# INFLUENCE OF VARIABLE FLUID PROPERTIES, THERMAL RADIATION AND CHEMICAL REACTION ON MHD SLIP FLOW OVER A FLAT PLATE

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**Abstract.** In the present study the magneto hydrodynamic (MHD) slip flow and heat transfer over a flat plate with convective surface heat flux at the boundary and temperature dependent fluid properties has been presented in presence of chemical reactions, thermal radiation and non-uniform heat source/sink. The transverse magnetic field is assumed as a function of the distance from the origin. Also it is assumed that the fluid viscosity and the thermal conductivity vary as an inverse function and linear function of temperature respectively. Using the similarity transformation, the governing system of equations are transformed into similarity non-linear ordinary differential equations which are solved numerically using symbolic software MATHEMATICA. As a result, the dimensionless velocity, temperature, concentration, the skin friction coefficient, the Nusselt number and the local Sherwood number are presented through graphs and tables for several sets of values of the involved parameters of the problem and discussed in details from the physical point of view.

**Keywords:** slip flow, variable viscosity, variable thermal conductivity, chemical reaction.

**2010 Mathematics Subject Classification:** 76W05, 76V05.

## 1. Introduction

The boundary layer flow for an electrically conducting fluid have been discussed by many authors [1]-[9] and historically Rossow [1] was the first to study the hydrodynamic behavior of the boundary layer on a semi-infinite plate in the presence of a uniform transverse magnetic field. Varshney and Kumar [10] studied magnetohydrodynamic boundary layer flow of non-Newtonian fluid past a flat plate.

The similarity solution for the thermal boundary layer for the case of constant surface temperature at the plate is well established [11]. Kays and Crawford [12] proposed that similarity solution does not exist for the boundary condition of constant heat flux at the plate. Bejan [13] disproved their claim by suggesting a different similarity temperature variable which reduced the energy equation to an ordinary differential equation. Aziz [14] has studied thermal boundary layer flow over a flat plate considering convective surface heat flux at the lower surface of the plate and established a condition for similarity solution. Later on Ishak [15], Yao and Zhong [16] developed the problem under different conditions and in the presence of various physical effects.

In all the aforementioned papers the thermo physical properties of the ambient fluid were assumed to be constant. However, it is well known [17]-[21] that these physical properties may change with temperature, especially fluid viscosity and thermal conductivity. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid and so the properties of the fluid are no longer assumed to be constant. The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer and so the heat transfer at the wall is also affected. Therefore to predict the flow and heat transfer rates, it is necessary to take into account the variable fluid properties.

Slip flow happens if the characteristic size of the flow system is small or the flow pressure is very low. In no-slip-flow, as a requirement of continuum physics, the fluid velocity is zero at a solid-fluid interface. When fluid flows in micro electro mechanical system (MEMS), the no slip condition at the solid-fluid interface is no longer applicable. Beavers and Joseph [22] were the first to investigate the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary conditions at the porous interface. The slip flows under different flow configurations have been studied in recent years [23]-[27]. Recently, Das [28] have considered the slip effects on heat and mass transfer in MHD micropolar fluid flow over an inclined plate with thermal radiation and chemical reaction.

However, the effect of thermal radiation on the flow and heat transfer have not been taken into account in the most of the investigations. The effect of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Cogley et al. [29] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Raptis [30] investigated the steady flow of a viscous fluid through a porous medium bounded by a porous plate subject to a constant suction velocity in presence of thermal radiation. Makinde [31] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Ibrahim et al. [32] discussed the case of mixed convection flow of a micropolar fluid past a semi infinite, steady moving porous plate with varying suction velocity normal to the plate in presence of thermal radiation and viscous dissipation. Recently, Das [33] investigated the impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydro metallurgical industries. Chamkha [34] investigated the problem of heat and mass transfer by steady flow of an electrically conducting fluid past a moving vertical surface in presence of first order chemical reaction. The problems involving chemical reactions can be found in the studies of Damseh et al. [35], Magyari and Chamkha [36] and Das [37]. Yazdi et al. [38] discussed slip MHD liquid flow and heat transfer over non-linear permeable stretching surface with chemical reaction.

