NEW EXTENDED (G'/G)-EXPANSION METHOD FOR TRAVELING WAVE SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS (NPDEs) IN MATHEMATICAL PHYSICS

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Abstract. The new extended (G'/G)-expansion method is proposed to construct abundant exact traveling wave solutions involving free parameters to the nonlinear partial differential equations (NPDEs) in mathematical physics. We highlight the power of the new extended (G'/G)-expansion method in providing generalized solitary wave solutions of different physical structures applying it in the right-handed noncommutative burgers and the $(1+1)$-dimensional compound KdVB equations. By this application, we enhanced new traveling wave solutions of the equations which can be used to exploit some practical physical and mechanical phenomena. Moreover,when the parameters are replaced by special values, the well-known solitary wave solutions of the equation rediscovered from the traveling waves that may imply some physical meaningful results in fluid mechanics, gas dynamics, traffic flow, nonlinear dispersion and dissipation effects.

Keywords: the new extended (G'/G)-expansion method; the right-handed noncommutative burgers equation; the $(1+1)$-dimensional compound KdVB equation; solitons wave solutions; traveling wave solutions.

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1. Introduction

The world around us is inherently nonlinear, and nonlinear evolution equations are widely used to describe the complex physical phenomena that come out in a broad range of scientific applications, such as the fluid dynamics, nuclear physics, high energy physics, plasma physics, solid state physics, condensed matter physics, elastic media, optical fibers, biology, chemical kinematics, chemical physics, and geochemistry, etc. The exact solutions of nonlinear partial differential equations (NPDEs) play a significant role in nonlinear science and engineering. Recently, a number of prominent mathematicians and physicists who are interested in the nonlinear physical phenomena have investigated exact solutions of NPDEs to understand the physical mechanism of the phenomena using symbolical computer programs such as Maple, Matlab, Mathematica that facilitate complex and tedious algebraical computations.

For example, the wave phenomena observed in fluid dynamics [4], [14], plasma and elastic media [5], [12] and optical fibers [11], [19] etc. Some of the existing powerful methods for deriving exact solutions of NLEEs are Backlund transformation method [10], Darboux Transformations [8], tanh-function method [18], Exp-function method [7] and so on. Wang et al. [17] firstly proposed the (G'/G)-expansion method, then many diverse group of researchers extended this method by different names like an improved (G'/G)-expansion method [3], improved (G'/G)-expansion method [24],extended (G'/G)-expansion method [2], [15], generalized (G'/G)-expansion method [13], modified simple equation method [6] with different auxiliary equations. Zayed [20] established extended (G'/G)-expansion method for solving the $(3+1)$-dimensional NLEEs in mathematical physics. We (Roshid et. al.) [15] also used this method to find new exact traveling wave solutions of nonlinear Klein-Gordon equation. Recently, Khan et al. [25]found traveling and soliton wave solutions of GZK-BBM and right-handed non-commutative burgers equations by Modified Simple Equations method.

In this article, our motivation is to add new more general traveling wave solutions of right-handed non-commutative burgers and the $(1+1)$-dimensional compound KdVB equations via new extended (G'/G)-expansion. The performances of the method will encourage other researchers to apply it in other nonlinear evolution equations.

2. Materials and method

For given nonlinear evolution equations with independent variables x and t, we consider the following form

$$F(u, u_t, u_x, u_{xt}, u_{tt}, ...) = 0. \tag{1}$$

By using the traveling wave transformation

$$u(x,t) = u(\xi), \xi = x - Vt, \tag{2}$$

where u is an unknown function depending on x and t, and is a polynomial F in $u(\xi) = u(x,t)$ and its partial derivatives and V is a constant to be determined later. The existing steps of method are as follows:

Step 1: Using equation (2) in equation (1), we can convert equation (1) to an ordinary differential equation

$$Q(u, -Vu\prime, u\prime, -Vu\prime\prime, V^2u\prime\prime, ...) = 0 \tag{3}$$

Step 2: Assume the solutions of equation (3) can be expressed in the form

$$u(\xi) = \sum_{i=-n}^{n} \left\{ \frac{a_i(G'/G)^i}{[1+\lambda G'/G]^i} + b_i(G'/G)^{i-1}\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]} \right\} \tag{4}$$

with $G = G(\xi)$ satisfying the differential equation

$$G'' + \mu G = 0 \tag{5}$$

in which the value of σ must be ± 1,$\mu \neq 0$, a_i, b_i $(i = -n, ..., n)$, and λ are constants to be determined later. We can evaluate n by balancing the highest-order derivative term with the nonlinear term in the reduced equation (3).

Step 3: Inserting equation (4) into equation (3) and making use of equation (5) and then, extracting all terms of like powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma[1+\frac{(G'/G)^2]}{\mu}}$ together, set each coefficient of them to zero yield an over-determined system of algebraic equations and then solving this system of algebraic equations for a_i, b_i $(i = -n, ..., n)$ and V, we obtain several sets of solutions.

Step 4: For the general solutions of Eq.(5), we have

$$\mu < 0,\ \frac{G'}{G} = \sqrt{-\mu}\left(\frac{Asinh(\sqrt{-\mu}\xi) + Bcosh(\sqrt{-\mu}\xi)}{Acosh(\sqrt{-\mu}\xi) + Bsinh(\sqrt{-\mu}\xi)}\right) = f_2(\xi) \tag{6}$$

$$\mu > 0,\ \frac{G'}{G} = \sqrt{\mu}\left(\frac{Acos(\sqrt{\mu}\xi) - Bsin(\sqrt{\mu}\xi)}{Asin(\sqrt{\mu}\xi) + Bcos(\sqrt{\mu}\xi)}\right) = f_1(\xi) \tag{7}$$

where A, B are arbitrary constants. At last, inserting the values of a_i, b_i $(i = -n, ..., n)$, V and (6), (7) into equation (4) and obtain required traveling wave solutions of equation (1).

