$\alpha A^*_{\mathcal{I}}$-SETS, $\alpha C_{\mathcal{I}}$-SETS, $\alpha C^*_{\mathcal{I}}$-SETS AND DECOMPOSITIONS OF α-$\mathcal{I}$-CONTINUITY

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Abstract. The aim of this paper is to introduce and study the notions of $\alpha A^*_{\mathcal{I}}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C^*_{\mathcal{I}}$-sets in ideal topological spaces. Properties of $\alpha A^*_{\mathcal{I}}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C^*_{\mathcal{I}}$-sets are investigated. Moreover, decompositions of α-$\mathcal{I}$-continuous functions and decompositions of $\alpha A^*_{\mathcal{I}}$-continuous functions via $\alpha A^*_{\mathcal{I}}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C^*_{\mathcal{I}}$-sets in ideal topological spaces are established.

Keywords: $\alpha A^*_{\mathcal{I}}$-set, $\alpha C_{\mathcal{I}}$-set, $\alpha C^*_{\mathcal{I}}$-set, pre-$\mathcal{I}$-regular set, ideal topological space, decomposition, $\star$-extremally disconnected ideal space, $\star$-hyperconnected ideal space, $\mathcal{I}$-submaximal ideal space.

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1. Introduction and preliminaries

In this paper, $\alpha A_{\mathcal{I}}^{*}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C_{\mathcal{I}}^{*}$-sets in ideal topological spaces are introduced and studied. The relationships and properties of $\alpha A_{\mathcal{I}}^{*}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C_{\mathcal{I}}^{*}$-sets are investigated. Furthermore, decompositions of α-$\mathcal{I}$-continuous functions and decompositions of $\alpha A_{\mathcal{I}}^{*}$-continuous functions via $\alpha A_{\mathcal{I}}^{*}$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C_{\mathcal{I}}^{*}$-sets in ideal topological spaces are provided.

Throughout this paper (X, τ), (Y, σ) (or simply X, Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, the closure and interior of A with respect to τ are denoted by cl(A) and int(A) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

1. A$\in\mathcal{I}$ and B$\subseteq$A$\Rightarrow$B$\in\mathcal{I}$ and
2. A$\in\mathcal{I}$ and B$\in\mathcal{I}\Rightarrow$A$\cup$B$\in\mathcal{I}$ [16].

If $\mathcal{I}$ is an ideal on X and X$\notin\mathcal{I}$, then $\mathcal{F}$ = {X\G : G$\in\mathcal{I}$} is a filter [14]. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^*$: $\wp$(X)$\rightarrow\wp$(X), called a local function [16] of A with respect to τ and $\mathcal{I}$ is defined as follows: for A$\subseteq$X, $A^*(\mathcal{I},\tau)$={x$\in$X | U$\cap$A$\notin\mathcal{I}$ for every U$\in\tau$(x)} where τ(x)={U$\in\tau$ | x$\in$U}. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I},\tau)$, called the $\star$-topology, finer than τ is defined by $cl^*(A)=A\cup A^*(\mathcal{I},\tau)$ [14]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I},\tau)$ and τ^* for $\tau^*(\mathcal{I},\tau)$. $int^*(A)$ will denote the interior of A in $(X,\tau^*,\mathcal{I})$.

Remark 1.1 [14] The $\star$-topology is generated by τ and by the filter $\mathcal{F}$. Also the family {H $\cap$ G : H $\in\tau$, G$\in\mathcal{F}$} is a basis for this topology.

Lemma 1.2 [13] *Let A be a subset of an ideal topological space $(X,\tau,\mathcal{I})$. If N is open, then $N\cap cl^*(A)\subseteq cl^*(N\cap A)$.*

Definition 1.3 A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be

1. pre-$\mathcal{I}$-open [4] if A $\subseteq$ int(cl*(A)).
2. semi-$\mathcal{I}$-open [11] if A $\subseteq$ cl*(int(A)).
3. α-$\mathcal{I}$-open [11] if A $\subseteq$ int(cl*(int(A))).
4. strongly β-$\mathcal{I}$-open [12] if A $\subseteq$ cl*(int(cl*(A))).
5. $\star$-dense [5] if cl*(A) = X.
6. t-$\mathcal{I}$-set [11] if int(A) = int(cl*(A)).
7. semi*-$\mathcal{I}$-open [8, 9] if A $\subseteq$ cl(int*(A)).

The family of all α-$\mathcal{I}$-open (resp. pre-$\mathcal{I}$-open) sets in an ideal topological space $(X, \tau, \mathcal{I})$ is denoted by $\alpha\mathcal{I}$O(X) (resp. P$\mathcal{I}$O(X)).

Remark 1.4 [8] For several subsets defined above, we have the following implications.

$$\begin{array}{ccccc} & & \text{pre-}\mathcal{I}\text{-open set} & \longrightarrow & \text{strongly } \beta\text{-}\mathcal{I}\text{-open set} \\ & & \uparrow & & \uparrow \\ \text{open set} & \longrightarrow & \alpha\text{-}\mathcal{I}\text{-open set} & \longrightarrow & \text{semi-}\mathcal{I}\text{-open set} \end{array}$$

The reverse implications are not true.

Lemma 1.5 [9] *Every semi-$\mathcal{I}$-open set is semi*-$\mathcal{I}$-open in an ideal topological space.*

Remark 1.6 The reverse implication of the above Lemma is not true in general as shown in [9].

