FUZZY HYPER KS-SEMIGROUPS

Bijan Davvaz

Department of Mathematics
Yazd University
Yazd
Iran
e-mail: davvaz@yazduni.ac.ir

Ann Leslie O. Vicedo

Jocelyn P. Vilela

Department of Mathematics and Statistics
MSU-Iligan Institute of Technology
Philippines
e-mails: annleslievicedo@gmail.com
jocelyn.vilela@g.msuiit.edu.ph

Abstract. Hyperstructure theory is applied to KS-semigroups, an algebra related to BCK-algebra and semigroup and thus, the notion of hyper KS-semigroups is introduced.

In this paper, the concept of fuzzy sets is applied to hyper KS-semigroups. In this fuzzification, the notions of fuzzy hyper subKS-semigroups and fuzzy hyper KS-ideals are introduced and relationships among them are investigated. Using the concept of upper level subsets, relationships between hyper subKS-semigroups (resp. hyper KS-ideals) and fuzzy hyper subKS-semigroups (resp. fuzzy hyper KS-ideals) are established. Finally, under a homomorphism $f : G \to H$ of hyper KS-semigroups, it is shown that the pre-image of a fuzzy hyper KS-ideal of H is a fuzzy hyper KS-ideal of G.

1. Introduction

KS-semigroups was introduced by K. H. Kim [6] which is a combination of BCK-algebra and semigroup.

Hyperstructure theory (also called multivalued algebras) was introduced by F. Marty at the 8th congress of Scandinavian Mathematicians in 1934. Recall that in a classical algebraic structure, the image of two elements of a set is an element of the set, while in an algebraic hyperstructure, the image of two elements is a set. Thus, it is considered as a generalization of classical algebraic structures.

In [8], hyperstructure theory was applied to KS-semigroups and so a new class of algebra, called hyper KS-semigroups, was introduced. In this paper, we apply the concept fuzzy sets to hyper KS-semigroups.

2. Preliminaries

Let H be a non-empty set endowed with a *hyperoperation* " $*$ " that is, " $*$ " is a function from $H \times H$ to $P^*(H) = P(H) \setminus \{\varnothing\}$. For two subsets A and B of H, $A * B = \bigcup_{a \in A, b \in B} a * b$. We shall use $x * y$ instead of $x * \{y\}, \{x\} * y$, or $\{x\} * \{y\}$. When $A \subseteq H$ and $x \in H$, we agree to write $A * x$ instead of $A * \{x\}$. Similarly, we write $x * A$ for $\{x\} * A$. In effect, $A * x = \bigcup_{a \in A} a * x$ and $x * A = \bigcup_{a \in A} x * a$. The structure $(H, \cdot)$ is called a *semihypergroup* if $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, for all $x, y, z \in H$.

Definition 2.1 [5] A *hyper BCK-algebra* is a nonempty set H endowed with a hyperoperation " $*$ " and a constant $\mathbf{0}$ satisfying the following axioms: for all $x, y, z \in H$,

(H1) $(x * z) * (y * z) < x * y$,

(H2) $(x * y) * z = (x * z) * y$,

(H3) $x * H \ll x$,

(H4) $x \ll y$ and $y \ll x$ imply $x = y$,

where (a) $x \ll y$ is defined by $\mathbf{0} \in x * y$, and (b) for every $A, B \subseteq H$, $A \ll B$ is defined as follows: for all $a \in A$, there exists $b \in B$ such that $a \ll b$. In such case, we call " $\ll$ " the *hyper order* in H.

A nonempty subset I of a hyper BCK-algebra $(H, *, \mathbf{0})$ is called a hyper BCK-ideal if $\mathbf{0} \in I$ and for any $x, y \in H$, $x * y \ll I$ and $y \in I$ imply that $x \in I$.

In any hyper BCK-algebra $(H, *, \mathbf{0})$, the following hold (see [5]) for all $x \in H$.

(A1) $x * H \ll x$ if and only if $x * y \ll \{x\}$ for all $y \in H$,

(A2) $x \ll x$ and

(A3) $x * \mathbf{0} = \{x\}$.