Remark 1. It is noteworthy to observe that if we put $\lambda = 0$ in the equation (5), then the proposed new extended (G'/G)-expansion coincide with the Guo and Zhou's extended (G'/G)-expansion [2]. On the other hand if we put $b_i = 0$ and $\lambda = 0$ in the equation (5), then the proposed method is identical to the improved (G'/G)-expansion method presented by Zhang et al. [24]. Again if we set $b_i = 0$ and $\lambda = 0$ and negative the exponents of (G'/G) are zero in equation (5), then the proposed method turn into the basic (G'/G)-expansion method introduced by Wang et al. [17]. Thus the methods presented in the Ref. [2], [17], [24] are only special cases of the proposed new extended (G'/G)-expansion method.

3. Application of our method

To test the validity of our method, let us consider two important equations of mathematical physics to construct exact traveling wave solutions:

Example 3.1. In this subsection, we will bring to bear the new extended (G'/G)-expansion method to find the traveling wave solutions to the right-handed nc-Burgers equation:

$$u_t = u_{xx} + 2uu_x \tag{8}$$

Using the traveling wave transformation (2), (8) is reduced to the following ODE:

$$u'' + 2uu' + Vu' = 0 \tag{9}$$

Integrating (9) with respect to ξ and setting the constant of integration to zero, we obtain

$$u' + u^2 + Vu = 0 \tag{10}$$

Balancing the highest order derivative and nonlinear term, we obtain$N = 1$. Now, the solutions of equation (10), according to equation (4) is

$$\begin{aligned} u(\xi) \;&=\; a_0 + \frac{a_1(G'/G)}{1+\lambda(G'/G)} + \frac{a_{-1}(1+\lambda(G'/G))}{(G'/G)} \\ &\quad + (b_0(G'/G)^{-1} + b_1 + b_{-1}(G'/G)^{-2})\sqrt{\sigma[1+(G'/G)^2/\mu]} \end{aligned} \tag{11}$$

where $G = G(\xi)$ satisfies equation (5). Substituting equations (11) and (5) into equation (10), collecting all terms with the like powers of $(G'/G)^j$ and $(G'/G)^j$ $\sqrt{\sigma[1+(G'/G)^2/\mu]}$, and setting them to zero, we obtain a over-determined system that consists of eighteen algebraic equations (which are omitted for convenience).

Solving this over-determined system with the assist of Maple and inserting in equation (11), we have the following results.

Set 1: $V = \pm\sqrt{-\mu}, a_0 = \mp\frac{1}{2}\sqrt{-\mu}, a_1 = \frac{1}{2}, b_1 = \pm\sqrt{\left(\frac{\mu}{4\sigma}\right)}, \lambda = a_{-1} = b_0 = b_{-1} = 0.$
Now, when $\mu > 0$, then using (7) and (11), we have

$$u_1(\xi) = -\frac{1}{2}\sqrt{-\mu} + \frac{1}{2}f_1(\xi) \pm \sqrt{\left(\frac{\mu}{4\sigma}\right)}\sqrt{\sigma(1+f_1^2(\xi)/\mu)}, \tag{12}$$

where $\xi = x - \sqrt{-\mu}t$

$$u_2(\xi) = \frac{1}{2}\sqrt{-\mu} + \frac{1}{2}f_1(\xi) \pm \sqrt{\left(\frac{\mu}{4\sigma}\right)}\sqrt{\sigma(1+f_1^2(\xi)/\mu)}, \tag{13}$$

where $\xi = x + \sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

$$u_3(\xi) = -\frac{1}{2}\sqrt{-\mu} + \frac{1}{2}f_2(\xi) \pm \sqrt{\left(\frac{\mu}{4\sigma}\right)}\sqrt{\sigma(1+f_2^2(\xi)/\mu)}, \tag{14}$$

where $\xi = x - \sqrt{-\mu}t$

(15) $$u_4(\xi) = \frac{1}{2}\sqrt{-\mu} + \frac{1}{2}f_2(\xi) \pm \sqrt{\left(\frac{\mu}{4\sigma}\right)}\sqrt{\sigma(1 + f_2^2(\xi)/\mu)},$$

where $\xi = x + \sqrt{-\mu}t$

Set 2: $V = \pm 2\sqrt{-\mu},\ a_0 = a_0,\ a_1 = \dfrac{a_0^2 \pm 2a_0\sqrt{-\mu}}{\mu},\ \lambda = \dfrac{-a_0 \mp \sqrt{-\mu}}{\mu}$, $a_1 = b_{-1} = b_0 = b_1 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

