ON FUZZY SOFT GRAPHS

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Abstract. Fuzzy sets and soft sets are two different soft computing models for representing vagueness and uncertainty. We apply these soft computing models in combination to study vagueness and uncertainty in graphs. We introduce the notions of fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, regular fuzzy soft graphs, and investigate some of their properties.

Keywords and phrases: fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, regular fuzzy soft graphs.
2000 Mathematics Subject Classification: 05C72, 05C99.

1. Introduction

Molodtsov [25] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. It has been demonstrated that soft sets have potential applications in various fields such as the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory [25], [27]. Since then research on soft sets has been very active and received much attention from researchers worldwide. Feng et al. [16], [18] combined soft sets with rough sets and fuzzy sets, obtaining three types of hybrid models: rough soft sets, soft rough sets, and soft-rough fuzzy sets. Ali et al. [8] discussed the fuzzy sets and fuzzy soft sets induced by soft sets. To extend the expressive power of soft sets, Jiang et al. [20] presented ontology-based soft sets, which extended soft sets with description logics. Ali et al. [9] proposed several new operations in soft set theory. Gong et al. [19] initiated the concept of bijective soft sets. Babitha and Sunil [10] extend the concepts of relations and functions in the context of soft set theory. Moreover, Maji et al. [24] presented the definition of fuzzy soft sets and Roy et al. [30] presented some applications of this notion to decision making problems.

Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Kaufmann's initial definition of a fuzzy graph [21] was based on Zadeh's fuzzy relations [32]. Bhattacharya [11] gave some remarks on fuzzy graphs. Mordeson and Peng [26] defined the concept of complement of fuzzy graph and studied some operations on fuzzy graphs. Akram *et al.* [1]-[6] introduced many new concepts, including bipolar fuzzy graphs, strong intuitionistic fuzzy graphs, intuitionistic fuzzy hypergraphs, and intuitionistic fuzzy trees. Thumbakara and George [31] discussed the concept of soft graphs in the specific way. On the other hand, Akram and Nawaz [7] have introduced the concepts of soft graphs and vertex-induced soft graphs in broad spectrum. In this paper, we introduce the notions of fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, regular fuzzy soft graphs, and investigate some of their properties.

2. Preliminaries

First we review some definitions which can be found in [32, 24, 28, 31]. By a *graph*, we mean a pair $G^* = (V, E)$, where V is the set and E is a relation on V. The elements of V are vertices of G^* and the elements of E are edges of G^*. We call $V(G^*)$ the vertex set and $E(G^*)$ the edge set of G^*. A *fuzzy set* A on a set V is characterized by its membership function $\mu_A : V \to [0, 1]$, where $\mu_A(u)$ is degree of membership of element u in fuzzy set A for each $u \in V$. A *fuzzy relation* on V is a fuzzy subset of $V \times V$. A *fuzzy relation* ν on V is a fuzzy relation on μ if $\nu(u, v) \leq \mu(u) \wedge \mu(v)$ for all u, v in V. A *fuzzy graph* $G' = (\mu, \nu)$ is a pair of functions $\mu : V \to [0, 1]$ and $\nu : V \times V \to [0, 1]$, where for all $u, v \in V$, we have $\nu(u, v) \leq \mu(u) \wedge \mu(v)$. The underlying crisp graph of a fuzzy graph $G' = (\mu, \nu)$ is denoted by $G'^* = (\mu^*, \nu^*)$, where $\mu^* = \{u \in V : \mu(u) > 0\}$ and $\nu^* = \{(u, v) \in V \times V : \nu(u, v) > 0\}$. The *strength of connectedness* between two nodes u, v is defined as the maximum of strengths of all paths between u and v and is denoted by $\mathrm{CONNG}(u, v)$. A fuzzy graph G' is *connected* if $\mathrm{CONNG}(u, v) > 0$ for all $u, v \in V$. The fuzzy graph $H = (\tau, \rho)$ is called a *fuzzy subgraph* of $G' = (\mu, \nu)$ if $\tau(u) \leq \mu(u)$ for all $u \in V$ and $\rho(u, v) \leq \nu(u, v)$ for all $u, v \in V$. A fuzzy graph $G' = (\mu, \nu)$ is a *strong* if $\nu(u, v) = \mu(u) \wedge \mu(v)$ for all $(u, v) \in E$ and is a *complete fuzzy graph* if $\nu(u, v) = \mu(u) \wedge \mu(v)$ for all $u, v \in V$. The *order* of fuzzy graph G is $O(G') = \sum_{u \in V} \mu(u)$. The *size* of fuzzy graph G is $S(G') = \sum_{(u,v) \in E} \nu(u, v)$. The *complement* of a fuzzy graph $G' = (\mu, \nu)$ is a fuzzy graph $\bar{G}' = (\bar{\mu}, \bar{\nu})$ where $\bar{\mu} = \mu$ and $\bar{\nu}(u, v) = \mu(u) \wedge \mu(v) - \nu(u, v)$ for all $u, v \in V$. The degree of a vertex u in fuzzy graph $G' = (\mu, \nu)$ is $\deg_{G'}(u) = \deg(u) = \sum_{u \neq v} \mu(u, v) = \sum_{uv \in E} \mu(u, v)$. A fuzzy graph $G' = (\mu, \nu)$ is said to be a *regular* if every vertex which is adjacent to vertices having same degrees.

Definition 2.1 [25] A pair $\mathfrak{S} = (F, A)$ is called a *soft set* over U, where $A \subseteq P$ is a parameter set and $F : A \to \mathscr{P}(U)$ is a set-valued mapping, called the *approximate function* of the soft set $\mathfrak{S}$. In other words, a *soft set* over U is a parameterized family of subsets of U. For any $\epsilon \in A$, $F(\epsilon)$ may be considered as set of ϵ-approximate elements of soft set (F, A).

Maji et al. [24] defined the fuzzy soft set in the following way.

Definition 2.2 Let U be an initial universe, P the set of all parameters, $A \subset P$ and $\mathcal{P}(U)$ the collection of all fuzzy subsets of U. Then $(\widetilde{F}, A)$ is called *fuzzy soft set*, where $\widetilde{F} : A \to \mathcal{P}(U)$ is a mapping, called *fuzzy approximate function* of the fuzzy soft set $(\widetilde{F}, A)$.

Definition 2.3 [14] Let $(\widetilde{F}_1, A)$ and $(\widetilde{F}_2, B)$ be two fuzzy soft sets over a common universal set U. Then a relation R of $(\widetilde{F}_1, A)$ to $(\widetilde{F}_2, B)$ can be defined as a fuzzy approximate function $R : A \times B \to P(U^2)$ such that $e_i \in A$, $e_j \in B$ and for all $u_p \in F_1(e_i)$, $u_q \in F_2(e_j)$, the relation R is characterized by the following membership function,

$$\nu_R(u_1, u_k) = \mu_{F_1(e_i)}(u_1) \times \mu_{F_2(e_j)}(u_k),$$

where $u_1 \in F_1(e_i)$, $u_k \in F_2(e_j)$.