In this paper, the work of Das [33] has been extended to investigate the effect of chemical reaction on the hydro-magnetic flow and heat transfer over an impermeable flat plate with variable fluid properties in presence of thermal radiation. The resulting governing equations have been transformed into a system of non-linear ordinary differential equations by applying a suitable similarity transformation.

## 2. Mathematical formulation of the problem

Consider a steady two dimensional laminar flow of an electrically conducting incompressible fluid moving over an impermeable flat plate under the influence of a transverse magnetic field $\vec{B}$ in the presence of non-uniform heat source/sink, chemical reaction and thermal radiation. The magnetic Reynolds number of the flow is taken to be small enough so that induced magnetic field is assumed to be negligible in comparison with applied magnetic field so that $\vec{B}=[0, B(x)]$, where $B(x)$ is the applied magnetic field acting normal to the plate and varies in strength as a function of $x$. The flow is assumed to be in the $x$-direction which is taken along the plate and $y$-axis is normal to it. The viscosity and thermal conductivity of the fluid are assumed to be functions of temperature.

Under the foregoing assumptions, the governing boundary layer equations [20, 33] for the present problem can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \sigma B^2(x)(u - U_\infty), \tag{2.2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y} + q''', \tag{2.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 T}{\partial y^2} - k_r C^m, \tag{2.4}$$

where $u$, $v$ are velocity components along $x$, $y$-axis respectively, $U_\infty$ is the free stream velocity, $\sigma$ is the electrical conductivity of the fluid, $T$ is the temperature of the fluid within the boundary layer, $\kappa$ is the thermal conductivity of the fluid, $c_p$ is the specific heat at constant pressure $p$, $\mu$ is the dynamic viscosity, $\rho$ is the constant fluid density, $C$ is the concentration of the fluid within the boundary layer, $D_m$ is the chemical molecular diffusivity, $k_r$ is the chemical reaction rate constant and $m$ is order of chemical reaction.

The radiative heat flux term $q_r$ by using the Rosseland approximation is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.5}$$

where $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that $T^4$ can be expressed as a linear combination of the temperature, we expand $T^4$ in Taylor's series about $T_\infty$ and neglecting higher order terms, we get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{2.6}$$

Thus we have

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{2.7}$$

Using equation (2.7) in equation (2.3), we obtain

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left[\left(\kappa + \frac{16T_\infty^3\sigma^*}{3k^*}\right)\frac{\partial T}{\partial y}\right] + q''', \tag{2.8}$$

The appropriate boundary conditions for the present problem are

$$\left.\begin{aligned} &u = L\frac{\partial u}{\partial y} \text{ (partial slip), v=0 (impermeable surface),}\\ &C = C_w - \kappa\frac{\partial T}{\partial y} = h_w(T_w - T) \text{ (convective surface heat flux) for } y = 0,\\ &u = U_\infty,\ T = T_\infty,\ C = C_\infty \text{ as } y \to \infty \end{aligned}\right\} \tag{2.9}$$

where $L$ is the slip length and $h_w$ is the convective heat transfer coefficient.

Now we transform the system of equations (2.2), (2.4), (2.8) and (2.9) into a dimensionless form. To this end, let the us introduce the following dimensionless variables:

$$\begin{aligned} &\eta = y\left(\frac{U_\infty}{v_\infty x}\right)^{1/2}, \quad f(\eta) = \frac{\psi}{(U_\infty v_\infty x)^{1/2}},\\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \tag{2.10}$$

where $\psi(x, y)$ is the stream function, $v_\infty = \mu_\infty/\rho$ is the kinematic viscosity of the ambient fluid. Since $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, we have from (2.10)

$$u = U_\infty f' \ \textit{and}\ v = -\frac{1}{2}\left(\frac{v_\infty U_\infty}{x}\right)^{1/2}(f - \eta f') \tag{2.11}$$

where $f$ is non-dimensional stream function and prime denotes differentiation with respect to $\eta$.