(16) $$u_5(\xi) = a_0 + \frac{a_0^2 + 2a_0\sqrt{-\mu}}{\mu} \times \frac{f_1(\xi)}{1 + \frac{-a_0 - \sqrt{-\mu}}{\mu} f_1(\xi)},$$

where $\xi = x - 2\sqrt{-\mu}t$

(17) $$u_6(\xi) = a_0 + \frac{a_0^2 - 2a_0\sqrt{-\mu}}{\mu} \times \frac{f_1(\xi)}{1 + \frac{-a_0 + \sqrt{-\mu}}{\mu} f_1(\xi)},$$

where $\xi = x + 2\sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

(18) $$u_7(\xi) = a_0 + \frac{a_0^2 + 2a_0\sqrt{-\mu}}{\mu} \times \frac{f_2(\xi)}{1 + \frac{-a_0 - \sqrt{-\mu}}{\mu} f_2(\xi)},$$

where $\xi = x - 2\sqrt{-\mu}t$

(19) $$u_8(\xi) = a_0 + \frac{a_0^2 - 2a_0\sqrt{-\mu}}{\mu} \times \frac{f_2(\xi)}{1 + \frac{-a_0 + \sqrt{-\mu}}{\mu} f_2(\xi)},$$

where $\xi = x + 2\sqrt{-\mu}t$

Set 3: $V = \pm 2\sqrt{-\mu}, a_{-1} = -\mu, a_0 = \pm\sqrt{-\mu}, \lambda = a_1 = b_1 = b_{-1} = b_0 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

(20) $$u_9(\xi) = -\sqrt{-\mu} - \mu f_1^{-1}(\xi),$$

where $\xi = x - 2\sqrt{-\mu}t$

(21) $$u_{10}(\xi) = \sqrt{-\mu} - \mu f_1^{-1}(\xi),$$

where $\xi = x + 2\sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

(22) $$u_{11}(\xi) = -\sqrt{-\mu} - \mu f_2^{-1}(\xi),$$

where $\xi = x - 2\sqrt{-\mu}t$

(23) $$u_{12}(\xi) = \sqrt{-\mu} - \mu f_2^{-1}(\xi),$$

where $\xi = x + 2\sqrt{-\mu}t$

Set 4: $V = \pm 4\sqrt{-\mu}, a_{-1} = -\mu, a_0 = \pm 2\sqrt{-\mu}, a_1 = 1, \lambda = b_1 = b_{-1} = b_0 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$u_{13}(\xi) = 2\sqrt{-\mu} + f_1(\xi) - \mu f_1^{-1}(\xi), \tag{24}$$

where $\xi = x - 4\sqrt{-\mu}t$

$$u_{14}(\xi) = -2\sqrt{-\mu} + f_1(\xi) - \mu f_1^{-1}(\xi), \tag{25}$$

where $\xi = x + 4\sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

$$u_{15}(\xi) = 2\sqrt{-\mu} + f_2(\xi) - \mu f_2^{-1}(\xi), \tag{26}$$

where $\xi = x - 4\sqrt{-\mu}t$

$$u_{16}(\xi) = -2\sqrt{-\mu} + f_2(\xi) - \mu f_2^{-1}(\xi), \tag{27}$$

where $\xi = x + 4\sqrt{-\mu}t$

Set 5: $V = \pm 2\sqrt{-\mu}, a_{-1} = -\mu, a_0 = a_0, \lambda = \dfrac{a_0 \pm \sqrt{-\mu}}{\mu}, b_{-1} = a_1 = b_1 = b_0 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$u_{17}(\xi) = a_0 - \mu(f_1(\xi))^{-1}\left[1 + \frac{a_0 + \sqrt{-\mu}}{\mu} f_1(\xi)\right], \tag{28}$$

where $\xi = x - 2\sqrt{-\mu}t$

$$u_{18}(\xi) = a_0 - \mu(f_1(\xi))^{-1}\left[1 + \frac{a_0 - \sqrt{-\mu}}{\mu} f_1(\xi)\right], \tag{29}$$

where $\xi = x + 2\sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

$$u_{19}(\xi) = a_0 - \mu(f_2(\xi))^{-1}\left[1 + \frac{a_0 + \sqrt{-\mu}}{\mu} f_2(\xi)\right], \tag{30}$$

where $\xi = x - 2\sqrt{-\mu}t$

$$u_{20}(\xi) = a_0 - \mu(f_2(\xi))^{-1}\left[1 + \frac{a_0 - \sqrt{-\mu}}{\mu} f_2(\xi)\right], \tag{31}$$

where $\xi = x + 2\sqrt{-\mu}t$

Set-6: $V = \pm\sqrt{-\mu}, a_{-1} = -\mu/2, a_0 = a_0, \lambda = \dfrac{2a_0 \pm \sqrt{-\mu}}{\mu}, b_0 = \pm\mu\sqrt{(}1/4\sigma)$, $a_1 = b_1 = b_{-1} = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$u_{21}(\xi) = a_0 - \frac{\mu + (2a_0 + \sqrt{-\mu}) f_1(\xi)}{2 f_1(\xi)} + \mu f_1^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_1^2(\xi)/\mu)}, \tag{32}$$

where $\xi = x - \sqrt{-\mu}t$

$$u_{22}(\xi) = a_0 - \frac{\mu + (2a_0 - \sqrt{-\mu})f_1(\xi)}{2f_1(\xi)} - \mu f_1^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_1^2(\xi)/\mu)}, \tag{33}$$

where $\xi = x + \sqrt{-\mu}t$

$$u_{23}(\xi) = a_0 - \frac{\mu + (2a_0 + \sqrt{-\mu})f_1(\xi)}{2f_1(\xi)} - \mu f_1^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_1^2(\xi)/\mu)}, \tag{34}$$

where $\xi = x - \sqrt{-\mu}t$

$$u_{24}(\xi) = a_0 - \frac{\mu + (2a_0 - \sqrt{-\mu})f_1(\xi)}{2f_1(\xi)} + \mu f_1^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_1^2(\xi)/\mu)}, \tag{35}$$

where $\xi = x + \sqrt{-\mu}t$ and when $\mu < 0$, then using (6) and (11), we have