Definition 2.2 [8] A *hyper KS-semigroup* is a nonempty set H together with two hyperoperations "$*$" and "$\cdot$" and a constant $\mathbf{0}$ satisfying the following conditions:

(i) $(H, *, \mathbf{0})$ is a hyper BCK-algebra.

(ii) $(H, \cdot)$ is a semihypergroup having zero as a bilaterally absorbing element, that is, $x \cdot \mathbf{0} = \mathbf{0} \cdot x = \{\mathbf{0}\}$ for all $x \in H$; and

(iii) " $\cdot$ " is left and right distributive over " $*$ ", that is, for any $x, y, z \in H$,

$$x \cdot (y * z) = (x \cdot y) * (x \cdot z) \text{ and } (x * y) \cdot z = (x \cdot z) * (y \cdot z).$$

From now on, a hyper KS-semigroup $(H, *, \cdot, \mathbf{0})$ shall be denoted by H and for all $x, y \in H$, we agree to write $x \cdot y$ as xy.

Example 2.3 [8] Let $H = \{0, 1, 2\}$. Then $(H, *, \cdot, 0)$ is a hyper KS-semigroup with hyperoperations " $*$ " and " $\cdot$ " defined as follows.

$*$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{0,1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1,2\}$

$\cdot$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{1\}$	$\{0,1\}$
2	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$

Let I be a nonempty subset of a hyper KS-semigroup $(H, *, \cdot, \mathbf{0})$. Then I is said to be a *hyper subKS-semigroup* of H if for all $x, y \in I$, $x * y \subseteq I$ and $xy \subseteq I$. I is said to be a *hyper left* (resp. *hyper right*) *stable* if $xa \subseteq I$ (resp. $ax \subseteq I$) for all $x \in H$ and for all $a \in I$. I is said to be a *hyper stable* if I is both hyper left and right stable. I is said to be a *hyper left* (resp. *hyper right*) *KS-ideal* if (i) I is a hyper left (resp. hyper right) stable and (ii) for any $x, y \in H$, $x * y \ll I$ and $y \in I$ imply that $x \in I$. I is a *hyper KS-ideal* if I is both a hyper left and a hyper right KS-ideal.

Remark 2.4 A hyper KS-ideal contains $\mathbf{0}$ and hence, it is a hyper BCK-ideal.

3. Fuzzy sets in hyper KS-semigroups

In this section, we introduce the notions of fuzzy hyper subKS-semigroups and fuzzy hyper KS-ideals and provide some characterizations with respect to their upper level subsets.

Throughout this paper, a hyper KS-semigroup $(H, *, \cdot, \mathbf{0})$ shall be denoted by H with hyper order denoted by $\ll$. Note that the symbols $\leq, \geq, <$ and $>$ refer to the usual inequalities in real numbers.

Definition 3.1 A fuzzy set μ in a hyper KS-semigroup H is called a *fuzzy hyper subKS-semigroup* of H if it satisfies the following conditions: for all $x, y \in H$,

(i) $\inf_{a \in x * y} \mu(a) \geq \min\{\mu(x), \mu(y)\}$ and

(ii) $\inf_{a \in xy} \mu(a) \geq \min\{\mu(x), \mu(y)\}$.

Lemma 3.2 *Let μ be a fuzzy hyper subKS-semigroup of a hyper KS-semigroup H. Then for all $x \in H$, $\mu(\mathbf{0}) \geq \mu(x)$. Moreover, if μ is onto, $\mu(\mathbf{0}) = 1$.*

Proof. Let μ be a fuzzy hyper subKS-semigroup of H and let $x \in H$. By (A2), $x \ll x$ and so $\mathbf{0} \in x * x$. Thus,

$$\mu(\mathbf{0}) \geq \inf_{t \in x*x} \mu(t) \geq \min\{\mu(x), \mu(x)\} = \mu(x).$$

If μ is onto, then $\mu(x) = 1$ for some $x \in H$. Hence, $1 = \mu(x) \leq \mu(\mathbf{0}) \leq 1$. Therefore, $\mu(\mathbf{0}) = 1$. ■

Definition 3.3 A fuzzy set μ in a hyper KS-semigroup H is called a *left* (resp. *right*) *fuzzy hyper KS-ideal* of H if it satisfies the following conditions: for all $x, y \in H$,

(F1) $x \ll y$ implies $\mu(x) \geq \mu(y)$,

(F2) $\mu(x) \geq \min\{\inf_{a \in x*y} \mu(a), \mu(y)\}$, and

(F3) $\inf_{a \in xy} \mu(a) \geq \mu(y)$ (resp. $\inf_{a \in xy} \mu(a) \geq \mu(x)$).