In order to predict the flow and heat transfer rates accurately, Ling and Dybbs [42] suggested a temperature dependent viscosity of the form

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] \tag{2.12}$$

where $\gamma$ is the thermal property of fluid, $T_\infty$ is the temperature of the fluid outside the boundary layer and $\mu_\infty$ is the dynamic viscosity at ambient temperature.

Equation (2.12) can be written as

$$\frac{1}{\mu} = A(T - T_r) \tag{2.13}$$

where $A = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$. In general, $A > 0$ corresponds to liquids and $A < 0$ to gases when the temperature at the plate is larger than that of the temperature at far away from the plate.

The non-uniform heat source/sink $q'''$ is given by [33]

$$q''' = \frac{\kappa_\infty U_0}{2\nu_\infty x}[Q(T - T_\infty) + Q^*(T_w - T_\infty)e^{-\alpha' y}] \tag{2.14}$$

where $\kappa_\infty$ is the thermal conductivity at ambient temperature, $Q$ and $Q^*$ are the coefficients of space and temperature dependent heat source/sink terms respectively and $\alpha'$ is the thermal property of fluid.

The dimensionless temperature $\theta$ can also be written as

$$\theta = \frac{T - T_r}{T_w - T_\infty} + \theta_r \tag{2.15}$$

where $\theta_r = T_r - T_\infty/(T_w - T_\infty) = -1/\delta(T_w - T_\infty)$. Using (2.15), equation (2.13) becomes

$$\mu = \mu_\infty \left(\frac{\theta_r}{\theta_r - \theta}\right) \tag{2.16}$$

Following Chiam [17], we consider the specific model for variable thermal conductivity as

$$\kappa = \kappa_\infty \left(1 + \varepsilon\frac{T - T_\infty}{\Delta T}\right) \tag{2.17}$$

where $\varepsilon$ is the thermal conductivity parameter and $\Delta T = T_w - T_\infty$. This relation can be written as

$$\kappa = \kappa_\infty(1 + \varepsilon\theta) \tag{2.18}$$

Now, introducing equations (2.16) and (2.18) into equations (2.2), (2.4) and (2.8), we obtain,

$$\left(\frac{\theta_r}{\theta_r-\theta}\right)f'''+\frac{1}{2}ff''+\frac{\theta_r}{(\theta_r-\theta)^2}f''\theta'-M(f'-1)=0, \tag{2.19}$$

$$(1+\varepsilon\theta+Nr)\theta''+\varepsilon\theta'^2+\frac{1}{2}Pr_\infty f\theta'+Q\theta+Q^*e^{-\alpha\eta}=0, \tag{2.20}$$

$$\phi''+Scf\phi'-Kr\phi^m=0 \tag{2.21}$$

where $M=\sigma' B^2(x)/\rho U_\infty$ is the magnetic field parameter, $Pr_\infty=\mu_\infty c_p/\kappa_\infty$ is the ambient Prandtl number, $Nr=16T_\infty^3\sigma^*/3k^*\kappa_\infty$ is the thermal radiation parameter, $\alpha=\alpha'(\frac{\nu_\infty x}{U_\infty})$ is the thermal property of fluid, $Sc=\nu/D_m$ is the Schmidt number and $Kr=kr\nu^2/D_m U_\infty^2$ is the chemical reaction rate parameter.

The corresponding boundary conditions (2.9) become

$$\left.\begin{array}{l} f=0, f'=\delta f'', \theta'=-a\left(\dfrac{1-\theta(0)}{1+\varepsilon\theta(0)}\right), \phi=1 \text{ for } \eta=0,\\ f'=1, \theta=0, \phi=0 \text{ as } \eta\to\infty \end{array}\right\} \tag{2.22}$$

where $a=\frac{h_w}{\kappa_\infty}\left(\frac{\nu_\infty x}{U_\infty}\right)^{1/2}$ is the surface convection parameter and $\delta=L\left(\frac{U_\infty}{\nu_\infty x}\right)^{1/2}$ is the slip parameter.

