] *Let N be a 3-prime near ring.*

(i) *If $z \in Z \setminus \{0\}$, then z is not a zero divisor.*

(ii) *If $Z \setminus \{0\}$ contains an element z for which $z + z \in Z$, then $(N, +)$ is abelian.*

(iii) *If $z \in Z \setminus \{0\}$ and x is an element of N such that $xz \in Z$ or $zx \in Z$, then $x \in Z$.*

Lemma 2.2 [2, Lemma 1.3] *Let N be a 3-prime near ring and U be a nonzero semigroup ideal of N.*

(i) *If* $x, y \in N$ *and* $xUy = \{0\}$, *then* $x = 0$ *or* $y = 0$.

(ii) *If* $x \in N$ *and* $xU = \{0\}$ *or* $Ux = \{0\}$, *then* $x = 0$.

Lemma 2.3 [2, Lemma 1.5] *If* N *is a* 3*-prime near ring and* Z *contains a nonzero semigroup left ideal or a semigroup right ideal, then* N *is a commutative ring.*

Lemma 2.4 [10, Lemma 2.4] *Let* N *be a near ring and let* $\Delta : N \times N \times N \longrightarrow N$ *be a permuting 3-derivation. Then we have*

$[\Delta(x,z,w)y + x\Delta(y,z,w)]v = \Delta(x,z,w)yv + x\Delta(y,z,w)v$ *for all* $v, w, x, y, z \in N$.

Lemma 2.5 *Let* N *be a* 3*-prime near ring and* U *be a nonzero semigroup ideal of* N. *If* Δ *is a nonzero* 3*-derivation on* N, *then* $\Delta \neq 0$ *on* U.

Proof. Suppose that $\Delta(U,U,U) = \{0\}$. For any $u, v, w \in U$, we have

$$\Delta(u, v, w) = 0. \tag{2.1}$$

Substituting ux for u in (2.1), we get

$$\Delta(u,v,w)x + u\Delta(x,v,w) = 0 \text{ for all } u,v,w \in U \text{ and } x \in N.$$

Using (2.1), we get $U\Delta(x,v,w) = \{0\}$. Invoking Lemma 2.2 (ii), we have

$$\Delta(x,v,w) = 0 \text{ for all } v,w \in U \text{ and } x \in N. \tag{2.2}$$

Substituting vy for v in (2.2), we get

$$\Delta(x,v,w)y + v\Delta(x,y,w) = 0 \text{ for all } v,w \in U \text{ and } x,y \in N.$$

Using (2.2), we find $U\Delta(x,y,w) = \{0\}$ and Lemma 2.2 (ii), yields that

$$\Delta(x,y,w) = 0 \text{ for all } w \in U \text{ and } x,y \in N. \tag{2.3}$$

Substituting wz for w in (2.3), we obtain $U\Delta(x,y,z) = \{0\}$. Another appeal to Lemma 2.2 (ii), yields that $\Delta(x,y,z) = 0$, for all $x,y,z \in N$, which is a contradiction. ∎

Lemma 2.6 *Let* N *be a* 3!*-torsion free near ring and* U *be a nonzero additive subgroup of* N. *If* Δ *is a permuting 3-additive map with trace* δ *such that* $\delta(x) = 0$ *for all* $x \in U$, *then* $\Delta = 0$ *on* U.

Proof. For any $x, y \in U$, we have the relation

$$\delta(x+y) = \delta(x) + 2\Delta(x,x,y) + \Delta(x,y,y) + \Delta(x,x,y) + 2\Delta(x,y,y) + \delta(y)$$

and so, by the hypothesis, we get

$$2\Delta(x,x,y) + \Delta(x,y,y) + \Delta(x,x,y) + 2\Delta(x,y,y) = 0 \text{ for all } x,y \in U. \tag{2.4}$$

Substituting $-x$ for x in (2.4), we obtain

$$2\Delta(x,x,y) - \Delta(x,y,y) + \Delta(x,x,y) - 2\Delta(x,y,y) = 0 \text{ for all } x,y \in U. \quad (2.5)$$

On the other hand, for any $x, y \in U$,

$$\delta(y+x) = \delta(y) + 2\Delta(y,y,x) + \Delta(y,x,x) + \Delta(y,y,x) + 2\Delta(y,x,x) + \delta(x)$$

and thus, by the hypothesis and using the fact that Δ is permuting, we have

$$2\Delta(x,y,y) + \Delta(x,x,y) + \Delta(x,y,y) + 2\Delta(x,x,y) = 0 \text{ for all } x,y \in U. \quad (2.6)$$

Comparing (2.4) and (2.5), we get

$$2\Delta(x,y,y) + \Delta(x,x,y) + \Delta(x,y,y) = \Delta(x,x,y) - 3\Delta(x,y,y)$$

which implies that

$$\begin{aligned} 2\Delta(x,y,y) + \Delta(x,x,y) + \Delta(x,y,y) + 2\Delta(x,x,y) \\ = \Delta(x,x,y) - 3\Delta(x,y,y) + 2\Delta(x,x,y). \end{aligned}$$

Hence it follows from (2.6) that

$$\Delta(x,x,y) - 3\Delta(x,y,y) + 2\Delta(x,x,y) = 0 \text{ for all } x,y \in U. \quad (2.7)$$

Substituting $-x$ for x in (2.7), we find

$$\Delta(x,x,y) + 3\Delta(x,y,y) + 2\Delta(x,x,y) = 0 \text{ for all } x,y \in U. \quad (2.8)$$

