COMMON FIXED POINT FOR SELF AND NONSELF-MAPS THROUGH AN IMPLICIT RELATION

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Abstract. Common fixed point theorems for three self and nonself-maps have been proved through the notions of property E.A., orbital completeness and weak compatibility under an implicit relation. The results of Singh and Mishra (1997), Singh and Asish Kumar (2006), Khan and Dolmo (2007) and Imdad and Ali (2008) are then particular cases.

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1. Introduction and preliminaries

Let (X, d) be a metric space. Self-maps S and A on X are *compatible* [2] if

$$\lim_{n\to\infty} d(SAx_n, ASx_n) = 0 \tag{1.1}$$

whenever $\langle x_n\rangle_{n=1}^{\infty} \subset X$ is such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = p \quad \text{for some } p \in X. \tag{1.2}$$

A point $x \in X$ is a *coincidence point* of self-maps S and A on X if $Sx = Ax$. Self-maps which commute at their coincidence points are called *weakly compatible* [3]. Thus S and A are weakly compatible if $SAx = ASx$ whenever $x \in X$ is such that $Ax = Sx$. It is obvious that every compatible pair is weakly compatible. One can find examples from [3] for weakly compatible maps which are not compatible. Let $\mathbb{R}_+$ be the space of nonnegative real numbers. A *contractive modulus* $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ with the choice $\phi(0) = 0$ and $\phi(t) < t$ for $t > 0$ is *upper semicontinuous* (abbriviated as *usc*) if for each $t_0 \geq 0$, $\limsup_{n\to\infty} \phi(t_n) \leq \phi(t_0)$ whenever $\langle t_n\rangle_{n=1}^{\infty} \subset \mathbb{R}_+$ is such that $\lim_{n\to\infty} t_n = 0$.

With these notations, Singh and Mishra, [11] proved the following result:

Theorem 1.1 *Let S, T and A be self-maps on X satisfying the inclusions*

$$S(X) \subset A(X) \quad \text{and} \quad T(X) \subset A(X) \tag{1.3}$$

and the contractive-type condition

$$\begin{aligned} &d(Sx, Ty) \\ &\leq \phi\left(\max\left\{d(Ax, Ay), d(Sx, Ax), d(Ty, Ay), \frac{d(Ty, Ax) + d(Sx, Ay)}{2}\right\}\right) \\ &\qquad \text{for all } x, y \in X, \end{aligned} \tag{1.4}$$

where ϕ is a nondecreasing, usc contractive modulus. Suppose that one of $S(X)$, $T(X)$ and $A(X)$ is a complete subspace of X. If

(a) *(A, S) and (A, T) are weakly compatible,*

then the three maps S, T and A will have a unique common fixed point.

From the definition of compatibility, it is clear that self-maps S and A are *noncompatible* on X if (1.2) holds good but $\lim_{n\to\infty} d(Sx_n, Ax_n)$ is either $\neq 0$ or $+\infty$ for some $\langle x_n\rangle_{n=1}^{\infty} \subset X$. It may be noted from [4] that both compatible and noncompatible maps are included in the class of maps with (1.2). Self-maps S and A on X are said to satify the *property E.A.* [1] if (1.2) holds good for some $\langle x_n\rangle_{n=1}^{\infty} \in X$, where the common limit p is called a *tangent point.* It is known that weak compatibility and property E.A. are independent.

With these ideas, the following theorem was proved in [10]:

Theorem 1.2 *Let S, T and $A : Y \to X$ satisfy the inequality*

$$\begin{aligned} &d(Sx, Ty) \\ &< \max\left\{d(Ax, Ay), \alpha d(Ax, Sx), \alpha d(Ay, Ty), \frac{d(Ax, Ty) + d(Sx, Ay)}{2}\right\}, \\ &\qquad \text{for all } \quad x, y \in X \text{ with } 0 < \alpha < 1 \end{aligned} \tag{1.5}$$

and both conditions

(b) $\overline{S(Y)} \subset A(Y)$,

(c) $\overline{T(Y)} \subset A(Y)$,

where Y an arbitrary nonempty subset of X. Suppose one of the pairs (S, A) *and* (T, A) *satisfies the property E.A. on Y. Then there is a coincidence point common to S, T and A in Y. Further, if Y=X and (a) of Theorem* 1.1 *holds good, then S, T and A will have a unique common fixed point in X.*

In this paper, we prove the generalizations of Theorem 1.1 and Theorem 1.2 using implicit relations.