Definition 1.7 The complement of a pre-$\mathcal{I}$-open (resp. semi-$\mathcal{I}$-open, α-$\mathcal{I}$-open, semi*-$\mathcal{I}$-open) set is called pre-$\mathcal{I}$-closed [4](resp. semi-$\mathcal{I}$-closed [11], α-$\mathcal{I}$-closed [11], semi*-$\mathcal{I}$-closed [8, 9]).

Definition 1.8 [9] The pre-$\mathcal{I}$-closure of a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, denoted by $p_{\mathcal{I}}$cl(A), is defined as the intersection of all pre-$\mathcal{I}$-closed sets of X containing A.

Lemma 1.9 [9] *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, $p_{\mathcal{I}}cl(A) = A \cup cl(int^*(A))$.*

Definition 1.10 A function f : $(X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be pre-$\mathcal{I}$-continuous [4] (resp. semi-$\mathcal{I}$-continuous [11], α-$\mathcal{I}$-continuous [11]) if $f^{-1}(V)$ is pre-$\mathcal{I}$-open (resp. semi-$\mathcal{I}$-open, α-$\mathcal{I}$-open) in X for each open set V in Y.

Definition 1.11 A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. an $\eta\zeta$-set [17] if A = L ∩ M, where L is open and M is clopen in X.
2. locally closed [3] if A = L ∩ M, where L is open and M is closed in X.
3. $\alpha_{\mathcal{I}}N_3$-set [2] if A = U ∩ V, where U ∈ $\alpha\mathcal{I}$O(X) and int(cl*(V)) = int(V).
4. semi-$\mathcal{I}$-regular [15] if A is a t-$\mathcal{I}$-set and semi-$\mathcal{I}$-open in X.
5. $B_{\mathcal{I}}$-set [11] if A = U ∩ V, where U is open and V is a t-$\mathcal{I}$-set.

The family of all $B_{\mathcal{I}}$-sets (resp. $\alpha_{\mathcal{I}}N_3$-sets) of X is denoted by $B_{\mathcal{I}}$(X) (resp. $\alpha_{\mathcal{I}}N_3$(X)).

Proposition 1.12 [1] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $V \in P\mathcal{I}O(X)$ and $A \in \alpha\mathcal{I}O(X)$, then $V \cap A \in P\mathcal{I}O(X)$.*

Definition 1.13 An ideal topological space $(X, \tau, \mathcal{I})$ is called

1. $\mathcal{I}$-submaximal if every $\star$-dense subset of X is open in X; [10]

2. $\star$-extremally disconnected if $\star$-closure of every open subset of X is open. [8]

Lemma 1.14 [8] *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is semi*-$\mathcal{I}$-open if and only if $cl(A) = cl(int^*(A))$.*

Lemma 1.15 [7] *For a subset A of an ideal topological space, $p_{\mathcal{I}}\mathrm{int}(A) = A \cap \mathrm{int}(\mathrm{cl}^*(A))$.*

Theorem 1.16 [2] *For an ideal topological space $(X, \tau, \mathcal{I})$, we have $\alpha\mathcal{I}O(X) = P\mathcal{I}O(X) \cap \alpha_{\mathcal{I}}N_3(X)$.*

Definition 1.17 [7] A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called pre-$\mathcal{I}$-regular if A is pre-$\mathcal{I}$-open and pre-$\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$.

Definition 1.18 [6], [7] A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called $A^*_{\mathcal{I}}$-set if A = L ∩ M, where L is an open and M = cl(int*(M)).

Proposition 1.19 [11] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For a subset A of $(X, \tau, \mathcal{I})$, the following conditions are equivalent:*

1. *A is open.*

2. *A is pre-$\mathcal{I}$-open and a $B_{\mathcal{I}}$-set.*

Lemma 1.20 [1] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. A subset A of X is α-$\mathcal{I}$-open if and only if it is semi-$\mathcal{I}$-open and pre-$\mathcal{I}$-open.*

Theorem 1.21 [10] *For an ideal topological space $(X, \tau, \mathcal{I})$, then the following properties are equivalent.*

1. *X is $\mathcal{I}$-submaximal.*

2. *Every pre-$\mathcal{I}$-open set is open.*

3. *Every pre-$\mathcal{I}$-open set is semi-$\mathcal{I}$-open and every α-$\mathcal{I}$-open set is open.*

Theorem 1.22 [7] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $K \subseteq X$. The following properties are equivalent.*

1. *K is an open set.*

2. *K is an α-$\mathcal{I}$-open set and an $A^*_{\mathcal{I}}$-set.*

3. *K is a pre-$\mathcal{I}$-open set and an $A^*_{\mathcal{I}}$-set.*

2. $\alpha A_{\mathcal{I}}^*$-sets, $\alpha C_{\mathcal{I}}$-sets and $\alpha C_{\mathcal{I}}^*$-sets

Definition 2.1 Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A $\subseteq$ X. A is said to be

1. an $\alpha C_{\mathcal{I}}^*$-set if A = L $\cap$ M, where L is an α-$\mathcal{I}$-open and M is a pre-$\mathcal{I}$-regular set in X.
2. an αA$B_{\mathcal{I}}$-set if A = L $\cap$ M, where L is α-$\mathcal{I}$-open and M is semi-$\mathcal{I}$-regular set in X.

Theorem 2.2 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then each $\alpha C_{\mathcal{I}}^*$-set in X is a pre-$\mathcal{I}$-open set.*

Proof. Let A be an $\alpha C_{\mathcal{I}}^*$-set in X. It follows that A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M is a pre-$\mathcal{I}$-regular set in X. Since M is a pre-$\mathcal{I}$-open set, then by Proposition 1.12, A = L $\cap$ M is a pre-$\mathcal{I}$-open set in X.