Comparing (2.7) and (2.8), we obtain

$$6\Delta(x,y,y) = 0 \text{ for all } x,y \in U.$$

Since N is 3!-torsion free, we get

$$\Delta(x,y,y) = 0 \text{ for all } x,y \in U. \quad (2.9)$$

Substituting $y+z$ for y in (2.9) and linearizing (2.9) we obtain

$$\Delta(x,y,z) = 0 \text{ for all } x,y \in U,$$

i.e., $\Delta = 0$ on U which completes the proof. ∎

Lemma 2.7 *Let N be a 3-prime near ring and U be a nonzero semigroup ideal of N. Let $\Delta : N \times N \times N \longrightarrow N$ be a 3-derivation. If for $x \in N$, $\Delta(U,U,U)x = \{0\}$ (or $x\Delta(U,U,U) = \{0\}$), then either $x = 0$ or $\Delta = 0$ on U.*

Proof. Let $\Delta(y,z,w)x = 0$ for all $y,z,w \in U$. Substituting $y = vy$, we get $\Delta(v,z,w)yx = 0$, for all $y,z,v,w \in U$. Hence $\Delta(v,z,w)Ux = \{0\}$ implies that either $x = 0$ or $\Delta = 0$ by Lemma 2.2 (i). ∎

Lemma 2.8 *Let N be a $3!$-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. If Δ is a permuting 3-derivation with trace δ and $x \in N$ such that $x\delta(y) = 0$ for all $y \in U$, then either $x = 0$ or $\Delta = 0$ on U.*

Proof. For any $y, z \in U$, we have

$$\delta(y+z) = \delta(y) + 2\Delta(y,y,z) + \Delta(y,z,z) + \Delta(y,y,z) + 2\Delta(y,z,z) + \delta(z).$$

By hypothesis

$$2x\Delta(y,y,z) + x\Delta(y,z,z) + x\Delta(y,y,z) + 2x\Delta(y,z,z) = 0 \text{ for all } y,z \in U. \quad (2.10)$$

Substituting $-y$ for y in (2.10), it follows that

$$2x\Delta(y,y,z) - x\Delta(y,z,z) + x\Delta(y,y,z) - 2x\Delta(y,z,z) = 0 \text{ for all } y,z \in U. \quad (2.11)$$

On the other hand,

$$\delta(z+y) = \delta(z) + 2\Delta(z,z,y) + \Delta(z,y,y) + \Delta(z,z,y) + 2\Delta(z,y,y) + \delta(y)$$

Again using hypothesis, we have

$$2x\Delta(z,z,y) + x\Delta(z,y,y) + x\Delta(z,z,y) + 2x\Delta(z,y,y) = 0.$$

Since Δ is permuting, we get

$$2x\Delta(y,z,z) + x\Delta(y,y,z) + x\Delta(y,z,z) + 2x\Delta(y,y,z) = 0 \text{ for all } y,z \in U. \quad (2.12)$$

Comparing (2.10) and (2.11), we get

$$2x\Delta(y,z,z) + x\Delta(y,y,z) + x\Delta(y,z,z) = x\Delta(y,y,z) - 3x\Delta(y,z,z)$$

i.e.,

$$\begin{aligned} 2x\Delta(y,z,z) + x\Delta(y,y,z) + x\Delta(y,z,z) + 2x\Delta(y,y,z) \\ = x\Delta(y,y,z) - 3x\Delta(y,z,z) + 2x\Delta(y,y,z). \end{aligned}$$

Now, from (2.12), we obtain

$$x\Delta(y,y,z) - 3x\Delta(y,z,z) + 2x\Delta(y,y,z) = 0 \text{ for all } y,z \in U. \quad (2.13)$$

Substituting $-y$ for y in (2.13), we find

$$x\Delta(y,y,z) + 3x\Delta(y,z,z) + 2x\Delta(y,y,z) = 0 \text{ for all } y,z \in U. \quad (2.14)$$

Comparing (2.13) and (2.14), we obtain

$$6x\Delta(y,z,z) = 0 \text{ for all } y,z \in U.$$

Since N is $3!$-torsion free, we get

$$x\Delta(y,z,z) = 0 \text{ for all } y,z \in U. \quad (2.15)$$

Substituting $z+w$ for z in (2.15), we have

$$x\Delta(w,y,z)=0 \text{ for all } w,y,z\in U. \tag{2.16}$$

Hence by Lemma 2.7 either $x=0$ or $\Delta=0$ on U. ∎

Lemma 2.9 *Let N be a $3!$-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. Let $\Delta : N\times N\times N \longrightarrow N$ be a 3-derivation.*

(i) *If F is a permuting 3-right generalized derivation of N associated with Δ and $x\in N$ such that $xf(y)=0$ for all $y\in U$, then either $x=0$ or $\Delta=0$ on U.*

(ii) *If F is a permuting 3-generalized derivation of N associated with Δ and $x\in N$ such that $xf(y)=0$ for all $y\in U$, then either $x=0$ or $F=0$ on U.*

Proof. (i) For any $y,z\in U$, we have

$$f(y+z)=f(y)+2F(y,y,z)+F(y,z,z)+F(y,y,z)+2F(y,z,z)+f(z).$$

Using hypothesis, we obtain

$$2xF(y,y,z)+xF(y,z,z)+xF(y,y,z)+2xF(y,z,z)=0 \text{ for all } y,z\in U. \tag{2.17}$$

Substituting $-y$ for y in (2.17), it follows that

$$2xF(y,y,z)-xF(y,z,z)+xF(y,y,z)-2xF(y,z,z)=0 \text{ for all } y,z\in U. \tag{2.18}$$

On the other hand, for any $y,z\in U$,

$$f(z+y)=f(z)+2F(z,z,y)+F(z,y,y)+F(z,z,y)+2F(z,y,y)+f(y)$$

and so, by the hypothesis, we have

$$2xF(z,z,y)+xF(z,y,y)+xF(z,z,y)+2xF(z,y,y)=0$$

Since F is permuting, we get

$$2xF(y,z,z)+xF(y,y,z)+xF(y,z,z)+2xF(y,y,z)=0 \text{ for all } y,z\in U. \tag{2.19}$$