3. Fuzzy soft graphs

Definition 2.1 A *fuzzy soft graph* $\widetilde{G} = (G^*, \widetilde{F}, \widetilde{K}, A)$ is a 4-tuple such that

(a) $G^* = (V, E)$ is a simple graph,

(b) A is a nonempty set of parameters,

(c) $(\widetilde{F}, A)$ is a fuzzy soft set over V,

(d) $(\widetilde{K}, A)$ is a fuzzy soft set over E,

(e) $(\widetilde{F}(a), \widetilde{K}(a))$ is a fuzzy (sub)graph of G^* for all $a \in A$. That is,

$$\widetilde{K}(a)(xy) \leq \min\{\widetilde{F}(a)(x), \widetilde{F}(a)(y)\}$$

for all $a \in A$ and $x, y \in V$. The fuzzy graph $(\widetilde{F}(a), \widetilde{K}(a))$ is denoted by $\widetilde{H}(a)$ for convenience.

On the other hand, a fuzzy soft graph is a parameterized family of fuzzy graphs.

The class of all fuzzy soft graphs of G^* is denoted by $\mathcal{F}(G^*)$.

Example 2.2 Consider a simple graph $G^* = (V, E)$ such that

$$V = \{a_1, a_2, a_3\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_1\}.$$

Let $A = \{e_1, e_2, e_3\}$ be a parameter set and $(\widetilde{F}, A)$ be a fuzzy soft set over V with its fuzzy approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ defined by

$$\widetilde{F}(e_1) = \{a_1|0.2, a_2|0.6, a_3|0.8\},$$
$$\widetilde{F}(e_2) = \{a_1|0.1, a_2|0.3, a_3|0.7\},$$
$$\widetilde{F}(e_3) = \{a_1|0.4, a_2|0.5, a_3|0.9\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its fuzzy approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ defined by

$$\widetilde{K}(e_1) = \{a_1a_2|0.1, a_2a_3|0.2, a_3a_1|0.1\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.1, a_2a_3|0.2, a_3a_1|0.1\},$$
$$\widetilde{K}(e_3) = \{a_1a_2|0.4, a_2a_3|0.4, a_3a_1|0.3\}.$$

Thus, $\widetilde{H}(e_1) = (\widetilde{F}(e_1), \widetilde{K}(e_1))$, $\widetilde{H}(e_2) = (\widetilde{F}(e_2), \widetilde{K}(e_2))$ and $\widetilde{H}(e_3) = (\widetilde{F}(e_3), \widetilde{K}(e_3))$ are fuzzy graphs of G^* as shown in Figure 1.

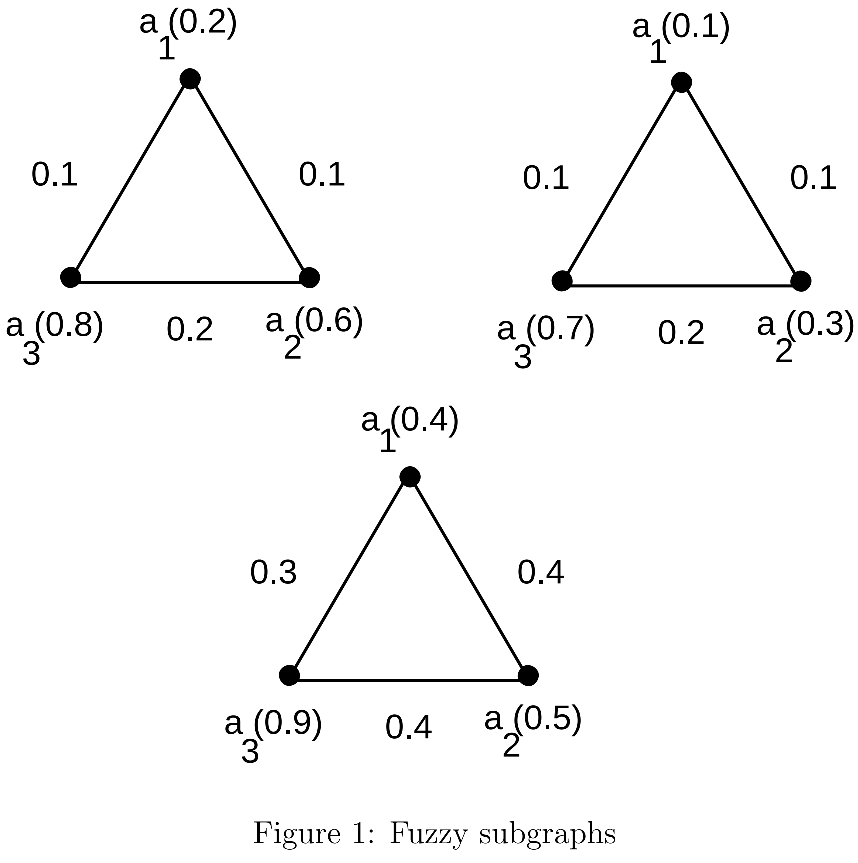

Figure 1: Fuzzy subgraphs

It is easy to verify that $\widetilde{G} = (G^*, \widetilde{F}, \widetilde{K}, A)$ is a fuzzy soft graph.

Example 2.3 Consider a crisp graph $G^* = (V, E)$ such that

$$V = \{a_1, a_2, a_3, a_4, a_5\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_5, a_5a_1, a_2a_5\}.$$

Let $A = \{e_1, e_3, e_5\}$ be a parameter set and $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ defined by

$$\tilde{F}(e_1) = \{a_1|0.5, a_2|0.7, a_3|0.0, a_4|0.0, a_5|0.4\},$$
$$\widetilde{F}(e_3) = \{a_1|0.0, a_2|0.9, a_3|0.8, a_4|0.6, a_4|0.0\},$$
$$\widetilde{F}(e_5) = \{a_1|0.1, a_2|0.5, a_3|0.0, a_4|0.7, a_5|0.8\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its fuzzy approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ defined by

$$\widetilde{K}(e_1) = \{a_1a_2|0.4, a_2a_3|0.0, a_3a_4|0.0, a_4a_5|0.0, a_1a_5|0.2, a_2a_5|0.3\},$$
$$\widetilde{K}(e_3) = \{a_1a_2|0.0, a_2a_3|0.5, a_3a_4|0.6, a_4a_5|0.0, a_5a_1|0.0, a_2a_5|0.0\},$$
$$\widetilde{K}(e_5) = \{a_1a_2|0.1, a_2a_3|0.0, a_3a_4|0.0, a_4a_5|0.6, a_1a_5|0.1, a_2a_5|0.4\}.$$

Thus, the fuzzy subgraphs are,

$$\widetilde{H}(e_1) = (\tilde{F}(e_1), \tilde{K}(e_1)),$$
$$\widetilde{H}(e_3) = (\widetilde{F}(e_3), \widetilde{K}(e_3)),$$
$$\widetilde{H}(e_5) = (\widetilde{F}(e_5), \widetilde{K}(e_5)).$$

It is clear that $\widetilde{H}(e_1)$, $\widetilde{H}(e_3)$ and $\widetilde{H}(e_5)$ are connected fuzzy graphs corresponding to the parameters e_1, e_3, e_5, respectively, as shown in Figure 2.