In the present study, both viscosity and thermal conductivity vary across the boundary layer so it is reasonable to consider the Prandtl number as a variable and is defined as (see Rahman [20] and Rahman et al. [21])

$$Pr=\frac{\mu c_p}{\kappa}=\frac{(\frac{\theta_r}{\theta_r-\theta})\mu_\infty c_p}{\kappa_\infty(1+\varepsilon\theta)}=\frac{1}{(1-\frac{\theta}{\theta_r})(1+\varepsilon\theta)}Pr_\infty \tag{2.23}$$

Using equation (2.23), the non-dimensional energy equation (2.20) can be written as

$$(1+\varepsilon\theta+Nr)\theta''+\varepsilon\theta'^2+Pr\left(1-\frac{\theta}{\theta_r}\right)(1+\varepsilon\theta)(f\theta'-f'\theta)+Q\theta+Q^*e^{-\alpha\eta}=0 \tag{2.24}$$

It should be noted that for large $\theta_r$ and small $\varepsilon$ i.e $\theta_r\to\infty$ and $\varepsilon\to 0$, the variable Prandtl number $Pr$ becomes the ambient Prandtl number $Pr_\infty$ and in that case equation (2.24) reduces to the equation (2.20).

The quantities of main physical interest are the skin friction coefficient (rate of shear stress), the Nusselt number (rate of heat transfer) and the Sherwood number (rate of mass transfer). The equation defining the wall shear stress is

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{2.25}$$

The local skin friction coefficient is defined as

$$C_f=2Re_x^{-1/2}\left[\left(\frac{\theta_r}{\theta_r-\theta(0)}\right)\right]f''(0) \tag{2.26}$$

or,

$$C_f^* = \left(\frac{\theta_r}{\theta_r - \theta(0)}\right) f''(0) \text{ where } C_f^* = \frac{1}{2} Re_x^{1/2} C_f \tag{2.27}$$

Knowing the temperature field, it is interesting to study the effect of the free convection and thermal radiation on the rate of heat transfer $q_w$, is given by

$$q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma^*}{3k^*} \left(\frac{\partial T^4}{\partial y}\right)_{y=0} \tag{2.28}$$

So the rate of heat transfer in terms of the dimensionless Nusselt number is defined as follows:

$$Nu = -\frac{1}{2} Re_x^{1/2} (1 + \varepsilon\theta(0) + Nr)\theta'(0) \tag{2.29}$$

or,

$$Nu^* = -(1 + \varepsilon\theta(0) + Nr)\theta'(0) \text{ where } Nu^* = 2Re_x^{-1/2} Nu \tag{2.30}$$

Similarly, the rate of mass transfer in terms of local Sherwood number is given by

$$Sh^* = -\phi'(0) \tag{2.31}$$

## 3. Method of solution

The non-linear differential equations (2.19), (2.21) and (2.24) with boundary conditions (2.22) have been solved in the symbolic computation software MATHEMATICA using finite difference code that implements the 3-stage Lobatto IIIa formula for partitioned Runge-Kutta method. For numerical computation infinity condition has been taken at a large but finite value of $\eta$ where no considerable variation in velocity, temperature etc. occur. To check the validity of the present code, the values of $-\theta'(0)$ have been calculated for different values of the surface convection parameter $a$ and Prandtl number $Pr$ in Table 1. From Table 1, it has been observed that the data produced by present code and those of Rahman [20] and Das [33] show excellent agreement and so justifies the use of the present numerical code.

## 4. Numerical results and discussions

In order to get a clear insight of the present problem, the numerical results for velocity, temperature, concentration etc. have been presented graphically in Figs. 1-10 and in Tables 1-3 for several sets of values of the pertinent parameters. In the simulation the default values of the parameters are considered as $\delta = 0.2$, $a = 0.2$, $\theta_r = 2.5$, $\varepsilon = 0.5$, $M = 0.5$, $Nr = 0.2$, $Pr = 0.71$, $\alpha = 1$, $Q = 0.2$, $Q^* = 0.3$, $Kr = 0.3$, $Sc = 0.4$ and $m = 1$ unless otherwise specified.