Comparing (2.17) and (2.18), we get

$$2xF(y,z,z)+xF(y,y,z)+xF(y,z,z)=xF(y,y,z)-3xF(y,z,z)$$

which implies that

$$\begin{aligned}2xF(y,z,z)&+xF(y,y,z)+xF(y,z,z)+2xF(y,y,z)\\&=xF(y,y,z)-3xF(y,z,z)+2xF(y,y,z).\end{aligned}$$

Now, from (2.19), we obtain

$$xF(y,y,z) - 3xF(y,z,z) + 2xF(y,y,z) = 0 \text{ for all } y,z \in U. \tag{2.20}$$

Substituting $-y$ for y in (2.20), we have

$$xF(y,y,z) + 3xF(y,z,z) + 2xF(y,y,z) = 0 \text{ for all } y,z \in U. \tag{2.21}$$

Combining (2.20) and (2.21), we obtain

$$6xF(y,z,z) = 0 \text{ for all } y,z \in U.$$

Since N is 3!-torsion free, we get

$$xF(y,z,z) = 0 \text{ for all } y,z \in U. \tag{2.22}$$

Substituting $z+w$ for z in (2.22), we find that

$$xF(w,y,z) = 0 \text{ for all } w,y,z \in U. \tag{2.23}$$

Replacing y by yv in (2.27) and using (2.23), we get $xy\Delta(v,z,w)$=0 for all $u,v,w,x,z \in U$. Hence $xU\Delta(v,z,w)$=0 implies that either $x = 0$ or $\Delta(v,z,w) = 0$ for all $u,v,w \in U$ by Lemma 2.2 (i).

(ii) Let x be a nonzero element of N. Thus from (i) we obtain Δ=0. Replacing y by yv in (2.23) and using hypothesis, we get $xyF(v,z,w)$=0, for all $v,w,x,y,z \in U$. Hence $xUF(v,z,w) = \{0\}$, again application of Lemma 2.2 (i) yields that $F = 0$ on U. ∎

Lemma 2.10 *Let N be a near ring and let $\Delta : N\times N\times N \longrightarrow N$ be a 3-derivation.*

(i) *If F is a 3-right generalized derivation of N associated with Δ, then*

$$[F(x,z,w)y + x\Delta(y,z,w)]v = F(x,z,w)yv + x\Delta(y,z,w)v, \quad \textit{for all } v,w,x,y,z \in N.$$

(ii) *If F is a 3-left generalized derivation of N associated with Δ , then*

$$[\Delta(x,z,w)y + xF(y,z,w)]v = \Delta(x,z,w)yv + xF(y,z,w)v, \quad \textit{for all } v,w,x,y,z \in N.$$

Proof. (i) Let F be a 3-right generalized derivation of N associated with Δ. Then

$$\begin{aligned} F((xy)v,z,w) &= F(xy,z,w)v + xy\Delta(v,z,w) \\ &= [F(x,z,w)y + x\Delta(y,z,w)]v + xy\Delta(v,z,w) \end{aligned} \tag{2.24}$$
$$\text{for all } v,w,x,y,z \in N.$$

On the other hand,

$$\begin{aligned} F(x(yv),z,w) &= F(x,z,w)yv + x\Delta(yv,z,w) \\ &= F(x,z,w)yv + x[\Delta(y,z,w)v + y\Delta(v,z,w)] \\ &= F(x,z,w)yv + x\Delta(y,z,w)v + xy\Delta(v,z,w) \\ &\qquad \text{for all } v,w,x,y,z \in N. \end{aligned} \tag{2.25}$$

Comparing (2.24) and (2.25), we get

$[F(x,z,w)y + x\Delta(y,z,w)]v = F(x,z,w)yv + x\Delta(y,z,w)v$ for all $v,w,x,y,z \in N$.

(ii) Let F be a 3-left generalized derivation of N associated with Δ. Then

$$\begin{aligned} F((xy)v,z,w) &= \Delta(xy,z,w)v + xyF(v,z,w) \\ &= [\Delta(x,z,w)y + x\Delta(y,z,w)]v + xyF(v,z,w) \\ &\qquad \text{for all } v,w,x,y,z \in N. \end{aligned} \tag{2.26}$$

On the other hand,

$$\begin{aligned} F(x(yv),z,w) &= \Delta(x,z,w)yv + xF(yv,z,w) \\ &= \Delta(x,z,w)yv + x[\Delta(y,z,w)v + yF(v,z,w)] \\ &= \Delta(x,z,w)yv + x\Delta(y,z,w)v + xyF(v,z,w) \\ &\qquad \text{for all } v,w,x,y,z \in N. \end{aligned} \tag{2.27}$$

Comparing (2.26) and (2.27), we get

$[\Delta(x,z,w)y + xF(y,z,w)]v = \Delta(x,z,w)yv + xF(y,z,w)v$ for all $v,w,x,y,z \in N$.

Lemma 2.11 *Let N be a 3-prime near ring and U be a nonzero semigroup ideal of N. If F is a nonzero 3-right (or left) generalized derivation of N associated with a nonzero 3-derivation Δ, then $F \neq 0$ on U.*

Proof. Let F be a nonzero 3-right generalized derivation of N such that

$$F(U,U,U) = \{0\}.$$

Then for all $x,y,z \in U$, we have $F(x,y,z) = 0$. Replacing x by xw, we get $F(x,y,z)w + x\Delta(w,y,z) = 0$ for $w,x,y,z \in U$. Hence $U\Delta(w,y,z) = \{0\}$ and Lemma 2.2 (*ii*) gives that $\Delta(w,y,z) = 0$, for all $w,y,z \in U$, a contradiction by Lemma 2.5 which completes the proof. ∎

3. The condition $F(U,U,U) \subseteq Z$

The theorems that we prove in this section will extend the results proved in [1, Theorem 2.1, 3.1 and 3.3], [2, Theorem 2.1 and 4.1], [3, Theorem 2 and 3], [8, Theorem 4 and 5] and [9, Theorem 3.1 and 3.2].

