PRIME IDEALS IN RIGHT TERNARY NEAR-RINGS AND RIGHT TERNARY N-GROUPS

A. Uma Maheswari

Department of Mathematics
Quaid-E-Millath Government College for Women (Autonomous)
Chennai – 600 002
Tamil Nadu
India
e-mail: umashiva2000@yahoo.com

C. Meera

Department of Mathematics
Bharathi Women's College (Autonomous)
Chennai – 600 108
Tamil Nadu
India
e-mail: eya65@rediffmail.com

Abstract. A Right Ternary Near-Ring (RTNR) is an algebraic system which is a group under binary addition and a ternary semigroup under ternary multiplication satisfying the right distributive law. In this paper ν-prime ideals where $\nu \in \{0,1,2,3\}$ and equiprime ideals of an RTNR are defined and the relationship among them are discussed. A comparison between ν-primitive ideals and ν-prime ideals where $\nu \in \{0,1,2\}$ is given. If N is an RTNR then ν-prime ideals where $\nu \in \{0,1,2,3,e,c\}$ in right ternary N-groups are defined and their relationships are studied. If Δ is an ideal of a right-lateral ternary N-group Γ, then the interrelationship between the ideal $(\Delta : \Gamma)$ of N and Δ are obtained. If $\nu = 0,1,2,3,c$ then ν-semiprime ideals in RTNR and right ternary N-groups are also studied.

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1. Introduction

A right ternary near-ring is a generalisation of a near-ring in ternary context. Algebraic structures when studied in n-ary context reveal their fundamental properties in depth.

Prime ideals in near-rings have been studied by several authors. In 1970, Holcombe [2] defined ν-prime ideals where $\nu \in \{0,1,2,3\}$. Booth and Groenewald [1] presented another generalization of prime rings, called equiprime or e-prime. Juglal, Groenewald and Lee [3] generalized the various notions of primeness that were defined in N to the N-group Γ. Tasdemir et al. [5] have studied about different prime N-ideals and IFP ideals in N-groups. The authors have defined prime ideal and c-prime ideal of an RTNR in [9].

In this paper, ν-prime ideals, where $\nu \in \{0,1,2,3\}$, and equiprime ideals of an RTNR are defined and the relationship among them are studied. If N is an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x,y,z \in N$ then it is established that ν-primitive ideals of N are ν-prime ideals of N where $\nu = 0,1,2$. If N is an RTNR then ν-prime ideals where $\nu \in \{0,1,2,3,c,e\}$ in right ternary N-groups are defined and their properties are studied. If Δ is an ideal of a right-lateral ternary N-group Γ, then the interrelationship between the ideal $(\Delta : \Gamma)$ of N and Δ are obtained. If $\nu = 0,1,2,3,c$ then ν-semiprime ideals in RTNR and right ternary N-groups are also studied.

2. Preliminaries

In this section, the basic definitions and results needed for the rest of the sections are given.

Definition 2.1. [11]

(a) Let N be a non-empty set together with a binary operation $+$ and a ternary operation $[\ ] : N \times N \times N \to N$. Then $(N,+,[\ ])$ is a right ternary near-ring (RTNR) if (i) $(N,+)$ is a group (ii) $[[x\ y\ z]\ u\ v] = [x\ [y\ z\ u]\ v] = [x\ y\ [z\ u\ v]] = [x\ y\ z\ u\ v]$ for every $x,\ y,\ z,\ u,\ v \in N$ (iii) $[(x+y)\ z\ w] = [x\ z\ w] + [y\ z\ w]$ for every $x,y,z,w \in N$. Similarly left ternary near-ring and lateral ternary near ring can be defined.

(b) Let N be a right ternary near-ring. Let I be a normal subgroup $(N,+)$. Then I is called (i) a right ideal of N if $[I\ N\ N] \subseteq I$ (ii) a left ideal if $[t\ t'(t''+i)] - [t\ t'\ t''] \in I$ (iii) a lateral ideal if $[t\ (t'+i)\ t''] - [t\ t'\ t''] \in I$ for every $t,t',t'' \in N$, $i \in I$. I is called a two-sided ideal if it is a left and right ideal of N and I is an ideal of N if it is a left, right and lateral ideal of N.

Definition 2.2. [10]

(a) A non-empty subset M of an RTNR N is called a right ternary subnear-ring (RTSNR) if (i) $(M,+)$ is a subgroup of N and (ii) $[M\ M\ M] \subseteq M$.

(b) A non-empty subset H of N is called an N-subgroup of N if (i) H is a subgroup of $(R,+)$ (ii) $[N\ N\ H] \subseteq H$ (iii) $[N\ H\ N] \subseteq H$ (iv) $[H\ N\ N] \subseteq H$. If (i) and (ii) hold, then H is called a left N-subgroup. If (i) and (iii) hold, then H is called a lateral N-subgroup. If (i) and (iv) hold, then H is called a right N-subgroup.

(c) An RTNR N is called a trio-RTNR if every one-sided N-subgroup is an N-subgroup of N.

Definition 2.3. [9]

(a) Let N be an RTNR. Then $N_0 = \{n \in N | [n\ 0\ 0] = 0\}$ is the zero-symmetric part of N. If $N = N_0$ then N is called a zero-symmetric RTNR.

(b) Let N be an RTNR. Then an ideal P of N is

  (i) a prime ideal if there are ideals X, Y, Z in N such that $[X\ Y\ Z] \subseteq P \Rightarrow$ either $X \subseteq P$ or $Y \subseteq P$ or $Z \subseteq P$.

  (ii) a completely prime ideal of N if for $x, y, z \in N$, $[x\ y\ z] \in P \Rightarrow$ either $x \in P$ or $y \in P$ or $z \in P$.

  (iii) a completely semi-prime ideal if $x^3 = [x\ x\ x] \in P \Rightarrow x \in P$.

Definition 2.4. [8]

(a) N is a zero RTNR if $N^3 = [N\ N\ N] = \{0\}$.

(b) If N is an RTNR then an element $e \in N$ is a left (resp. right, lateral) unital element if $[e\ e\ x] = x$ ($[x\ e\ e] = x$, $[e\ x\ e] = x$) for every $x \in N$. If $[e\ e\ x] = x = [x\ e\ e]$ then e is called a bi-unital element.

(c) If N is an RTNR then $N_c = \{t \in N | [t\ 0\ 0] = t\}$ is called the constant part of N and N is called a constant RTNR if $N = N_c$.

(d) An ideal I of N is an IFP ideal if $[a\ b\ c] \in I \Rightarrow [a\ u\ b\ v\ c] \in I \ \ \forall\ \ u, v \in N$.

Theorem 2.5. [8] *An ideal I of N is maximal iff N/I is simple.*

Definition 2.6. [7]

(a) Let $(N, +, [\ ])$ be an RTNR and $(\Gamma, +)$ be a group with additive identity o. Then Γ is said to be a right ternary N-group if there exists a mapping $[\ ]_\Gamma : N \times N \times \Gamma \to \Gamma$ such that (i) $[n + m\ x\ \gamma]_\Gamma = [n\ x\ \gamma]_\Gamma + [m\ x\ \gamma]_\Gamma$ (ii) $[[n\ m\ u]\ x\ \gamma]_\Gamma = [n\ [m\ u\ x]\ \gamma]_\Gamma = [n\ m\ [u\ x\ \gamma]_\Gamma]_\Gamma$ for all $\gamma \in \Gamma$ and $n, m, u \in N$.

(b) A subgroup Δ of ${}_N\Gamma$ is said to be

  (i) an N-subgroup of ${}_N\Gamma$ if $[N\ N\ \Delta]_\Gamma \subseteq \Delta$.

  (ii) a normal subgroup of ${}_N\Gamma$ if $\gamma + \delta - \gamma \in \Delta\ \forall\ \gamma \in \Gamma$, $\delta \in \Delta$.

  (iii) an N-ideal of ${}_N\Gamma$ if Δ is a normal subgroup of ${}_N\Gamma$ and $[n\ x\ (\gamma + \delta)]_\Gamma - [n\ x\ \gamma]_\Gamma \in \Delta\ \forall\ \gamma \in \Gamma$, $\forall\ \delta \in \Delta$ and $\forall\ n, x \in N$.

(c) A right ternary N-group ${}_N\Gamma$ is said to be

  (i) simple if $\{o\}$ and Γ are the only ideals of ${}_N\Gamma$ where o is the identity element of Γ.

(ii) N-simple if Ω and Γ are the only N-subgroups of ${}_N\Gamma$, where $\Omega = [N_c\, 0\, o]$.

(iii) faithful if $(o : \Gamma) = \{0\}$.

(d) Let ${}_N\Gamma$ be a right ternary N-group of an RTNR N. Then for $x \in N$ there exists $\gamma \in \Gamma$, N is monogenic by γ w.r.to x if $[N\ x\ \gamma]_\Gamma = \Gamma$ and N is monogenic by γ if there exists $\gamma \in \Gamma$ and for every $x \in N$, $[N\ x\ \gamma]_\Gamma = \Gamma$ and Γ is a generator of Γ. A right ternary N-group ${}_N\Gamma$ is strongly monogenic if Γ is monogenic and $[N\ x\ \gamma]_\Gamma = \{o\}$ or Γ for every $x \in N$ and $\gamma \in \Gamma$.