### 4.1. Computational results for velocity profiles

In Figs. 1-4 we presented the behavior of the fluid velocity for various material parameters. It can be easily seen from Fig. 1 that the fluid velocity within the boundary layer increases with the increase of $\delta$ and, as a result, thickness of momentum boundary layer increases. From Fig. 2 we see that $f'(\eta)$ is considerably increased with an increase in the surface convection parameter $a$ but effect is not significant for higher values of $a$. The variations of the velocity profiles against transverse coordinate $\eta$ are shown in Figs.3 for various values of viscosity parameter $\theta_r$. The results indicate that with increase in the parameter $\theta_r$, the velocity profiles increases within the boundary region. Fig. 4 illustrates the effect of thermal radiation parameter $Nr$ on velocity profiles. From figure we see that the velocity increases as $\eta$ increases for a fixed value of $Nr$. For a non-zero fixed value of $\eta$, the velocity distribution across the boundary layer increases with the increasing values of $Nr$. Table 2 shows that surface convection parameter $a$ enhances the skin friction coefficient $C_f^*$. It is evident from the table 3 that the skin friction coefficient $C_f^*$ decreases on increasing $\delta$ and $\theta_r$.

### 4.2. Computational results for temperature profiles

The effect of various physical parameters on the fluid temperature are illustrated in Figs. 5-8. Fig. 5 shows that the fluid temperature is the maximum near the boundary layer region and it decreases on increasing boundary layer coordinate $\eta$ to approach free stream value. Also fluid temperature decreases on increasing $\delta$ in the boundary layer region and, as a consequence, thickness of the thermal boundary layer decreases. Fig. 6 demonstrates the effects of $a$ on fluid temperature in the presence of non-uniform heat source/sink. It is observed from the figure that temperature $\theta(\eta)$ decreases on increasing $a$ in the boundary layer region and is maximum at the surface of the plate. The solution approaches to the solution for constant surface temperature for large values of $a$, i.e., $a \to \infty$. For a non-zero fixed value of $\eta$, temperature distribution across the boundary layer decreases with the increasing values of $Nr$ and hence the thickness of thermal boundary layer decreases as shown in Fig. 7. The influence of viscosity parameter $\theta_r$ on temperature distribution are highlighted in Fig. 8. It is seen that as $\theta_r$ increases, the thickness of the thermal boundary layer decreases with a consequent reduction of the temperature in the boundary layer. From Table 2, we observed that $Nu^*$ increases with increasing $a$ and $Nr$. The influence of variable viscosity parameter $\theta_r$ on $Nu^*$ is presented in Table 3. It is observed from this table that $\theta_r$ enhances the dimensionless Nusselt number.

### 4.3. Computational results for concentration profiles

Fig. 9 illustrates the variation of the concentration distribution across the boundary layer for various values of the chemical reaction parameter $Kr$. It is seen that the effect of increasing values of the chemical reaction parameter results in decreasing concentration distribution across the boundary layer. Fig. 10 shows the variation of concentration profiles for different values of reaction order parameter $m$. It is observed from this figure that the concentration profiles increase with increasing $m$ but effect is not significant for higher order reaction. It is found from Table 2 that an increase in $Kr$ leads to increase in the values of the dimensionless Sherwood number $Sh^*$. It is observed from Table 3 that as reaction-order parameter $m$ increases, the dimensionless Sherwood number $Sh^*$ decreases.

## 5. Conclusions

The effects of chemical reaction and thermal radiation on steady two dimensional boundary layer flow of an incompressible electrically conducting fluid over a flat plate with partial slip at the surface of the boundary with temperature dependent fluid viscosity as well as with variable thermal conductivity have been studied in the present paper. Numerical results are presented to illustrate the details of the flow, heat and mass transfer characteristics and their dependence on material parameters. Following conclusions can be drawn from the present investigation:

(i) The velocity distribution are increasing for increasing values of slip parameter $\delta$, surface convection parameter $a$ and variable viscosity parameter $\theta_r$.

(ii) The temperature profile decreases with a increasing of slip parameter $\delta$, surface convection parameter $a$ and thermal radiation parameter $Nr$ while the opposite effect is observed for variable viscosity parameter $\theta_r$.

(iii) The chemical species concentration decreases with increase of $Kr$ but reverse effect occurs for $m$.

