ON L–FUZZY TOPOLOGICAL TM–SUBSYSTEM

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Abstract. Recently, in 2010, Tamilarasi and Megalai introduced a new class of algebras known as TM-algebras. In this paper, we discuss the notion of an L-fuzzy topological TM-subsystem.

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Introduction

To evaluate the modern concept of uncertainty in real physical world, L.A. Zadeh [8] introduced the notion of fuzzy sets, in which the boundaries are not crisp or sharp but flexible. In [4], Goguen generalized the notion of fuzzy sets into L-fuzzy sets where L- can be a complete lattice.

Recently, in 2010, Tamilarasi and Megalai introduced a new class of algebras, called TM-algebras [6]. In their paper they investigated the relationship between TM-algebras and other algebras. They claimed that the TM-algebra is a generalization of BCH/BCI/BCK and Q algebras. In [1], the authors, while studying L-fuzzy structures on TM-algebras, brought out the fact that the TM-algebra is not a generalization of BCH/BCI/BCK algebras by giving counter examples.

The notion of a fuzzy set provides a natural framework for generalizing many of the concepts of general topology. The theory of fuzzy topological spaces is developed by Chang [3], Wong [7], Lowen [5] and others. In our paper [2], we have studied the notion of Fuzzy Topological subsystem on a TM-algebra. In this paper, we introduce the notion of an L-fuzzy topological TM-subsystem and investigate some simple but elegant results.

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2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1 Let X be a non-empty set. A mapping $\mu : X \to L$ is called an L-fuzzy set of X,where L is a complete lattice, with sup 1 and inf 0.

Definition 2.2 Let A and B be any two fuzzy sets in a non-empty set X.

1. The union of A and B, denoted by, $A \cup B$ is defined to be the L-fuzzy set $(A \cup B)(x) = \mu_A(x) \vee \mu_B(x)\ for\ all\ x \in X$.
2. The intersection of A and B, denoted by, $A \cap B$ is defined to be the L-fuzzy set $(A \cap B)(x) = \mu_A(x) \wedge \mu_B(x)\ for\ all\ x \in X$.
3. $A \subset B \ \Rightarrow\ A(x) \leq B(x)\ for\ all\ x \in X$.
4. The complement of A is defined to be $A'(x) = 1 - A(x)\ for\ all\ x \in X$.

Definition 2.3 A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions

1. $\phi, X \in T$
2. If $A, B \in T$ then $A \cap B \in T$
3. If $A_i \in T$ for each $i \in I$ then $\cup_I A_i \in T$ where I is an indexing set.

Remark 2.1 If X is a set with a fuzzy topology T then (X, T) is called a fuzzy topological space and any element in T is called a T-open fuzzy set in X.

Definition 2.4 Let f be a function from X to Y. Let σ be a fuzzy set in Y. The inverse image of σ under f is defined as $\sigma_{f^{-1}}(x) = \sigma(f(x))\ \forall x\ \in X$. Let μ be a fuzzy set in X. The image of μ under f is defined as

$$\mu_f(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} \mu(z), & f^{-1}(y) \text{ is not empty}, \quad \forall y \in Y. \\ 0 & \text{otherwise,} \end{cases}$$

Definition 2.5 A TM-Algebra $(X, *, 0)$ is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:

1. $X * 0 = X$
2. $(X * Y) * (X * Z) = Z * Y\ for\ all\ x, y, z \in X$.

Definition 2.6 L-Fuzzy TM-Subalgebra. Let L be a complete lattice with sup 1 and inf 0. An L-fuzzy subset μ of a TM-Algebra $(X, *, 0)$ is called an L-fuzzy TM-Subalgebra of X if, for all $x, y\ \in X$, $\mu(x * y) \geq \mu_(x) \wedge \mu_(y)$

Definition 2.7 Let $(X, *)$ be a TM-algebra. X is said to be an L-fuzzy topological TM-system if there is a family (X, L_τ) of L-fuzzy subsets in X which satisfies the following conditions

1. $\phi, X \in L_\tau$
2. If $A, B \in L_\tau$ then $A \cap B \in L_\tau$
3. If $A_i \in L_\tau$ for each $i \in I$ then $\cup_I A_i \in L_\tau$ where I is an indexing set.