Theorem 3.1 *Let N be a 3-prime near ring and U be a nonzero semigroup ideal of N which is closed under addition. Suppose F is a nonzero 3-right generalized derivation of N associated with a nonzero 3-derivation Δ. If $F(U,U,U) \subseteq Z$, then $(N,+)$ is abelian.*

Proof. Suppose that $x,y,z \in U$ such that $0 \neq F(x,y,z) \in Z \setminus \{0\}$. Then $F(x,y,z+z) = F(x,y,z) + F(x,y,z) \in Z$, since F is nonzero on U by Lemma 2.11. Hence $(N,+)$ is abelian by Lemma 2.1 (ii). ∎

Theorem 3.2 *Let N be a 3!-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. Suppose Δ is a permuting 3-derivation and F is a nonzero permuting 3-generalized derivation of N associated with Δ such that $f(U) \subseteq U$, where f is the trace of F. If $F(U,U,U) \subseteq Z$, then N is a commutative ring.*

Proof. Suppose that $\Delta = 0$ on U. Then $F(xy,z,w) = F(x,z,w)y \in Z$ for all $w,x,y,z \in U$ gives that $F(x,z,w)yv = vF(x,z,w)y$ for all $w,x,y,z \in U$ and $v \in N$ this implies that $[y,v]f(x) = 0$ for all $v,x,y \in U$. Applying Lemma 2.9 (ii), we get $[y,v] = 0$ for all $y \in U$ and $v \in N$ and $U \subseteq Z$. Hence N is a commutative ring by Lemma 2.3.

Now, let us consider the case $\Delta \neq 0$ on U. Using the hypothesis, we get $F(x,y,z)w = wF(x,y,z)$ for all $x,y,z \in U$ and $w \in N$. Replacing x by xv, we obtain

$$F(x,y,z)vw + x\Delta(v,y,z)w = wF(x,y,z)v + wx\Delta(v,y,z) \quad \text{for all } v,x,y,z \in U \text{ and } w \in N. \tag{3.1}$$

Replacing x by $rf(x)$ in (3.1) and using (3.1), we have

$$r\Delta(f(x),y,z)[v,w] = 0 \text{ for all } r,v,x,y,z \in U \text{ and } w \in N. \tag{3.2}$$

Thus $U\Delta(f(x),y,z)[v,w] = 0$ and by Lemma 2.2 (ii) $\Delta(f(x),y,z)[v,w] = 0$. Substituting ry for y, we obtain $\Delta(f(x),r,z)U[v,w] = \{0\}$. Using Lemma 2.2 (i), we get either $\Delta(f(x),y,z) = 0$ or $[v,w] = 0$. Later yields that N is a commutative ring by Lemma 2.3.

Now, suppose that

$$\Delta(f(x),y,z) = 0 \text{ for all } x,y,z \in U. \tag{3.3}$$

Substituting $x+v$ for x in (3.3) and using the fact that N is 3!-torsion free, we get that

$$\Delta(F(x,v,v),y,z) = 0 \text{ for all } v,x,y,z \in U. \tag{3.4}$$

Replacing v by $v+r$ in (3.4), we get

$$\Delta(F(x,v,r),y,z) = 0 \text{ for all } r,v,x,y,z \in U. \tag{3.5}$$

Taking xt instead of x in (3.5) and using (3.5), we have

$$F(x,v,r)\Delta(t,y,z)+\Delta(x,y,z)\Delta(t,v,r)+x\Delta(\Delta(t,v,r),y,z)=0 \quad \text{for all } r,t,v,x,y,z\in U. \tag{3.6}$$

Substituting $f(r)$ for r in (3.6) and using (3.3), we get

$$F(f(r),x,v)\Delta(t,y,z)=0 \text{ for all } r,v,t,x,y,z\in U.$$

Substituting uv for v, we obtain

$$F(f(r),x,u)U\Delta(t,y,z)=\{0\}.$$

Again, by Lemma 2.2 (i), we get either

$$F(f(r),x,u)=0 \text{ or } \Delta(t,y,z)=0.$$

If $\Delta(t,y,z)=0$, for all $t,y,z\in U$, then $\Delta=0$ on U, a contradiction. Thus we find that

$$F(f(r),x,v)=0 \text{ for all } r,v,x\in U. \tag{3.7}$$

Taking $r+s$ instead of r in (3.7) and using (3.7) and the fact that N is 3!-torsion free, we obtain

$$F(F(r,r,s),x,v)=0 \text{ for all } r,s,x,v\in U. \tag{3.8}$$

Substituting $r+z$ for r in (3.8) and using (3.8), we have

$$F(F(r,z,s),x,v)=0 \text{ for all } r,s,v,x,z\in U. \tag{3.9}$$

Replacing r by rt in (3.9) and using (3.9), we get

$$F(r,z,s)\Delta(t,z,v)+F(r,x,v)\Delta(t,z,s)+r\Delta(\Delta(t,z,s),x,v)=0 \quad \text{for all } r,s,v,t,x,z\in U. \tag{3.10}$$

Substituting $f(r)$ for r in (3.10) and using (3.7), we get

$$f(r)\Delta(\Delta(t,z,s),x,v)=0 \text{ for all } r,s,v,t,x,z\in U,$$

i.e.,

$$\Delta(\Delta(t,z,s),x,v)f(r)=0 \text{ for all } r,s,v,t,x,z\in U.$$

Applying Lemma 2.9 (ii), we obtain

$$\Delta(\Delta(t,z,s),x,v)=0 \text{ for all } s,v,t,x,z\in U. \tag{3.11}$$

Taking wy instead of w in (3.11) and using (3.11), we get

$$\Delta(t,z,s)\Delta(y,x,v)+\Delta(t,x,v)\Delta(y,z,s)=0 \text{ for all } s,v,t,x,y,z\in U. \quad (3.12)$$