Remark 2.3 The converse of the Theorem 2.2 need not be true in general as shown in the following Example.

Example 2.4 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\mathcal{I}$=$\{\emptyset, \{a\}\}$. Then A=$\{a, b\}$ is a pre-$\mathcal{I}$-open set but not $\alpha C_{\mathcal{I}}^*$-set.

Remark 2.5 In an ideal topological space, every α-$\mathcal{I}$-open set and every pre-$\mathcal{I}$-regular set is an $\alpha C_{\mathcal{I}}^*$-set. The converses are not true in general as shown in the following Examples.

Example 2.6 In Example 2.4, A = {a} is $\alpha C_{\mathcal{I}}^*$-set but not a pre-$\mathcal{I}$-regular set.

Example 2.7 Let X=$\{a, b, c\}$, τ=$\{\emptyset, \{a, b\}, X\}$ and $\mathcal{I}$=$\{\emptyset\}$. Then A=$\{a\}$ is $\alpha C_{\mathcal{I}}^*$-set but not an α-$\mathcal{I}$-open set.

Remark 2.8 By Remark 2.5 and Theorem 2.2, the following diagram holds for a subset A of an ideal topological space $(X, \tau, \mathcal{I})$.

$$\begin{array}{ccc} & & \text{pre-}\mathcal{I}\text{-open set} \\ & & \uparrow \\ \text{pre-}\mathcal{I}\text{-regular set} & \longrightarrow & \alpha\text{C}_{\mathcal{I}}^*\text{-set} \end{array}$$

Definition 2.9 A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. an $\alpha C_{\mathcal{I}}$-set if A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M is a pre-$\mathcal{I}$-closed set in X.
2. an $\alpha\eta_{\mathcal{I}}$-set if A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M is an α-$\mathcal{I}$-closed set in X.
3. an $\alpha A_{\mathcal{I}}^*$-set if A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M = cl(int*(M)).

Remark 2.10 Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A $\subseteq$ X. The following diagram holds for A.

$$\begin{array}{ccccc} & & \alpha C_{\mathcal{I}}^{*}\text{-set} & \longrightarrow & \alpha C_{\mathcal{I}}\text{-set} \\ & & & & \uparrow \\ A_{\mathcal{I}}^{*}\text{-set} & \longrightarrow & \alpha A_{\mathcal{I}}^{*}\text{-set} & \longrightarrow & \alpha \eta_{\mathcal{I}}\text{-set} \end{array}$$

The following Examples show that these implications are not reversible in general.

Example 2.11 Let X = {a, b, c, d}, τ = {$\emptyset$, {a}, {b}, {a, b}, {a, b, c}, X} and $\mathcal{I} = \{\emptyset\}$. Then A = {a, b, d} is $\alpha A_{\mathcal{I}}^{*}$-set but not an $A_{\mathcal{I}}^{*}$-set.

Example 2.12 In Example 2.11, A = {c} is $\alpha\eta_{\mathcal{I}}$-set but not an $\alpha A_{\mathcal{I}}^{*}$-set.

Example 2.13 In Example 2.11, A = {c} is $\alpha C_{\mathcal{I}}$-set but not an $\alpha C_{\mathcal{I}}^{*}$-set.

Example 2.14 In Example 2.4, A = {c} is $\alpha C_{\mathcal{I}}$-set but not an $\alpha\eta_{\mathcal{I}}$-set.

Theorem 2.15 *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent.*

1. *A is an $\alpha C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.*
2. *A = L $\cap$ cl(int*(A)) for an α-$\mathcal{I}$-open set L.*

Proof. (1) $\Rightarrow$ (2): Suppose that A is an $\alpha C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X. Since A is $\alpha C_{\mathcal{I}}$-set, then we have A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M is a pre-$\mathcal{I}$-closed set in X. We have A $\subseteq$ M, so cl(int*(A)) $\subseteq$ cl(int*(M)). Since M is a pre-$\mathcal{I}$-closed set in X, we have cl(int*(M)) $\subseteq$ M. Since A is a semi*-$\mathcal{I}$-open set in X, We have A $\subseteq$ cl(int*(A)). It follows that A = A $\cap$ cl(int*(A))=L $\cap$ M $\cap$ cl(int*(A)) = L $\cap$ cl(int*(A)).

(2) $\Rightarrow$ (1): Let A = L $\cap$ cl(int*(A)) for an α-$\mathcal{I}$-open set L. We have A $\subseteq$ cl(int*(A)). It follows that A is a semi*-$\mathcal{I}$-open set in X. Since cl(int*(A)) is a closed set, then cl(int*(A)) is a pre-$\mathcal{I}$-closed set in X. Hence, A is an $\alpha C_{\mathcal{I}}$-set in X.

Theorem 2.16 *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent.*

1. *A is an $\alpha A_{\mathcal{I}}^{*}$-set in X.*
2. *A is an $\alpha\eta_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.*
3. *A is an $\alpha C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.*

Proof. (1) ⇒ (2): Suppose that A is an $\alpha A^*_{\mathcal{I}}$-set in X. It follows that $A{=}L\cap M$, where L is an α-$\mathcal{I}$-open set and M = cl(int*(M)). This implies $A = L \cap M =$ cl[int(cl*(int(L))) ∩ int*(M)] ⊆ cl[cl*(int(L)) ∩ int*(M)] ⊆ cl[cl*(int(L) ∩ int*(M))] ⊆ cl[cl(int(L) ∩ int*(M))] ⊆ cl[int(L) ∩ int*(M)] ⊆ cl[int*(L) ∩ int*(M)] = cl(int*(L ∩ M)). Thus A ⊆ cl(int*(A)) and hence A is a semi*-$\mathcal{I}$-open set in X. Moreover, by Remark 2.10, A is an $\alpha\eta_{\mathcal{I}}$-set in X.