Remark 2.2 If X is a set with an L-fuzzy topology L_τ then (X, L_τ) is called an L-fuzzy topological TM-system and any element in L_τ is called an L_τ-open fuzzy set in X.

3. On L-Fuzzy Topological TM-Subsystem

Definition 3.8 Let $(X, *)$ be a TM-Algebra. Let (X, L_τ) be an L-Fuzzy Topological TM-System. Let A be an L-fuzzy set in X. Then the induced L-fuzzy topological TM-system is the intersection of the L-fuzzy set A with L_τ-open fuzzy sets of X. The induced L-fuzzy topological TM-system is denoted by L_{τ_A}. (A, L_{τ_A}) is called an L-fuzzy topological TM-subsystem.

Example 3.1 Consider the set $X = \{0, 1, 2, 3, 4, 5\}$ with the following cayley table

*	0	1	2	3	4	5
0	0	3	4	1	2	5
1	1	0	2	3	5	4
2	2	4	0	5	1	3
3	3	1	5	0	4	2
4	4	5	3	2	0	1
5	5	2	1	4	3	0

Then $(X, *)$ is a TM-algebra.

Let L be a complete lattice with sup 1 and inf o. Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq t_7 \leq 1$. Let the fuzzy subsets $\mu_i : X \to L$, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ be given by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1, 3 \\ t_3 & \text{if } x = 2, 4, 5 \end{cases} \qquad \mu_2(x) = \begin{cases} t_7 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_3(x) = \begin{cases} t_3 & \text{if } x = 0, 5 \\ 0 & \text{if } x = 1, 2, 3, 4 \end{cases} \qquad \mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_4 & \text{if } x = 1, 3 \\ t_3 & \text{if } x = 2, 4, 5 \end{cases}$$

$$\mu_5(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_6(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_6 & \text{if } x = 1, 2 \\ t_4 & \text{if } x = 3 \end{cases}$$

$$\mu_7(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad \mu_8(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases}$$

$$\mu_9(x) = \begin{cases} t_6 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad \mu_{10}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_7 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases}$$

Then the collection $L_\tau = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_9, \mu_{10}\}$ is an L-Fuzzy Topology on X. Hence (X, L_τ) is an L-fuzzy topological TM-system. Choose $A = \mu_8$. Then $L_{\tau_A} = \{\phi, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_7, \mu_8, \mu_9\}$ and $A = (\mu_8, L_{\tau_A})$ is an L-fuzzy topological TM-subsystem.

Definition 3.9 Let $(X, *)$, $(Y, *)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems. A mapping f of (X, L_T) into (Y, L_U) is L-fuzzy continuous iff the inverse image of each L_U-open fuzzy set is an L_T-open fuzzy set.

Example 3.2 Consider the TM-algebra $(X, *)$ as in the 3.1. Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq t_7 \leq 1$. Consider the L-fuzzy subsets $\mu_i : X \to L$, $i = 1,2,3,4,5,6,7,8$ be given by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1,3 \\ t_3 & \text{if } x = 2,4,5 \end{cases} \quad \mu_2(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_4 & \text{if } x = 1,3 \\ t_3 & \text{if } x = 2,4,5 \end{cases}$$

$$\mu_3(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1,3 \\ 0 & \text{if } x = 2,4,5 \end{cases} \quad \mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_6 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases}$$

$$\mu_5(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases} \quad \mu_6(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1,2 \\ 0 & \text{if } x = 3,4 \end{cases}$$

$$\mu_7(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_7 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases} \quad \mu_8(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_3 & \text{if } x = 1,3 \\ 0 & \text{if } x = 2,4,5 \end{cases}$$

Then the collection $L_T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8\}$ is an L-Fuzzy Topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. Consider the set $Y = \{0, a, b, c, d, e\}$ with the following Cayley table

$*$	0	a	b	c	d	e
0	0	e	d	c	b	a
a	a	0	e	d	c	b
b	b	a	0	e	d	c
c	c	b	a	0	e	d
d	d	c	b	a	0	e
e	e	d	c	b	a	0

Then $(Y, *)$ is a TM-algebra.

Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq 1$. Consider the L-fuzzy subsets $\sigma_i : Y \to L, i = 1, 2, 3, 4, 5$ be given by

$$\sigma_1(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_3 & \text{if } y = a, b, d, e \\ t_5 & \text{if } y = c \end{cases} \qquad \sigma_2(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_4 & \text{if } y = a, b, d, e \\ t_6 & \text{if } y = c \end{cases}$$

$$\sigma_3(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{if } y = a, b, d, e \\ t_2 & \text{if } y = c \end{cases} \qquad \sigma_4(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_3 & \text{if } y = a, b, d, e \\ t_4 & \text{if } y = c \end{cases}$$

$$\sigma_5(y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{if } y = a, b, d, e \\ t_1 & \text{if } y = c \end{cases}$$

Then the collection $L_U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$ is an L-Fuzzy Topology on Y. Hence (Y, L_U) is an L-fuzzy topological TM-system. Here an L-fuzzy topological TM-systems (X, L_T) and (Y, L_U) have the same values of t_i's $i = 1, 2, 3, 4, 5, 6$.

Let $f : X \to Y$ be the function given by, $f(0) = 0$, $f(1) = c$, $f(2) = e$, $f(3) = c$, $f(4) = b$, $f(5) = b$, $\sigma_{f^{-1}}(x) = \sigma(f(x))$ for all x in X for any L-fuzzy set σ in Y,

$$(\sigma_1)_{f^{-1}}(x) = \mu_1(x) \quad (\sigma_2)_{f^{-1}}(x) = \mu_4(x) \quad (\sigma_3)_{f^{-1}}(x) = \mu_3(x)$$
$$(\sigma_4)_{f^{-1}}(x) = \mu_2(x), (\sigma_5)_{f^{-1}}(x) = \mu_6(x), \quad x \in X$$

Hence the inverse image of each L_U-open fuzzy set is L_T-open and hence the function f is an L-fuzzy continuous.

Definition 3.10 Let $(X, *)$, $(Y, *)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems . A mapping f of (X, L_T) into (Y, L_U) is an L-fuzzy open iff the image of each L_T-open fuzzy set is L_U-open fuzzy set.

Example 3.3 Consider the TM-algebra $(X, *)$ as in the example 3.1, Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq 1$. Consider the L-fuzzy subsets $\mu_i : X \to L, i = 1, 2, 3, 4$ be given by

$$\mu_1(x) = \begin{cases} t_5 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_2(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_3(x) = \begin{cases} t_4 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

Then the collection $L_T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4\}$ is an L-Fuzzy Topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. Consider the TM-algebra

$(Y, *)$ as in Example 3.2. Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq 1$.

Consider the L-fuzzy subsets $\sigma_i : Y \to L, i = 1, 2, 3, 4, 5, 6$ be given by

$$\sigma_1(y) = \begin{cases} t_5 & \text{if } y = 0 \\ 0 & \text{if } y = a, b, d, e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_2(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_1 & \text{if } y = a, b, d, e \\ t_4 & \text{if } y = c \end{cases}$$

$$\sigma_3(y) = \begin{cases} t_6 & \text{if } y = 0 \\ 0 & \text{if } y = a, b, d, e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_4(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_4 & \text{if } y = a, b, d, e \\ t_6 & \text{if } y = c \end{cases}$$

$$\sigma_5(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_2 & \text{if } y = a, b, d, e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_6(y) = \begin{cases} t_4 & \text{if } y = 0 \\ 0 & \text{if } y = a, b, d, e \\ t_3 & \text{if } y = c \end{cases}$$

Then the collection $L_U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_5, \sigma_6\}$ is an L-Fuzzy Topology on Y. Hence (Y, L_U) is an L-fuzzy topological TM-system. Here an L-fuzzy topological TM-systems (X, L_T) and (Y, L_U) have the same values of t_i's $i = 1, 2, 3, 4, 5$.

Let $f : X \to Y$ be given by, $f(0) = 0$, $f(1) = b$, $f(2) = d$, $f(3) = e$, $f(4) = a$, $f(5) = c$. Then $f^{-1}(0) = 0$, $f^{-1}(b) = 1$, $f^{-1}(d) = 2$, $f^{-1}(e) = 3$, $f^{-1}(a) = 4$, $f^{-1}(c) = 5$.

$$f(\mu_1) = \sigma_1 \quad f(\mu_2) = \sigma_5 \quad f(\mu_3) = \sigma_6 \quad f(\mu_4) = \sigma_2$$

Hence the image of each L_T-open fuzzy set is L_U-open and hence the function f is L-Fuzzy Open.