(e) A monogenic N-group Γ with $\Gamma \neq \{o\}$ is said to be of type 0 if Γ is simple, type 1 if Γ is simple and strongly monogenic and type 2 if Γ is N_0-simple.

(f) If $[x\ y\ \gamma]_\Gamma = 0 \Rightarrow [x\ u\ y\ v\ \gamma]_\Gamma = 0\ \ \forall\ \ x, y, u, v \in N$, $\gamma \in \Gamma$ then ${}_N\Gamma$ is called an IFP RTNR.

Theorem 2.7. [7]

(i) *If ${}_N\Gamma$ is monogenic by γ_0 w.r. to $x \in N$ and L is a left ideal of N, then $[L\ x\ \gamma_0]_\Gamma$ is an ideal of ${}_N\Gamma$*

(ii) *If ${}_N\Gamma$ is monogenic by γ_0 and if e is a left unital element of N, then $\forall\ \gamma \in \Gamma$, $[e\ e\ \gamma]_\Gamma = \gamma$.*

Proposition 2.8. [7] *Let Γ be a right ternary N-group. Then ${}_N\Gamma$ is of type 2 $\Rightarrow$ ${}_N\Gamma$ is of type 1 $\Rightarrow$ ${}_N\Gamma$ is of type 0.*

Definition 2.9. [6] An ideal I of an RTNR N is called a direct summand of N if there exists an ideal J of N such that $N = I + J$. The ideal J is called a direct complement of I in N. N is called ν-primitive on ${}_N\Gamma$ if $(o : \Gamma) = \{0\}$ i.e., Γ is faithful right ternary N-group and Γ is of type ν. N is ν-primitive if there exists an N-group ${}_N\Gamma$ such that N is ν-primitive on ${}_N\Gamma$. If in N $[x\ y\ z] = [y\ x\ z]$ then an ideal I of N is called ν-primitive ideal of N if N/I is ν-primitive.

Definition 2.10. [6]

(a) Let N be a right and a lateral ternary near ring. Let I be an ideal of N. Then a group $(\Gamma, +)$ is called a right-lateral ternary N-group if $[\ ]_\Gamma : N \times N \times \Gamma \to \Gamma$ such that

(i) $[n + m\ x\ \gamma]_\Gamma = [n\ x\ \gamma]_\Gamma + [m\ x\ \gamma]_\Gamma$

(ii) $[n\ m + x\ \gamma]_\Gamma = [n\ m\ \gamma]_\Gamma + [n\ x\ \gamma]_\Gamma$

(iii) $[[n\ m\ u]\ x\ \gamma]_\Gamma = [n\ [m\ u\ x]\ \gamma]_\Gamma = [n\ m\ [u\ x\ \gamma]_\Gamma]_\Gamma$ for every $n, m, x, u \in N$ and $\gamma \in \Gamma$.

(b) If Δ_1 and Δ_2 are any two non-empty subsets of a right-lateral ternary N-group then $(\Delta_1 : \Delta_2) = \{n \in N | [n\ x\ \delta_2]_\Gamma \in \Delta_1$ and $[x\ n\ \delta_2]_\Gamma \in \Delta_1\ \forall\ x \in N$, $\delta_2 \in \Delta_2\}$. Also if for every $x, y, z \in N$, $[x\ y\ z] = [y\ x\ z]$ and Δ is an N-subgroup then $(o : \Delta)$ is an ideal of N.

3. Prime ideals in RTNR

In this section, ν-prime ideals where $\nu \in \{0, 1, 2, 3, c\}$ and equiprime ideals are defined and the relationship among them are studied. In an RTNR with $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$, it is shown that ν-primitive ideals of N are ν-prime ideals of N where $\nu = 0, 1, 2$.

Definition 3.1. Let P be an ideal of an RTNR N. Then

(a) if $[A\ B\ C] \subseteq P \Rightarrow$ either $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ holds for all

(i) ideals A, B, C of N then P is 0-prime.

(ii) left ideals A, B, C of N then P is 1-prime.

(iii) N-subgroups A, B, C of N then P is 2-prime.

(b) P is 3-prime if $a, b, c \in N$ and $[a\ N\ b\ N\ c] \subseteq P \Rightarrow$ either $a \in P$ or $b \in P$ or $c \in P$.

(c) P is completely prime (c-prime) if $a, b, c \in N$ and $[a\ b\ c] \subseteq P \Rightarrow$ either $a \in P$ or $b \in P$ or $c \in P$.

(d) P is equiprime (e-prime) if $a \in N - P$ and $x, y \in N$ with $[a\ n\ x] - [a\ n\ y] \in P$ $\forall\ \ n \in N \Rightarrow x - y \in P$.

Equivalently if $a \notin P$, $b \notin P$ and $[a\ n\ b\ m\ x] - [a\ n\ b\ m\ y] \in P\ \ \forall\ \ n, m \in N \Rightarrow x - y \in P$ then P is e-prime.

Definition 3.2. An RTNR N is said to be ν-prime if $\{0\}$ is a ν-prime ideal where $\nu \in \{0, 1, 2, 3, c, e\}$.

Example 3.3. Let $N = \{0, x, y, z\}$ be as given in [4, Scheme 1, p. 408]. Define $+$ as in Table 1 and the ternary operation $[\ \ ]$ on N by $[x\ y\ z] = (x.y).z$ for every $x, y, z \in N$ where $\cdot$ is defined as in Table 2.

Table 1:

$+$	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	z	0

Table 2:

$\cdot$	0	x	y	z
0	0	0	0	0
x	0	x	x	x
y	0	y	y	y
z	0	z	z	z

Since $[x\ y\ z] = 0$ holds only if either $x = 0$ or $y = 0$ or $z = 0$ and $[A\ B\ C] = 0$ holds only if either $A = \{0\}$ or $B = \{0\}$ or $C = \{0\}$, N is ν-prime where $\nu = 0, 1, 2, c$.

Table 3:

$+$	0	x	y
0	0	x	y
x	x	y	0
y	y	0	x

Table 4:

$\cdot$	0	x	y
0	0	0	0
x	0	x	x
y	0	y	y

Example 3.4. Let $N = \{0, x, y\}$ be as given in [4, Scheme 3, p. 407]. Define $+$ as in Table 3 and the ternary operation $[\ \ ]$ on N by $[xyz] = (x.y).z$ for every $x, y, z \in N$ where $\cdot$ defined as in Table 4.

Since $[a\ N\ b\ N\ c] \subseteq \{0\}$ holds only if either $a = 0$ or $b = 0$ or $c = 0$, N is a 3-prime RTNR.

Example 3.5. Let $\Gamma = \{0, 1, 2\} = \mathbb{Z}_3$ and $N = M_0(\Gamma) = \{\alpha_1, \alpha_2,\ \alpha_3,\ \alpha_4,\ \alpha_5,\ \alpha_6,$ $\alpha_7, \alpha_8, \alpha_9\}$ where α_i's are defined as given in Table 5.

Table 5:

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	2	1	2	1	2
2	0	1	2	0	0	1	2	2	1

Note that $\{\alpha_1\}$ (α_1 is the zero mapping) is an e-prime ideal of N. For if $\alpha_j - \alpha_k \notin \{\alpha_1\}$ i.e., $\alpha_j \neq \alpha_k$ then for some $t \in \Gamma$, $\alpha_j(t) \neq \alpha_k(t)$ and also for any $0 \neq \alpha_l \in N$ there exists $0 \neq x \in \Gamma$ such that $\alpha_l(x) \neq 0$. Define $\theta : \Gamma \to \Gamma$ by $\theta(z) = x$ if $z = \alpha_j(t)$ and 0 otherwise. Then $\theta \in N$ and also $[\alpha_l\ \theta\ \alpha_j] \neq [\alpha_l\ \theta\ \alpha_k]$ i.e., $[\alpha_l\ \theta\ \alpha_j] - [\alpha_l\ \theta\ \alpha_k] \notin \{\alpha_1\}$. Thus $\{\alpha_1\}$ is an e-prime ideal of N. Hence $(N.+, [\ ])$ is an e-prime RTNR.

Proposition 3.6. *If P is a left ideal of an RTNR N then P is a left N-subgroup of N iff $[n\ x\ 0] \in P$ for all $n, x \in N$.*

Proof. It is obvious that if P is a left N-subgroup of N then for all $n, x \in N$, $[n\ x\ 0] \in P$. Conversely let $[n\ x\ 0] \in P$ for all $n, x \in N$. Now $[n\ x\ i] = [n\ x\ i+0] - [n\ x\ 0] + [n\ x\ 0] \Rightarrow [n\ x\ i] \in P\ \ \forall\ \ i \in P$. Hence P is a left N-subgroup of N. ∎

Proposition 3.7. *If P is an e-prime ideal then P is a left N-subgroup of N.*

Proof. In view of the above proposition it suffices to prove that $[n\ x\ 0] \in P$ for all $n, x \in N$. Now $[[n\ 0\ 0]\ m\ [n\ x\ 0]] - [[n\ 0\ 0]\ m\ 0] = [n\ 0\ 0] - [n\ 0\ 0] = 0 \in P$ by associativity in N. Since P is an e-prime ideal $[n\ x\ 0] - 0 \in P$. i.e., $[n\ x\ 0] \in P$. Hence the proof. ∎

The following lemmas describe the relationship between any two ν-prime ideals where $\nu \in \{0, 1, 2, 3, e, c\}$.