2. Main results

Given $x_0 \in X$ and S, T and A, self-maps on X, if there exist points $x_1, x_2, x_3, ...$ in X such that

$$Sx_{2n-2} = Ax_{2n-1}, Tx_{2n-1} = Ax_{2n}, \quad \text{for} \quad n = 1, 2, 3, ... \tag{2.1}$$

then the sequence $\langle Ax_n \rangle_{n=1}^{\infty} \subset X$ is an (S, T, A)-*orbit* or simply an *orbit* at x_0. The metric space X is (S, T, A)-*orbitally complete* or *orbitally complete* [7], [8] at x_0 if every Cauchy sequence in some orbit at $x_0 \in X$ converges in X. It easily follows that every complete metric space is orbitally complete at each of its points. However the converse is not true (cf. [7], [8]).

Let $\mathbb{R}_+^6$ denote the space of 6-tuples of nonnegative real numbers in this paper, and $\psi : \mathbb{R}_+^6 \to \mathbb{R}$, lower semicontinuous in each coordinate variable such that

(C_1) ψ is nondecreasing in the fifth and sixth coordinate variables,

(C_2) For every $l \geq 0, m \geq 0$, there is a constant $0 \leq \omega < 1$ such that

$$\min\{\psi(l, m, m, l, l+m, 0), \psi(l, m, l, m, 0, l+m)\} \leq 0 \ \Rightarrow \ l \leq \omega m, \tag{2.2}$$

(C_3) $\psi(l, l, 0, 0, l, l) > 0$ for all $l > 0$.

Such implicit-type relations were first introduced by Popa [9], which covers several contractive conditions in proving fixed point theorems.

Theorem 2.1 *Let S, T and A be self-maps on X satisfying the implicit-type inequality*

$$\psi(d(Sx, Ty), d(Ax, Ay), d(Sx, Ax), d(Ty, Ay), d(Ty, Ax), d(Sx, Ay)) \leq 0 \quad \textit{for all} \quad x, y \in X. \tag{2.3}$$

Suppose that either (S, A) *or* (T, A) *satisfies the property E.A. and that one of the following statements holds good:*

1. *$A(X)$ is orbitally complete at some $x_0 \in X$,*
2. $\overline{S(X)} \subset A(X)$,
3. $\overline{T(X)} \subset A(X)$.

Then there is a coincidence point u common to S, T and A. Further, if

(a°) *(S, A) or (T, A) is weakly compatible,*

then the point of coincidence of S, T and A with respect to u will be their unique common fixed point.

Proof. Suppose that (S, A) satisfies the property E.A. Then (1.2) holds good for some $\langle x_n\rangle_{n=1}^{\infty} \subset X$. We claim that $q = \lim\limits_{n\to\infty} Tx_n = p$. Writing $x = y = x_n$ in (2.3), we find that

$$\psi(d(Sx_n, Tx_n), d(Ax_n, Ax_n), d(Ax_n, Sx_n), d(Ax_n, Tx_n), d(Ax_n, Tx_n), d(Ax_n, Sx_n)) < 0.$$

Applying the limit as $n \to \infty$ in this, using (1.2) and the lower semicontinuity of ψ, we get

$$\psi(d(p, q), 0, 0, d(p, q), d(p, q), 0) \leq 0,$$

which gives (2.2) with $l = d(p, q)$ and $m = 0$. Therefore, by (C_2), we get $d(p, q) \leq 0$ or $p = q$. Thus

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Ax_n = p \quad \text{for some} \quad p \in X. \tag{2.4}$$

While if (T, A) satisfies the property E.A., there is some $\langle y_n\rangle_{n=1}^{\infty} \subset X$ such that

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Ay_n = r \quad \text{for some} \quad r \in X. \tag{2.5}$$

Then (2.3) with $x = y = y_n$ gives

$$\psi(d(Sy_n,Ty_n), d(Ay_n, Ay_n), d(Ay_n, Sy_n), d(Ay_n,Ty_n), d(Ay_n,Ty_n), d(Ay_n, Sy_n)) < 0.$$

As $n \to \infty$, this along with (2.5), $s = \lim\limits_{n\to\infty} Sx_n$ and the lower semicontinuity of ψ imply that

$$\psi(d(s, r), 0, d(r, s), 0, 0, d(r, s)) \leq 0.$$

So that (C_2) with $l = d(s, r)$ and $m = 0$ gives $d(s, r) \leq 0$ or $s = r$, proving (2.4).