Definition 3.11 Let $(X, *)$, $(Y, *)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems. Let $(A, L_{T_A}), (B, L_{U_B})$ be an L-fuzzy topological subsystems on X and Y. f is said to be a mapping of (A, L_{T_A}) into (B, L_{U_B}) if $f(A) \subset B$.

Definition 3.12 Let $(X, *)$, $(Y, *)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems. Let $(A, L_{T_A}), (B, L_{U_B})$ be an L-fuzzy topological subsystems on X and Y. A mapping f from (A, L_{T_A}) into (B, L_{U_B}) is relatively L-fuzzy continuous if and only if $f^{-1}(\sigma_B) \wedge A$ is in L_{T_A} where $\sigma_B \in L_{U_B}$.

Example 3.4 Consider the TM-algebra $(X, *)$ as in Example 3.1. Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4 \leq t_5 \leq t_6 \leq t_7 \leq 1$. Consider the L-fuzzy subsets $\mu_i : X \to L$, $i = 1, 2, 3, 4, 5, 6, 7, 8$ be given by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1, 3 \\ t_3 & \text{if } x = 2, 4, 5 \end{cases} \qquad \mu_2(x) = \begin{cases} t_6 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_3(x) = \begin{cases} t_3 & \text{if } x = 0, 5 \\ 0 & \text{if } x = 1, 2, 3, 4 \end{cases} \qquad \mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_5(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_3 & \text{if } x = 1,2,3,4 \\ t_4 & \text{if } x = 5 \end{cases} \quad \mu_6(x) = \begin{cases} t_5 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_7(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad \mu_8(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_7 & \text{if } x = 1,3 \\ t_4 & \text{if } x = 2,4,5 \end{cases}$$

Then the collection $L_T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_6, \mu_7, \mu_8\}$ is an L-Fuzzy Topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. Choose $A = \mu_5$. Then $L_{T_A} = \{\phi, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7\}$ and $A = (\mu_5, L_{T_A})$ is an L-fuzzy topological TM-subsystem. Consider the TM-algebra $(Y, *)$ as in Example 3.2.

Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le 1$. Consider the L-fuzzy subsets $\sigma_i : Y \to L, i = 1,2,3,4,5$ be given by

$$\sigma_1(y) = \begin{cases} t_6 & \text{if } y = 0 \\ 0 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases} \quad \sigma_2(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_2 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases}$$

$$\sigma_3(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_1 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases} \quad \sigma_4(y) = \begin{cases} t_5 & \text{if } y = 0 \\ 0 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases}$$

$$\sigma_5(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_4 & \text{if } y = a,b,d,e \\ t_6 & \text{if } y = c \end{cases}$$

Then the collection $L_U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ is an L-Fuzzy Topology on Y. Hence (Y, L_U) is an L-fuzzy topological TM-system. Choose $B = \sigma_5$. Then $L_{U_B} = \{\phi, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ and $B = (\sigma_5, L_{U_B})$ is an L-fuzzy topological TM-subsystem. Here an L-fuzzy topological TM-subsystem (A, L_{T_A}) and (B, L_{U_B}) have the same values of t_i's $i = 1,2,3,4,5,6$.

Let $f : X \to Y$ be given by $f(0) = 0,\ f(1) = b,\ f(2) = d,\ f(3) = e,$ $f(4) = a,\ f(5) = c$. Then $f^{-1}(0) = 0,\ f^{-1}(b) = 1,\ f^{-1}(d) = 2,\ f^{-1}(e) = 3,$ $f^{-1}(a) = 4,\ f^{-1}(c) = 5$

$$f(A) = \begin{cases} 1 & \text{if } x = 0 \\ t_3 & \text{if } x = a,b,d,e \\ t_4 & \text{if } x = c \end{cases} \subset B$$

$\sigma_{f^{-1}}(x) = \sigma(f(x))$ for all x in X for any L-fuzzy set σ in Y.