(2) ⇒ (3): It follows from the fact that every $\alpha\eta_{\mathcal{I}}$-set is an $\alpha C_{\mathcal{I}}$-set in X by Remark 2.10.

(3) ⇒ (1): Suppose that A is an $\alpha C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X. By Theorem 2.15, A = L ∩ cl(int*(A)) for an α-$\mathcal{I}$-open set L. We have cl(int*(cl(int*(A)))) = cl(int*(A)). It follows that A is an $\alpha A^*_{\mathcal{I}}$-set in X.

Remark 2.17

1. The notions of $\alpha\eta_{\mathcal{I}}$-set and semi*-$\mathcal{I}$-open set are independent of each other.
2. The notions of $\alpha C_{\mathcal{I}}$-set and semi*-$\mathcal{I}$-open set are independent of each other.

Example 2.18

1. In Example 2.11, A = {c, d} is $\alpha C_{\mathcal{I}}$-set as well as $\alpha\eta_{\mathcal{I}}$-set but not semi*-$\mathcal{I}$-open set.
2. In Example 2.4, A = {a, c} is a semi*-$\mathcal{I}$-open set but it is neither $\alpha C_{\mathcal{I}}$-set nor $\alpha\eta_{\mathcal{I}}$-set.

Theorem 2.19 [7] *A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is semi*-$\mathcal{I}$-closed if and only if A is a t-$\mathcal{I}$-set.*

Theorem 2.20 *Let $(X,\tau,\mathcal{I})$ be an $\mathcal{I}$-submaximal and ⋆-extremally disconnected ideal topological space. Then $B_{\mathcal{I}}(X) = \alpha_{\mathcal{I}}N_3(X)$.*

Proof. It follows from Lemma 1.14 and Theorem 1.21.

Theorem 2.21 *Let $(X,\tau,\mathcal{I})$ be an $\mathcal{I}$-submaximal and ⋆-extremally disconnected ideal topological space and A ⊆ X. The following properties are equivalent.*

1. *A is an open set in X.*
2. *A is an α-$\mathcal{I}$-open set and a $A^*_{\mathcal{I}}$-set.*
3. *A is a pre-$\mathcal{I}$-open and an $\alpha A^*_{\mathcal{I}}$-set.*

Proof. (1) ⇔ (2): It follows from Theorem 1.22.

(2) ⇒ (3): It follows from the fact that every α-$\mathcal{I}$-open set is pre-$\mathcal{I}$-open and every $A^*_{\mathcal{I}}$-set is $\alpha A^*_{\mathcal{I}}$-set.

(3) ⇒ (1): Suppose that A is a pre-$\mathcal{I}$-open set and an $\alpha A^*_{\mathcal{I}}$-set. Since A is an $\alpha A^*_{\mathcal{I}}$-set, then we have A = L ∩ M, where L is an α-$\mathcal{I}$-open set and M = cl(int*(M)). It follows that int(cl*(M)) ⊆ cl*(M) ⊆ cl(M) = cl(int*(M)) = M.

Since int(cl*(M)) $\subseteq$ M, then M is a semi*-$\mathcal{I}$-closed set. By Theorem 2.19, M is a t-$\mathcal{I}$-set. Hence, A is an $\alpha_{\mathcal{I}}$N$_3$-set. Since A is an $\alpha_{\mathcal{I}}$N$_3$-set and a pre-$\mathcal{I}$-open set, then by Theorem 2.20, A is a B$_{\mathcal{I}}$-set and a pre-$\mathcal{I}$-open set. By Proposition 1.19, A is an open set in X.

Theorem 2.22 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. The following properties are equivalent.*

1. *A is an α-$\mathcal{I}$-open set in X.*
2. *A is a pre-$\mathcal{I}$-open and an $\alpha A^*_{\mathcal{I}}$-set.*

Proof. (1) $\Rightarrow$ (2): It follows from the fact that every α-$\mathcal{I}$-open set is pre-$\mathcal{I}$-open and every α-$\mathcal{I}$-open set is an $\alpha A^*_{\mathcal{I}}$-set.

(2) $\Rightarrow$ (1): Suppose that A is a pre-$\mathcal{I}$-open set and an $\alpha A^*_{\mathcal{I}}$-set. Since A is an $\alpha A^*_{\mathcal{I}}$-set, then we have A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M = cl(int*(M)). It follows that int(cl*(M)) $\subseteq$ cl*(M) $\subseteq$ cl(M) = cl(int*(M)) = M. Since int(cl*(M)) $\subseteq$ M, then M is a semi*-$\mathcal{I}$-closed set. By Theorem 2.19, M is a t-$\mathcal{I}$-set. Hence, A is an $\alpha_{\mathcal{I}}$N$_3$-set. Since A is an $\alpha_{\mathcal{I}}$N$_3$-set and a pre-$\mathcal{I}$-open set, then by Theorem 1.16, A is an α-$\mathcal{I}$-open set in X.

Remark 2.23 The notions of pre-$\mathcal{I}$-open set and $\alpha A^*_{\mathcal{I}}$-set are independent of each other.