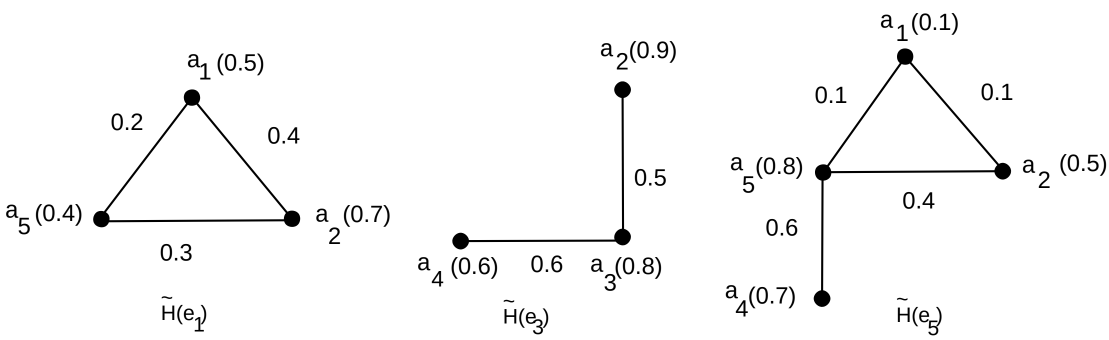

Figure 2: Fuzzy subgraphs $\widetilde{H}(e_1)$,$\widetilde{H}(e_3)$,$\widetilde{H}(e_5)$

Hence, $\widetilde{G} = \{\widetilde{H}(e_1), \widetilde{H}(e_3), \widetilde{H}(e_5)\}$ is a fuzzy soft graph of G^*.

Definition 2.4 The *order* of a fuzzy soft graph is $Ord(\widetilde{G}) = \sum\limits_{e_i \in A}(\sum\limits_{a \in V} \tilde{F}(e_i)(a))$.

Definition 2.5 *The size of a fuzzy soft graph is* $Siz(\widetilde{G}) = \sum\limits_{e_i \in A}(\sum\limits_{ab \in E} \tilde{K}(e_i)(ab))$.

In Example 2.3, the order of fuzzy soft graph is $= \sum\limits_{e_i \in A}(\sum\limits_{a \in V} \tilde{F}(e_i)(a))$ $= \sum\limits_{e_i \in A}(\tilde{F}(e_i)(a_1) + \tilde{F}(e_i)(a_2) + \tilde{F}(e_i)(a_3) + \tilde{F}(e_i)(a_4) + \tilde{F}(e_i)(a_5) = (0.5 + 0.7 + 0.4) + (0.6 + 0.9 + 0.8) + (0.1 + 0.5 + 0.7 + 0.8) = 1.6 + 2.3 + 2.1 = 6.0$

The size of fuzzy soft graph is $= \sum\limits_{e_i \in A}(\sum\limits_{ab \in E} \tilde{K}(e_i)(ab)) = \sum\limits_{e_i \in A}(\tilde{K}(e_i)(a_1a_2) + \tilde{K}(e_i)(a_2a_3) + \tilde{K}(e_i)(a_3a_4) + \tilde{K}(e_i)(a_4a_5) + \tilde{K}(e_i)(a_5a_1) + \tilde{K}(e_i)(a_2a_5)) = (0.4 + 0.3 + 0.2) + (0.5 + 0.6) + (0.1 + 0.4 + 0.1 + 0.6) = 0.9 + 1.1 + 1.2 = 3.2.$

Definition 2.6 A fuzzy soft graph $\widetilde{G}$ is a *strong fuzzy soft graph* if $\tilde{H}(e)$ is a strong fuzzy graph for all $e \in A$, i.e.,

$$\tilde{K}(e)(ab) = \min\{\tilde{F}(e)(a), \tilde{F}(e)(b)\}$$

for all $ab \in E$.

A fuzzy soft graph $\widetilde{G}$ is a *complete fuzzy soft graph* if $\tilde{H}(e)$ is a complete fuzzy graph for all $e \in A$. That is,

$$\tilde{K}(e)(ab) = \min\{\tilde{F}(e)(a), \tilde{F}(e)(b)\}$$

for all $a, b \in V$.

Example 2.7 Consider the crisp graph $G^* = (V, E)$ where

$$V = \{a_1, a_2, a_3, a_4\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1\}.$$

Let $A = \{e_1, e_2\}$ be a parameter set. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\tilde{F} : A \to \mathcal{P}(V)$ given by

$$\tilde{F}(e_1) = \{a_1|0.5, a_2|0.3, a_3|0.2, a_4|0.9\},$$
$$\tilde{F}(e_2) = \{a_1|0.7, a_2|0.5, a_3|0.1, a_4|0.8\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\tilde{K} : A \to \mathcal{P}(E)$ given by

$$\tilde{K}(e_1) = \{a_1a_2|0.3, a_2a_3|0.2, a_3a_4|0.2, a_4a_1|0.5\},$$
$$\tilde{K}(e_2) = \{a_1a_2|0.5, a_2a_3|0.1, a_3a_4|0.1, a_4a_1|0.7\}.$$

It is easy to see that $\tilde{H}(e_1) = (\tilde{F}(e_1), \tilde{K}(e_1))$ and $\tilde{H}(e_2) = (\tilde{F}(e_2), \tilde{K}(e_2))$ are strong fuzzy graphs. Hence $\widetilde{G}$ is a strong fuzzy soft graph of G^* as shown in Figure 3.