$$(\sigma_1)_{f^{-1}}(x) = \begin{cases} t_6 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad (\sigma_1)_{f^{-1}}(x) \wedge A = \mu_2 \in (A, L_{T_A})$$

$$(\sigma_2)_{f^{-1}}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad (\sigma_2)_{f^{-1}}(x) \wedge A = \mu_4 \in (A, L_{T_A})$$

$$(\sigma_3)_{f^{-1}}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \quad (\sigma_3)_{f^{-1}}(x) \wedge A = \mu_7 \in (A, L_{T_A})$$

Hence the inverse image of each L_{U_B}-open fuzzy set is L_{T_A}-open. Therefore, the function f is relatively L-fuzzy continuous.

Definition 3.13 Let $(X,*)$, $(Y,*)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems. Let $(A, L_{T_A}), (B, L_{U_B})$ be an L-fuzzy topological TM-subsystems. A mapping f from (A, L_{T_A}) into (B, L_{U_B}) is relatively L-fuzzy open if and only iff $f(\mu_A) \in L_{U_B}$ where $\mu_A \in L_{T_A}$.

Example 3.5 Consider the TM-algebra $(X,*)$ as in Example 3.1, Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le 1$. Consider the L-fuzzy subsets $\mu_i : X \to L$, $i = 1,2,3,4,5$ be given by

$$\mu_1(x) = \begin{cases} t_6 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_2(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_3(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_3 & \text{if } x = 1,2,3,4 \\ t_4 & \text{if } x = 5 \end{cases} \qquad \mu_4(x) = \begin{cases} t_5 & \text{if } x = 0 \\ 0 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_5(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1,2,3,4 \\ t_3 & \text{if } x = 5 \end{cases}$$

Then the collection $L_T = \{\phi, X, \mu_1, \mu_2, \mu_4, \mu_5\}$ is an L-Fuzzy Topology on X. Hence (X, L_T) is an L-fuzzy topological TM-system. Choose $A = \mu_3$.

$L_{T_A} = \{\phi, \mu_1, \mu_2, \mu_4, \mu_5\}$ and $A = (\mu_3, L_{T_A})$ is an L-fuzzy topological TM-subsystem. Consider the TM-algebra $(Y,*)$ as in Example 3.2. Let L be a complete lattice with sup 1 and inf 0. Let $t_1, t_2, t_3, t_4, t_5, t_6 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le 1$. Consider the L-fuzzy subsets $\sigma_i : Y \to L$, $i = 1,2,3,4,5,6$ be given by

$$\sigma_1(y) = \begin{cases} t_5 & \text{if } y = 0 \\ 0 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_2(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_1 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases}$$

$$\sigma_3(y) = \begin{cases} t_6 & \text{if } y = 0 \\ 0 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_4(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_4 & \text{if } y = a,b,d,e \\ t_6 & \text{if } y = c \end{cases}$$

$$\sigma_5(y) = \begin{cases} 1 & \text{if } y = 0 \\ t_2 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases} \qquad \sigma_6(y) = \begin{cases} t_4 & \text{if } y = 0 \\ 0 & \text{if } y = a,b,d,e \\ t_3 & \text{if } y = c \end{cases}$$

Then the collection $L_U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_5, \sigma_6\}$ is an L-Fuzzy Topology on Y. Hence (Y, L_U) is an L-fuzzy topological TM-system. Choose $B = \sigma_4$. Then $L_{U_B} =$

$\{\phi, \sigma_1, \sigma_2, \sigma_3, \sigma_5, \sigma_6\}$ and $B = (\sigma_4, L_{U_B})$ is an L-fuzzy topological TM-subsystem. Here an L-fuzzy topological TM-subsystems (A, L_{T_A}) and (B, L_{U_B}) have the same values of t_i's $i = 1, 2, 3, 4, 5, 6$.