Lemma 3.8. *Every e-prime ideal of an RTNR N is a 3-prime ideal of N.*

Proof. Let P be an e-prime ideal of an RTNR N. Suppose for $a, b, c \in N$, $[a\ N\ b\ N\ c] \subseteq P$ but $a \notin P$, $b \notin P$ and $c \notin P$. Since $[a\ [n\ b\ m]\ c] \in P$ and $[a\ [n\ b\ m]\ 0] \in P\ \ \forall\ \ n, m \in N$, $[a\ [n\ b\ m]\ c]$ - $[a\ [n\ b\ m]\ 0] \in P \Rightarrow c - 0 \in P$, which contradicts the hypothesis. Hence P is a 3-prime ideal of N. ∎

Remark 3.9. The converse of the above lemma is not true. For if N is as in Example 3.4 then $\{0\}$ is 3-prime but is not e-prime as $[x\ y\ x] - [x\ y\ y] \in \{0\}$ but $x - y \notin \{0\}$.

Lemma 3.10. *Let N be an RTNR. Then every e-prime ideal is a c-prime ideal of N.*

Proof. Let $a, b, c \in N$ be such that $[a\ b\ c] \in P$. Suppose $a \notin P$ and $b \notin P$. Since P is e-prime, $[a\ b\ c] \in P$. Hence $[a\ b\ c] - [a\ b\ 0] \in P \Rightarrow c \in P$, proving that P is a c-prime ideal. ∎

Lemma 3.11. *If N is an RTNR with $[x\ y\ z] = [y\ x\ z]\ \ \forall\ \ x, y, z \in N$ then every c-prime ideal is an e-prime ideal of N.*

Proof. Let P be a c-prime ideal of N. Let $a \in N - P$ and $x, y \in N$ be such that $[a\ n\ x] - [a\ n\ y] \in P$ for all $n \in N$. Let $q \in N - P$. Now for $t \in N$, $[([a\ n\ x] - [a\ n\ y])\ t\ q] \in P \Rightarrow [[x\ n\ t]\ a\ q] - [[y\ n\ t]\ a\ q] \in P$ by associativity and hypothesis. This implies that $[(x - y)\ n\ t] \in P$ as $a, q \notin P$. Hence $x - y \in P$, for otherwise $[(x - y)\ n\ t] \notin P$, a contradiction. Hence P is an e-prime ideal of N. ∎

Lemma 3.12. *Let N be an RTNR. Then every 3-prime ideal is a 2-prime ideal of N.*

Proof. Let P be a 3-prime ideal of an RTNR N. Let $[A\ B\ C] \subseteq P$ for all N-subgroups A, B, C of N. If either $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ then the lemma follows. Suppose not. Then there exists $a \in A$, $b \in B$, $c \in C$ and $a, b, c \notin P$ with $[a\ N\ b\ N\ c] \subseteq [a\ N\ B\ N\ c] \subseteq [a\ B\ C] \subseteq [A\ B\ C] \subseteq P$. Since P is a 3-prime ideal this implies that $a \in P$ or $b \in P$ or $c \in P$, a contradiction. Hence P is a 2-prime ideal of N. ∎

Remark 3.13. The converse of the above lemma is not true. For if $N = \{0, x, y\}$ as in [4, Scheme 2, p. 407] and $+$ is defined as given in Table 6 and the ternary multiplication $[\ \ ]$ is defined as $[a\ b\ c] = (a \cdot b) \cdot c$ where $\cdot$ is defined as in Table 7.

Table 6:

$+$	0	x	y
0	0	x	y
x	x	y	0
y	y	0	x

Table 7:

$\cdot$	0	x	y
0	0	0	0
x	0	0	x
y	0	0	y

Then N is an RTNR and $\{0\}$ is a 2-prime ideal but is not a 3-prime ideal as $[x\ N\ x\ N\ x] \subseteq \{0\}$ but $x \neq 0$.

Lemma 3.14. *If N is an RTNR with unital element e then every 2-prime ideal is a 3-prime ideal.*

Proof. Let P be a 2-prime ideal of N and $[a\ N\ b\ N\ c] \subseteq P\ \ \forall\ \ a, b, c \in N$. Then

$$[N\ N\ [a\ N\ b\ N\ c]] \subseteq [N\ N\ P] \subseteq P$$

Now

$$\begin{aligned} &[N\ e\ a\ N\ b\ c\ N\ e\ c] \subseteq [N\ n\ a\ N\ b\ N\ c] \subseteq P \\ &\Rightarrow\ [N\ e\ a] \subseteq P \text{ or } [N\ b\ e] \subseteq P \text{ or } [N\ e\ c] \subseteq P \\ &\Rightarrow\ [e\ e\ a] \in P \text{ or } [e\ b\ e] \in P \text{ or } [e\ e\ c] \in P \\ &\Rightarrow\ a \in P \text{ or } b \in P \text{ or } c \in P \end{aligned}$$

Hence P is a 3-prime ideal of N. ∎

Lemma 3.15. *If N is a trio-RTNR then every 2-prime ideal is a 1-prime ideal of N.*

Proof. Let N be a trio-RTNR and P be a 2-prime ideal of N. Then N is zero-symmetric. Let A, B, C be left ideals of N such that $[A\ B\ C] \subseteq P$. Then A, B, C are left N-subgroups of N and hence are N-subgroups of N as N is a trio-RTNR. Since P is 2-prime ideal and $[A\ B\ C] \subseteq P$, $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$. Thus P is a 1-prime ideal of N. ∎

Remark 3.16. The converse of the above lemma is not true. For if $N = \{0, x, y, z\}$ as given in [ ] where $+$ is defined as in Table 8 and the ternary multiplication [ ] is defined as $[a\ b\ c] = (a \cdot b) \cdot c$ where $\cdot$ is as given in Table 9. Then N is a trio RTNR and $\{0\}$ is a 1-prime ideal but is not a 2-prime ideal as $\{0, x\} = A$ (say) is an N-subgroup of N such that $[A\ A\ A] \subseteq \{0\}$ but $A \not\subseteq \{0\}$.

Table 8:

$+$	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	z	0

Table 9:

$\cdot$	0	x	y	z
0	0	0	0	0
x	0	0	0	x
y	0	0	0	y
z	0	0	0	z

Lemma 3.17. *If N is an RTNR then every 1-prime ideal of N is a 0-prime ideal of N.*

Proof. Let A, B, C be ideals of N such that $[A\ B\ C] \subseteq P$. Since every ideal is a left ideal and P is 1-prime $[A\ B\ C] \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$. Thus P is a 0-prime ideal of N. ∎

Remark 3.18. The converse of the above lemma holds if N is commutative.

Lemma 3.19. *If N is an RTNR then every c-prime ideal is a 3-prime ideal of N.*

Proof. Let P be a c-prime ideal of an RTNR N. Let $[a\ N\ b\ N\ c] \subseteq P$. Then $[a\ u\ b\ v\ c] \in P$ for all $u, v \in N$. Since P is a c-prime ideal,

$$\begin{aligned} [a\ u\ b\ v\ c] \in P \quad &\Rightarrow [a\ u\ b] \in P \text{ or } v \in P \text{ or } c \in P \\ &\Rightarrow a \in P \text{ or } u \in P \text{ or } b \in P \text{ or } v \in P \text{ or } c \in P \\ &\Rightarrow a \in P \text{ or } b \in P \text{ or } c \in P \end{aligned}$$

as $N \nsubseteq P$. Hence P is a 3-prime ideal of N. ∎

Remark 3.20. The converse holds if P is an IFP ideal of N.

Lemma 3.21. *If N is a zero-symmetric RTNR then every 2-prime ideal is a 0-prime ideal of N.*

Proof. Let P be a 2-prime ideal of a zero-symmetric RTNR. Let A, B, C be ideals of N such that $[A\ B\ C] \subset P$. Then A, B, C are N-subgroups of N and as P is a 2-prime ideal $[A\ B\ C] \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$. Hence P is a 0-prime ideal. ∎

Proposition 3.22. *If P is an e-prime ideal of an RTNR N and H is an N-subgroup of N then $H \cap P$ is an e-prime ideal of H.*

Proof. If P is an ideal and H is an N-subgroup of N then it can be easily seen that $H \cap P$ is an ideal of H. Now let $x, y \in H$ and $h \notin H \cap P$ be such that $[h\ a\ x] - [h\ a\ y] \in H \cap P\ \forall\ a \in H$. If $x - y \notin H \cap P$ then $[h\ v\ x] - [h\ v\ y] \notin P\ \forall\ v \in N$ as P is e-prime in N. This implies that $[h\ u\ [h\ v\ x]] - [h\ u\ [h\ v\ y]] \notin P$ or $[h\ [u\ h\ v]\ x] - [h\ [u\ h\ v]\ y] \notin P$ which contradicts the assumption. Therefore $x - y \in H \cap P$. Hence $H \cap P$ is e-prime in H. ∎

Remark 3.23.

(i) If N is an e-prime RTNR then every N-subgroup is an e-prime RTSNR.

(ii) If N is zero-symmetric e-prime RTNR then every ideal is an e-prime RTSNR.