A fuzzy set μ is a *fuzzy hyper KS-ideal* if it is both a left and a right fuzzy hyper KS-ideal of H.

In a hyper BCK-algebra, a fuzzy hyper BCK-ideal satisfies (F1) and (F2).

Example 3.4 Let H be the hyper KS-semigroup in Example 2.3. Define a fuzzy set μ in H by $\mu(0) = t_1$, $\mu(1) = t_2$ and $\mu(2) = t_3$ where $t_1, t_2, t_3 \in [0, 1]$ and $t_1 > t_2 > t_3$. Then it can be shown that μ is a fuzzy hyper KS-ideal of H.

Theorem 3.5 *A fuzzy hyper KS-ideal of a hyper KS-semigroup is a fuzzy hyper subKS-semigroup.*

Proof. Let μ be a fuzzy hyper KS-ideal in H and let $x, y \in H$. By (A1), $x * y \ll \{x\}$. Thus, for all $a \in x * y$, $a \ll x$. By Definition 3.3(F1), $\mu(x) \leq \mu(a)$ for all $a \in x * y$ and so $\mu(x) \leq \inf_{a \in x*y} \mu(a)$. This implies that

$$\min\{\inf_{a \in x*y} \mu(a), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$$

and so it follows that $\inf_{a \in x*y} \mu(a) \geq \min\{\inf_{a \in x*y} \mu(a), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$. Also, $\inf_{a \in xy} \mu(a) \geq \mu(y)$ and $\inf_{a \in xy} \mu(a) \geq \mu(x)$. Thus, $\inf_{a \in xy} \mu(a) \geq \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy hyper subKS-semigroup in H. ■

The converse of the preceding theorem may not be true in general. Consider the following example.

Example 3.6 Consider the hyper KS-semigroup $(H, *, \cdot, 0)$ with hyperoperations "$*$" and "$\cdot$" defined as follows.

$*$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{0,1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1,2\}$

$\cdot$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{1\}$	$\{0,1,2\}$
2	$\{0\}$	$\{0,1\}$	$\{0,2\}$

Define a fuzzy set μ in H by $\mu(0) = t_1$, $\mu(1) = t_2$ and $\mu(2) = t_3$, where $t_1, t_2, t_3 \in [0,1]$ and $t_1 > t_2 > t_3$. Then by routine calculations, we can show that μ is a fuzzy hyper subKS-semigroup but not a fuzzy hyper KS-ideal since $t_3 = \mu(2) = \inf_{a \in 1\cdot 2=\{0,1,2\}} \mu(a) < \mu(1) = t_2$.

For two fuzzy sets μ and ν in H, $(\mu \cap \nu)(x) = \min\{\mu(x), \nu(x)\}$ for all $x \in H$.

Theorem 3.7 *If μ and ν are fuzzy hyper subKS-semigroups of a hyper KS-semigroup H, then $\mu \cap \nu$ is also a fuzzy hyper subKS-semigroup of H.*

Proof. Since μ and ν are fuzzy hyper subKS-semigroups of H, it follows that for all $x, y \in H$ and $a \in x * y$, $\mu(a) \geq \inf_{a \in x*y} \mu(a) \geq \min\{\mu(x), \mu(y)\}$ and $\nu(a) \geq \inf_{a \in x*y} \nu(a) \geq \min\{\nu(x), \nu(y)\}$ Thus, for all $a \in x * y$, we have

$$\begin{aligned}(\mu \cap \nu)(a) &= \min\{\mu(a), \nu(a)\} \\ &\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\nu(x), \nu(y)\}\} \\ &= \min\{\min\{\mu(x), \nu(x)\}, \min\{\mu(y), \nu(y)\}\} \\ &= \min\{(\mu \cap \nu)(x), (\mu \cap \nu)(y)\}\end{aligned}$$