(iv) The skin friction coefficient decreases with increase of thermal radiation parameter $Nr$, slip parameter $\delta$ and variable viscosity parameter $\theta_r$ but effect is reverse for surface convection parameter $a$.

(v) Nusselt number $Nu^*$ increases for increasing of surface convection parameter $a$, thermal radiation parameter $Nr$ and variable viscosity parameter $\theta_r$ while it decreases for increasing slip parameter $\delta$.

(vi) Sherwood number $Sh^*$ decreases with increase of reaction-order parameter $m$ but effect is opposite for chemical reaction parameter $Kr$.

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Table 1. Comparison of the values of $-\theta'(0)$ for various values of $a$ in the absence of mass transfer

| | Rahman [20] | | Das [33] | | Present results | |
|---|---|---|---|---|---|---|
| $a$ | $Pr = 0.1$ | $Pr = 0.71$ | $Pr = 0.1$ | $Pr = 0.71$ | $Pr = 0.1$ | $Pr = 0.71$ |
| 0.05 | 0.036900 | 0.042781 | 0.036866 | 0.042767 | 0.036863 | 0.042762 |
| 0.2 | 0.082477 | 0.119358 | 0.082473 | 0.119295 | 0.082483 | 0.119288 |
| 0.6 | 0.113688 | 0.198155 | 0.113741 | 0.198051 | 0.113722 | 0.198051 |
| 1.0 | 0.122999 | 0.228303 | 0.123074 | 0.228178 | 0.123039 | 0.228178 |
| 5.0 | 0.136400 | 0.279283 | 0.136515 | 0.279131 | 0.136519 | 0.279135 |

Table 2. Effects of $a$, $Kr$ and $Q, Q^*$ on $C_f^*$, $Nu^*$ and $Sh^*$.

| $a$ | $Kr$ | $Q = Q^*$ | $C_f^*$ | $Nu^*$ | $Sh^*$ |
|---|---|---|---|---|---|
| 0.0 | 0.3 | 0.5 | 0.583515 | 0.000000 | |
| 0.4 | | | 0.605034 | 0.122074 | |
| 1.0 | | | 0.612411 | 0.158639 | |
| 0.2 | 0.0 | 0.5 | | | 0.407903 |
| | 0.6 | | | | 0.82648 |
| | 0.9 | | | | 0.984198 |
| 0.2 | 0.3 | 0.0 | 0.576377 | 0.183116 | |
| | | 0.5 | 0.788644 | -0.890034 | |
| | | 1.0 | 0.476148 | 0.876308 | |

Table 3. Effects of $\theta_r$, $\delta$ and $m$ on $C_f^*$, $Nu^*$ and $Sh^*$

| $\theta_r$ | $\delta$ | $m$ | $C_f^*$ | $Nu^*$ | $Sh^*$ |
|---|---|---|---|---|---|
| 2.0 | 0.3 | 1.0 | 0.677068 | 0.0882478 | |
| 5.0 | | | 0.598672 | 0.0934037 | |
| $\infty$ | | | 0.559764 | 0.0964655 | |
| 2.0 | 0.0 | 1.0 | 0.732151 | | |
| | 0.6 | | 0.58307 | | |
| | 1.2 | | 0.476659 | | |
| 2.0 | 0.3 | 1.0 | | | 0.640209 |
| | | 2.0 | | | 0.577629 |
| | | 3.0 | | | 0.543216 |