$$u_{25}(\xi) = a_0 - \frac{\mu + (2a_0 + \sqrt{-\mu})f_2(\xi)}{2f_2(\xi)} + \mu f_2^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_2^2(\xi)/\mu)}, \tag{36}$$

where $\xi = x - \sqrt{-\mu}t$

$$u_{26}(\xi) = a_0 - \frac{\mu + (2a_0 - \sqrt{-\mu})f_2(\xi)}{2f_2(\xi)} - \mu f_2^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_2^2(\xi)/\mu)}, \tag{37}$$

where $\xi = x + \sqrt{-\mu}t$

$$u_{27}(\xi) = a_0 - \frac{\mu + (2a_0 + \sqrt{-\mu})f_2(\xi)}{2f_2(\xi)} - \mu f_2^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_2^2(\xi)/\mu)}, \tag{38}$$

where $\xi = x - \sqrt{-\mu}t$

$$u_{28}(\xi) = a_0 - \frac{\mu + (2a_0 - \sqrt{-\mu})f_2(\xi)}{2f_2(\xi)} + \mu f_2^{-1}(\xi)\sqrt{\frac{1}{4}(1 + f_2^2(\xi)/\mu)}, \tag{39}$$

where $\xi = x + \sqrt{-\mu}t$

Remark 2. Comparison between Khan et al. [25] solutions and new solutions:

(i) If $C_1 = v, C_2 = 1$ in the paper of Khan et al. [25], from sub section 3.2 in example 2, the exact solution (35) turns into the solitary wave solution (36) $-\frac{v}{2}\{1 - coth(v(x - vt)/2)\}$ but if we put $v/2 = \sqrt{-\mu}, A = 1, B = 0$ in our solution u_{11} becomes $-\frac{v}{2}\{1 - coth(v(x - vt)/2)\}$

(ii) If $C_1 = -v, C_2 = 1$ in the paper of Khan et al. [25], from Section 3.2 in Example 2, the exact solution (35) turns into the solitary wave solution (37) $-\frac{v}{2}\{1 - tanh(v(x - vt)/2)\}$ but if we put $v/2 = \sqrt{-\mu}, A = 0, B = 1$ in our solution u_{11} becomes $-\frac{v}{2}\{1 - tanh(v(x - vt)/2)\}$

Example 3.2. In this subsection, we will bring to bear the new extended (G'/G)-expansion method to find the traveling wave solutions of the (1+1)-dimensional compound KdVB equation in the form:

$$u_t + \alpha u u_x + \beta u^2 u_x + \gamma u_{xx} - \delta u_{xxx} = 0, \tag{40}$$

where α, β, γ and δ are constants.

This equation can be thought of as a generalization of KdV-mKdV and Burgers equations involving nonlinear dispersion and dissipation effects. The traveling wave solutions of equation (40) have been found in [23] using (G'/G)-expansion method. To this end, we are going to find new traveling solution of the equation by our proposed method. Using traveling wave transformation (2), equation (40) is reduced to the following ODE:

$$C - Vu + \tfrac{1}{2}\alpha u^2 + \tfrac{1}{3}\beta u^3 + \gamma u' - \delta u'' = 0, \tag{41}$$

where C is an integration constant. Considering the homogeneous balance between the highest order derivative and nonlinear term, we obtain$N = 1$. Now, the solutions of equation (40), according to equation (4) is same of the equation (11). Substituting equation (11) and equation (5) into equation (41), collecting all terms with the like powers of $(G'/G)^j$and $(G'/G)^j\sqrt{\sigma[1 + (G'/G)^2/\mu]}$, and setting them to zero, we obtain a over-determined system that consists of eighteen algebraic equations (which are omitted for convenience).

Solving this over-determined system with the assist of Maple and inserting in equation (11), we have the following results.

Set 1. $C = \frac{1}{72\delta\beta}\{(288\alpha\mu\delta^2\beta + 3\alpha^3\delta - 6\alpha\beta\gamma^2) \mp (8\beta\gamma^3 + 1152\beta\gamma\mu\delta^2)/\sqrt{6\delta/\beta}\}$, $V = -\frac{96\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}$, $\lambda = 0$, $a_1 = \mp\sqrt{\frac{6\delta}{\beta}}$, $a_{-1} = \pm\mu\sqrt{\frac{6\delta}{\beta}}$, $a_0 = -\frac{\alpha}{2\beta} \mp \frac{\gamma}{\sqrt{6\delta\beta}}$, $b_1 = b_0 = b_{-1} = 0$.

Now, when $\mu > 0$, then using (7) and (11), we have

$$u_{2,1}^{\pm}(\xi) = -\frac{\alpha}{2\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \mu\sqrt{\frac{6\delta}{\beta}} f_1^{-1}(\xi) \pm \sqrt{\frac{6\delta}{\beta}} f_1(\xi), \tag{42}$$

and when $\mu < 0$, then using (6) and (11), we have

$$u_{2,2}^{\pm}(\xi) = -\frac{\alpha}{2\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \mu\sqrt{\frac{6\delta}{\beta}} f_2^{-1}(\xi) \pm \sqrt{\frac{6\delta}{\beta}} f_2(\xi), \tag{43}$$

where $\xi = x + \frac{96\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}t$

Set 2. $C = \frac{1}{72\delta\beta^2}\{(72\alpha\mu\delta^2\beta + 3\alpha^3\delta - 6\alpha\beta\gamma^2) \pm (8\beta\gamma^2 + 288\beta\gamma\mu\delta^2)/\sqrt{6\delta/\beta}\}$,