Example 2.24

1. In Example 2.4, A = {b, c} is $\alpha A^*_{\mathcal{I}}$-set but not a pre-$\mathcal{I}$-open set.
2. In Example 2.4, A = {a, c} is pre-$\mathcal{I}$-open set but not an $\alpha A^*_{\mathcal{I}}$-set.

Theorem 2.25 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. The following properties are equivalent.*

1. *A is an α-$\mathcal{I}$-open set.*
2. *A is an $\alpha C^*_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set.*

Proof. (1) $\Rightarrow$ (2): It follows from the fact that every α-$\mathcal{I}$-open set is an $\alpha C^*_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set by Remark 1.4 and Lemma 1.5.

(2) $\Rightarrow$ (1): Let A be an $\alpha C^*_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set. Since A is an $\alpha C^*_{\mathcal{I}}$-set, then A is an $\alpha C_{\mathcal{I}}$-set. Since A is an $\alpha C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X, then by Theorem 2.16, A is an $\alpha A^*_{\mathcal{I}}$-set. Moreover, since A is an $\alpha C^*_{\mathcal{I}}$-set, then A is a pre-$\mathcal{I}$-open by Theorem 2.2. Hence, by Theorem 2.22, A is an α-$\mathcal{I}$-open set in X.

Remark 2.26 The notions of $\alpha C^*_{\mathcal{I}}$-set and semi*-$\mathcal{I}$-open set are independent of each other.

Example 2.27

1. Let X=$\{a,b,c\}$, τ = $\{\emptyset,\{a\},\{b\},\{a,b\},X\}$ and $\mathcal{I}$=$\{\emptyset\}$. Then A=$\{a,c\}$ is semi*-$\mathcal{I}$-open set but not an $\alpha C^*_{\mathcal{I}}$-set.

2. In Example 2.7, A = {a, c} is $\alpha C^*_{\mathcal{I}}$-set but not semi*-$\mathcal{I}$-open set.

Theorem 2.28 *Let $(X,\tau,\mathcal{I})$ be an ideal topological space and $A \subseteq X$. The following properties are equivalent.*

1. *A is an α-$\mathcal{I}$-open set.*

2. *A is a semi-$\mathcal{I}$-open set and an $\alpha C^*_{\mathcal{I}}$-set.*

3. *A is a semi-$\mathcal{I}$-open set and a pre-$\mathcal{I}$-open set.*

Proof. (1) ⇒ (2): It is obvious.

(2) ⇒ (3): It follows from the fact that every $\alpha C^*_{\mathcal{I}}$-set is a pre-$\mathcal{I}$-open set by Theorem 2.2.

(3) ⇒ (1): It follows from Lemma 1.20.

Remark 2.29 The notions of semi-$\mathcal{I}$-open set and $\alpha C^*_{\mathcal{I}}$-set are independent of each other.

Example 2.30

1. In Example 2.27(1), A = {a, c} is semi-$\mathcal{I}$-open set but not an $\alpha C^*_{\mathcal{I}}$-set.

2. In Example 2.7, A = {a, c} is $\alpha C^*_{\mathcal{I}}$-set but not a semi-$\mathcal{I}$-open set.

Definition 2.31 A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be αgp$_{\mathcal{I}}$-open if N ⊆ p$_{\mathcal{I}}$int(A) whenever N ⊆ A and N is an α-$\mathcal{I}$-closed set in X.

Definition 2.32 A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be α-generalized pre-$\mathcal{I}$-closed (αgp$_{\mathcal{I}}$-closed) in X if X \ A is αgp$_{\mathcal{I}}$-open.

Theorem 2.33 *For a subset A of an ideal topological space $(X,\tau,\mathcal{I})$, A is $\alpha gp_{\mathcal{I}}$-closed if and only if $p_{\mathcal{I}}cl(A) \subseteq N$ whenever $A \subseteq N$ and N is an α-$\mathcal{I}$-open set in $(X,\tau,\mathcal{I})$.*

Proof. Let A be an αgp$_{\mathcal{I}}$-closed set in X. Suppose that A ⊆ N and N is an α-$\mathcal{I}$-open set in $(X,\tau,\mathcal{I})$. Then $X \setminus A$ is αgp$_{\mathcal{I}}$-open and $X \setminus N \subseteq X \setminus A$ where $X \setminus N$ is α-$\mathcal{I}$-closed. Since $X \setminus A$ is αgp$_{\mathcal{I}}$-open, then we have $X \setminus N \subseteq p_{\mathcal{I}}\text{int}(X \setminus A)$, where $p_{\mathcal{I}}\text{int}(X \setminus A) = (X \setminus A) \cap \text{int}(\text{cl}^*(X \setminus A))$. Since $(X \setminus A) \cap \text{int}(\text{cl}^*(X \setminus A))$ $= (X \setminus A) \cap (X \setminus \text{cl}(\text{int}^*(A))) = X \setminus (A \cup \text{cl}(\text{int}^*(A)))$, then by Lemma 1.9, $(X \setminus A) \cap \text{int}(\text{cl}^*(X \setminus A)) = X \setminus (A \cup \text{cl}(\text{int}^*(A))) = X \setminus p_{\mathcal{I}}\text{cl}(A)$. It follows that $p_{\mathcal{I}}\text{int}(X \setminus A) = X \setminus p_{\mathcal{I}}\text{cl}(A)$. Thus $p_{\mathcal{I}}\text{cl}(A) = X \setminus p_{\mathcal{I}}\text{int}(X \setminus A) \subseteq N$ and hence $p_{\mathcal{I}}\text{cl}(A) \subseteq N$. The converse is similar.