Theorem 3.24. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \ \forall\ \ x, y, z \in N$. Then*

(i) *every 0-primitive ideal of N is a 0-prime ideal of N.*

(ii) *every 1-primitive ideal of N is a 1-prime ideal of N.*

(iii) *every 2-primitive ideal of N is a 2-prime ideal of N.*

Proof. (i) Let P be a 0-primitive ideal of N. Then N/P is 0-primitive. This implies that there exists a right-lateral ternary N-group ${}_N\Gamma$ such that $(o : \Gamma) = P$ and Γ is of type 0. Also for $\gamma_0 \in \Gamma$, $\Gamma = [N\ x\ \gamma_0]_\Gamma \neq \{o\}\ \ \forall\ \ x \in N$. Suppose for ideals A, B, C of N, $[A\ B\ C] \subseteq P$ but $A \nsubseteq P$, $B \nsubseteq P$ and $C \nsubseteq P$. It is noted that as C is an ideal of $N\ \ \forall\ \ x \in N$, $[C\ x\ \Gamma]_\Gamma = [C\ x\ [N\ x\ \Gamma_0]_\Gamma]_\Gamma \subseteq [C\ x\ \gamma_0]_\Gamma \subseteq [C\ x\ \Gamma]_\Gamma\ \ \forall\ \ x \in N$ and hence $[C\ x\ \Gamma]_\Gamma = [C\ x\ \gamma_0]_\Gamma$. Now as

$C \nsubseteq (o : \Gamma)$, $[C\ x\ \Gamma]_\Gamma = [C\ x\ \Gamma_0]_\Gamma \neq \{0\}$. Since C is an ideal $[C\ x\ \gamma_0]_\Gamma$ is an ideal of ${}_N\Gamma$ and ${}_N\Gamma$ is type 0 it follows that $[C\ x\ \Gamma]_\Gamma = [C\ x\ \gamma_0]_\Gamma = \Gamma$. Thus, $[A\ B\ [C\ x\ \Gamma]_\Gamma]_\Gamma = [A\ B\ \Gamma]_\Gamma = \Gamma \neq \{o\}$. $\Rightarrow [A\ B\ C] \nsubseteq (o : \Gamma) = P$ which contradicts the assumption that $[A\ B\ C] \subseteq P$. Hence P is a 0-prime ideal of N.

(ii) Let P be a 1-primitive ideal of N. Then there exists a right-lateral ternary N-group such that $P = (o : \Gamma)$ where Γ is of type 1. If A, B, C are left ideal of N such that $[A\ B\ C] \subseteq P$ with $A \nsubseteq P$ and $B \nsubseteq P$ and $C \nsubseteq P$. Then $[C\ x\ \Gamma]_\Gamma \neq \{0\}$ and Γ is strongly monogenic $[C\ x\ \Gamma]_\Gamma = \Gamma$. Hence $[A\ B\ [C\ x\ \Gamma]_\Gamma]_\Gamma = [A\ B\ \Gamma]_\Gamma = \Gamma \neq \{o\} \Rightarrow [A\ B\ C] \nsubseteq P$, a contradiction to the assumption. Hence P is a 1-prime ideal.

(iii) Let P be a 2-primitive ideal of N. Then there exists a right-lateral ternary N-group ${}_N\Gamma$ such that $(o : \Gamma) = P$ where Γ is of type 2. Since Γ is of type 2, $\Gamma = [N\ x\ \gamma_0]_\Gamma \neq \{o\}\ \forall\ x \in N$. Suppose for N-subgroups A, B, C of N, $[A\ B\ C] \subseteq P$ but $A \nsubseteq P$, $B \nsubseteq P$ and $C \nsubseteq P$. Then as $C \nsubseteq \Gamma$, $[C\ x\ \Gamma]_\Gamma \neq \{o\}$. Also $[C\ x\ \Gamma]_\Gamma = [C\ x\ [N\ x\ \gamma_0]_\Gamma]_\Gamma \subseteq [C\ x\ \gamma_0]_\Gamma \subseteq [C\ x\ \Gamma]_\Gamma$. Thus $[C\ x\ \Gamma]_\Gamma = [C\ x\ \gamma_0]_\Gamma \neq \{o\}$. Since C is an N-subgroup of N, $[C\ x\ \gamma_0]_\Gamma$ is an N-subgroup of ${}_\Gamma N$ and since Γ is N_0-simple, $[C\ x\ \Gamma]_\Gamma = \Gamma$. Thus $[A\ B\ C\ x\ \Gamma]_\Gamma = [A\ B\ \Gamma]_\Gamma = \Gamma \neq \{o\}$ which contradicts $[A\ B\ C] \subseteq P$. Hence P is of type 2. ∎

Proposition 3.25. *If an ideal I of N is a direct summand and P is a ν-prime ideal in N then $P \cap I$ is a ν-prime ideal in I, where $\nu = 0$.*

Proof. Let A, B, C be ideals of I. Then as I is a direct summand of N, A, B, C are ideals of N. Now let $[A\ B\ C] \subseteq P \cap I \Rightarrow [A\ B\ C] \subseteq P$ and $[A\ B\ C] \subseteq I \Rightarrow A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$. Thus $A \subseteq P \cap I$ or $B \subseteq P \cap I$ or $C \subseteq P \cap I$ and hence $P \cap I$ is a 0-prime ideal in I. A similar argument holds for 1-prime ideal in N. ∎

Proposition 3.26. *If $N = N_c$ then every normal subgroup of $(N, +)$ is a 0-prime ideal of N.*

Proof. Every normal subgroup of $(N_c, +)$ is clearly an ideal of N_c. Let P be an ideal of N such that $[A\ B\ C] \subseteq P$ for all ideals A, B, C of N. Consider, $[a\ b\ c] = [a\ b\ c\ 0\ 0] = [a\ b\ 0] = a$, where $a \in A$, $b \in B$, $c \in C$. Thus $A \subseteq P$. Hence P is a 0-prime ideal of N. ∎

Corollary 3.27. *N_c is a 0-prime RTSNR.*

Proof. Since $\{0\}$ is a normal subgroup of N_c and is an ideal of N_c, by the above proposition N_c is 0-prime RTSNR. ∎

Proposition 3.28. *If N is simple then either N is 0-prime RTNR or N is a zero RTNR.*

Proof. If N is simple then $\{0\}$ and N are the only ideals of N. If $\{0\}$ is a prime ideal of N then N is a prime RTNR. If $\{0\}$ is not a prime ideal of N then $N^3 \subseteq \{0\}$ but $N \nsubseteq \{0\}$. Thus $N^3 = \{0\}$ and hence N is a zero RTNR. **Proof.**

Proposition 3.29. *If I is a maximal ideal in N then I is 0-prime or $N^3 \subseteq I$.*

Proof. If I is a maximal ideal in N then N/I is simple. This implies that N/I is 0-prime RTNR or N/I is zero-RTNR. i.e., I is 0-prime ideal or $N^3 \subseteq I$. In the following the definition of ν-semiprime ideals of an RTNR N is given. ∎

Definition 3.30. Let N be an RTNR and I be an ideal of N. Then

(a) $A^3 \subseteq I \Rightarrow A \subseteq I$ holds for all

  (i) ideals A of N then I is 0-semiprime

  (ii) left ideals A of N then I is 1-semiprime

  (iii) N-subgroups A of N then I is 2-semiprime

(b) I is 3-semiprime if $[a\ N\ a\ N\ a] \subseteq I \Rightarrow a \in I$ for all $a \in I$

(c) I is c-semiprime if $a^3 \in I \Rightarrow a \in I$ for all $a \in I$

Definition 3.31. An RTNR N is called a semiprime RTNR if $\{0\}$ is a semiprime ideal of N.

Note 3.32. In an RTNR N, every ν-prime ideal is a ν-semiprime ideal where $\nu \in \{0, 1, 2, 3, c\}$.

Proposition 3.33. *The intersection of ν-prime ideals of an RTNR N is a ν-semiprime ideal of N, $\nu \in \{0, 1, 2\}$.*

Proof. Let H denote the intersection of ν-prime ideals P. Then H is an ideal of N and for any ideal A (or N-subgoup or left ideal) of N, $A^3 \subseteq H \Rightarrow A^3 \subseteq P$ for every $P \Rightarrow A \subseteq P$. Hence H is ν-semiprime ideal of N. ∎

Proposition 3.34. *If an ideal I of N is a direct summand of N and S is a ν-semiprime ideal of N then $S \cap I$ is a ν-semiprime ideal in I where $\nu \in \{0, 1\}$.*

Proof. Let A be an ideal (left ideal) of I. Then as I is a direct summand of N, A is an ideal (left ideal) of N. Now let $A^3 \subseteq S \cap I \Rightarrow A^3 \subseteq S$ and $A^3 \subseteq I \Rightarrow A \subseteq S \cap I$. Thus $S \cap I$ is a ν-semiprime ideal of N where $\nu = 0, 1$. ∎

4. Prime ideals in right ternary N-groups

In this section ν-prime ideals in right ternary N-groups are defined and their interrelationship are studied.

Definition 4.1. Let N be an RTNR and ${}_N\Gamma$ be a right ternary N-group. Let Δ be an N-ideal of Γ such that $[N\ N\ \Gamma]_\Gamma \not\subseteq \Delta$. Then

(a) if $[A\ B\ \Delta']_\Gamma \subseteq \Delta \Rightarrow$ either $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta\ \forall\ x \in N$ holds for all

  (i) ideals A, B of N and an N-ideal Δ' of Γ then Δ is called 0-prime.