Replacing x,z,s,v by t in (3.12), we get

$$\delta(t)\Delta(y,t,t)=0 \text{ for all } t,y\in U. \quad (3.13)$$

Taking yr instead of y in (3.13) and using (3.13), we get

$$\delta(t)y\Delta(r,t,t)=0$$

and so

$$\delta(t)U\delta(t)=\{0\} \text{ for all } t\in U.$$

By Lemma 2.2 (i), we get $\delta(t)=0$, for all $t\in U$. Applying Lemma 2.6, we get $\Delta=0$ on U, a contradiction. ∎

4. Traces of permuting 3-generalized derivations

Ozturk and Yazarli [7, Theorem 3] proved that if N is a 2-torsion free 3-prime near ring and D_1,D_2 are nonzero symmetric (σ,τ)-biderivations of N with trace d_1 and d_2 respectively such that $d_2(y),d_2(y)+d_2(y)\in C(D_1(x,z))$, the centralizer of $(D_1(x,z)$, for all $x,y,z\in N$, then $(N,+)$ is abelian and $d_2(N)\subseteq Z$. Later in [8, Theorem 5] they obtained that if N is 2, 3-torsion free 3-prime near ring and F is a nonzero permuting 3-right generalized derivation of N with associated 3-derivation D such that $f(x),f(x)+f(x)\in C(D(y,z,w))$ for all $w,x,y,z\in N$, where f is the trace of F, then $(N,+)$ is abelian and $f(x)\in Z$ for all $x\in N$. Now we prove the following:

Theorem 4.1 *Let N be a $3!$-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. Suppose Δ is a nonzero 3-derivation of N and F is a nonzero permuting 3-right generalized derivation of N associated with Δ. If $f(x),f(x)+f(x)\in C(\Delta(y,z,w))$, for all $w,x,y,z\in U$, where f is the trace of F, then $(N,+)$ is abelian. Moreover, if $\delta(U)\subseteq U$, where δ is the trace of Δ, then $f(U)\subseteq Z$.*

Proof. For all $v,w,x,y,z\in U$

$$\begin{aligned}
&\Delta(v+y,z,w)(f(x)+f(x))\\
&\quad=(f(x)+f(x))\Delta(v+y,z,w)\\
&\quad=(f(x)+f(x))[\Delta(v,z,w)+\Delta(y,z,w)]\\
&\quad=(f(x)+f(x))\Delta(v,z,w)+(f(x)+f(x))\Delta(y,z,w) \qquad (4.1)\\
&\quad=\Delta(v,z,w)(f(x)+f(x))+\Delta(y,z,w)(f(x)+f(x))\\
&\quad=\Delta(v,z,w)f(x)+\Delta(v,z,w)f(x)+\Delta(y,z,w)f(x)+\Delta(y,z,w)f(x)\\
&\quad=f(x)\Delta(v,z,w)+f(x)\Delta(v,z,w)+f(x)\Delta(y,z,w)+f(x)\Delta(y,z,w).
\end{aligned}$$

On the other hand,

$$\begin{aligned}
&\Delta(v+y,z,w)(f(x)+f(x)) \\
&\quad = \Delta(v+y,z,w)f(x)+\Delta(v+y,z,w)f(x) \\
&\quad = f(x)\Delta(v+y,z,w)+f(x)\Delta(v+y,z,w) \\
&\quad = f(x)[\Delta(v,z,w)+\Delta(y,z,w)]+f(x)[\Delta(v,z,w)+\Delta(y,z,w)] \\
&\quad = f(x)\Delta(v,z,w)+f(x)\Delta(y,z,w)+f(x)\Delta(v,z,w)+f(x)\Delta(y,z,w), \\
&\quad \text{for all } v,w,x,y,z \in U.
\end{aligned} \tag{4.2}$$

Comparing (4.1) and (4.2), we obtain

$$f(x)\Delta((v,y),z,w) = 0 \text{ for all } v,w,x,y,z \in U.$$

By hypothesis we get

$$\Delta((v,y),z,w)f(x) = 0 \text{ for all } v,w,x,y,z \in U.$$

Hence it follows from Lemma 2.9 (i), that

$$\Delta((v,y),z,w) = 0 \text{ for all } v,w,x,y,z \in U.$$

Replacing v by sv and y by sy, we get

$$\Delta(s,z,w)(v,y) = 0, \text{ for all } s \in U.$$

Replacing v by vr and y by vp, we get

$$\Delta(s,z,w)v(r,p) = 0, \text{ for all } s,v,w,z \in U \text{ and } r,p \in N,$$

i.e.,

$$\Delta(s,z,w)U(r,p) = \{0\}.$$

Using Lemma 2.2 (i) we get either $\Delta(s,z,w) = 0$ or $(r,p) = 0$. If $\Delta(s,z,w) = 0$, for all $s,z,w \in U$, then $\Delta = 0$ on U, a contradiction by Lemma 2.4. Hence $(r,p) = 0$, for $r,p \in N$ and $(N,+)$ is abelian.