Theorem 2.34 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $V \subseteq X$. Then V is an $\alpha C_{\mathcal{I}}$-set in X if and only if $V = G \cap p_{\mathcal{I}}cl(V)$ for an α-$\mathcal{I}$-open set G in X.*

Proof. If V is an $\alpha C_{\mathcal{I}}$-set, then V = G ∩ M for an α-$\mathcal{I}$-open set G and a pre-$\mathcal{I}$-closed set M. But then V ⊆ M and so V ⊆ $p_{\mathcal{I}}$cl(V)⊆ M. It follows that V = V ∩ $p_{\mathcal{I}}$cl(V) = G ∩ M ∩ $p_{\mathcal{I}}$cl(V) = G ∩ $p_{\mathcal{I}}$cl(V). Conversely, it is enough to prove that $p_{\mathcal{I}}$cl(V) is a pre-$\mathcal{I}$-closed set. But $p_{\mathcal{I}}$cl(V) ⊆ M, for any pre-$\mathcal{I}$-closed set M containing V. So, cl(int*($p_{\mathcal{I}}$cl(V))) ⊆ cl(int*(M)) ⊆ M. It follows that cl(int*($p_{\mathcal{I}}$cl(V))) $\subseteq \cap_{V \subseteq M,\ M\ is\ pre-\mathcal{I}-closed}$ M = $p_{\mathcal{I}}$cl(V).

Theorem 2.35 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. The following properties are equivalent.*

1. *A is a pre-$\mathcal{I}$-closed set in X.*
2. *A is an $\alpha C_{\mathcal{I}}$-set and an $\alpha gp_{\mathcal{I}}$-closed set in X.*

Proof. (1) ⇒ (2): It follows from the fact that any pre-$\mathcal{I}$-closed set in X is an $\alpha C_{\mathcal{I}}$-set and an $\alpha gp_{\mathcal{I}}$-closed set in X.

(2) ⇒ (1): Suppose that A is an $\alpha C_{\mathcal{I}}$-set and an $\alpha gp_{\mathcal{I}}$-closed set in X. Since A is an $\alpha C_{\mathcal{I}}$-set, then by Theorem 2.34, A = G ∩ $p_{\mathcal{I}}$cl(A) for an α-$\mathcal{I}$-open set G in $(X, \tau, \mathcal{I})$. Since A ⊆ G and A is $\alpha gp_{\mathcal{I}}$-closed set in X, then $p_{\mathcal{I}}$cl(A) ⊆ G. It follows that $p_{\mathcal{I}}$cl(A) ⊆ G ∩ $p_{\mathcal{I}}$cl(A) = A. Thus, A = $p_{\mathcal{I}}$cl(A) and hence A is pre-$\mathcal{I}$-closed.

Theorem 2.36 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. If A is an $\alpha C_{\mathcal{I}}$-set in X, then $p_{\mathcal{I}}cl(A) \setminus A$ is a pre-$\mathcal{I}$-closed set and $A \cup (X \setminus p_{\mathcal{I}}cl(A))$ is a pre-$\mathcal{I}$-open set in X.*

Proof. Suppose that A is an $\alpha C_{\mathcal{I}}$-set in X. By Theorem 2.34, we have A = L ∩ $p_{\mathcal{I}}$cl(A) for an α-$\mathcal{I}$-open set L in X. It follows that $p_{\mathcal{I}}$cl(A) \ A = $p_{\mathcal{I}}$cl(A) \ (L ∩ $p_{\mathcal{I}}$cl(A)) = $p_{\mathcal{I}}$cl(A) ∩ (X \ (L ∩ $p_{\mathcal{I}}$cl(A))) = $p_{\mathcal{I}}$cl(A) ∩ ((X \ L) ∪ (X \ $p_{\mathcal{I}}$cl(A))) = ($p_{\mathcal{I}}$cl(A) ∩ (X \ L)) ∪ ($p_{\mathcal{I}}$cl(A) ∩ (X \ $p_{\mathcal{I}}$cl(A))) = ($p_{\mathcal{I}}$cl(A) ∩ (X \ L)) ∪ ϕ = $p_{\mathcal{I}}$cl(A) ∩ (X \ L). Thus $p_{\mathcal{I}}$cl(A) \ A = $p_{\mathcal{I}}$cl(A) ∩ (X \ L) and hence $p_{\mathcal{I}}$cl$(A) \setminus A$ is pre-$\mathcal{I}$-closed set. Moreover, since $p_{\mathcal{I}}$cl$(A) \setminus A$ is a pre-$\mathcal{I}$-closed set in X, then X \ ($p_{\mathcal{I}}$cl(A) \ A) = (X \ ($p_{\mathcal{I}}$cl(A) ∩ (X \ A)) = (X \ $p_{\mathcal{I}}$cl(A)) ∪ A is a pre-$\mathcal{I}$-open set. Thus, X \ ($p_{\mathcal{I}}$cl(A) \ A) = (X \ $p_{\mathcal{I}}$cl(A)) ∪ A is a pre-$\mathcal{I}$-open set in X.

3. Further properties

Definition 3.1 [7] Let $(X, \tau, \mathcal{I})$ be an ideal topological space. $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}$-connected if X can not be expressed as the disjoint union of two nonvoid pre-$\mathcal{I}$-open sets.