(ii) left ideals A, B of N and an N-ideal Δ' of Γ then Δ is called 1-prime.

(iii) N-subgroups A, B of N and an N-subgroup Δ' of Γ then Δ is called 2-prime.

(b) Δ is 3-prime if $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta \Rightarrow$ either $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Gamma\ \forall\ a, b \in N$ and $\gamma \in \Gamma$

(c) Δ is c-prime (completely prime) if $[a\ b\ \gamma]_\Gamma \in \Delta \Rightarrow$ either $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Delta\ \forall\ a, b \in N$ and $\gamma \in \Gamma$

(d) Δ is e-prime (equiprime) if $a \in N$, $[a\ x\ \Gamma]_\Gamma \not\subseteq \Delta\ \forall\ x \in N$ and $\gamma_1, \gamma_2 \in \Gamma$ then $[a\ n\ \gamma_1]_\Gamma - [a\ n\ \gamma_2]_\Gamma \in \Delta\ \forall\ n \in N \Rightarrow \gamma_1 - \gamma_2 \in \Delta$.

Definition 4.2. A right ternary N-group ${}_N\Gamma$ is said to be ν-prime where $\nu \in \{0, 1, 2, 3, c, e\}$ if $[N\ N\ \Gamma]_\Gamma \neq \{o\}$ and $\{o\}$ is a ν-prime ideal of ${}_N\Gamma$.

Proposition 4.3. *If Δ is an ideal of a right ternary N-group ${}_N\Gamma$ then Δ is an N-subgroup of ${}_N\Gamma$ iff $[n\ x\ o]_\Gamma \in \Delta$ for all $n, x \in N$.*

Proof. It is obvious that if Δ is an N-subgroup of ${}_N\Gamma$ then for all $n, x \in N$, $[n\ x\ o]_\Gamma \in \Delta$. Conversely let $[n\ x\ o]_\Gamma \in \Delta$ for all $n, x \in N$. Now $[n\ x\ \delta] = [n\ x\ \delta + o]_\Gamma - [n\ x\ o]_\Gamma + [n\ x\ o]_\Gamma \Rightarrow [n\ x\ \delta]_\Gamma \in \Delta$. Hence Δ is an N-subgroup of ${}_N\Gamma$. ∎

Proposition 4.4. *If Δ is an e-prime ideal of ${}_N\Gamma$ then Δ is an N-subgroup of ${}_N\Gamma$.*

Proof. In view of the above proposition it suffices to prove that $[n\ x\ o]_\Gamma \in \Delta$ for all $n, x \in N$. Now $[[n\ 0\ 0]\ m[n\ x\ o]_\Gamma]_\Gamma - [[n\ 0\ 0]\ m\ o]_\Gamma = [n\ 0\ o]_\Gamma - [n\ 0\ o]_\Gamma = o \in \Delta$. Since Δ is an e-prime ideal, $[n\ x\ o]_\Gamma - o \in \Delta$. i.e., $[n\ x\ o]_\Gamma \in \Delta$. Hence the proof. ∎

Theorem 4.5. *Let Δ be an N-ideal of ${}_N\Gamma$ such that $[N\ N\ \Gamma]_\Gamma \not\subseteq \Delta\ \forall\ x \in N$ and $\gamma \in \Gamma$. Then*

(i) Δ *is e-prime* $\Rightarrow$ Δ *is* 3*-prime*

(ii) Δ *is e-prime* $\Rightarrow$ Δ *is c-prime*

(iii) Δ *is c-prime* $\Rightarrow$ Δ *is* 3*-prime*

(iv) Δ *is* 3*-prime* $\Rightarrow$ Δ *is* 2*-prime*

(v) Δ *is* 2*-prime* $\Rightarrow$ Δ *is* 0*-prime if N is zero-symmetric*

(vi) Δ *is* 2*-prime* $\Rightarrow$ Δ *is* 1*-prime if N is a trio-RTNR*

(vii) Δ *is* 1*-prime* $\Rightarrow$ Δ *is* 0*-prime*

(viii) Δ *is* 3*-prime* $\Rightarrow$ Δ *is c-prime if Δ is an IFP ideal of* ${}_N\Gamma$

(ix) Δ *is* 0*-prime* $\Rightarrow$ Δ *is* 1*-prime if N is commutative.*

Proof. Let Δ be an N-ideal of ${}_N\Gamma$ such that $[N\ N\ \Gamma]_\Gamma \not\subseteq \Delta$

(i) Let $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta$ where $a, b \in N$, $\gamma \in \Gamma$. Then if either $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Delta$ then there is nothing to prove. Suppose not. Then since Δ is e-prime, $[a\,u\,b\,v\,\gamma]_\Gamma - [a\,u\,b\,v\,0]_\Gamma \in \Delta \Rightarrow [b\ v\ \gamma]_\Gamma - [b\ v\ 0]_\Gamma \in \Delta \Rightarrow \gamma \in \Gamma$ which is not true. Hence Δ is 3-prime.

(ii) For $a, b \in N$ let $[a\ b\ \gamma]_\Gamma \in \Delta$. Then if either $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Delta$ then there is nothing to prove. Suppose not. Then as $[a\ b\ \gamma]_\Gamma - [a\ b\ 0]_\Gamma \in \Delta$ and Δ is e-prime, $\gamma \in \Delta$ a contradiction. Hence Δ is c-prime.

(iii) For $a, b \in N$ and $\gamma \in \Gamma$ consider $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta$. Then $[a\ u\ b\ v\ \gamma]_\Gamma \in \Delta \forall\ u, v \in N$. Since Δ is c-prime, $[a[u\ b\ v]\gamma]_\Gamma \in \Delta$, either $[a\,x\,\Gamma]_\Gamma \subseteq \Delta$ or $[b\,x\,\Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Delta\ \forall\ x \in N$ as $[N\,N\,\Gamma]_\Gamma \not\subseteq \Delta$. Thus Δ is 3-prime.

(iv) Let Δ be 3-prime in an RTNR N. Let A, B be N-subgroups of N. Let $[A\ B\ \Delta']_\Gamma \subseteq \Delta$. Now for every $a \in A, b \in B$ and $\delta' \in \Delta'$, $[a\ N\ b\ N\ \delta']_\Gamma = [a\ [N\ b\ N]\ \delta']_\Gamma \subseteq [A\ B\ \Delta']_\Gamma \subseteq \Delta$. Since Δ is 3-prime this implies that either $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\delta' \in \Delta$ i.e., $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta$. Hence Δ is 2-prime.

(v) Let A and B be ideals of a zero-symmetric RTNR N and Δ' be an ideal of ${}_N\Gamma$ such that $[A\ B\ \Delta']_\Gamma \subseteq \Delta$. Then by hypothesis it follows that $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta\ \forall\ x \in N$. Hence Δ is 0-prime.

(vi) Let N be a trio-RTNR. Then N is zero-symmetric. Let A and B be left ideals of N and Δ' be an ideal of N such that $[A\ B\ \Delta']_\Gamma \subseteq \Delta$. Then $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta$ as Δ is 2-prime and N is zero-symmetric. Hence Δ is 1-prime.

(vii) Let A and B be ideals of an RTNR N such that $[A\ B\ \Delta']_\Gamma \subseteq \Delta$. Since A, B are left ideals and Δ is 1-prime, either $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta$. Hence Δ is 0-prime.

(viii) Let $a, b \in N$ and $\gamma \in \Gamma$ such that $[a\ b\ \gamma]_\Gamma \in \Delta \Rightarrow [a\ u\ b\ v\ \gamma]_\Gamma \in \Delta$ for every $u, v \in N$. Thus $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma \in \Gamma$. Hence Δ is c-prime.

(ix) Let A and B be left ideals of N and Δ' be an N-ideal of ${}_N\Gamma$ such that $[a\ b\ \Delta']_\Gamma \subseteq \Delta$. Since N is commutative every left ideal of N is an ideal of N and since Δ is 0-prime, either $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $\Delta' \subseteq \Delta$, $\forall\ \ x \in N$. Hence Δ is 1-prime. ∎

Proposition 4.6. *If N is an RTNR with unital element e and ${}_N\Gamma$ is monogenic then every 2-prime ideal is a 3-prime ideal.*

Proof. Let Δ be a 2-prime ideal of ${}_N\Gamma$ and $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta\ \ \forall\ \ \gamma \in \Gamma$. Consider

$$\begin{aligned}
&[N\ e\ a\ N\ b\ c\ N\ e\ \gamma]_\Gamma \subseteq [N\ N\ a\ N\ b\ N\ \gamma]_\Gamma \subseteq \Delta\\
&\qquad \Rightarrow [[N\ e\ a]\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } [[N\ b\ e]\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } [N\ e\ \gamma]_\Gamma \subseteq \Delta\\
&\qquad \Rightarrow [[e\ e\ a]\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } [[e\ b\ e]\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } [e\ e\ \gamma]_\Gamma \in \Delta\\
&\qquad \Rightarrow [a\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } [b\ x\ \Gamma]_\Gamma \subseteq \Delta \text{ or } \gamma \in \Delta, \text{ by Theorem 2.7 (ii).}
\end{aligned}$$

Hence P is a 3-prime ideal of N. ∎

Proposition 4.7. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Let Δ be an N-ideal of a right-lateral ternary N-group. Then $Q = (\Delta : \Gamma) = \{x \in N \mid [x\ n\ \gamma]_\Gamma \in \Delta$ and $[n\ x\ \gamma]_\Gamma \in \Delta,\ \forall\ \gamma \in \Gamma\}$ is an ideal of N.*