Case 2.1: Suppose that $A(X)$ orbitally complete at $x_0 \in X$. Then (2.4) implies that $p \in A(X)$ so that $p = Au$ for some $u \in X$. From (2.3) with $x = u$ and $y = p_n$, we would get

$$\psi(d(Su, Tp_n), d(Au, Ap_n), d(Au, Su), d(Ap_n, Tp_n), d(Au, Tp_n), d(Ap_n, Su)) < 0.$$

Employing the limit as $n \to \infty$ and using (2.4), this gives

$$\psi(d(Su, Au), 0, d(Au, Su), 0, 0, d(Au, Su)) \leq 0,$$

which in view of (2.2) with $l = d(Au, Su)$ and $m = 0$ together with (C_2) gives $d(Au, Su) = 0$ or

$$Au = Su = p. \tag{2.6}$$

Again from (2.3) with $x = u = y$ and (2.6), it follows that

$$\psi(d(Su, Tu), d(Au, Au), d(Au, Su), d(Au, Tu), d(Au, Su), d(Au, Tu)) < 0$$

or $\psi(d(p, Tu), 0, 0, d(p, Tu), 0, d(p, Tu)) < 0$ so that from (C_2) with $l = d(p, Tu)$ and $m = 0$, we get $d(Ap, Tu) = 0$ or $Tu = p$, that is

$$Au = Su = Tu = p. \tag{2.7}$$

Case 2.1: Suppose that (2.1) holds good. Then $\langle Sx_n \rangle_{n=1}^{\infty} \subset S(X)$ and hence lies in $\overline{S(X)}$ so that $p \in \overline{S(X)} \subset A(X)$. It follows that u is a common coincidence from Case (2.1).

Case 2.1: Suppose that (2.1) holds good. Then as above, we get that $\langle Tx_n \rangle_{n=1}^{\infty}$ converges in $\overline{T(X)}$ so that $p \in \overline{S(X)} \subset A(X)$ and (2.7) follows from Case (2.1).

To prove the second part of Theorem 2.1, we suppose that (S, A) is weakly compatible. From (2.6), we find that $SAu = ASu$ or $Ap = Sp$. But then from (2.3) with $x = y = p$, we get

$$\psi(d(Sp, Tp), d(Ap, Ap), d(Ap, Sp), d(Ap, Tp), d(Ap, Tp), d(Ap, Sp)) < 0 \text{ or}$$
$$\psi(d(Sp, Tp), 0, 0, d(Sp, Tp), d(Sp, Tp), 0) < 0,$$

which, in view of (C_2) with $l = d(Sp, Tp)$ and $m = 0$, gives $d(Sp, Tu) = 0$ or $Sp = Tp$, that is

$$Ap = Sp = Tp. \tag{2.8}$$

If (T, A) is weakly compatible, from (2.7), it follows that $Tp = Ap$. Then taking $x = y = p$ in (2.3) and using this and (C_2) with $l = d(Sp, Tp)$ and $m = 0$ and simplifying, we get (2.8).

Now, p is a fixed point of S. In fact, from (2.3) with $x = u$ and $y = p$, we get

$$\psi(d(Su, Tp), d(Au, Ap), d(Au, Su), d(Ap, Tp), d(Au, Tp), d(Ap, Su)) < 0 \text{ or}$$
$$\psi(d(p, Sp), d(p, Sp), 0, 0, d(p, Sp), d(Sp, p)) < 0.$$

Again using (C_2) in this, we get $d(p, Sp) = 0$. Hence p is a common fixed point of S, T and A, in view of (2.8).