$V=-\frac{24\delta^2\mu\beta+3\delta\alpha^2-2\beta\gamma^2}{12\delta\beta}$, $\lambda=0$, $a_1=\pm\sqrt{\frac{6\delta}{\beta}}$, $a_0=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}$, $a_{-1}=b_1=b_0=b_{-1}=0$. Now, when $\mu>0$, then using (7) and (11), we have

$$u^{\pm}_{2,3}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}\pm\sqrt{\frac{6\delta}{\beta}}f_1(\xi), \tag{44}$$

and when $\mu<0$, then using (6) and (11), we have

$$u^{\pm}_{2,4}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}\pm\sqrt{\frac{6\delta}{\beta}}f_2(\xi), \tag{45}$$

where $\xi=x+\frac{24\delta^2\mu\beta+3\delta\alpha^2-2\beta\gamma^2}{12\delta\beta}t$

Set 3. $C=\frac{1}{216\delta^2\beta^2}\{\pm(72\gamma\mu\beta^2\delta^2+8\beta^2\gamma^3)\sqrt{3\delta/2\beta}+54\alpha\beta\mu\delta^3-18\alpha\beta\gamma^2\delta\}$, $V=-\frac{6\delta^2\mu\beta+3\delta\alpha^2-2\beta\gamma^2}{12\delta\beta}$, $\lambda=0$, $a_1=\pm\sqrt{\frac{3\delta}{2\beta}}$, $a_0=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}$, $b_1=\pm\sqrt{\frac{3\delta\mu}{2\beta\sigma}}$, $a_{-1}=b_{-1}=b_0=0$. Now, when $\mu>0$, then using (7) and (11), we have

$$u^{\pm}_{2,5}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}\pm\sqrt{\frac{3\delta}{2\beta}}f_1(\xi)\mp\sqrt{\frac{3\delta\mu}{2\beta\sigma}}\sqrt{\sigma(1+f_1^2(\xi)/\mu)}, \tag{46}$$

and when $\mu<0$, then using (6) and (11), we have

$$u^{\pm}_{2,6}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma}{\sqrt{6\delta\beta}}\pm\sqrt{\frac{3\delta}{2\beta}}f_2(\xi)\mp\sqrt{\frac{3\delta\mu}{2\beta\sigma}}\sqrt{\sigma(1+f_2^2(\xi)/\mu)}, \tag{47}$$

where $\xi=x+\frac{6\delta^2\mu\beta+3\delta\alpha^2-2\beta\gamma^2}{12\delta\beta}t$

Set 4. $C=\frac{1}{72\delta\beta^2}\{(72\alpha\mu\delta^2\beta+3\alpha^3\delta-6\alpha\beta\gamma^2)\mp(8\beta\gamma^2+288\beta\gamma\mu\delta^2)/\sqrt{6\delta/\beta}\}$, $V=-\frac{24\delta^2\mu\beta+3\delta\alpha^2-2\beta\gamma^2}{12\delta\beta}$, $\lambda=\lambda$, $a_{-1}=\pm\mu\sqrt{\frac{6\delta}{\beta}}$, $a_0=-\frac{\alpha}{2\beta}\pm\frac{\gamma+6\delta\mu\lambda}{\sqrt{6\delta\beta}}$, $a_1=b_{-1}=b_0=b_1=0$. Now, when $\mu>0$, then using (7) and (11), we have

$$u^{\pm}_{2,7}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma+6\delta\mu\lambda}{\sqrt{6\delta\beta}}\mp\mu\sqrt{\frac{6\delta}{\beta}}\frac{1+\lambda f_1(\xi)}{f_1(\xi)}, \tag{48}$$

and when $\mu<0$, then using (6) and (11), we have

$$u^{\pm}_{2,8}(\xi)=-\frac{\alpha}{2\beta}\pm\frac{\gamma+6\delta\mu\lambda}{\sqrt{6\delta\beta}}\mp\mu\sqrt{\frac{6\delta}{\beta}}\frac{1+\lambda f_2(\xi)}{f_2(\xi)}, \tag{49}$$

where $\xi = x + \frac{24\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}t$

Set 5. $C = \frac{1}{72\delta\beta^2}\{(72\alpha\mu\delta^2\beta + 3\alpha^3\delta - 6\alpha\beta\gamma^2) \pm (8\beta\gamma^2 + 288\beta\gamma\mu\delta^2)/\sqrt{6\delta/\beta}\}$, $V = -\frac{24\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}$, $\lambda = \lambda$, $a_1 = \pm(\lambda^2\mu+1)\sqrt{\frac{6\delta}{\beta}}$, $a_0 = -\frac{\alpha}{2} \pm \frac{\gamma - 6\delta\mu\lambda}{\sqrt{6\delta\beta}}$, $a_{-1} = b_{-1} = b_0 = b_1 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$u^{\pm}_{2,9}(\xi) = -\frac{\alpha}{2} \pm \frac{\gamma - 6\delta\mu\lambda}{\sqrt{6\delta/\beta}} \pm (\lambda^2\mu + 1)\sqrt{\frac{6\delta}{\beta}}\frac{f_1(\xi)}{1 + \lambda f_1(\xi)}, \tag{50}$$

and when $\mu < 0$, then using (6) and (11), we have

$$u^{\pm}_{2,10}(\xi) = -\frac{\alpha}{2} \pm \frac{\gamma - 6\delta\mu\lambda}{\sqrt{6\delta/\beta}} \pm (\lambda^2\mu + 1)\sqrt{\frac{6\delta}{\beta}}\frac{f_2(\xi)}{1 + \lambda f_2(\xi)}, \tag{51}$$

where $\xi = x + \frac{24\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}t$

