ON OD-CHARACTERIZABILITY OF A CERTAIN ALTERNATING AND SYMMETRIC GROUP

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Abstract. In this paper we will show that if G is a finite group with the same order and degree pattern as the alternating group on 27 letters, then G is isomorphic to $\mathbb{A}_{27}$. Furthermore we will show that there are three non-isomorphic finite groups with the same order and degree pattern as the symmetric group on 27 letters.

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1. Introduction and preliminaries

Let G be a finite group. The set of all the prime divisors of G is denoted by $\pi(G)$ and the set of elements order in G is denoted by $\pi_e(G)$. The prime graph or the Gruenberg–Kegel graph of G is a simple graph denoted by $\Gamma(G)$ or $GK(G)$ and is defines as follows. Vertices of $\Gamma(G)$ are elements of $\pi(G)$and two distinct vertices p and q are joined by an edge if G possesses an element of order pq. In this case, we will write $p \sim q$, otherwise $p \nsim q$ means that G does not have an element of order pq. The number of connected components of $\Gamma(G)$ is denoted by $t(G)$ and we write $\pi_i = \pi_i(G)$, $1 \leq i \leq t(G)$ for these connected components. In the case that $2 \in \pi(G)$ we choose $\pi_1(G)$ for the component containing 2. For a natural number n we write $\pi(n)$ for the set of all primes dividing n. With this notation $|G|$ can be expressed as a product of the numbers $m_1, m_2, ..., m_{t(G)}$ where m_is are positive integers with $\pi(m_i) = \pi_i$. The numbers m_i, $1 \leq i \leq t(G)$, are called the order components of G. We write $OC(G) = \{m_1, m_2, ..., m_{t(G)}\}$ and call it the set of order components of G. The set of prime graph components of G is denoted by $T(G) = \left\{\pi_1(G), \pi_2(G), ..., \pi_{t(G)}(G)\right\}$. Throughout this paper we use standard notation, in particular $\mathbb{P}$ denotes the set of all the prime numbers. All groups considered in this paper are finite.

Definition 1. Let G be a finite group with $|G| = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k}$, where $p_i \in P$ and $\alpha_i \in N$. Let $\Gamma(G)$ be the prime graph of G. For $p \in \pi(G)$, the degree of the vertex p in $\Gamma(G)$ is the number of vertices adjacent to p and it is denoted by $\deg(p)$, i.e., $\deg(p) = |\{q \in \pi(G) \mid p \sim q\}|$. We choose an ordering such as $p_1 < p_2 < ... < p_k$ in $\pi(G)$ and define $D(G) = (\deg(p_1), \deg(p_2), ..., \deg(p_k))$. We call $D(G)$ the degree pattern of G.

Definition 2. *Let G be a natural number. A group M is called k-fold OD-characterizable if there are exactly k non-isomorphic groups G such that $|G| = |M|$ and $D(G) = D(M)$. A $1-$fold OD-characterizable group is called an OD-characterizable group. In particular, an OD-cha-racterizable group is uniquely determined by its order and the degrees of its vertices.*

Definition 3. Let $n \in \mathbb{N}$. A finite non-abelian simple group G is called a simple K_n-group if $|\pi(G)| = n$.

The Gruenberg–Kegel graph (prime graph) of a finite group G for the first time was defined in [5] and its significance can be found in many recent researches. Characterization of finite groups by their orders and degree pattern will help us to know the properties of almost simple groups. In [1] it is shown that the alternating group $\mathbb{A}_p$, where p and $p-2$ are primes, and all sporadic simple groups are OD-characterizable. It is also shown that certain groups of Lie type are OD-characterizable, but the projective symplectic group $SP_6(3)$ is $2-$fold OD-characterizable. In [4] it is proved that all the simple K_4-groups except $\mathbb{A}_{10}$ are OD-characterizable whereas $\mathbb{A}_{10}$ is $2-$fold OD-characterizable. In [2] it is proved that all simple groups of order less than 10^8 except $\mathbb{A}_{10}$ and $U_4(2)$ are OD-characterizable while $\mathbb{A}_{10}$ and $U_4(2)$ are $2-$fold OD-characterizable. In [3] some almost simple groups related to $L_2(49)$ are considered and it is proved that if $G = L_2(49) : V$, where V is isomorphic to the Klein fourgroup, is the full automorphism group of $L_2(49)$, then G is $9-$fold OD-characterizable, while if W is a proper subgroup of V, then $H = L_2(49) : W$ is OD-characterizable. In [6] the authors proved that the alternating group of degree 16 is OD-characterizable. In this article our aim is to consider the OD-characterizability of the alternating and symmetric groups $\mathbb{A}_{27}$ and $\mathbb{S}_{27}$. The importance of these groups is that both of them have connected prime graph. Figure 1 shows the prime graph of $\mathbb{A}_{27}$.