Since $f(x) \in C(\Delta(y,z,w))$, for all $x,y,z,w \in U$, we have

$$f(x)\Delta(y,z,w) = \Delta(y,z,w)f(x) \text{ for all } x,y,z,w \in U. \tag{4.3}$$

Replacing y by yv in (4.3), we obtain

$$f(x)\Delta(y,z,w)v + f(x)y\Delta(v,z,w) = \Delta(y,z,w)vf(x) + y\Delta(v,z,w)f(x) \tag{4.4}$$

Replacing y by $\delta(y)$ in (4.4), and using the hypothesis, we get

$$\Delta(\delta(y),z,w)[v,f(x)] = 0 \text{ for all } v,w,x,y,z \in U. \tag{4.5}$$

Substituting zt for z in (4.5), we get

$$\Delta(\delta(y), z, w)t[v, f(x)] = 0 \text{ for all } t, v, w, x, y, z \in U$$

i.e.,

$$\Delta(\delta(y), z, w)U[v, f(x)] = 0 \text{ for all } v, w, x, y, z \in U.$$

By Lemma 2.2 (i), we get either $\Delta(\delta(y), z, w)$=0 or $[f(x), v]$=0 for $v, w, x, y, z \in U$. Suppose that $\Delta(\delta(y), z, w) = 0$ for all $y, z, w \in U$. Taking $y+v$ instead of y, we get

$$\Delta(\delta(y), z, w) + \Delta(\delta(v), z, w) + 3\Delta(\Delta(y, y, v), z, w) + 3\Delta(\Delta(y, v, v), z, w) = 0.$$

Since $\Delta(\delta(y), z, w) = 0$ and N is 3!-torsion free, we get

$$\Delta(\Delta(y, y, v), z, w) + \Delta(\Delta(y, v, v), z, w) = 0 \text{ for all } v, w, y, z \in U. \tag{4.6}$$

Replacing y by $-y$ in (4.6), we have

$$\Delta(\Delta(y, y, v), z, w) - \Delta(\Delta(y, v, v), z, w) = 0 \text{ for all } v, w, y, z \in U. \tag{4.7}$$

Combining (4.6) and (4.7), we obtain

$$\Delta(\Delta(y, v, v), z, w) = 0 \text{ for all } v, w, y, z \in U. \tag{4.8}$$

Replacing y by yx in (4.8) and using (4.8), we have

$$\Delta(y, v, v)\Delta(x, z, w) + \Delta(y, z, w)\Delta(x, v, v) = 0 \text{ for all } v, w, x, y, z \in U.$$

Taking xt instead of x in the above relation, we get

$$\Delta(y, v, v)x\Delta(t, z, w) + \Delta(y, z, w)x\Delta(t, v, v) = 0 \text{ for all } t, v, w, y, z \in U. \tag{4.9}$$

Replacing t, w, y, z by v in (4.9), we get $\delta(v)x\delta(v) = 0$, for all $x, v \in U$ i.e., $\delta(v)U\delta(v) = \{0\}$. By Lemma 2.2 (i), we get $\delta(v) = 0$, for all $v \in U$. Lemma 2.6 yields that $\Delta = 0$ on U, a contradiction by Lemma 2.5. Thus $[f(x), v] = 0$, for all $v, x \in U$. Replacing v by vr, for all $r \in N$, we get $v[f(x), r] = 0$, i.e., $U[f(x), r] = 0$ for all $x \in U$ and $r \in N$. Again by Lemma 2.2 (ii), we get $[f(x), r] = 0$ for all $x \in U$ and $r \in N$. Hence $f(U) \subseteq Z$. ■

Theorem 4.2 *Let N be a 3!-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. Let Δ be a 3-derivation on N. If F is a nonzero permuting 3-generalized derivation of N associated with Δ such that $f(x), f(x) + f(x) \in C(F(u, v, w))$, for all $u, v, w, x \in U$, where f is the trace of F, then $(N, +)$ is abelian.*

Proof. For all $p, u, v, w, x \in U$

$$\begin{aligned} F(u+p,v,w)&(f(x)+f(x)) \\ &= (f(x)+f(x))F(u+p,v,w) \\ &= (f(x)+f(x))[F(u,v,w)+F(p,v,w)] \\ &= (f(x)+f(x))F(u,v,w)+(f(x)+f(x))F(p,v,w) \\ &= F(u,v,w)(f(x)+f(x))+F(p,v,w)(f(x)+f(x)) \\ &= F(u,v,w)f(x)+F(u,v,w)f(x)+F(p,v,w)f(x)+F(p,v,w)f(x) \\ &= f(x)F(u,v,w)+f(x)F(u,v,w)+f(x)F(p,v,w)+f(x)F(p,v,w). \end{aligned} \tag{4.10}$$

On the other hand,

$$\begin{aligned} F(u+p,v,w)&(f(x)+f(x)) \\ &= F(u+p,v,w)f(x)+F(u+p,v,w)f(x) \\ &= f(x)F(u+p,v,w)+f(x)F(u+p,v,w) \\ &= f(x)[F(u,v,w)+F(p,v,w)]+f(x)[F(u,v,w)+F(p,v,w)] \\ &= f(x)F(u,v,w)+f(x)F(p,v,w)+f(x)F(u,v,w)+f(x)F(p,v,w) \\ &\quad \text{for all } p,u,v,w,x \in U. \end{aligned} \tag{4.11}$$

Comparing (4.10) and (4.11), we obtain

$$f(x)F((u,p),v,w) = 0 \text{ for all } p,u,v,w,x \in U.$$

By hypothesis we get

$$F((u,p),v,w)f(x) = 0 \text{ for all } p,u,v,w,x \in U.$$

Hence it follows from Lemma 2.10 (ii), that

$$F((u,p),v,w) = 0 \text{ for all } p,u,v,w \in U. \tag{4.12}$$

First, let us consider the case $\Delta = 0$. Substituting uz for u and up for p in (4.12), we get

$$F(u,v,w)(z,p) + u\Delta((z,p),v,w) = 0 \text{ for all } p,u,v,w,z \in U. \tag{4.13}$$

Thus

$$F(u,v,w)(z,p) = 0 \text{ for all } p,u,v,w,z \in U. \tag{4.14}$$

Replacing z by zr and p by zs in (4.14), we get

$$F(u,v,w)z(r,s) = 0 \text{ for all } u,v,w,z \in U \text{ and } r,s \in N$$

i.e.,

$$F(u,v,w)U(r,s) = 0 \text{ for all } u,v,w \in U \text{ and } r,s \in N. \quad (4.15)$$

Invoking Lemma 2.2 (i), either $(r,s) = 0$ or $F(u,v,w) = 0$. Later yields a contradiction by Lemma 2.11. Hence $(r,s) = 0$ for all $r,s \in N$ and $(N,+)$ is abelian.