Theorem 3.2 *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. The following properties are equivalent.*

1. $(X, \tau, \mathcal{I})$ *is pre-$\mathcal{I}$-connected.*

2. $(X, \tau, \mathcal{I})$ *can not be expressed as the disjoint union of two nonvoid $\alpha C^*_{\mathcal{I}}$-sets.*

Proof. (1) $\Rightarrow$ (2): Suppose that $(X, \tau, \mathcal{I})$ can be expressed as the disjoint union of two nonvoid $\alpha C^*_{\mathcal{I}}$-sets. Since any $\alpha C^*_{\mathcal{I}}$-set is a pre-$\mathcal{I}$-open set, then $(X, \tau, \mathcal{I})$ can be expressed as the disjoint union of two nonvoid pre-$\mathcal{I}$-open sets. So, $(X, \tau, \mathcal{I})$ is not pre-$\mathcal{I}$-connected. This is a contradiction.

(2) $\Rightarrow$ (1): Suppose that $(X, \tau, \mathcal{I})$ is not pre-$\mathcal{I}$-connected. Then, X can be expressed as the disjoint union of two nonvoid pre-$\mathcal{I}$-open sets. It follows that X has a nontrivial pre-$\mathcal{I}$-regular subset A. Moreover, A and B = X $\setminus$ A are pre-$\mathcal{I}$-regular. Then A and B are $\alpha C^*_{\mathcal{I}}$-sets. Hence $(X, \tau, \mathcal{I})$ can be expressed as the disjoint union of two nonvoid $\alpha C^*_{\mathcal{I}}$-sets. This is a contradiction.

Theorem 3.3 *In an $\mathcal{I}$-submaximal ideal space $(X, \tau, \mathcal{I})$, the following properties holds.*

1. *Any $\alpha C^*_{\mathcal{I}}$-set is an $\eta\zeta$-set and an $\alpha AB_{\mathcal{I}}$-set.*

2. *Any $\alpha\eta_{\mathcal{I}}$-set is a locally closed set.*

Proof. (1) Suppose that A is an $\alpha C^*_{\mathcal{I}}$-set in X. It follows that A = L $\cap$ M, where L is an α-$\mathcal{I}$-open set and M is a pre-$\mathcal{I}$-regular set in X. By Theorem 1.21, M is semi-$\mathcal{I}$-open and semi-$\mathcal{I}$-closed. It follows from Lemma 1.5 that M is semi-$\mathcal{I}$-open and semi*-$\mathcal{I}$-closed. By Theorem 2.19, M is semi-$\mathcal{I}$-open and a t-$\mathcal{I}$-set in X. Hence M is semi-$\mathcal{I}$-regular set. Thus, A is an αA$B_{\mathcal{I}}$-set in X. Furthermore, by Theorem 1.21, A is an $\eta\zeta$-set.

(2) It follows from Theorem 1.21.

Definition 3.4 [9] An ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\star$-hyperconnected if A is $\star$-dense for every open subset A $\neq \phi$ of X.

Theorem 3.5 [9] *The following properties are equivalent for an ideal topological space $(X, \tau, \mathcal{I})$.*

1. *X is $\star$-hyperconnected.*

2. *A is $\star$-dense for every strongly β-$\mathcal{I}$-open subset $\phi \neq A \subseteq X$.*

Theorem 3.6 *For an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent.*

1. $(X, \tau, \mathcal{I})$ *is $\star$-hyperconnected.*

2. *any $\alpha C^*_{\mathcal{I}}$-set in X is $\star$-dense.*

Proof. (1) ⇒ (2): Let A be an $\alpha C^*_{\mathcal{I}}$-set in X. By Theorem 2.2, A is pre-$\mathcal{I}$-open. By Remark 1.4, A is strongly β-$\mathcal{I}$-open set. Since $(X, \tau, \mathcal{I})$ is a $\star$-hyperconnected ideal topological space, then by Theorem 3.5, A is $\star$-dense.

(2) ⇒ (1): Suppose that any $\alpha C^*_{\mathcal{I}}$-set in $(X, \tau, \mathcal{I})$ is $\star$-dense in X. Since an open set A in X is an α-$\mathcal{I}$-open set and every α-$\mathcal{I}$-open set A is an $\alpha C^*_{\mathcal{I}}$-set, then A is $\star$-dense. Thus, $(X, \tau, \mathcal{I})$ is $\star$-hyperconnected.

4. Decompositions of α-$\mathcal{I}$-continuity and $\alpha A^*_{\mathcal{I}}$-continuity

Definition 4.1 A function f : $(X, \tau, \mathcal{I}) \to$ (Y, σ) is said to be

1. $\alpha C^*_{\mathcal{I}}$-continuous if f^{-1}(A) is an $\alpha C^*_{\mathcal{I}}$-set in X for every open set A in Y.
2. P$R_{\mathcal{I}}$-continuous [7] if f^{-1}(A) is a pre-$\mathcal{I}$-regular set in X for every open set A in Y.

Remark 4.2 For a function f : $(X, \tau, \mathcal{I}) \to$ (Y, σ), the following diagram holds. The reverses of these implications are not true in general as shown in the following Examples.

$$\begin{array}{ccc} \text{pre-}\mathcal{I}\text{-continuity} & & \\ \uparrow & & \\ \alpha C^*_{\mathcal{I}}\text{-continuity} & \longleftarrow & \text{P}R_{\mathcal{I}}\text{-continuity} \end{array}$$

Example 4.3 Let X = {a, b, c}, τ = {∅, {a}, X}, Y = {p, q, r}, σ = {∅, {p}, {p, q}, {p, r}, Y}, $\mathcal{I}$ = {∅} and $\mathcal{J}$ = {∅}. Define f : $(X, \tau, \mathcal{I}) \to$ (Y, σ, $\mathcal{J}$) by f(a) = p; f(b) = q and f(c) = r. Then f is pre-$\mathcal{I}$-continuous but not $\alpha C^*_{\mathcal{I}}$-continuous.