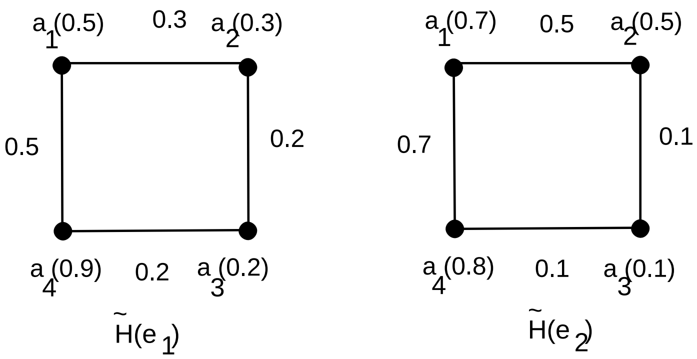

Figure 3: Strong fuzzy soft graph

Example 2.8 Consider the simple graph $G^* = (V, E)$ where

$$V = \{a_1, a_2, a_3, a_4\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1, a_1a_3, a_2a_4\}.$$

Let $A = \{e_1, e_2\}$. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ defined by

$$\widetilde{F}(e_1) = \{a_1|0.5, a_2|0.3, a_3|0.2, a_4|0.9\},$$
$$\widetilde{F}(e_2) = \{a_1|0.4, a_2|0.3, a_3|0.2, a_4|0.7\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ defined by

$$\widetilde{K}(e_1) = \{a_1a_2|0.3, a_2a_3|0.2, a_3a_4|0.2, a_4a_1|0.5, a_1a_3|0.2, a_2a_4|0.3\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.3, a_2a_3|0.2, a_3a_4|0.2, a_4a_1|0.4, a_1a_3|0.2, a_2a_4|0.3\}.$$

It is clear that $\widetilde{H}(e_1) = (\widetilde{F}(e_1), \widetilde{K}(e_1))$ and $\widetilde{H}(e_2) = (\widetilde{F}(e_2), \widetilde{K}(e_2))$ are complete fuzzy graphs. Hence $\widetilde{G}$ is a complete fuzzy soft graph as shown in Figure 4.

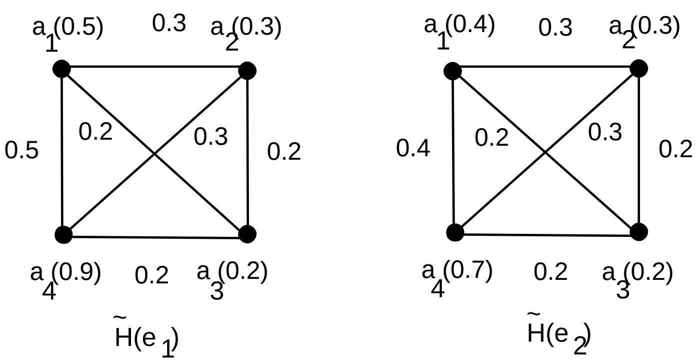

Figure 4: Complete fuzzy soft graph

Definition 2.9 Let $G^* = (V, E)$ be a crisp graph and $\widetilde{G}$ be a fuzzy soft graph of G^*. Then $\widetilde{G}$ is said to be a *regular fuzzy soft graph* if $\widetilde{H}(e)$ is a regular fuzzy graph for all $e \in A$. If $\widetilde{H}(e)$ is a regular fuzzy graph of degree r for all $e \in A$, then $\widetilde{G}$ is a r-regular fuzzy soft graph.

Example 2.10 Consider a crisp graph such that

$$V = \{a_1, a_2, a_3, a_4\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_1\}.$$

Let $A = \{e_1, e_2, e_3, e_4\}$ be a parameter set and let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\begin{aligned}\widetilde{F}(e_1) &= \{a_1|0.3, a_2|0.4, a_3|0.5, a_4|0.2\},\\ \widetilde{F}(e_2) &= \{a_1|0.5, a_2|0.4, a_3|0.6, a_4|0.7\},\\ \widetilde{F}(e_3) &= \{a_1|0.3, a_2|0.5, a_3|0.3, a_4|0.7\},\\ \widetilde{F}(e_4) &= \{a_1|0.5, a_2|0.6, a_3|0.7, a_4|0.8\}.\end{aligned}$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\begin{aligned}\widetilde{K}(e_1) &= \{a_1a_2|0.1, a_2a_3|0.2, a_3a_4|0.1, a_4a_1|0.2\},\\ \widetilde{K}(e_2) &= \{a_1a_2|0.2, a_2a_3|0.4, a_3a_4|0.2, a_4a_1|0.4\},\\ \widetilde{K}(e_3) &= \{a_1a_2|0.2, a_2a_3|0.3, a_3a_4|0.2, a_4a_1|0.3\},\\ \widetilde{K}(e_4) &= \{a_1a_2|0.5, a_2a_3|0.4, a_3a_4|0.5, a_4a_1|0.4\}.\end{aligned}$$

By routine computations, it is easy to see that fuzzy graphs

$$\begin{aligned}\widetilde{H}(e_1) &= (\widetilde{F}(e_1), \widetilde{K}(e_1)),\\ \widetilde{H}(e_2) &= (\widetilde{F}(e_2), \widetilde{K}(e_2)),\\ \widetilde{H}(e_3) &= (\widetilde{F}(e_3), \widetilde{K}(e_3)),\\ \widetilde{H}(e_4) &= (\widetilde{F}(e_4), \widetilde{K}(e_4)),\end{aligned}$$

are regular are shown in Figure 5. Hence $\widetilde{G}$ is a regular fuzzy soft graph of G^*.

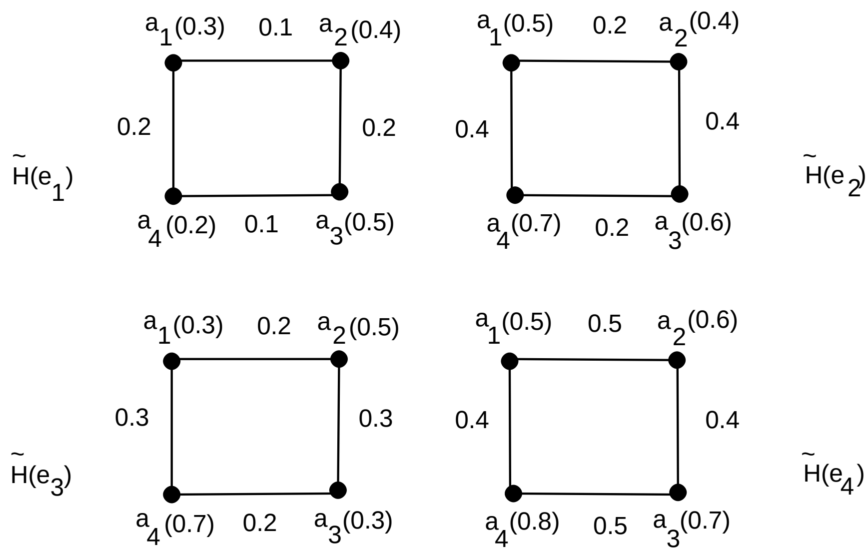

Figure 5: Regular fuzzy soft graph

Definition 2.11 Let $G^* = (V, E)$ be a simple graph and $\widetilde{G}$ be a fuzzy soft graph of G^*. Then $\widetilde{G}$ is said to be a *totally regular fuzzy soft graph* if $\widetilde{H}(e)$ is a totally regular fuzzy graph for all $e \in A$. If $\widetilde{H}(e)$ is a totally regular fuzzy graph of total degree r for all $e \in A$, then $\widetilde{G}$ is called r-totally regular fuzzy soft graph.