Set 6. $C = \frac{1}{72\delta\beta^2}\{(18\alpha\mu\delta^2\beta + 3\alpha^3\delta - 6\alpha\beta\gamma^2) \mp (2\gamma^3 + 18\gamma\mu\delta^2)/\sqrt{6\delta\beta}\}$, $V = -\frac{6\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}$, $\lambda = \lambda$, $a_{-1} = \pm\mu\sqrt{\frac{3\delta}{2\beta}}$, $a_0 = -\frac{\alpha}{2\beta} \mp \frac{\gamma + 3\delta\mu\lambda}{\sqrt{6\delta\beta}}$, $b_0 = \pm\mu\sqrt{\frac{3\delta}{2\beta\sigma}}$, $a_1 = b_{-1} = b_1 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$\begin{aligned} u_{2,11}(\xi) &= -\frac{\alpha}{2\beta} - \frac{\gamma+3\delta\mu\lambda}{\sqrt{6\delta\beta}} + \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1+\lambda f_1(\xi)}{f_1(\xi)} \\ &\quad \mp \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1+f_1^2(\xi)/\mu)}, \end{aligned} \tag{52}$$

$$\begin{aligned} u_{2,12}(\xi) &= -\frac{\alpha}{2\beta} + \frac{\gamma + 3\delta\mu\lambda}{\sqrt{6\delta\beta}} - \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1 + \lambda f_1(\xi)}{f_1(\xi)} \\ &\quad \mp \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1 + f_1^2(\xi)/\mu)}, \end{aligned} \tag{53}$$

and when $\mu < 0$, then using (6) and (11), we have

$$\begin{aligned} u_{2,13}(\xi) &= -\frac{\alpha}{2\beta} - \frac{\gamma + 3\delta\mu\lambda}{\sqrt{6\delta\beta}} + \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1 + \lambda f_2(\xi)}{f_2(\xi)} \\ &\quad \mp \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1 + f_2^2(\xi)/\mu)}, \end{aligned} \tag{54}$$

$$
\begin{aligned}
u_{2,14}(\xi) &= -\frac{\alpha}{2\beta} + \frac{\gamma + 3\delta\mu\lambda}{\sqrt{6\delta\beta}} - \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1+\lambda f_2(\xi)}{f_2(\xi)} \\
&\quad \mp \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1+f_2^2(\xi)/\mu)},
\end{aligned}
\tag{55}
$$

where $\xi = x + \dfrac{6\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}t$

Set 7. $C = -\dfrac{1}{216\delta^2\beta^2}\{\pm(72\gamma\mu\beta^2\delta^2 + 8\beta^2\gamma^3)\sqrt{3\delta/2\beta} - 9\delta^2\alpha^3 - 54\alpha\beta\mu\delta^3 + 18\alpha\beta\gamma^2\delta\}$, $V = -\dfrac{6\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}$, $\lambda = -\dfrac{\gamma}{3\mu\delta}$, $a_{-1} = \pm\mu\sqrt{\dfrac{3\delta}{2\beta}}$, $a_0 = -\dfrac{\alpha}{2\beta}$, $b_0 = \pm\mu\sqrt{\dfrac{3\delta}{2\beta\sigma}}$, $a_1 = b_{-1} = b_1 = 0$. Now, when $\mu > 0$, then using (7) and (11), we have

$$
u_{2,15}(\xi) = -\frac{\alpha}{2\beta} \pm \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1 - \frac{\gamma}{3\mu\delta}f_1(\xi)}{f_1(\xi)} \pm \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1+f_1^2(\xi)/\mu)},
\tag{56}
$$

and when $\mu < 0$, then using (6) and (11), we have

$$
u_{2,16}(\xi) = -\frac{\alpha}{2\beta} \pm \mu\sqrt{\frac{3\delta}{2\beta}}\frac{1 - \frac{\gamma}{3\mu\delta}f_2(\xi)}{f_2(\xi)} \pm \mu\sqrt{\frac{3\delta}{2\beta\sigma}}\sqrt{\sigma(1+f_2^2(\xi)/\mu)},
\tag{57}
$$

where $\xi = x + \dfrac{6\delta^2\mu\beta + 3\delta\alpha^2 - 2\beta\gamma^2}{12\delta\beta}t$

Note. For correct solutions, we have one solution taking upper signs and another solution taking lower sign but there is no restriction on the signs of b_1, b_0, b_{-1}.

Remark 3. Comparison between Zayed [23] solutions and new solutions:

In the paper of Zayed [23], from Section 3 in Example 2, the exact solution (23) turns our solution (45) when $\gamma = 0$ and $AB = -\mu/2$. After then, if we use the same conditions on A, B like [23], we can get all the solitary solutions of [23].

Remark 4. We have verified all the achieved solutions by putting them back into the original equation (8) of Example 3.1 and into the original equation (40) of Example 3.2 with the aid of Maple 13.

4. Results and discussion

We have constructed twenty eight exact traveling wave solutions for the nc-Burger equations and thirty two solutions for the $(1+1)$-dimensional compound KdVB equation including solitons, periodic solutions via the new extended (G'/G)-expansion method. It is important to state that one (for each equation) of our obtained solutions is in good agreement with the existing results which are shown in Remarks 2, 3. Beyond Remarks 2, 3, we have constructed new exact traveling wave solutions u_1 to u_{28} for ncBurgers equation and solutions $u_{1,2}^{\pm}$ to $u_{2,16}^{\pm}$ for (1+1)-dimensional compound KdVB equation which have not been reported in the previous literature. In addition, the graphical representations of some obtained traveling wave solutions are shown in Figure 1 to Figure 13.