Finally, suppose that q is also a common fixed point of S, T and A with $p \neq q$ so that $d(p,q) > 0$, then writing $x = p$ and $y = q$ in (2.3), we obtain

$$\psi(d(p,q), d(p,q), d(p,p), d(q,q), d(p,q), d(q,p)) < 0,$$

which is a contradiction to the choice (C_3). This shows that $p = q$, that is the common fixed point of S, T and A is unique. ∎

The following example emphasizes that none of the conditions (2.1), (2.1) and (2.1) can be dropped in Theorem 2.1 to ensure a common coincidence point for the three self-maps:

Example 2.1 Let

$$\psi(l_1, l_2, l_3, l_4, l_5, l_6) = l_1 - q \max\left\{l_2, l_3, l_4, \frac{l_5 + l_6}{2}\right\},$$

where $0 \leq q < 1$. Then ψ satisfies (C_1) and

$$\psi(l, m, m, l, l + m, 0) = l - q \max\{l, m\}) = \psi(l, m, l, m, 0, l + m)$$

so that from (2.2) we get

$$l \leq q \max\{l, m\}. \tag{2.9}$$

It may be noted that (2.9) is trivial if $l = 0$. Therefore let $l > 0$.

Again, if $\max\{l, m\} = l$, (2.9) would give a contradiction that $l \leq ql < l$, since $q < 1$. Thus $\max\{l, m\} = m$ so that $l \leq qm$), proving (C_2).

Finally, $\psi(l, l, 0, 0, l, l) = (1 - q)l \leq 0$, which is a contradiction to the choice of ψ if $l > 0$, and (C_3) follows.

Let $X = \mathbb{R}_+$ with $d(x,y) = |x - y|$ for all $x, y \in X$. Define

$$Sx = \begin{cases} \frac{1}{2} & (x = 0) \\ \frac{x}{2} & (x > 0), \end{cases} \quad Tx = \begin{cases} \frac{1}{2} & (x = 0) \\ \frac{2x}{3} & (x > 0) \end{cases} \quad \text{and} \quad Ax = \begin{cases} 1 & (x = 0) \\ \frac{3x}{4} & (x > 0). \end{cases}$$

Note that $S(X) = T(X) = A(X) = (0, \infty)$. But $\overline{S(X)} = \overline{T(X)} = [0, \infty)$ so that (2.1) and (2.1) are not satisfied. For $x_0 = 0$, we choose

$$x_1 = \frac{2}{3}, \; x_{2n} = 3\left(\frac{16}{27}\right)^n, \; x_{2n+1} = 2\left(\frac{16}{27}\right)^n \text{ for } n \geq 1,$$

the orbit is

$$\left\{\frac{1}{2}, \frac{4}{3}, \frac{8}{9}, \ldots, \frac{9}{4}\left(\frac{16}{27}\right)^n, \frac{3}{2}\left(\frac{16}{27}\right)^n, \ldots\right\}.$$

While, for $x_0 > 0$, choose

$$x_1 = \frac{2}{3}x_0, x_{2n} = \left(\frac{16}{27}\right)^n x_0, x_{2n+1} = \frac{2}{3}\left(\frac{16}{27}\right)^n x_0,\ n = 1, 2, 3, ...,$$

so that its orbit is

$$\left\{\frac{x_0}{2}, \left(\frac{2}{3}\right)^2 x_0, ..., \frac{3}{4}\left(\frac{16}{27}\right)^n x_0, \frac{1}{2}\left(\frac{16}{27}\right)^n x_0, ...\right\}.$$

In each case, the orbit converges to 0; (S, A) and (T, A) satisfy the property E.A. However, since the limit 0 does not belong to $A(X)$, the subspace $A(X)$ cannot not be orbitally complete at each x_0. Thus, (2.1) fails and the three maps do not have a common coincidence point, though X is complete. In other words, at least one of (2.1), (2.1) and (2.1) is necessary in Theorem 2.1 to obtain a common coincidence point.