Example 2.12 Consider a simple graph $G^* = (V, E)$ where

$$V = \{a_1, a_2, a_3, a_4\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4\}.$$

Let $A = \{e_1, e_2\}$ be a parameter set. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\widetilde{F}(e_1) = \{a_1|0.3, a_2|0.2, a_3|0.2, a_4|0.3\},$$
$$\widetilde{F}(e_2) = \{a_1|0.5, a_2|0.4, a_3|0.5, a_4|0.6\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\widetilde{K}(e_1) = \{a_1a_2|0.1, a_2a_3|0.1, a_3a_4|0.1\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.2, a_2a_3|0.1, a_3a_4|0.1\}.$$

Fuzzy graphs are $\widetilde{H}(e_1) = (\widetilde{F}(e_1), \widetilde{K}(e_1))$ and $\widetilde{H}(e_2) = (\widetilde{F}(e_2), \widetilde{K}(e_2))$ as shown in Figure 6. By routine computations, we have

$$\text{tdeg}(a_1) = 0.4, \quad \text{tdeg}(a_2) = 0.4,$$
$$\text{tdeg}(a_3) = 0.4, \quad \text{tdeg}(a_4) = 0.4,$$

in fuzzy graph $\widetilde{H}(e_1)$, so $\widetilde{H}(e_1)$ is a totally regular fuzzy graph.

Also,

$$\text{tdeg}(a_1) = 0.7, \quad \text{tdeg}(a_2) = 0.7,$$
$$\text{tdeg}(a_3) = 0.7, \quad \text{tdeg}(a_4) = 0.7,$$

in fuzzy graph $\widetilde{H}(e_2)$, so $\widetilde{H}(e_2)$ is a totally regular fuzzy graph. Hence $\widetilde{G}$ is totally regular fuzzy soft graph. But $\deg(a_1) = 0.1$, $\deg(a_2) = 0.2$ in fuzzy subgraph $\widetilde{H}(e_1)$. Since $\deg(a_1) \neq \deg(a_2)$, so $\widetilde{H}(e_1)$ is not regular fuzzy graph. Hence $\widetilde{G}$ is not regular fuzzy soft graph.

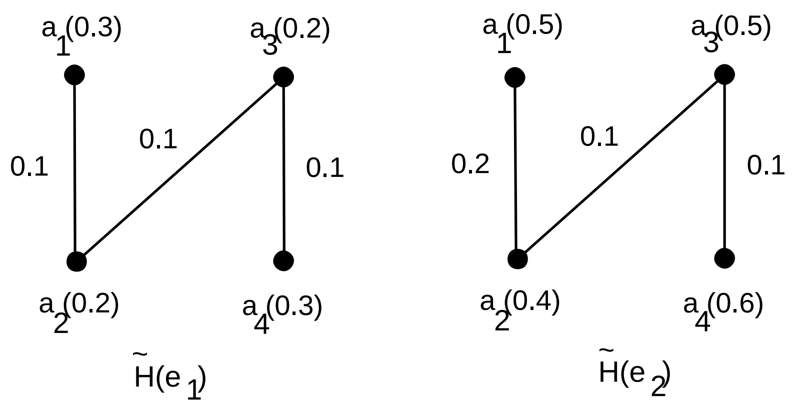

Figure 6: Fuzzy subgraphs

Example 2.13 Consider a simple graph $G^* = (V, E)$ as taken in Example 2.2.

Let $A = \{e_1, e_2, e_3\}$. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its fuzzy approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\begin{aligned}\widetilde{F}(e_1) &= \{a_1|0.5, a_2|0.6, a_3|0.2\},\\ \widetilde{F}(e_2) &= \{a_1|0.2, a_2|0.1, a_3|0.4\},\\ \widetilde{F}(e_3) &= \{a_1|0.5, a_2|0.6, a_3|0.7\}.\end{aligned}$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\begin{aligned}\widetilde{K}(e_1) &= \{a_1a_2|0.2, a_2a_3|0.2, a_3a_1|0.2\},\\ \widetilde{K}(e_2) &= \{a_1a_2|0.1, a_2a_3|0.1, a_3a_1|0.1\},\\ \widetilde{K}(e_3) &= \{a_1a_2|0.5, a_2a_3|0.5, a_3a_1|0.5\}.\end{aligned}$$

By routine calculations, it is easy to see that $\widetilde{H}(e_1)$, $\widetilde{H}(e_2)$ and $\widetilde{H}(e_3)$ are regular fuzzy graphs of G^*. Hence $\widetilde{G}$ is a regular fuzzy soft graph. But $\widetilde{H}(e)$ is not a totally regular fuzzy graph for all $e \in A$. Hence $\widetilde{G}$ is not a totally regular fuzzy soft graph.

Example 2.14 Consider a simple graph $G^* = (V, E)$ where

$$V = \{a_1, a_2, a_3, a_4, a_5\} \text{ and } E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_5, a_5a_1\}.$$

Let $A = \{e_1, e_2\}$ and $(\widetilde{F}, A)$ be a fuzzy soft set over V with its fuzzy approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\begin{aligned}\widetilde{F}(e_1) &= \{a_1|0.5, a_2|0.5, a_3|0.5, a_4|0.5, a_5|0.5\},\\ \widetilde{F}(e_2) &= \{a_1|0.3, a_2|0.3, a_3|0.3, a_4|0.3, a_5|0.3\}.\end{aligned}$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its fuzzy approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\begin{aligned}\widetilde{K}(e_1) &= \{a_1a_2|0.4, a_2a_3|0.4, a_3a_4|0.4, a_4a_5|0.4, a_5a_1|0.4\},\\ \widetilde{K}(e_2) &= \{a_1a_2|0.2, a_2a_3|0.2, a_3a_4|0.2, a_4a_5|0.2, a_5a_1|0.2\}.\end{aligned}$$

Clearly, $\deg(a_i) = 0.8$ in fuzzy graph $\widetilde{H}(e_1)$ and $\deg(a_i) = 0.4$ in fuzzy graph $\widetilde{H}(e_2)$ for $i = 1, 2, 3, 4, 5$. So $\widetilde{G}$ is a regular fuzzy soft graph. Also $\mathrm{tdeg}(a_i) = 1.3$ in $\widetilde{H}(e_1)$ and $\mathrm{tdeg}(a_i) = 0.7$ in $\widetilde{H}(e_2)$ for $i = 1, 2, 3, 4, 5$. Hence $\widetilde{G}$ is totally regular fuzzy soft graph.

We have seen, in the above examples, there is no relationship between regular and totally regular fuzzy soft graph. So we proof the following theorems.