Graphical representations of the solutions: The graphical illustrations of the solutions are depicted in the figures with the aid of Maple. Solutions u_3, u_4, u_7, u_8, $u_{25}, u_{28}, u_{2,4}^{\pm}, u_{2,6}^{\pm}, u_{2,8}^{\pm}, u_{2,10}^{\pm}, u_{2,13}, u_{2,14}$ and $u_{2,16}^{\pm}$ describes the kink wave. Kink waves are traveling waves which arise from one asymptotic state to another. The kink solutions are approach to a constant at infinity. Fig. 1 and Fig. 2 below shows the shape of the exact Kink-type solution of u_3 and u_{28} the right-handed noncommutative burgers equation (8). The shape of figures of solutions $u_4, u_7, u_8, u_{25}, u_{2,8}^{\pm}, u_{2,10}^{\pm}, u_{2,13}, u_{2,14}$ are similar to the figure of solution of u_3 and $u_4^{\pm}, u_6^{\pm}, u_{16}^{\pm}$ are similar to Fig. 2. So, the figures of these solutions are omitted for convenience.

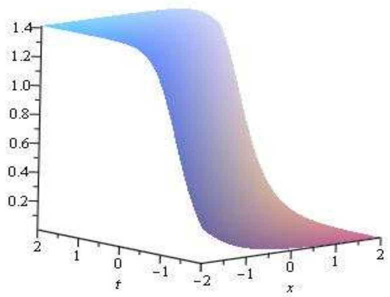

Fig-1: Kink solution for Eq (14) with μ= -2, σ = -1, B=1, A=2

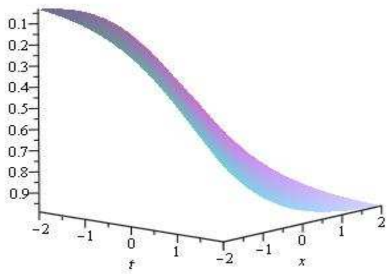

Fig-2: Kink solution for Eq (39) with μ= -1, σ =B=1, A=2, a0= -2

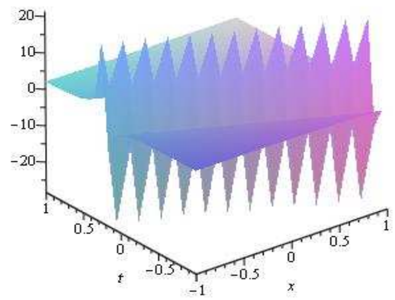

Fig-3: Kink solution for Eq (31) with μ= -1, σ =B=1, A=2, a0= -1

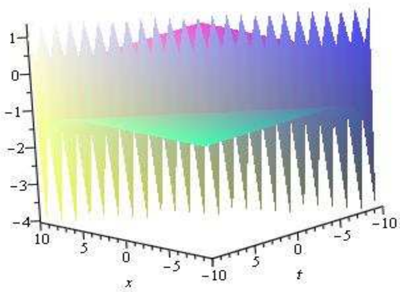

Fig-4: Singular Kink solution for Eq (38) with μ= -1, σ =B=1, A=2, a0= -

Solutions $u_{11}, u_{12}, u_{15}, u_{16}, u_{19}, u_{20}$ and u_{27} comes infinity as in hyperbolic function, are singular Kink solution. Fig. 3 and Fig. 4 shows the shape of the exact singular Kink-type solution of u_{20} and u_{27} respectively. The shape of figures of solutions $u_{11}, u_{12}, u_{15}, u_{16}, u_{19}$ are similar to the figure of solution u_{20}, and so, the figures of these solutions are omitted for convenience. Another type of singular kink type figure is expressed by the solution u_{26} whose figure is described by Fig. 5 and singular kink type figure of the of KdVB equation is expressed by solutions $u^{\pm}_{2,2}$ whose figure is described by Fig. 10.