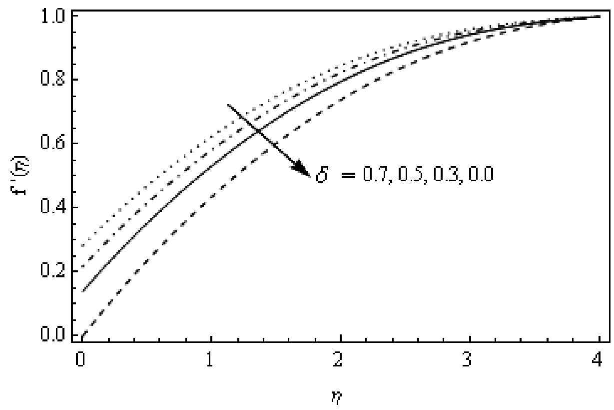


Figure 1: Velocity profiles for various values of $\delta$

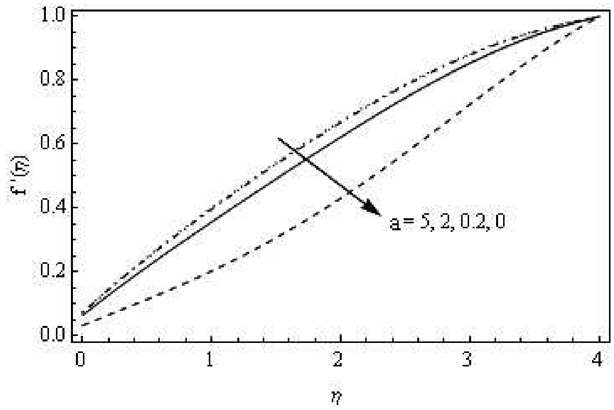


Figure 2: Velocity profiles for various values of $a$

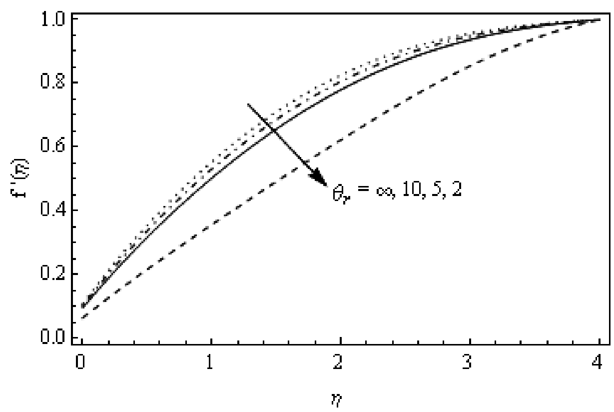


Figure 3: Velocity profiles for various values of $\theta_r$.

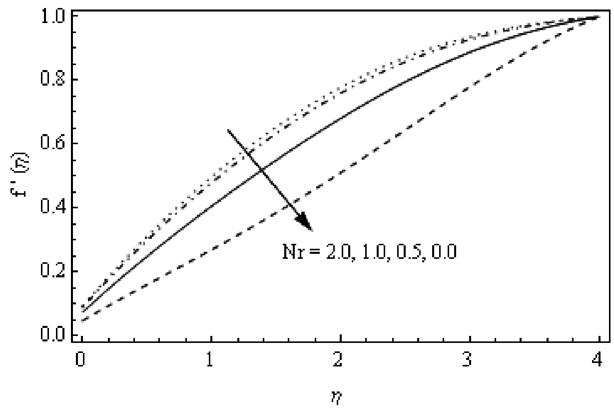


Figure 4: Velocity profiles for various values of $Nr$.

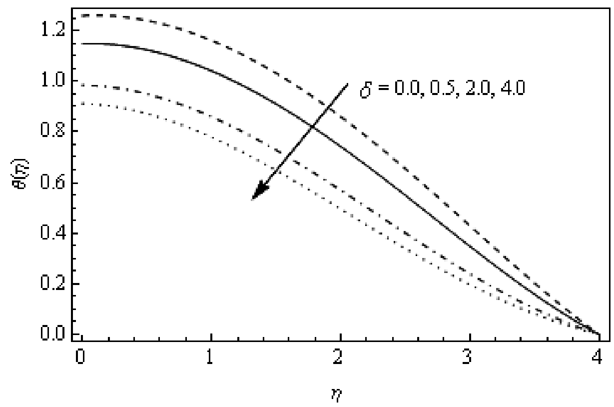


Figure 5: Temperature profiles for various values of $\delta$.