Theorem 2.15 *Let $G^* = (V, E)$ be a simple graph and $\widetilde{G}$ be a fuzzy soft graph of G^*. If $\widetilde{G}$ is a regular fuzzy soft graph and $\widetilde{F}$ is a constant function in fuzzy graph $\widetilde{H}(e_i)$ of G^* for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Then $\widetilde{G}$ is a totally regular fuzzy soft graph.*

Proof. Suppose that $\widetilde{G}$ is a regular fuzzy soft graph and $\widetilde{F}$ is a constant function. Then $\widetilde{F}(e_i)(a) = c_i$, c_i is a constant, $c_i \in [0,1]$, $\forall\, a \in V$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and $\deg(a) = r_i$ in fuzzy graphs $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and $\forall\, a \in V$. Since $\mathrm{tdeg}(a) = \deg(a) + \widetilde{F}(e_i)(a)$. This implies $\mathrm{tdeg}(a) = r_i + c_i$ in fuzzy graphs $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. Hence $\widetilde{G}$ is a totally regular fuzzy soft graph. ∎

Theorem 2.16 *Let $G^* = (V,E)$ be a simple graph and $\widetilde{G}$ be a fuzzy soft graph of G^*. If $\widetilde{G}$ is a totally regular fuzzy soft graph and $\widetilde{F}$ is a constant function in fuzzy graph $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1,2,3...,n$. Then $\widetilde{G}$ is a regular fuzzy soft graph.*

Proof. Suppose that $\widetilde{G}$ is a totally regular fuzzy soft graph and $\widetilde{F}$ is a constant function. Then $\widetilde{F}(e_i)(a) = c_i$, c_i is a constant, $c_i \in [0,1]$, $\forall\, a \in V$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and $\mathrm{tdeg}(a) = r_i$ in $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. As $\mathrm{tdeg}(a) = \deg(a) + \widetilde{F}(e_i)(a)$ in $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. This implies $\deg(a) = \mathrm{tdeg}(a) - \widetilde{F}(e_i)(a)$ in $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. This implies $\deg(a) = r_i - c_i$ in $\widetilde{H}(e_i)$, $\forall\, e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. Hence $\widetilde{G}$ is a regular fuzzy soft graph. ∎

Theorem 2.17 *If $\widetilde{G}$ is both regular and totally regular fuzzy soft graph. Then $\widetilde{F}$ is a constant function in $\widetilde{H}(e_i)$ of G^* for all $e_i \in A$ for $i = 1,2,3,...,n$.*

Proof. Let $\widetilde{G}$ be both regular and totally regular fuzzy soft graph. Then $\deg(a) = r_i$ and $\mathrm{tdeg}(a) = s_i$ in fuzzy subgraphs $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. This implies $\deg(a) + \widetilde{F}(e_i)(a) = s_i$ in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. This implies $r_i + \widetilde{F}(e_i)(a) = s_i$ in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. This implies $\widetilde{F}(e_i)(a) = s_i - r_i$ in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1,2,3,...,n$ and for all $a \in V$. Hence $\widetilde{F}$ is a constant function in $\widetilde{H}(e_i)$ of G^* for all $e_i \in A$ for $i = 1,2,3,...,n$. ∎

The converse of the above theorem is not true in general, that is, if $\widetilde{F}(e)$ is a constant function then $\widetilde{G}$ need not be both regular and totally regular fuzzy soft graph.

Example 2.18 Consider a simple graph $G^* = (V,E)$ as taken in Example 2.7. Let $A = \{e_1, e_2\}$. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\widetilde{F}(e_1) = \{a_1|0.6, a_2|0.6, a_3|0.6, a_4|0.6\},$$
$$\widetilde{F}(e_2) = \{a_1|0.4, a_2|0.4, a_3|0.4, a_4|0.4\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\widetilde{K}(e_1) = \{a_1a_2|0.1, a_2a_3|0.2, a_3a_4|0.5, a_4a_1|0.3\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.2, a_2a_3|0.4, a_3a_4|0.3, a_4a_1|0.1\}.$$

Clearly, $\widetilde{F}(e_i)$ is constant in fuzzy graphs $\widetilde{H}(e_i)$ for $i = 1, 2$. But $\widetilde{G}$ is neither regular nor totally regular fuzzy soft graph.

Theorem 2.19 *Let $\widetilde{G}$ be a fuzzy soft graph over an odd cycle $G^* = (V, E)$. Then $\widetilde{G}$ is regular fuzzy soft graph if and only if $\widetilde{K}$ is a constant function in fuzzy subgraph $\widetilde{H}(e_i)$ over $H^*(e_i)$, where $H^*(e_i)$ is an odd cycle for all $e_i \in A$ for $i = 1, 2, 3, ..., n$.*

Proof. Suppose that $\widetilde{K}$ is a constant function. Then $\widetilde{K}(e_i)(ab) = c_i$, a constant, $c_i \in [0, 1]$, for all $e_i \in A$ for $i = 1, 2, 3, ..., n$ in fuzzy graph $\widetilde{H}(e_i)$ and for all $ab \in E$. So $\deg(a) = 2c_i$, in fuzzy graph $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$ and for all $a \in V$. Hence $\widetilde{G}$ is regular fuzzy soft graph.

Conversely, assume that $\widetilde{G}$ is a regular fuzzy soft graph of G^*. Let $d_1, d_2, d_3, ...,$ d_{2n+1} be the edges of G^* in that order. Let $\widetilde{K}(e_i)(d_1) = r_i$ in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Since $\widetilde{H}(e_i)$ is s_i- regular fuzzy graphs for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Then $\widetilde{K}(e_i)(d_2) = s_i - r_i$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. $\widetilde{K}(e_i)(d_3) = s_i - (s_i - r_i) = r_i$ and so on. Therefore,

$$\widetilde{K}(e_i)(d_j) = \begin{cases} r_i, & \text{if } j \text{ is odd} \\ s_i - r_i, & \text{if } j \text{ is even} \end{cases}$$

So $\widetilde{K}(e_i)(d_1) = \widetilde{K}(e_i)(d_{2n+1}) = r_i$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Thus, if d_1 and d_{2n+1} incident at vertex v, then $\deg(v) = s_i$ $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Then $\widetilde{K}(e_i)(d_1) + \widetilde{K}(e_i)(d_{2n+1}) = s_i$ for all $e_i \in A$ for $i = 1, 2, ..., n$.

$$r_i + r_i = s_i, \qquad 2r_i = s_i, \qquad r_i = \frac{s_i}{2}$$

So

$$s_i - r_i = s_i - \frac{s_i}{2} = \frac{s_i}{2}.$$

Therefore, $\widetilde{K}(e_i)(d_j) = \frac{s_i}{2}$ in fuzzy graphs $\widetilde{H}(e_i)$ for all j and for $i = 1, 2, ..., n$. Hence $\widetilde{K}$ is a constant function. ∎

Theorem 2.20 *Let $\widetilde{G}$ be a fuzzy soft graph over an even cycle $G^* = (V, E)$. Then $\widetilde{G}$ is regular fuzzy soft graph if and only if $\widetilde{K}$ is a constant function or alternate edges have same membership degrees in fuzzy subgraph $\widetilde{H}(e_i)$ over $H^*(e_i)$, where $H^*(e_i)$ is an even cycle for all $e_i \in A$ for $i = 1, 2, ..., n$.*

Proof. If either $\widetilde{K}$ is a constant function or alternate edges have same membership degrees. Then $\widetilde{G}$ is regular fuzzy soft graph.