and so $\inf_{a \in x*y} (\mu \cap \nu)(a) \geq \min\{(\mu \cap \nu)(x), (\mu \cap \nu)(y)\}$. Also, for all $b \in xy$,

$$\begin{aligned}(\mu \cap \nu)(b) &= \min\{\mu(b), \nu(b)\} \\ &\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\nu(x), \nu(y)\}\} \\ &= \min\{\min\{\mu(x), \nu(x)\}, \min\{\mu(y), \nu(y)\}\} \\ &= \min\{(\mu \cap \nu)(x), (\mu \cap \nu)(y)\}\end{aligned}$$

and so $\inf_{b \in xy} (\mu \cap \nu)(b) \geq \min\{(\mu \cap \nu)(x), (\mu \cap \nu)(y)\}$. Therefore, $\mu \cap \nu$ is a fuzzy hyper subKS-semigroup of H.

For a fuzzy set μ in H, $H_\mu = \{x \in H | \mu(x) = \mu(\mathbf{0})\}$.

Theorem 3.8 *Let μ be a fuzzy set in a hyper KS-semigroup H.*

(i) *If μ is a fuzzy hyper subKS-semigroup of H, then H_μ is a hyper subKS-semigroup of H.*

(ii) *If μ is a fuzzy hyper KS-ideal of H, then H_μ is a hyper KS-ideal of H.*

Proof. Let $H_\mu = \{x \in H | \mu(x) = \mu(\mathbf{0})\}$.

(i) Since $\mathbf{0} \in H_\mu$, $H_\mu \neq \varnothing$. Clearly $H_\mu \subseteq H$. Suppose that μ is a fuzzy hyper subKS-semigroup of H. Let $x, y \in H_\mu$. Then $\mu(x) = \mu(\mathbf{0}) = \mu(y)$ and so for all $a \in x * y, b \in xy$,

$$\mu(a) \geq \inf_{a \in x*y} \mu(a) \geq \min\{\mu(x), \mu(y)\} = \min\{\mu(\mathbf{0}), \mu(\mathbf{0})\} = \mu(\mathbf{0}) \text{ and}$$
$$\mu(b) \geq \inf_{b \in xy} \mu(b) \geq \min\{\mu(x), \mu(y)\} = \min\{\mu(\mathbf{0}), \mu(\mathbf{0})\} = \mu(\mathbf{0}).$$

By Lemma 3.2, $\mu(a) \leq \mu(\mathbf{0})$ and $\mu(b) \leq \mu(\mathbf{0})$. Hence, $\mu(a) = \mu(\mathbf{0})$ and $\mu(b) = \mu(\mathbf{0})$. Thus, $a, b \in H_\mu$. Therefore, $x * y, xy \subseteq H_\mu$ and so H_μ is a hyper subKS-semigroup of H.

(ii) Suppose that μ is a fuzzy hyper KS-ideal of H. From [4], H_μ satisfies condition (ii) of a hyper KS-ideal. We only need to show that H_μ is hyper stable. Let $x \in H$ and $a \in H_\mu$. Then $\mu(a) = \mu(\mathbf{0})$. Let $b \in ax$ and $c \in xa$. Then

$$\mu(b) \geq \inf_{b \in ax} \mu(b) \geq \mu(a) = \mu(\mathbf{0}) \text{ and } \mu(c) \geq \inf_{c \in xa} \mu(c) \geq \mu(a) = \mu(\mathbf{0}).$$

By Lemma 3.2, $\mu(\mathbf{0}) \geq \mu(b)$ and $\mu(\mathbf{0}) \geq \mu(c)$. Thus, $\mu(\mathbf{0}) = \mu(b)$ and $\mu(\mathbf{0}) = \mu(c)$ and so, $b, c \in H_\mu$. Hence, $ax, xa \subseteq H_\mu$ so that H_μ is hyper stable. Therefore, H_μ is a hyper KS-ideal of H. ■

For a fuzzy set μ in H and $t \in [0, 1]$, the *upper level subset* of μ is given by $\mu_t = \{x \in H | \mu(x) \geq t\}$.