Now, let us consider the case $\Delta \neq 0$. Again substituting uz for u and up for p in (4.12), we get

$$\Delta(u,v,w)(z,p) + uF((z,p),v,w) = 0 \text{ for all } p,u,v,w,z \in U.$$

Using (4.12) we have

$$\Delta(u,v,w)(z,p) = 0 \text{ for all } p,u,v,w,z \in U. \quad (4.16)$$

Replacing z by zr and p by zs in (4.16), we obtain

$$\Delta(u,v,w)z(r,s) = 0 \text{ for all } u,v,w,z \in U \text{ and } r,s \in N$$

i.e.,

$$\Delta(u,v,w)U(r,s) = \{0\} \text{ for all } u,v,w \in U \text{ and } r,s \in N. \quad (4.17)$$

Applying Lemma 2.2 (i), we get either $(r,s) = 0$ or $\Delta(u,v,w) = 0$. Later yields a contradiction by Lemma 2.5. Hence $(r,s) = 0$ for all $r,s \in N$ and $(N,+)$ is abelian. ■

Theorem 4.3 *Let N be a $3!$-torsion free 3-prime near ring and U be a nonzero additive subgroup and a semigroup ideal of N. Suppose Δ is a 3-derivation on N and F is a nonzero permuting 3-generalized derivation of N associated with Δ such that $f(U) \subseteq U$ and $\delta(U) \subseteq U$, where f and δ are the trace of F and Δ respectively. If $f(x), f(x)+f(x) \in C(F(u,v,w))$, for all $u,v,w,x \in U$, then N is a commutative ring.*

Proof. First, let us consider the case $\Delta = 0$. Then

$$F(xy,z,t) = F(x,z,t)y + x\Delta(y,z,t)$$

and we have

$$F(xy,z,t) = F(x,z,t)y \text{ for all } t,x,y,z \in U. \quad (4.18)$$

On the other hand,

$$F(xy,z,t) = \Delta(x,z,t)y + xF(y,z,t),$$

and we have

$$F(xy,z,t) = xF(y,z,t) \text{ for all } t,x,y,z \in U. \quad (4.19)$$

Comparing (4.18) and (4.19) we obtain

$$F(x,z,t)y = xF(y,z,t) \text{ for all } t,x,y,z \in U. \tag{4.20}$$

Replacing x by y in (4.20), we obtain

$$[F(y,z,t),y] = 0 \text{ for all } t,y,z \in U. \tag{4.21}$$

Substituting zr for z in (4.21), we get

$$[F(y,z,t)r,y] = 0 \text{ for all } t,y,z \in U \text{ and } r \in N.$$

Using (4.21), we find

$$F(y,z,t)[r,y] = 0 \text{ for all } t,y,z \in U \text{ and } r \in N. \tag{4.22}$$

Substituting rs for r in (4.22) and using (4.22), we get

$$F(y,z,t)r[s,y] = 0 \text{ for all } t,y,z \in U \text{ and } r,s \in N,$$

i.e.,

$$F(y,z,t)N[s,y] = 0 \text{ for all } t,y,z \in U \text{ and } s \in N.$$

Since N is a 3-prime near ring, then either $[s,y]=0$ or $F(y,z,t)=0$. Later yields a contradiction by Lemma 2.11. Thus $[s,y]=0$ for all $y \in U$ and $s \in N$. Hence $U \subseteq Z$ and N is a commutative ring by Lemma 2.3.

Now, let $\Delta \neq 0$. By hypothesis

$$[f(x),F(u,v,w)] = 0 \text{ for all } u,v,w,x \in U. \tag{4.23}$$

Replacing x by $x+y$ in (4.23) and applying Theorem 4.2, we obtain

$$[F(x,x,y),F(u,v,w)]+[F(x,y,y),F(u,v,w)] = 0 \text{ for all } u,v,w,x,y \in U. \tag{4.24}$$

Setting $y=-y$ in (4.24) and comparing the result with (4.24), we obtain

$$[F(x,y,y),F(u,v,w)] = 0 \text{ for all } u,v,w,x,y \in U. \tag{4.25}$$

Replacing y by $y+z$ in (4.25) and using (4.25) and the fact that F is permuting, we have

$$[F(x,y,z),F(u,v,w)] = 0 \text{ for all } u,v,w,x,y,z \in U$$

-i.e.,

$$F(x,y,z)F(u,v,w) = F(u,v,w)F(x,y,z) \text{ for all } u,v,w,x,y,z \in U. \tag{4.26}$$

Substituting ut for u in (4.26) and applying Lemma 2.7 (ii), we obtain

$$\begin{aligned}\Delta(u,v,w)tF(x,y,z) - F(x,y,z)\Delta(u,v,w)t + uF(t,v,w)F(x,y,z)\\ -F(x,y,z)uF(t,v,w) = 0 \text{ for all } t,u,v,w,x,y,z \in U.\end{aligned} \tag{4.27}$$