Conversely, assume that $\widetilde{G}$ is a regular fuzzy soft graph of G^*. Let $d_1, d_2, d_3, ...,$ d_{2n} be the edges of G^* in that order. Let $\widetilde{K}(e_i)(d_1) = r_i$ in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1, 2, 3, ..., n$. Since $\widetilde{H}(e_i)$ is s_i- regular fuzzy graphs, for $i = 1, 2, 3, ..., n$. Then $\widetilde{K}(e_i)(d_2) = s_i - r_i$, for $i = 1, 2, 3, ..., n$. $\widetilde{K}(e_i)(d_3) = s_i - (s_i - r_i) = r_i$ and so on. Therefore,

$$\widetilde{K}(e_i)(d_j) = \begin{cases} r_i, & \text{if } j \text{ is odd} \\ s_i - r_i, & \text{if } j \text{ is even} \end{cases}$$

Proceeding as theorem 2.19, if $r_i = s_i - r_i$, then $\widetilde{K}$ is a constant function. If $r_i \neq s_i - r_i$, then alternate edges have same membership degrees. ∎

Note that the above theorems do not hold for totally regular fuzzy soft graphs. To illustrate we consider the following examples.

For example, consider an odd cycle $G^* = (V, E)$, where $V = \{a_1, a_2, a_3, a_4, a_5\}$ and $E = \{a_1a_2, a_2a_3, a_3a_4, a_4a_5, a_5a_1\}$. Let $A = \{e_1, e_2\}$ and $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ defined by

$$\widetilde{F}(e_1) = \{a_1|0.5, a_2|0.4, a_3|0.3, a_4|0.3, a_5|0.5\},$$
$$\widetilde{F}(e_2) = \{a_1|0.5, a_2|0.4, a_3|0.4, a_4|0.5, a_5|0.4\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ by

$$\widetilde{K}(e_1) = \{a_1a_2|0.2, a_2a_3|0.2, a_3a_4|0.3, a_4a_5|0.2, a_5a_1|0.1\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.1, a_2a_3|0.3, a_3a_4|0.1, a_4a_5|0.2, a_5a_1|0.2\}.$$

By routine calculations, it is easy to see that $\widetilde{H}(e_1) = (\widetilde{F}(e_1), \widetilde{K}(e_1))$ and $\widetilde{H}(e_2) = (\widetilde{F}(e_2), \widetilde{K}(e_2))$ are totally regular fuzzy graphs of G^*. Hence $\widetilde{G}$ is totally regular fuzzy soft graph. But $\widetilde{K}$ is not a constant function.

Now, we take an even cycle $G^* = (V, E)$ as taken in Example 2.7.

Let $A = \{e_1, e_2\}$. Let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\widetilde{F}(e_1) = \{a_1|0.7, a_2|0.8, a_3|0.6, a_4|0.5\},$$
$$\widetilde{F}(e_2) = \{a_1|0.8, a_2|0.6, a_3|0.4, a_4|0.6\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its fuzzy approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ given by

$$\widetilde{K}(e_1) = \{a_1a_2|0.2, a_2a_3|0.2, a_3a_4|0.4, a_4a_1|0.3\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.1, a_2a_3|0.4, a_3a_4|0.3, a_4a_1|0.2\}.$$

$\widetilde{H}(e_1)$ and $\widetilde{H}(e_2)$ are totally regular fuzzy graphs of G^*. Hence $\widetilde{G}$ is totally regular fuzzy soft graph. But in $\widetilde{H}(e)$, neither $\widetilde{K}$ is a constant function nor alternate edges have same membership degrees for all $e \in A$.

Theorem 2.21 *If $\widetilde{G}$ is a regular fuzzy soft graph and $\widetilde{F}$ is a constant function, then $\widetilde{G}^c$ is a regular fuzzy soft graph.*

Theorem 2.22 *If $\widetilde{G}$ is a totally regular fuzzy soft graph and $\widetilde{F}$ is a constant function, then $\widetilde{G}^c$ is a totally regular fuzzy soft graph.*

Theorem 2.23 *A regular fuzzy soft graph $\widetilde{G}$ on G^* with $|V| \geq 3$ and $\widetilde{H}(e_i)$ is regular fuzzy graph of degree $s_i > 0$, $i = 1, 2, \cdots, n$ have no end node.*

Proof. Since $\widetilde{H}(e_i)$ is regular fuzzy graph of degree s_i, so $\deg_{\widetilde{H}(e_i)}(a) = s_i$ for all $a \in V$, for all $e_i \in A$ for $i = 1, 2, ..., n$. As $s_i > 0$, $\deg_{\widetilde{H}(e_i)}(a) > 0$ for all $a \in V$. That is, every node is adjacent to at least one other node. On contrary, suppose that b is an end node, then $\deg_{\widetilde{H}(e_i)}(b) = s_i = \widetilde{K}_{\widetilde{H}(e_i)}(ab)$. Since $\widetilde{H}(e_i)$ is regular fuzzy graph with $|V| \geq 3$ for $i = 1, 2, ..., n$ then a must be adjacent to an other node $c \neq b$. Then $\deg_{\widetilde{H}(e_i)}(a) = \widetilde{K}_{\widetilde{H}(e_i)}(ab) + \widetilde{K}_{\widetilde{H}(e_i)}(ac) > \widetilde{K}_{\widetilde{H}(e_i)}(ab)$ for $i = 1, 2, ..., n$. $\Rightarrow \deg_{\widetilde{H}(e_i)}(a) > s_i$, which is a contradiction to the fact that $\widetilde{H}(e_i)$ is regular fuzzy graph of degree s_i for $i = 1, 2, ..., n$. Hence $\widetilde{G}$ have no end node. ■

Definition 2.24 Let $\widetilde{G}$ be a fuzzy soft graph on G^*. Then $\widetilde{G}$ is called a *partially regular fuzzy soft graph* if $\widetilde{H}(e)$ is partially regular fuzzy graph for all $e \in A$.