Proof. Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Then N is a right and lateral ternary near-ring and hence $[0\ n\ \gamma]_\Gamma = o = [n\ 0\ \gamma]_\Gamma$. Thus $0 \in Q$. As Δ is a subgroup of Γ, Q is a subgroup of N. Let $u \in N$ and $x \in Q$. Then $[u + x - u\ n\ \gamma]_\Gamma = [u\ n\ \gamma]_\Gamma + [x\ n\ \gamma]_\Gamma - [u\ n\ \gamma]_\Gamma$ and $[n\ u + x - u\ \gamma] = [n\ u\ \gamma]_\Gamma + [n\ x\ \gamma]_\Gamma - [n\ u\ \gamma]_\Gamma\ \ \forall\ \ \gamma \in \Gamma$ are in Q, showing that Q is normal in N. Let $H = [Q\ N\ N]$. Then $h = [x\ w\ v]$ where $x \in Q$ and $w, v \in N$. Now $[h\,n\,\gamma]_\Gamma = [x\,w\,v\,n\,\gamma]_\Gamma = [x\,w\,\gamma']_\Gamma \in \Delta$ and $[n\,h\,\gamma]_\Gamma = [n\,x\,w\,v\,\gamma]_\Gamma = [n\,x\,\gamma']_\Gamma \in \Delta$. Thus Q is a right ideal of N.

Since $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$ and $[Q\ N\ N] \subseteq Q$ it follows that $[N\ Q\ N] \subseteq Q$. Thus Q is a lateral ideal of N. For $u, v, w \in N$ and $x \in Q$

$$\begin{aligned}
&[[u\ v\ w + x] - [u\ v\ w]\ n\ \gamma]_\Gamma\\
&\qquad = [u\ v\ [w + x\ n\ \gamma]_\Gamma]_\Gamma - [u\ v\ w\ n\ \gamma]_\Gamma\\
&\qquad = [u\ v\ [w\ n\ \gamma]_\Gamma + [x\ n\ \gamma]_\Gamma]_\Gamma - [u\ v\ [w\ n\ \gamma]_\Gamma]_\Gamma \in \Delta,
\end{aligned}$$

as Δ is an ideal of ${}_N\Gamma$. Also

$$\begin{aligned}
[n\ ([u\ v\ w + x] - [u\ v\ w])\ \gamma]_\Gamma &= [[n\ u\ v]\ w + x\ \gamma]_\Gamma - [n\ u\ v\ w\ \gamma]_\Gamma\\
&= [[n\ u\ v]\ w\ \gamma]_\Gamma + [[n\ u\ v]\ x\ \gamma]_\Gamma - [[n\ u\ v]\ w\ \gamma]_\Gamma \in \Delta,
\end{aligned}$$

as $x \in Q$ and Δ is normal in Γ. Thus Q is an ideal of N. ∎

In the following lemma, the interrelationship between Δ and the ideal quotient Q is given.

Lemma 4.8. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Let Δ be an N-ideal of a right-lateral ternary N-group. Then*

(i) Δ *is* 2*-prime* $\Rightarrow Q$ *is* 2*-prime*

(ii) Δ *is* 3*-prime* $\Rightarrow Q$ *is* 3*-prime*

(iii) Δ *is c-prime* $\Rightarrow Q$ *is c-prime*

(iv) Δ *is e-prime* $\Rightarrow Q$ *is e-prime*

(v) Δ *is* 0*-prime* $\Rightarrow Q$ *is* 0*-prime if* ${}_N\Gamma$ *is monogenic.*

Proof.

(i) Let A, B, C be N-subgroups of N such that $[A\ B\ C] \subseteq Q$. If either $A \subseteq Q$ or $B \subseteq Q$ or $C \subseteq Q$ then (i) follows. Suppose $[A\ x\ \Gamma]_\Gamma \nsubseteq Q$, $[B\ x\ \Gamma]_\Gamma \nsubseteq Q$ and $[C\ x\ \Gamma]_\Gamma \nsubseteq Q$. Since $[[A\ B\ C]\ x\ \Gamma]_\Gamma \subseteq \Delta$, $[A\ B\ [C\ x\ \gamma]\ \Gamma]_\Gamma \subseteq \Delta\ \forall\ \gamma \in \Gamma$ and $x \in N$. Also as Δ is 2-prime either $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$, or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[C\ x\ \gamma]_\Gamma \subseteq \Delta\ \forall\ \gamma \in \Gamma$ which contradicts the assumption. Hence Q is 2-prime.

(ii) Let $a, b, c \in N$ be such that $[a\ N\ b\ N\ c] \subseteq Q$. If $a \in Q$ or $b \in Q$ or $c \in Q$ then (ii) follows. Suppose not. Also $[a\ N\ b\ N\ [c\ x\ \gamma]_\Gamma]_\Gamma \in \Delta \Rightarrow [a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[c\ x\ \gamma]_\Gamma \in \Delta\ \ \forall\ \ x \in N, \gamma \in \Gamma$. $\Rightarrow a \in Q$ or $b \in Q$ or $c \in Q$, a contradiction to the assumption and hence Q is 3-prime.

(iii) Let $a, b, c \in N$ be such that $[a\ b\ c] \in Q$. Suppose $a \notin Q$, $b \notin Q$ and $c \notin Q$. Then for $x \in N$, $[a\ x\ \Gamma]_\Gamma \nsubseteq \Delta$, $[b\ x\ \Gamma]_\Gamma \nsubseteq \Delta$ and $[c\ x\ \Gamma]_\Gamma \nsubseteq \Delta$.

Now $[a\ b\ c] \in Q \Rightarrow [a\ b\ c\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ b\ c\ x\ \gamma]_\Gamma \in \Delta\ \forall\ \gamma \in \Gamma \Rightarrow [a\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[b\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[c\ x\ \gamma]_\Gamma \subseteq \Delta$ which is a contradiction to the assumption. Hence Q is c-prime.

(iv) Let for $a, x, y \in N$ with $a \notin Q$, $[a\ n\ x] - [a\ n\ y] \in Q\ \forall\ n \in N$.
$\Rightarrow [[a\ n\ x] - [a\ n\ y]\ t\ \gamma]_\Gamma \in \Delta,\ \forall\ \ t \in N \Rightarrow [a\ n\ [x\ t\ \gamma]_\Gamma]_\Gamma - [a\ n[y\ t\ \gamma]_\Gamma]_\Gamma \in \Delta$
$\Rightarrow [a\ n\ \gamma_1]_\Gamma - [a\ n\ \gamma_2]_\Gamma \in \Delta$ where $\gamma_1 = [x\ t\ \gamma]_\Gamma, \gamma_2 = [y\ t\ \gamma]_\Gamma$.
$\Rightarrow \gamma_1 - \gamma_2 \in \Delta$ as $[a\ x\ \Gamma]_\Gamma \nsubseteq \Delta \Rightarrow [x\ t\ \gamma]_\Gamma - [y\ t\ \gamma]_\Gamma \in \Delta$
$\Rightarrow [x - y\ t\ \gamma]_\Gamma \in \Delta \Rightarrow x - y \in Q$.
Hence Q is an e-prime ideal of N.

(v) Let A, B, C be ideals of N such that $[A\ B\ C] \subseteq Q$ if either $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[B\ x\ \Gamma]_\Gamma \subseteq \Delta$ or $[C\ x\ \Gamma]_\Gamma \subseteq \Delta$ then there is nothing to prove. Suppose $A \nsubseteq Q$, $B \nsubseteq Q$ and $C \nsubseteq Q$. Since Γ is monogenic there exists $\gamma_0 \in \Gamma$ such that $[N\ x\ \gamma_0]_\Gamma = \Gamma$. Since $[C\ x\ \Gamma]_\Gamma \nsubseteq \Delta$, $[C\ x\ \gamma_0]_\Gamma \nsubseteq \Delta$. Since $[C\ x\ \gamma_0]_\Gamma$ is an ideal of Γ with $[C\ x\ \gamma_0]_\Gamma \nsubseteq \Delta$ and $[A\ x\ \Gamma]_\Gamma \nsubseteq \Delta$ and $[B\ x\ \Gamma]_\Gamma \nsubseteq \Delta$ and Δ is 0-prime it follows that $[A\ B\ [C\ x\ \gamma_0]_\Gamma]_\Gamma \nsubseteq \Delta$. This implies that $[A\ B\ C] \nsubseteq Q$ which contradicts the assumption. Thus Q is 0-prime. ∎

Proposition 4.9. *Let N be a zero-symmetric RTNR such that $[x\ y\ z] = [y\ x\ z]$ $\forall x, y, z \in N$ and let the right lateral ternary N-group ${}_N\Gamma$ be monogenic. Then an N-ideal Δ of ${}_N\Gamma$ is an IFP N-ideal of ${}_N\Gamma$.*

Proof. Let $a, x \in N$ and $\gamma \in \Gamma$ be such that $[a\ x\ \gamma]_\Gamma \in \Delta$. Since ${}_N\Gamma$ is monogenic there exists $\gamma_0 \in \Gamma$ and $u \in N$ such that $[N\ y\ \gamma_0]_\Gamma = \Gamma \Rightarrow \gamma = [n\ y\ \gamma_0]_\Gamma$ for some $n \in N$. Now $[a\ u\ x\ v\ \gamma]_\Gamma = [a\ u\ x\ v\ n\ y\ \gamma_0]_\Gamma = [u\ a\ v\ x\ n\ y\ \gamma_0]_\Gamma = [u\ a\ v\ x\ \gamma]_\Gamma = [u\ v\ [a\ x\ \gamma]_\Gamma]_\Gamma \in [N\ N\ \Delta]_\Gamma \subseteq \Delta$. Thus $[a\ x\ \gamma]_\Gamma \in \Delta \Rightarrow [a\ u\ x\ v\ \gamma]_\Gamma \in \Delta\ \forall u, v \in N$. Hence Δ is an IFP N-ideal of ${}_N\Gamma$. ∎

Theorem 4.10. *Let N be a zero-symmetric RTNR such that $[x\ y\ z] = [y\ x\ z]$ $\forall\ x, y, z \in N$. Let Δ be an N-ideal of right-lateral monogenic ternary N-group ${}_N\Gamma$. Then Δ is e-prime ideal iff Δ is c-prime ideal of ${}_N\Gamma$.*

Proof. If Δ is e-prime, by Theorem 4.5 (i) and (viii) and the above proposition Δ is c-prime.