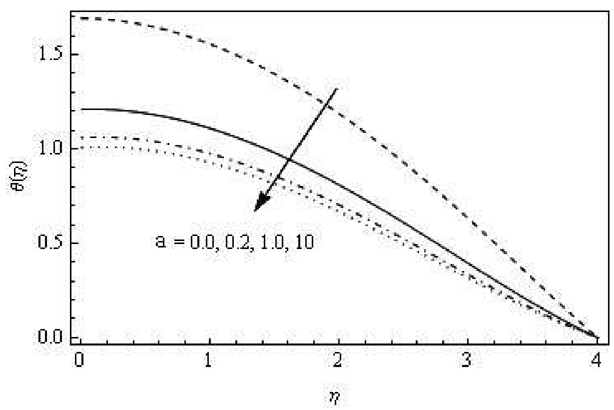


Figure 6: Temperature profiles for various values of $a$.

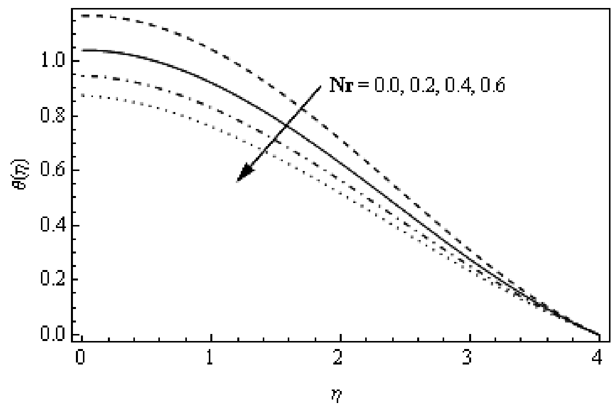


Figure 7: Temperature profiles for various values of $Nr$.

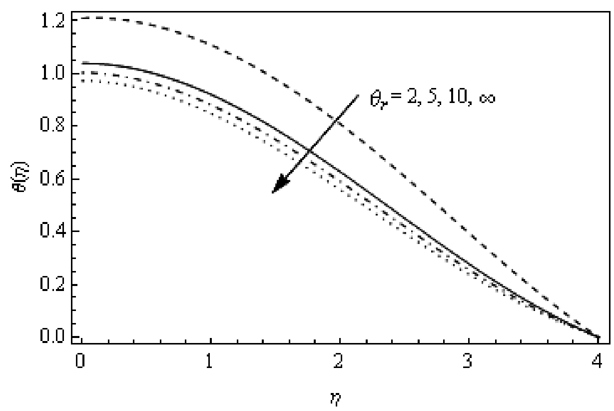


Figure 8: Temperature profiles for various values of $\theta_r$.

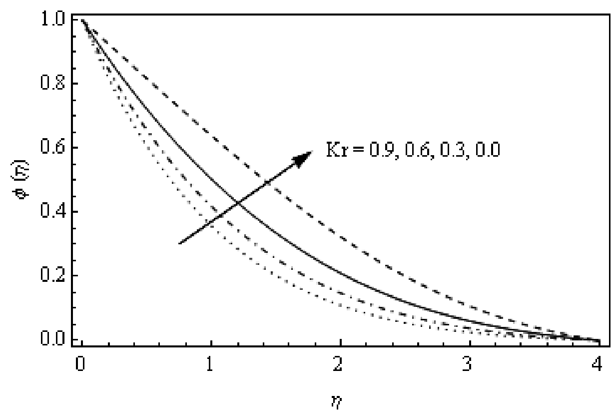


Figure 9: Concentration profiles for various values of $Kr$.

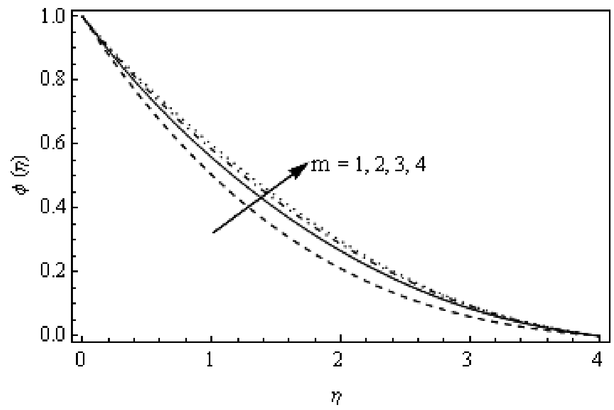


Figure 10: Concentration profiles for various values of $m$.

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