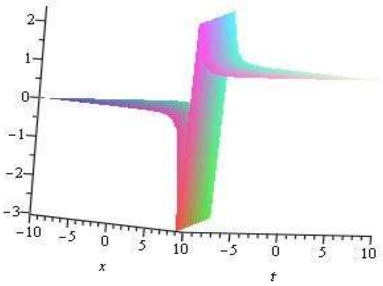

Fig-5: Singular Kink solution for Eq (37) with μ= -1, σ =B=1, A=2, a0= -2

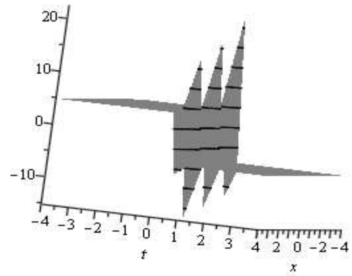

Fig-10: singular kink solution to Eq(47) for α=β=γ=δ=σ =1, μ= -1, A=2, B=3

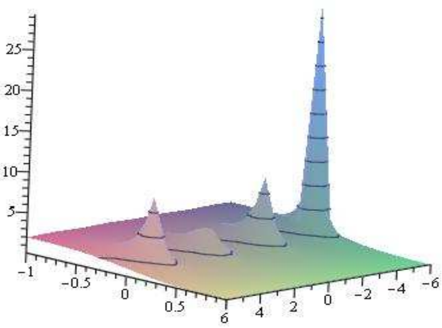

Fig-6: Singular Kink solution for (21) with μ=σ =B=1, A=3

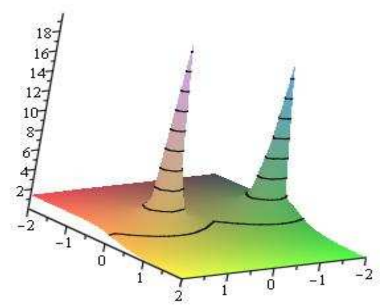

Fig-7: Singular Kink solution for (33) with A=μ=2, σ =B=1, a0=-2

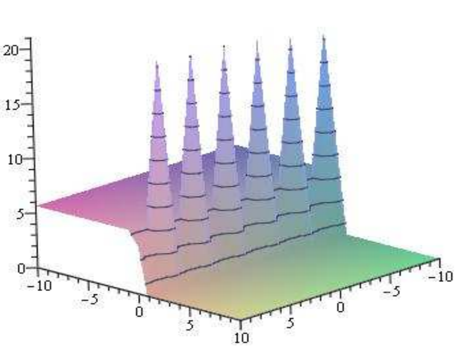

Fig-8: Singular Kink solution for (25) with μ=2, σ =B=1, A=3

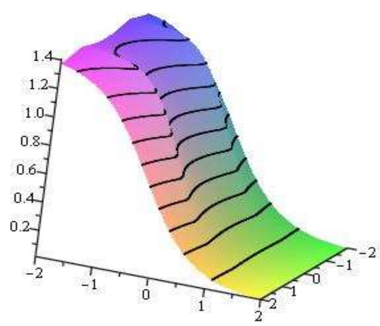

Fig-9: Singular Kink solution for (32) with A=μ=2, σ =B=1, a0=-2

Solutions $u_1, u_2, u_5, u_6, u_9, u_{10}, u_{13}, u_{14}, u_{17}, u_{18}, u_{21}, u_{22}, u_{23}$ and u_{24} comes infinity as in trigonometric function, are singular Kink solution. Fig. 6, Fig. 7, Fig. 8 and Fig. 9 shows the shape of the exact singular Kink-type solution of u_{10}, u_{22}, u_{14}, and u_{21}, respectively. The shape of figures of solutions $u_1, u_2, u_5, u_6, u_9, u_{18}$ are similar to the figure of solution u_{10}, the shape of figure of solution u_{13} is similar to the figure of solution u_{25} and the shape of figure of solution u_{24} is similar to the figure of solution u_{21}, and so, the figures of these solutions are omitted for convenience.

Solutions $u^{\pm}_{2,1}, u^{\pm}_{2,3}, u^{\pm}_{2,5}, u^{\pm}_{2,7}, u^{\pm}_{2,9}, u_{2,11}, u_{2,12}$ and $u^{\pm}_{2,15}$ of KdVB equation (trigonometric functions as $cos(x-t)$) are periodic solutions. Fig. 11 Fig. 12 and Fig. 13 shows the shape of the periodic solution of $u^{-}_{2,1}$, $u^{-}_{2,5}$ and $u_{2,15}$ (when b_0 take $-$, a_{-1} take $+$ sign), respectively. The shape of figures of solutions $u^{+}_{2,1}$, $u^{\pm}_{2,3}$, $u^{\pm}_{2,7}$, $u^{\pm}_{2,9}$, $u_{2,11}$, $u_{2,12}$ are similar to the figure of solution $u^{-}_{2,1}$, the shape of figure of solution $u_{2,15}$ (b_0 take $+$, a_{-1} take $-$), $u_{2,11}$ (when sign of a_0 take $-$, a_{-1} take $+$, b_1 take $-$) and $u_{2,12}$ (when sign of a_0 take $+$, a_{-1} take $-$, b_1 take $+$) is similar to the Fig. 13. Others are similar to the Fig. 11, and so, the figures of these solutions are omitted for convenience.

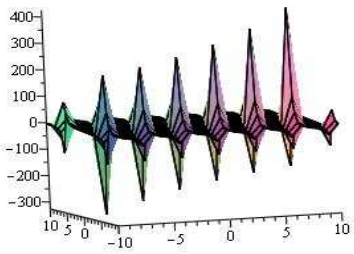

Fig-11: Periodic solution of Eq.(43) for
α=β=γ=δ=σ =1, A=μ=2, B=3

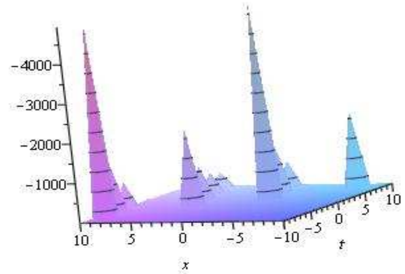

Fig-12: Periodic solution of Eq.(46) for
α=β=γ=δ=1, σ = -1, A=μ=2, B=3

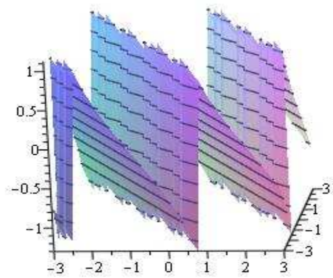

Fig-13: Periodic solution of Eq.(56) for
α=β=γ=δ=σ =μ=1, A=2, B=3

5. Conclusion

The new extended (G'/G)-expansion method is presented to search exact traveling wave solutions for NPDEs. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation using a computer by the help of symbolic programs such as Maple, Mathematica, Matlab, and so on. We apply it to the right-handed non-commutative burgers and the $(1+1)$-dimensional compound KdVB equations. As a results, many plentiful new hyperbolic functions and periodic solutions with free parameters including soliton solutions are obtained. Overall, the results reveal that the presented method is effective, productive and more powerful than the original (G'/G)-expansion method and it can be applied for other kind of nonlinear evolution equations in mathematical physics.

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