Conversely let Δ be c-prime. Let $a \in N$ with $[a\ x\ \Gamma]_\Gamma \nsubseteq \Delta$ and $\gamma_1, \gamma_2 \in \Gamma$ be such that $[a\ n\ \gamma_1]_\Gamma - [a\ n\ \gamma_2]_\Gamma \in \Delta\ \forall\ n \in N$. Since Γ is monogenic there exists $\gamma_0 \in \Gamma$ such that $[N\ x\ \gamma_0]_\Gamma = \Gamma \Rightarrow u, v \in N$ such that $\gamma_1 = [u\ x\ \gamma_0]_\Gamma$ and $\gamma_2 = [v\ x\ \gamma_0]_\Gamma$. Consider, $[a\ n\ \gamma_1]_\Gamma - [a\ n\ \gamma_2]_\Gamma = [([n\ a\ u] - [n\ a\ v])\ x\ \gamma_0]_\Gamma \in \Delta \Rightarrow [([n\ a\ u] - [n\ a\ v])\ y\ \Gamma]_\Gamma \subseteq \Delta$ or $\gamma_0 \in \Delta\ \forall\ y \in N$ by the hypothesis.

If $\gamma_0 \in \Delta$ then $\Gamma = [N\ x\ \gamma_0]_\Gamma \subseteq [N\ N\ \Delta]_\Gamma \subseteq \Delta$ which is not true as $[N\ N\ \Gamma]_\Gamma \nsubseteq \Delta$. Thus $[[n\ a\ u] - [n\ a\ v]\ y\ \gamma]_\Gamma \subseteq \Delta \Rightarrow [n\ a\ u] - [n\ a\ v] \in (\Delta : \Gamma)$.

Also since $(\Delta : \Gamma) \neq N$ there exists $q \in N$ and $q \notin (\Delta : \Gamma)$. Consider $[[n\ a\ u] - [n\ a\ v]\ z\ q] \in (\Delta : \Gamma) \Rightarrow [[n\ u\ z] - [n\ v\ z]\ a\ q] \in (\Delta : \Gamma)$, by the given condition.

Since Δ is c-prime by Lemma 4.8 (iii) $(\Delta : \Gamma)$ is c-prime.

Hence $[n\ u\ z] - [n\ v\ z] \in (\Delta : \Gamma)$ or $a \in (\Delta : \Gamma)$ or $q \in (\Delta : \Gamma)$. Since $q \notin (\Delta : \Gamma)$ and $[a\ x\ \Gamma]_\Gamma \nsubseteq \Delta$, $[n\ u\ z] - [n\ v\ z] \in (\Delta : \Gamma)$. This implies that $[[n\ u\ z]\ t\ q] - [[n\ v\ z]\ t\ q] \in (\Delta : \Gamma)\ \forall\ t \in N \Rightarrow [u\ n\ z\ t\ q] - [v\ n\ z\ t\ q] \in (\Delta : \Gamma) \Rightarrow [u - v\ n\ z\ t\ q] \in (\Delta : \Gamma) \Rightarrow u - v \in (\Delta : \Gamma)$ or $[n\ z\ t] \in (\Delta : \Gamma)$ or $q \in (\Delta : \Gamma)$. If $[n\ z\ t] \subseteq (\Delta : \Gamma)$ then $N \subseteq (\Delta : \Gamma)$ which is not true. Also $q \notin (\Delta : \Gamma)$. Hence $u - v \in (\Delta : \Gamma) \Rightarrow [u - v\ x\ \gamma_0]_\Gamma \in \Delta \Rightarrow [u\ x\ \gamma_0]_\Gamma - [v\ x\ \gamma_0]_\Gamma \in \Delta \Rightarrow \gamma_1 - \gamma_2 \in \Delta$. Thus Δ is e-prime. Hence the proof. ∎

Proposition 4.11. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Let Δ be an IFP N-ideal of a right-lateral ternary N-group ${}_N\Gamma$. Then $(\Delta : \Gamma)$ is an IFP ideal of N.*

Proof. Let $[a\ b\ c] \in (\Delta : \Gamma)$ for $a, b, c \in N$.
$\Rightarrow [[a\ b\ c]\ x\ \gamma]_\Gamma \in \Delta\ \forall\ \gamma \in \Gamma,\ x \in N \Rightarrow [a\ u\ b\ v\ [c\ x\ \gamma]_\Gamma]_\Gamma \in \Delta\ \forall\ u, v \in N$
$\Rightarrow [[a\ u\ b\ v\ c]\ x\ \gamma]_\Gamma \in \Delta \Rightarrow [a\ u\ b\ v\ c] \in (\Delta : \Gamma)$.
Thus $(\Delta : \Gamma)$ is an IFP ideal of N. ∎

Lemma 4.12. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Let Δ be an N-ideal of right-lateral monogenic ternary N-group such that $Q = (\Delta : \Gamma)$ is 3-prime. Then Δ is 3-prime in ${}_N\Gamma$.*

Proof. For $a, b \in N$ and $\gamma \in \Gamma$ let $[a\ N\ b\ N\ \gamma]_\Gamma \subseteq Q$. Suppose Δ is not 3-prime. Then $a \notin Q$, $b \notin Q$ and $\gamma \notin \Delta$. Since ${}_N\Gamma$ is monogenic there exists $\gamma_0 \in \Gamma$, $x \in N$ such that $[N\ x\ \gamma_0]_\Gamma = \Gamma \Rightarrow \gamma = [n\ x\ \gamma_0]_\Gamma$ for some $n \in N$. Since $\gamma \notin \Delta$, $[n\ x\ \gamma_0]_\Gamma \notin \Delta \Rightarrow n \notin (\Delta : \Gamma)$. Also since $a \notin Q$, $b \notin Q$ and $n \notin Q$,

$$\begin{aligned}[a\ N\ b\ N\ n] \nsubseteq Q &\Rightarrow [a\ N\ b\ N\ n] \nsubseteq [\Delta : [N\ x\ \gamma_0]_\Gamma]\\ &\Rightarrow [a\ N\ b\ N\ n\ t\ [u\ x\ \gamma_0]_\Gamma]_\Gamma \nsubseteq \Delta \text{ for all } u \in N\\ &\Rightarrow [a\ N\ b\ N[t\ u\ \gamma]_\Gamma]_\Gamma \nsubseteq \Delta, \text{ as } [x\ y\ z] = [y\ x\ z]\\ &\Rightarrow [a\ N\ b\ N\ \gamma]_\Gamma \nsubseteq Q,\end{aligned}$$

which contradicts the hypothesis. Hence Δ is 3-prime in ${}_N\Gamma$. ∎

Corollary 4.13. *If N is an RTNR such that $[x\ y\ z] = [y\ x\ z]$ $\forall$ $x, y, z \in N$ and Δ is an ideal of right-lateral monogenic N-group such that $(\Delta : \Gamma)$ is c-prime then Δ is 3-prime in ${}_N\Gamma$.*

Proof. If Q is c-prime then by Lemma 3.19 Q is 3-prime and hence by the above lemma the corollary follows. ∎

5. Semiprime ideals in right ternary N-groups

In this section, ν-semiprime ideals of a right ternary N-group are defined where $\nu \in \{0, 1, 2, 3, c\}$ and their properties are discussed.

Definition 5.1. Let N be an RTNR and Δ be an N-ideal of a right ternary N-group ${}_N\Gamma$ such that $[N\ N\ \Gamma]_\Gamma \nsubseteq \Delta$. Then

(a) if $[A^3\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow [A\ x\ \Gamma]_\Gamma \subseteq \Delta$ $\forall$ $x \in N$ holds for all

- (i) ideals A of N then Δ is 0-semiprime.
- (ii) left ideals A of N then Δ is 1-semiprime.
- (iii) N-subgroups A of N then Δ is 2-semiprime.

(b) Δ is 3-semiprime if $[a\ N\ a\ N\ \gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ x\ \gamma]_\Gamma \in \Delta$ $\forall$ $a, x \in N$ and $\gamma \in \Gamma$

(c) Δ is c-semiprime if $[a\ a\ [a\ a\ \gamma]_\Gamma]_\Gamma \in \Delta \Rightarrow [a\ x\ \gamma]_\Gamma \in \Delta$ $\forall$ $x, a \in N$, $\gamma \in \Gamma$.