Remark 3.9 $\mu_t = H$ if $t = 0$.

Theorem 3.10 *Let μ be a fuzzy set in a hyper KS-semigroup H. Then μ is a fuzzy hyper subKS-semigroup of H if and only if the upper level subset μ_t is a hyper subKS-semigroup of H whenever $\mu_t \neq \varnothing$ for $t \in [0, 1]$.*

Proof. Suppose that μ is a fuzzy hyper subKS-semigroup of H and assume that $\mu_t \neq \varnothing$ where $t \in [0, 1]$. Let $x, y \in \mu_t$. Then $\mu(x) \geq t$ and $\mu(y) \geq t$. Thus, for all $a \in x * y$ and for all $b \in xy$,

$$\mu(a) \geq \inf_{a \in x*y} \mu(a) \geq \min\{\mu(x), \mu(y)\} \geq \min\{t, t\} = t \text{ and}$$
$$\mu(b) \geq \inf_{b \in xy} \mu(b) \geq \min\{\mu(x), \mu(y)\} \geq \min\{t, t\} = t.$$

Hence, μ_t is a hyper subKS-semigroup of H.

Conversely, assume that for each $t \in [0, 1]$, $\mu_t \neq \varnothing$ is a hyper subKS-semigroup of H. Let $x, y \in H$ and $t = \min\{\mu(x), \mu(y)\}$. Then $\mu(x) \geq t$ and $\mu(y) \geq t$. Thus, $x, y \in \mu_t$. Since μ_t is a hyper subKS-semigroup of H, $x*y, xy \subseteq \mu_t$ and so for all $a \in x * y$ and for all $b \in xy$, $\mu(a) \geq t$ and $\mu(b) \geq t$. Hence, $\inf_{a \in x*y} \mu(a) \geq t = \min\{\mu(x), \mu(y)\}$ and $\inf_{b \in xy} \mu(b) \geq t = \min\{\mu(x), \mu(y)\}$. Therefore, μ is a fuzzy hyper subKS-semigroup of H. ■

Theorem 3.11 *Let μ be a fuzzy set in a hyper KS-semigroup H. Then μ is a fuzzy hyper KS-ideal of H if and only if the upper level subset μ_t is a hyper KS-ideal of H whenever $\mu_t \neq \varnothing$ for $t \in [0,1]$.*

Proof. Suppose that μ is a fuzzy hyper KS-ideal of H and assume that $\mu_t \neq \varnothing$ where $t \in [0,1]$. From [3], μ_t satisfies condition (ii) of a hyper KS-ideal. Thus, we only need to show that $ax, xa \subseteq \mu_t$ for all $x \in H$ and for all $a \in \mu_t$. Let $x \in H$ and $a \in \mu_t$. Then $\mu(a) \geq t$. Now, by Definition 3.3(F3), we have $\mu(z) \geq \inf\limits_{z \in ax} \mu(z) \geq \mu(a) \geq t$ and $\mu(w) \geq \inf\limits_{w \in xa} \mu(w) \geq \mu(a) \geq t$. Hence, $z, w \in \mu_t$ and so $ax, xa \subseteq \mu_t$. Thus, μ_t is hyper stable. Therefore, μ_t is a hyper KS-ideal of H.

Conversely, assume that for each $t \in [0,1]$, $\mu_t \neq \varnothing$ is a hyper KS-ideal of H. From Remark [?] and [3], μ satisfies (F1) and (F2). We only need to show Definition 3.3(F3). Let $x, y \in H$ and take $t = \mu(y)$. Then $y \in \mu_t$ and since μ_t is hyper stable, it follows that $xy \in \mu_t$. Thus, for all $a \in xy$, $a \in \mu_t$. That is, $\mu(a) \geq t = \mu(y)$. Hence, $\inf\limits_{a \in xy} \mu(a) \geq t = \mu(y)$. Similarly, take $t = \mu(x)$ so that $xy \subseteq \mu_t$ and thus, for all $a \in xy$, $a \in \mu_t$, that is, $\mu(a) \geq t = \mu(x)$. Hence, $\inf\limits_{a \in xy} \mu(a) \geq t = \mu(x)$. Therefore, μ is a fuzzy hyper KS-ideal of H. ∎

For any nonempty subset I of a hyper KS-semigroup H, we define a fuzzy set μ_I in H by

$$\mu_I(x) = \begin{cases} 1, & \text{if } x \in I, \\ 0, & \text{otherwise} \end{cases}$$

that is, μ_I is the *characteristic function* of I.