Example 4.4 Let X = {a, b, c}, τ = {∅, {a}, X}, Y = {p, q, r}, σ = {∅, {p}, Y}, $\mathcal{I}$ = {∅} and $\mathcal{J}$ = {∅}. Define f : $(X, \tau, \mathcal{I}) \to$ (Y, σ, $\mathcal{J}$) by f(a) = p; f(b) = q and f(c) = r. Then f is $\alpha C^*_{\mathcal{I}}$-continuous but not P$R_{\mathcal{I}}$-continuous.

Definition 4.5 A function f : $(X, \tau, \mathcal{I}) \to$ (Y, σ) is said to be

1. $\alpha C_{\mathcal{I}}$-continuous if f^{-1}(A) is an $\alpha C_{\mathcal{I}}$-set in X for every open set A in Y.
2. $\alpha A^*_{\mathcal{I}}$-continuous if f^{-1}(A) is an $\alpha A^*_{\mathcal{I}}$-set in X for every open set A in Y.
3. $\alpha \eta_{\mathcal{I}}$-continuous if f^{-1}(A) is an $\alpha \eta_{\mathcal{I}}$-set in X for every open set A in Y.
4. $A^*_{\mathcal{I}}$-continuous [7] if f^{-1}(A) is an $A^*_{\mathcal{I}}$-set in X for every open set A in Y.

Remark 4.6 For a function f : $(X, \tau, \mathcal{I}) \to$ (Y, σ), the following diagram holds. The reverses of these implications are not true in general as shown in the following Examples.

$$\begin{array}{ccccc} \alpha C_{\mathcal{I}}\text{-continuity} & \longleftarrow & \alpha C^*_{\mathcal{I}}\text{-continuity} & & \\ \uparrow & & & & \\ \alpha \eta_{\mathcal{I}}\text{-continuity} & \longleftarrow & \alpha A^*_{\mathcal{I}}\text{-continuity} & \longleftarrow & A^*_{\mathcal{I}}\text{-continuity} \end{array}$$

Example 4.7 Let $X=\{a,b,c,d\}$, $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$, $Y=\{p,q,r,s\}$, $\sigma=\{\emptyset,\{p\},\{q\},\{p,q,s\},Y\}$, $\mathcal{I}=\{\emptyset\}$ and $\mathcal{J}=\{\emptyset\}$. Define $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma,\mathcal{J})$ by $f(a)=p$, $f(b)=q$, $f(c)=r$ and $f(d)=s$. Then f is $\alpha A_{\mathcal{I}}^*$-continuous but not $A_{\mathcal{I}}^*$-continuous.

Example 4.8 Let X = {a, b, c, d}, τ = {$\emptyset$, {a}, {b}, {a, b}, {a, b, c}, X}, Y = {p, q, r, s}, σ = {$\emptyset$, {r}, {s}, {r, s}, Y}, $\mathcal{I}=\{\emptyset\}$ and $\mathcal{J}=\{\emptyset\}$. Define f : $(X,\tau,\mathcal{I})\rightarrow$ (Y, σ, $\mathcal{J}$) by f(a) = p, f(b) = q, f(c) = r and f(d) = s. Then f is $\alpha\eta_{\mathcal{I}}$-continuous but not $\alpha A_{\mathcal{I}}^*$-continuous.

Example 4.9 In Example 4.8, f is $\alpha C_{\mathcal{I}}$-continuous but not $\alpha C_{\mathcal{I}}^*$-continuous.

Example 4.10 Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},X\}$, $Y=\{p,q,r\}$, $\sigma=\{\emptyset,\{q\},Y\}$, $\mathcal{I}=\{\emptyset\}$ and $\mathcal{J}=\{\emptyset\}$. Define $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma,\mathcal{J})$ by $f(a)=p$; $f(b)=q$ and $f(c)=r$. Then f is $\alpha C_{\mathcal{I}}$-continuous but not $\alpha\eta_{\mathcal{I}}$-continuous.

Definition 4.11 [7] A function f : $(X,\tau,\mathcal{I})\rightarrow$ (Y, σ) is said to be semi*-$\mathcal{I}$-continuous if f^{-1}(V) is a semi*-$\mathcal{I}$-open set in X for every open set V in Y.

Theorem 4.12 *The following properties are equivalent for a function* $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$*:*

1. *f is $\alpha A_{\mathcal{I}}^*$-continuous.*
2. *f is $\alpha\eta_{\mathcal{I}}$-continuous and semi*-$\mathcal{I}$-continuous.*
3. *f is $\alpha C_{\mathcal{I}}$-continuous and semi*-$\mathcal{I}$-continuous.*

Proof. *It follows from Theorem 2.16.*

Theorem 4.13 *The following properties are equivalent for a function* $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$*:*

1. *f is α-$\mathcal{I}$-continuous.*
2. *f is pre-$\mathcal{I}$-continuous and $\alpha A_{\mathcal{I}}^*$-continuous.*
3. *f is semi*-$\mathcal{I}$-continuous and $\alpha C_{\mathcal{I}}^*$-continuous.*
4. *f is semi-$\mathcal{I}$-continuous and $\alpha C_{\mathcal{I}}^*$-continuous.*
5. *f is semi-$\mathcal{I}$-continuous and pre-$\mathcal{I}$-continuous.*

Proof. It follows from Theorems 2.22, 2.25 and 2.28.

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