Definition 5.2. A right ternary N-group ${}_N\Gamma$ is called ν-semiprime ($\nu = 0, 1, 2, 3, c$) if $[N\ N\ \Gamma]_\Gamma \neq \{o\}$ and $\{o\}$ is ν-semiprime ideal of ${}_N\Gamma$.

Lemma 5.3. *Let N be an RTNR and ${}_N\Gamma$ be a right ternary N-group. Then every 0-prime N-ideal of ${}_N\Gamma$ is 0-semiprime N-ideal of ${}_N\Gamma$.*

Proof. Let A be an ideal of N such that $[A^3\ x\ \Gamma]_\Gamma \subseteq \Delta$. Suppose $[A\ x\ \Gamma]_\Gamma \nsubseteq \Delta$ $\forall$ $x \in N$. Then there exists $\gamma \in \Gamma$ and $\gamma \notin \Delta$ such that $[A\ x\ \gamma]_\Gamma \nsubseteq \Delta$.

Since $[A\ x\ \gamma]_\Gamma$ is an N-ideal of ${}_N\Gamma$ and $[A\ A\ [A\ x\ \gamma]_\Gamma]_\Gamma \subseteq \Delta$ and Δ is 0-prime, $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ which contradicts the assumption. Hence ${}_N\Gamma$ is 0-semiprime N-ideal of ${}_N\Gamma$. ∎

Remark 5.4. Using a similar argument given in the proof of the above lemma it can be easily proved that every ν-prime N-ideal of a right ternary N-group is ν-semiprime N-ideal if $\nu = 1, 2$. The case for $\nu = 3$, c follows from the definition ν-prime and ν semiprime N-ideal of ${}_N\Gamma$.

Proposition 5.5. *Let Δ be an N-ideal of a ternary N-group ${}_N\Gamma$. Then*

- (i) Δ *is c-semiprime* $\Rightarrow$ Δ *is 3-semiprime*
- (ii) Δ *is 3-semiprime* $\Rightarrow$ Δ *is 2-semiprime*
- (iii) Δ *is 2-semiprime* $\Rightarrow$ Δ *is 1-semiprime*
- (iv) Δ *is 1-semiprime* $\Rightarrow$ Δ *is 0-semiprime if N is a zero-symmetric RTNR.*

Proof.

(i) Let Δ be a c-semiprime N-ideal. Let $a \in N$ and $\gamma \in \Gamma$ be such that $[a\ N\ a\ N\ \gamma]_\Gamma \subseteq \Delta$. Then $[a\ a\ [a\ a\ \gamma]_\Gamma]_\Gamma \in \Delta$. Since Δ is c-semiprime, $[a\ x\ \gamma]_\Gamma \in \Delta\ \forall\ x \in N$. Thus Δ is 3- semiprime.

(ii) Suppose Δ is 3-semiprime. Let A be an N-subgroup of N such that $[A^3\, x\, \Gamma] \subseteq \Delta$. Then $[a^3\ x\ \gamma]_\Gamma \in \Delta\ \forall\ a \in A$, $\gamma \in M$.

Consider $[a\ N\ a\ N\ \gamma]_\Gamma \subseteq [A\ N\ A\ N\ \Gamma]_\Gamma \subseteq [A\ A\ \Gamma]_\Gamma \subseteq \Delta$. Since Δ is 3-semiprime this implies that $[a\ x\ \gamma]_\Gamma \in \Delta\ \forall\ x \in N$. Hence $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$. Thus Δ is 2-semiprime.

(iii) Suppose Δ is 2-semiprime. Let A be a left ideal of N such that $[A^3\ x\ \Gamma]_\Gamma \subseteq \Delta$. Suppose $[A\ x\ \Gamma]_\Gamma \nsubseteq \Delta\ \forall\ x \in N$. Then there exists $\gamma \in \Gamma$ such that $[A\ x\ \gamma]_\Gamma \nsubseteq \Delta$. Since A is left ideal of N then a is a left N-subgroup of N and hence $[A\ x\ \gamma]_\Gamma$ is an N-subgroup of $_\Gamma$. Also $[A\ A\ [A\ x\ \gamma]_\Gamma]_\Gamma \subseteq \Delta$ and as Δ is 2-semiprime $\Rightarrow [A\ x\ \Gamma]_\Gamma \subseteq \Delta$ which contradicts the assumption. Hence Δ is 1-semiprime.

(iv) Let Δ be 1-semiprime. Let A be an ideal of N such that $[A^3\ x\ \Gamma]_\Gamma \subseteq \Delta$ $\forall\ x \in N$. Suppose $[A\ x\ \Gamma]_\Gamma \nsubseteq\ \forall\ x \in N$. Then there exists $\gamma \in \Gamma$ such that $[A\ x\ \gamma]_\Gamma \nsubseteq \Delta$. Since $[A\ x\ \gamma]_\Gamma$ is a left ideal of $_N\Gamma$ and $[A\ A\ [A\ x\ \gamma]_\Gamma]_\Gamma \subseteq \Delta$ and Δ is 1-semiprime, $[A\ x\ \Gamma]_\Gamma \subseteq \Delta$ which contradicts the assumption. Hence Δ is 0-semiprime. ∎

Proposition 5.6. *If Δ is an IFP N-ideal of $_N\Gamma$ then Δ is 3-semiprime $\Rightarrow \Delta$ is c-semiprime.*

Proof. Let $a \in N$ and $\gamma \in \Gamma$ be such that $[a^3\ x\ \Gamma]_\Gamma \in \Delta \Rightarrow [a^3\ u\ x\ v\ \gamma]_\Gamma \in \Delta$ for every $u, v \in N$. Thus $[a^3\ N\ x\ N\ \gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ [m\ a\ n\ a\ u]\ x\ v\ x\ \gamma]_\Gamma \subseteq \Delta$ or $[a\ N\ a\ N\ \gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow [a\ x\ \gamma]_\Gamma \in \Delta$. Hence Δ is c-semiprime. ∎

Lemma 5.7. *Let N be an RTNR such that $[x\ y\ z] = [y\ x\ z]\ \forall\ x, y, z \in N$. Let Δ be an N-ideal of a right ternary N-group of $_N\Gamma$. Let $Q = (\Delta : \Gamma)$. Then*

(i) *Δ is a 0-semiprime N-ideal of $_N\Gamma \Leftrightarrow Q$ is a 0-semiprime ideal of N*

(ii) *Δ is a 1-semiprime N-ideal of $_N\Gamma \Leftrightarrow Q$ is a 1-semiprime ideal of N*

(iii) *Δ is a 2-semiprime N-ideal of $_N\Gamma \Leftrightarrow Q$ is a 2-semiprime ideal of N*

(iv) *If Δ is a 3-semiprime N-ideal of $_N\Gamma$ then $Q = (\Delta : \Gamma)$ is a 3-semiprime ideal of N.*

(v) *If Δ is a c-semiprime N-ideal of $_N\Gamma$ then Q is a c-semiprime ideal of $_N\Gamma$.*

Proof. (i) Let A be an ideal of N such that $A^3 \subseteq Q \Rightarrow [A^3\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow$ $[A\ A\ [A\ x\ \Gamma]_\Gamma]_\Gamma \subseteq \Delta$. Since Δ is 0-semiprime $[A\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow A \subseteq Q$. Conversely let A be an ideal of N such that $[A^3\ x\ \Gamma]_\Gamma \subseteq \Delta\ \forall\ x \in N$, $A^3 \subseteq Q$. Since A is 0-semiprime ideal $A \subseteq Q$. This implies that $[A\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow \Delta$ is 0-semiprime.

(ii) and (iii) A similar argument as in (i) can be used to prove (ii) and (iii).

(iv) Let $a \in N$ be such that $[a\,N\,a\,N\,a] \subseteq Q$. Then $[a\,N\,a\,N\,a\,x\,\gamma]_\Gamma \subseteq \Delta$ $\forall\, x \in N$ and $\gamma \in \Gamma$. This implies that $[a\ N\ a\ N\ \gamma']_\Gamma \subseteq \Delta\ \forall\ \gamma' \in \Gamma$. Since Δ is 3-semiprime, $[a\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow a \in Q$. Hence Q is 3-semiprime ideal of N.

(v) Let $a \in N$ be such that $a^3 \in Q$. Then $[a^3\ x\ \gamma]_\Gamma \in \Delta\ \forall\ x \in N$ and $\gamma \in \Gamma$. Now by hypothesis $[a\,x\,\gamma]_\Gamma \in \Delta\ \forall\ \gamma \in \Gamma$. This implies that $[a\ x\ \Gamma]_\Gamma \subseteq \Delta \Rightarrow a \in Q$. Thus Q is a c-semiprime ideal of N. ∎

6. Conclusion

In this paper different prime idals in RTNR and right ternary N-groups were defined and their structural properties were investigated. Semiprime ideals were also defined in both the contexts. The interconnection between $(\Delta : \Gamma)$ and Δ were discussed. For future work radicals of an RTNR may be considered and their properties can be studied. Furthermore multiplication modules as in the binary case may be defined in the ternary context and the converse of the results involving $(\Delta : \Gamma)$ may be explored.

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