Corollary 3.12 *Let I be a nonempty subset I of a hyper KS-semigroup H and μ_I be the characteristic function of I.*

(i) *I is a hyper subKS-semigroup of H if and only if μ_I is a fuzzy hyper subKS-semigroup of H.*

(ii) *I is a hyper KS-ideal of H if and only if μ_I is a fuzzy hyper KS-ideal of H.*

Proof. Observe the level subset of μ_I

$$(\mu_I)_t = \begin{cases} I, & \text{if } t \in (0,1], \\ H, & \text{if } t = 0. \end{cases}$$

The results follow directly from Theorems 3.10 and 3.11. ∎

Let $(H_1, *_1, \cdot_1, 0_1)$ and $(H_2, *_2, \cdot_2, 0_2)$ be hyper KS-semigroups and $f : H_1 \to H_2$ be a map. Then f is called a *hyper KS-semigroup homomorphism* if $f(x *_1 y) = f(x) *_2 f(y)$ and $f(x \cdot_1 y) = f(x) \cdot_2 f(y)$ for all $x, y \in H_1$.

Theorem 3.13 *Let $f : G \to H$ be an epimorphism of hyper KS-semigroups. If ν is a fuzzy hyper KS-ideal of H, then the homomorphic pre-image μ of ν under f is a fuzzy hyper KS-ideal of G.*

Proof. Let $f : G \to H$ be an epimorphism of hyper KS-semigroups. From Remark 2.4 and [4], μ satisfies (F1) and (F2). So, we only need to show (F3). Let $x, y \in G$. Then

$$\inf_{a \in xy} \mu(a) = \inf_{f(a) \in f(x)f(y)} \nu(f(a)) \geq \nu(f(x)) = \mu(x) \text{ and}$$

$$\inf_{a \in xy} \mu(a) = \inf_{f(a) \in f(x)f(y)} \nu(f(a)) \geq \nu(f(y)) = \mu(y).$$

Thus, μ is a fuzzy hyper KS-ideal of G. ■

References

[1] Corsini, P. and Leoreanu, V., *Applications of Hyperstructure Theory*, Advances in Mathematics, Kluwer Academic Publishers, Dordrecht, 2003.

[2] Imai, Y. and Iseki, K., *On Axiom systems of Propositional Calculi XIV*, Proc. Japan Academy, 42 (1966), 19-22.

[3] Jun, Y.B., Xin, X.L., *Fuzzy Hyper BCK-ideals of Hyper BCK-algebras*, Scientiae Mathematicae Japonicae Online, 4 (2000), 415-422.

[4] Jun, Y.B., Shim, W.H., *Fuzzy Implicative Hyper BCK-ideals of Hyper BCK-algebras*, International Journal of Mathematics and Mathematical Sciences, 29 (2), (2002), 63-70.

[5] Jun, Y.B., Zahedi, M.M., Xin, X.L., Borzooei, R.A., *On hyper BCK-algebras*, Italian Journal of Pure and Applied Mathematics, 8 (2000), 127-136.

[6] Kim, K.H., *On structure of KS-semigroups*, International Mathematical Forum, 1 (2006), 67-76.

[7] Saeid, A.B., Resvani M.A., *On Fuzzy BF-algebras*, International Mathematical Forum, 4 (1) (2009), 13-25.

[8] B. Davvaz, Vicedo, A.O., Vilela, J.P., *Some Structural Properties of Hyper KS-semigroups*, Italian Journal of Pure and Applied Mathematics, 33 (2014) (to appear).

[9] Zadeh, L.A., *Fuzzy Sets*, Information and Control, 8 (1965), 338-353.

Accepted: 24.11.2014