If $\widetilde{G}$ is both regular and partially regular fuzzy soft graph, then $\widetilde{G}$ is called a *full regular fuzzy soft graph.*

Example 2.25 Consider a simple graph $G^* = (V, E)$ as taken in Example 2.7. Let $A = \{e_1, e_2\}$ and let $(\widetilde{F}, A)$ be a fuzzy soft set over V with its approximate function $\widetilde{F} : A \to \mathcal{P}(V)$ given by

$$\widetilde{F}(e_1) = \{a_1|0.4, a_2|0.5, a_3|0.7, a_4|0.3\},$$
$$\widetilde{F}(e_2) = \{a_1|0.9, a_2|0.6, a_3|0.8, a_4|0.4\}.$$

Let $(\widetilde{K}, A)$ be a fuzzy soft set over E with its approximate function $\widetilde{K} : A \to \mathcal{P}(E)$ by

$$\widetilde{K}(e_1) = \{a_1a_2|0.3, a_2a_3|0.4, a_3a_4|0.1, a_4a_1|0.2\},$$
$$\widetilde{K}(e_2) = \{a_1a_2|0.5, a_2a_3|0.4, a_3a_4|0.2, a_4a_1|0.3\}.$$

Fuzzy subgraphs are $\widetilde{H}(e_1) = (\widetilde{F}(e_1), \widetilde{K}(e_1))$ and $\widetilde{H}(e_2) = (\widetilde{F}(e_2), \widetilde{K}(e_2))$. Since the underlying graphs of $\widetilde{H}(e_1)$ and $\widetilde{H}(e_2)$ are regular so $\widetilde{H}(e_1)$ and $\widetilde{H}(e_2)$ are partially regular fuzzy graphs as shown in Figure 7.

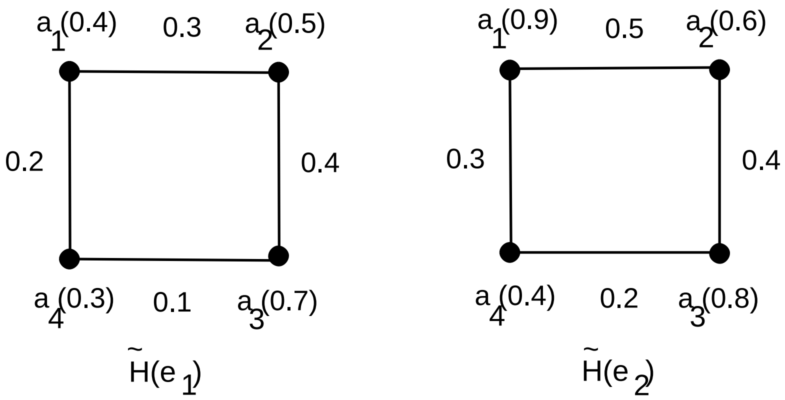

Figure 7: Fuzzy subgraphs

Hence $\widetilde{G}$ is a partially regular fuzzy soft graph of G^*.

Remark 1 Every regular fuzzy soft graph may not be a partially regular fuzzy soft graph.

Remark 2 Every partially regular fuzzy soft graph may not be a regular fuzzy soft graph.

Theorem 2.26 *Let $\widetilde{G}$ be a fuzzy soft graph such that $\widetilde{K}$ is constant in $\widetilde{H}(e_i)$ for all $e_i \in A$ for $i = 1, 2, ..., n$. Then $\widetilde{G}$ is a regular fuzzy soft graph if and only if $\widetilde{G}$ is a partially regular fuzzy soft graph.*

Proof. Suppose that $\widetilde{K}(e_i)(ab) = c_i$, a constant for all $ab \in E$ and for all $e_i \in A$ for $i = 1, 2, .., n$. Now, $\deg_{\widetilde{H}(e_i)}(a) = \sum\limits_{ab \in E} \widetilde{K}(e_i)(ab) = \sum\limits_{ab \in E} c_i = c_i \deg_{H^*(e_i)}(a)$ for all $a \in V$, $e_i \in A$ for $i = 1, 2, ..., n$. Let $\widetilde{G}$ be a regular fuzzy soft graph. Then $\deg_{\widetilde{H}(e_i)}(a) = c_i \deg_{H^*(e_i)}(a) = t_i$ for all $a \in V$ and for all $e_i \in A$ for $i = 1, 2, ..., n$. $\Rightarrow \deg_{H^*(e_i)}(a) = \frac{t_i}{c_i}$ for all $a \in V$, $e_i \in A$ for $i = 1, 2, ..., n$. $\Rightarrow H^*(e_i)$ is regular graph for all $e_i \in A$ for $i = 1, 2, ..., n$. So $\widetilde{H}(e_i)$ is a partially regular fuzzy graph and hence $\widetilde{G}$ is a partially regular fuzzy soft graph.

Conversely, suppose that $\widetilde{G}$ is a partially regular fuzzy soft graph. Assume that $H^*(e_i)$ is regular of degree $'s_i'$ for all $e_i \in A$ for $i = 1, 2, ..., n$. Then $\deg_{\widetilde{H}(e_i)}(a) = c_i \deg_{H^*(e_i)}(a) = c_i s_i$ for all $a \in V$ and for all $e_i \in A$ for $i = 1, 2, ..., n$. Hence $\widetilde{G}$ is a regular fuzzy soft graph. ∎

Remark 3 A regular or partially regular fuzzy soft graph need not be a full regular fuzzy soft graph.

Theorem 2.27 *Let $\widetilde{G}$ be a strong fuzzy soft graph such that $\widetilde{F}$ is a constant function. Then $\widetilde{G}$ is a regular fuzzy soft graph if and only if $\widetilde{G}$ is a partially regular fuzzy soft graph.*

Proof. Suppose that $\widetilde{F}(e_i)(a) = c_i$, where c_i is a constant for all $e_i \in A$ and for all $a \in V$ for $i = 1, 2, ..., n$. Since $\widetilde{G}$ is a strong fuzzy soft graph, then $\widetilde{H}(e_i)$ is a strong fuzzy graph for all $e_i \in A$ for $i = 1, 2, ..., n$. This implies $\widetilde{K}(e_i)(ab) = \min(\widetilde{F}(e_i)(a), \widetilde{F}(e_i)(b)) = c_i$ for all $ab \in E$. Thus $\widetilde{K}$ is a constant function. Proceeding in the same way as in Theorem 2.26 we proof the theorem. ∎

4. Conclusions

Fuzzy graph has numerous applications in modern sciences and technology, especially in research areas of computer science including database theory, data mining, neural networks, expert systems, cluster analysis, control theory, and image capturing. Fuzzy sets and soft sets are two different soft computing models for representing vagueness and uncertainty. We have applied these soft computing models in combination to study vagueness and uncertainty in graphs. We have investigated some properties of regular fuzzy soft graphs. We plan to extend our research of fuzzification to (1) Interval-valued fuzzy soft graphs; (2) Bipolar fuzzy soft regular graphs.

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Accepted: 22.05.2015