Let $f : X \to Y$ be given by $f(0) = 0$, $f(1) = b$, $f(2) = d$, $f(3) = e$, $f(4) = a$, $f(5) = c$. Then, $f^{-1}(0) = 0$, $f^{-1}(b) = 1$, $f^{-1}(d) = 2$, $f^{-1}(e) = 3$, $f^{-1}(a) = 4$, $f^{-1}(c) = 5$

$$f(A) = \begin{cases} 1 & \text{if } x = 0 \\ t_3 & \text{if } x = a, b, d, e \quad \subset B \\ t_4 & \text{if } x = c \end{cases}$$

$$f(\mu_1) = \sigma_3 \quad f(\mu_2) = \sigma_5 \quad f(\mu_4) = \sigma_1 \quad f(\mu_5) = \sigma_2$$

Hence the image of each L_{T_A}-open fuzzy set is L_{U_B}-open and hence the function f is relatively L-Fuzzy Open.

Theorem 3.1 *Let $(X, *)$, $(Y, *)$ be two TM-Algebras. Let (X, L_T),(Y, L_U) be two an L-fuzzy topological TM-systems. Let (A, L_{T_A}), (B, L_{U_B}) be an L-fuzzy topological TM-subsystems.Let f be an L-fuzzy continuous mapping of (X, L_T) in to (Y, L_U) such that $f(A) \subset B$. Then f is a relatively L-fuzzy continuous mapping of (A, L_{T_A}) into (B, L_{U_B}).*

Proof. Let $\sigma' \in L_{U_B}$. Then there exists $\sigma \in L_U$ such that $\sigma' = \sigma \wedge B$. Hence

$$f^{-1}(\sigma') \wedge A = f^{-1}(\sigma \wedge B) \wedge A = (f^{-1}(\sigma) \wedge f^{-1}(B)) \wedge A = f^{-1}(\sigma) \wedge A \because f(A) \subset B.$$

Since $\sigma \in L_U, f$ is an L-fuzzy continuous and $f^{-1}(\sigma) \in L_T$.

Therefore $f^{-1}(\sigma) \wedge A$ is open in L_{T_A}. Therefore, f is relatively L-fuzzy continuous mapping of (A, L_{T_A}) into (B, L_{U_B}).

Theorem 3.2 *Let f be a L-fuzzy continuous mapping of an L-fuzzy topological TM-system (X, L_T) in to (Y, L_U). Let g be a mapping of (Y, L_U) in to an L-fuzzy topological TM-system (Z, L_V). Then the composition mapping $g \circ f$ is an L-fuzzy continuous mapping of (X, L_T) in to (Z, L_V).*

Proof. Since the functions f, g are L-fuzzy continuous, by Definition 3.9, the inverse image of each L_U-open fuzzy set σ is L_T-open.

The inverse image of each open fuzzy set χ of L_V is an L_U-open fuzzy set.

$$(g \circ f)^{-1}(\chi) = (f^{-1} \circ g^{-1})(\chi) = f^{-1}(g^{-1}(\chi)) = f^{-1}(\sigma)$$

Since $f^{-1}(\sigma)$ is L_T-open fuzzy set, $(g \circ f)^{-1}(\chi)$ is L_T-open fuzzy set.

Hence $(g \circ f)$ is an L-fuzzy continuous.

Theorem 3.3 *Let (A, L_{T_A}),(B, L_{U_B}),(C, L_{V_C}) be an L-fuzzy topological TM-subsystems. Let f, g be the relatively L-fuzzy continuous mappings of (A, L_{T_A}) into (B, L_{U_B}) and (B, L_{U_B}) into (C, L_{V_C}). Then, the composition $g \circ f$ is a relatively L-fuzzy continuous mapping of (A, L_{T_A}) into (C, L_{V_C}).*

Proof. Let $\chi' \in L_{V_C}$. Then, there exists $\chi \in L_V$ such that $\chi' = \chi \wedge C$.

Since g is relatively L-fuzzy continuous from (B, L_{U_B}) into (C, L_{V_C}) by Definition 3.12, $g^{-1}(\chi') \wedge B$ is open in L_{U_B}.

Since f is relatively L-fuzzy continuous from (A, L_{T_A}) into (B, L_{U_B}) by Definition 3.12, $f^{-1}((g^{-1}(\chi') \wedge B)) \wedge A$ is open in L_{T_A}

But $(g \circ f)^{-1}(\chi') \wedge A = f^{-1}(g^{-1}(\chi') \wedge B) \wedge A$ implying that $(g \circ f)^{-1}(\chi') \wedge A$ is open in L_{T_A}.

Hence $f(A) \subset B$, $g \circ f$ is relatively L-fuzzy continuous.

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