Remark 2.1 Set

$$\psi(l_1, l_2, l_3, l_4, l_5, l_6) = l_1 - \phi\left(\max\left\{l_2, l_3, l_4, \frac{l_5 + l_6}{2}\right\}\right),$$

where ϕ is a nondecreasing usc contractive modulus. Then (C_1) holds good. Ignoring the triviality $l = 0$, we assume that $l > 0$, and $m \geq 0$ so that

$$\psi(l, m, m, l, l + m, 0) = l - \phi(\max\{l, m\}) = \psi(l, m, l, m, 0, l + m)$$

and from (2.2) we get

$$l - \phi(\max\{l, m\}) \leq 0 \quad \text{or} \quad l \leq \phi(\max\{l, m\}),$$

which would give a contradiction that $l \leq \phi(l) < l$ if $\max\{l, m\} = l$. Thus $\max\{l, m\} = m$ so that $l \leq \phi(m)$ and hence $l \leq qm$ for some $0 < q < 1$ proving (C_2). Finally the choice of ϕ gives

$$\psi(l, l, 0, 0, l, l) = l - \phi\left(\max\left\{l, 0, 0, \tfrac{l+l}{2}\right\}\right) = l - \phi(l) > 0,$$

and hence (C_3).

The choice of ψ reveals that (1.4) is a special case of (2.3). Given $x_0 \in X$, using (1.3), one can define an orbit $\langle Ax_n\rangle_{n=1}^{\infty}$ at x_0 with the choice (2.1). But, from [11], $\langle Ax_n\rangle_{n=1}^{\infty}$ is a Cauchy sequence in $A(X)$. Condition (1) holds good whenever if $A(X)$ is a complete subspace of X and hence $Ax_n \to p$ as $n \to \infty$ for some $p \in A(X)$ so that $p = Au$ for some $u \in X$. Then $\langle Sx_{2n-2}\rangle_{n=1}^{\infty}$ and $\langle Tx_{2n-1}\rangle_{n=1}^{\infty}$ being its subsequences also converge to $p = Au$. That is

$$\lim_{n\to\infty} Sx_{2n-2} = \lim_{n\to\infty} Tx_{2n-1} = \lim_{n\to\infty} Ax_{2n} = p = Au. \tag{2.10}$$

Write $x = y = x_{2n}$ in (1.4). Then applying the limit as $n \to \infty$, $s = \lim_{n\to\infty} Tx_{2n}$ and using (2.10), we find that

$$d(p,s) \le \phi\left(\max\left\{0, 0, d(s,p), \frac{d(s,p)+0}{2}\right\}\right),$$

so that $d(p,s) = 0$ or $p = s$. This proves that (T, A) satisfies the property E.A. While taking $x = y = x_{2n-1}$ in (1.4), applying the limit as $n \to \infty$, using (2.10), $t = \lim_{n\to\infty} Sx_{2n-1}$ and proceeding as above, it follows that (S, A) satisfies the property E.A. If $S(X)$ is orbitally complete, then $p \in S(X)$ and (1.3) gives, $p \in A(X)$ and the conclusion follows from the earlier case. Similarly the case that $T(X)$ is orbitally complete can be handled. The conclusion follows from Theorem 2.1. Thus Theorem 1.1 is a particular case of Theorem 2.1.

Remark 2.2 When $T = S$ in (2.1), we get an (S, A)-orbit [6] at $x_0 \in X$ given by

(2.11) $$Sx_{n-1} = Ax_n \quad \text{for} \quad n = 1, 2, 3, ...$$

and hence the (S, A)-orbital completeness of X at x_0. Therefore, taking $T = S$ in Theorem 2.1, we get

Corollary 2.1 (Theorem 3.1, [4]) *Let S and A be self-maps on X satisfying the property E.A. and the inequality*

(2.12) $$\psi(d(Sx,Sy), d(Ax,Ay), d(Sx,Ax), d(Sy,Ay), d(Sx,Ay), d(Sy,Ax)) \le 0 \quad \text{for all} \quad x, y \in X,$$

If $A(X)$ is complete and (S, A) is a weakly compatible pair, then S and A have a unique common fixed point.

Let Y an arbitrary nonempty subset of X. Imitating the proof of Theorem 2.1, we can establish the following result for nonself-maps:

Theorem 2.2 *Let S, T and $A : Y \to X$ satisfy (2.3) for all $x, y \in Y$. Suppose that either (S, A) or (T, A) satisfies the property E.A. on Y, and any one of the following conditions holds good:*

(1°) $A(Y)$ is closed subspace of Y,

(2°) $\overline{S(Y)} \subset A(Y)$,

(3°) $\overline{T(Y)} \subset A(Y)$.