Substituting $f(u)$ for u in (4.27) and then using hypothesis and (4.27), we get

$$\Delta(f(u), v, w)tF(x, y, z) - F(x, y, z)\Delta(f(u), v, w)t = 0 \tag{4.28}$$

Now replacing t by $f(t)$, we have

$$\Delta(f(u), v, w)f(t)F(x, y, z) - F(x, y, z)\Delta(f(u), v, w)f(t) = 0 - i.e.,$$

$$\Delta(f(u), v, w)F(x, y, z)f(t) - F(x, y, z)\Delta(f(u), v, w)f(t) = 0.$$

Applying Lemma 2.4, we get

$$[\Delta(f(u), v, w)F(x, y, z) - F(x, y, z)\Delta(f(u), v, w)]f(t) = 0$$

Using Lemma 2.9 (ii)

$$\Delta(f(u), v, w)F(x, y, z) = F(x, y, z)\Delta(f(u), v, w) \text{ for all } u, v, w, x, y, z \in U$$

and (4.28) yields that

$$\Delta(f(u), v, w)[t, F(x, y, z)] = 0 \text{ for all } t, u, v, w, x, y, z \in U. \tag{4.29}$$

Replacing w by ww' in (4.29), we have

$$\Delta(f(u), v, w)w'[t, F(x, y, z)] = 0 \text{ for all } t, u, v, w, w', x, y, z \in U$$

i.e.,

$$\Delta(f(u), v, w)U[t, F(x, y, z)] = 0 \text{ for all } t, u, v, w, x, y, z \in U.$$

From Lemma 2.2 (i), either $\Delta(f(u), v, w) = 0$ or $[t, F(x, y, z)] = 0$. If $\Delta(f(u), v, w) = 0$ for all $u, v, w \in U$, then arguing in the similar manner as in the proof of Theorem 3.2, we arrive at a contradiction. Thus

$$[t, F(x, y, z)] = 0 \text{ for all } t, x, y, z \in U. \tag{4.30}$$

Substituting tr' for t in (4.30), we get

$$t[r', F(x, y, z)] = 0 \text{ for all } t, x, y, z \in U \text{ and } r' \in N$$

i.e.,

$$U[r', F(x, y, z)] = 0 \text{ for all } x, y, z \in U \text{ and } r' \in N. \tag{4.31}$$

By Lemma 2.2 (ii), we get $[r', F(x, y, z)] = 0$ for all $x, y, z \in U$ and $r' \in N$ and $F(x, y, z) \in Z$ for all $x, y, z \in U$ and N is a commutative ring by Theorem 3.2. ■

The following examples show that the conditions in the hypothesis of Theorems 4.1, 4.2 and 4.3 are not superfluous.

Example 4.1 In Example 1.1 if we take U to be the set of all matrices of the type$\begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$, then U is an additive subgroup but not a semigroup ideal of N. N

is not a 3-prime near ring. It can be easily verified that $\delta(U) \subseteq U$, $f(x), f(x)+f(x) \in C(\Delta(u,v,w))$, for all $u,v,w,x \in U$. But $(N,+)$ is not abelian and $f(U) \nsubseteq Z$. Hence N to be 3-prime near ring and U to be a semigroup ideal are essential in the hypothesis of Theorem 4.1.

Example 4.2 In Example 1.3 if we take U to be the set of all matrices of the type $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$. Then U is an additive subgroup but not a semigroup ideal of N. N is not a 3-prime near ring and $\delta(U) \subseteq U$, $f(U) \subseteq U$, $f(x), f(x)+f(x) \in C(F(u,v,w))$, for all $u,v,w,x \in U$ and $F(U,U,U) \subseteq Z$. But neither $(N,+)$ is abelian nor N under multiplication is commutative. Hence N to be 3-prime near ring and U to be a semigroup ideal are essential in the hypothesis of Theorems 4.2 and 4.3.

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References

[1] Bell, H.E., *On derivations in near rings II*, Kluwer Academic Publ. Math. Appl. Dordr., 426 (1997), 191-197.

[2] Bell, H.E., *On prime near rings with generalized derivation*, Internat. J. Math. and Math. Sci., (2008), Article ID 490316.

[3] Bell, H.E., Mason, G., *On derivations in near rings*, Near rings and Near fields, North-Holland Mathematical Studies, 137 (1987), 31-35.

[4] Ceven, Y., Ozturk, M.A., *Some properties of symmetric bi-(σ,τ)-derivations in near-rings*, Commun. Korean Math. Soc., 22 (4) (2007), 487-491.

[5] Jung, Y.S., Park, K.H., *On prime and semiprime rings with permuting 3-derivations*, Bull. Korean Math. Soc., 44 (4) (2007), 789-794.

[6] Ozturk, M.A., *Permuting tri-derivations in prime and semiprime rings*, East Asian Math. J., 15 (2) (1999), 177-190.

[7] Ozturk, M.A., Yazarli, H., *Some results on symmetric bi-(σ,τ)-derivations in near rings*, Miskolc Mathematical Notes, 11 (2010), 169-173.

[8] Ozturk, M.A., Yazarli, H., *A note on permuting tri-derivation in near rings*, Gazi Univ. J. Sci., 24 (4) (2011), 723-729.

[9] Park, K.H., Jung, Y.S., *On symmetric generalized 3-derivations and commutativity in prime near rings*, Honam Math. J., 31 (2) (2009), 203-217.

[10] Park, K.H., Jung, Y.S., *On permuting 3-derivations and commutativity in prime near rings*, Commun. Korean Math. Soc., 25 (1) (2010), 1-9, DOI 10.4134/CKMS.2010.25.1001.

[11] Posner, E.C., *Derivations in prime rings*, Proc. Amer. Math. Soc., 8 (1957), 1093-1100.

[12] Uckun, M., Ozturk, M.A., *On trace of symmetric bi-gamma derivations in gamma near rings*, Houston J. Math., 33 (2) (2007), 323-339.

[13] Vukman, J., *Symmetric biderivations on prime and semiprime rings*, Aequationes Math., 38 (1989), 245-254.

[14] Vukman, J., *Two results concerning symmetric biderivations on prime rings*, Aequationes Math., 40 (1990), 181-189.

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