Then there is a coincidence point u common to S, T and A in Y. Further if the point of common coincidence of S, T and A with respect to u lies in Y, it will be their unique common fixed point, provided either (S, A) or (T, A) is a weakly compatible pair.

Remark 2.3 Write

$$\psi(l_1, l_2, l_3, l_4, l_5, l_6) = l_1 - \max\left\{l_2, \alpha l_3, \alpha l_4, \frac{l_5 + l_6}{2}\right\}$$

in Theorem 2.2, where $0 < \alpha < 1$. Then (C_1)-(C_3) hold good and (1.5) is a particular case of (2.3). Interestingly, Theorem 1.2 required both the containments (2°) and (3°) and weak compatibility of both the pairs (S, A) and (T, A). Also a common fixed point was obtained under the condition that $Y = X$. However, the proof of our second results suggests that weak compatibility and property E.A. of either pair is sufficient to obtain a common fixed point under a generalized inequality, even without the condition that $Y = X$.

Finally taking

$$\psi(l_1, l_2, l_3, l_4, l_5, l_6) = l_1 - \max\left\{l_2, \beta l_3 + \alpha l_4, \frac{l_5 + l_6}{2}\right\},$$

where $\beta \geq 0$ and $0 < \alpha < 1$ and $T = S$ in Theorem 2.2, we get

Corollary 2.2 (Theorem 3.1, [5]) *Let $S, A : Y \to X$ satisfying*

$$\begin{aligned} &d(Sx, Sy) \\ (2.13) \quad &< \max\left\{d(Ax, Ay), \beta d(Sx, Ax) + \alpha d(Sy, Ay), \frac{d(Sy, Ax) + d(Sx, Ay)}{2}\right\} \\ &\qquad \text{for all } x, y \in Y \text{ with } x \neq y, \end{aligned}$$

Suppose that either $A(Y)$ is a complete subspace of Y or $S(Y)$ is a complete subspace of Y with $S(Y) \subset A(Y)$. Then (S, A) have a coincidence point u in Y. Further if the point of coincidence of S and A with respect to u is in Y, then S and A will have a unique unique common fixed point in Y, provided S and A are weakly compatible.

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References

[1] Aamri, M.A., Moutawaki, D. El., *Some new common fixed point theorems under strict contractive conditions,* J. Math. Anal. Appl., 270 (2002), 181-188.

[2] Gerald Jungck, *Compatible maps and common fixed points,* Int. J. Math. & Math. Sci., 9 (1986), 771-779.

[3] Gerald Jungck, Rhoades, B.E., *Fixed point for set valued functions without continuity,* Indian J. Pure Appl. Math., 29 (1998), 227-238.

[4] Imdad, M., Javid Ali, *Jungck's common fixed point theorem and E.A. property,* Acta Mathematica Sinica, English Ser., 24 (2008), 87-94.

[5] Khan, A.R., Domlo, A.A., *Coincidence and fixed points of nonself contractive maps with applications,* Ind. Jour. Math., 49 (2007), 17-13.

[6] Phaneendra, T., *Coincidence Points of Two Weakly Compatible Self-Maps and Common Fixed Point Theorem through Orbits,* Ind. Jour. Math., 46 (2-3) (2003), 173-180.

[7] Phaneendra, T., *Asymptotic regularity and common fixed point,* Pure and Appl. Math. Sci., 59 (1-2) (2004), 45-49.

[8] Phaneendra, T., *Orbital continuity and common fixed point,* Buletini Shkencor, 3 (4) (2011), 375-380.

[9] Popa, V., *Some fixed point theorems for compatible mappings satisfying an implicit relation,* Demon. Math., 32 (1999), 157-163.

[10] Singh, S.L., Ashish Kumar, *Common fixed point theorems for contractive maps,* Math. Vesnik., 58 (2006), 85-90.

[11] Singh, S.L., Mishra, S.N., *Remarks on Jachymski's fixed point theorems for compatible maps*, Ind. J. Pure Appl. Math., 28 (1997), 611-615.

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