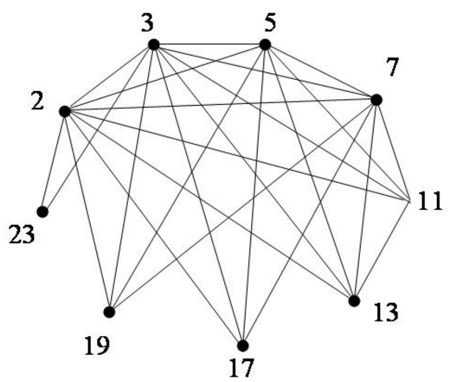

Figure 1. The prime graph of $\mathbb{A}_{27}$

For the proof we need to know about finite non-abelian simple groups of order at most 19. In general for a given prime p, we denote by $\mathfrak{S}_p$ the set of all non-abelian simple groups G such that $p \in \pi(G) \subseteq \{1, 2, ..., p\}$. Therefore Table 1 lists all $\mathfrak{S}_{19}$ groups. These are given in [7] and we reproduce them in Table 1 below.

Table 1: Non-abelian simple groups G with $\pi(G) \subseteq \{1, 2, 3, ..., 19\}$

G	$\|G\|$	$\|Out(G)\|$
$\mathfrak{S}_5$	$\pi(G) \subseteq \{2, 3, 5\}$	
$\mathbb{A}_5 \cong L_2(4) \cong L_2(5)$	$2^2.3.5$	2
$\mathbb{A}_6 \cong L_2(9)$	$2^3.3^2.5$	4
$S_4(3) \cong U_4(2)$	$2^6.3^4.5$	2
$\mathfrak{S}_7$	$7 \in \pi(G) \subseteq \{2, 3, 5, 7\}$	
$L_2(7) \cong L_3(2)$	$2^3.3.7$	2
$L_2(8)$	$2^3.3^2.7$	3
$U_3(3)$	$2^5.3^3.7$	2
$\mathbb{A}_7$	$2^3.3^2.5.7$	2
$L_2(49)$	$2^4.3.5.7^2$	4
$U_3(5)$	$2^4.3^2.5^3.7$	6
$L_3(4)$	$2^6.3^2.5.7$	12
$\mathbb{A}_8 \cong L_4(2)$	$2^6.3^2.5.7$	2
$\mathbb{A}_9$	$2^6.3^4.5.7$	2
J_2	$2^7.3^3.5^2.7$	2
$\mathbb{A}_{10}$	$2^7.3^4.5^2.7$	2
$U_4(3)$	$2^7.3^6.5.7$	8
$S_4(7)$	$2^8.3^2.5^2.7^4$	2
$S_6(2)$	$2^9.3^4.5.7$	1
$O_8^+(2)$	$2^{12}.3^5.5^2.7$	6
$\mathfrak{S}_{11}$	$11 \in \pi(G) \subseteq \{2, 3, 5, 7, 11\}$	$\|Out(G)\|$
$L_2(11)$	$2^2.3.5.11$	2
M_{11}	$2^4.3^2.5.11$	1
M_{12}	$2^6.3^3.5.11$	2
$U_5(2)$	$2^{10}.3^5.5.11$	2
M_{22}	$2^7.3^2.5.7.11$	2
$\mathbb{A}_{11}$	$2^7.3^4.5^2.7.11$	2
McL	$2^7.3^6.5^3.7.11$	2
HS	$2^9.3^2.5^3.7.11$	2
$\mathbb{A}_{12}$	$2^9.3^5.5^2.7.11$	2
$U_6(2)$	$2^{15}.3^6.5.7.11$	6
$\mathfrak{S}_{13}$	$13 \in \pi(G) \subseteq \{2, 3, 5, ..., 13\}$	$\|Out(G)\|$
$L_3(3)$	$2^4.3^3.13$	2
$L_2(25)$	$2^3.3.5^2.13$	4
$U_3(4)$	$2^6.3.5^2.13$	4
$S_4(5)$	$2^6.3^2.5^4.13$	2
$L_4(3)$	$2^7.3^6.5.13$	4

${}^2F_4(2)'$	$2^{11}.3^3.5^2.13$	2
$L_2(13)$	$2^2.3.7.13$	2
$L_2(27)$	$2^2.3^3.7.13$	6
$G_2(3)$	$2^6.3^6.7.13$	2
${}^3D_4(2)$	$2^{12}.3^4.7^2.13$	3
$Sz(8)$	$2^6.5.7.13$	3
$L_2(64)$	$2^6.3^2.5.7.13$	6
$U_4(5)$	$2^7.3^4.5^6.7.13$	4
$L_3(9)$	$2^7.3^6.5.7.13$	4
$S_6(3)$	$2^9.3^9.5.7.13$	2
$O_7(3)$	$2^9.3^9.5.7.13$	2
$G_2(4)$	$2^{12}.3^3.5^2.7.13$	2
$S_4(8)$	$2^{12}.3^4.5.7^2.13$	6
$O_8^+(3)$	$2^{12}.3^{12}.5^2.7.13$	24
$L_5(3)$	$2^9.3^{10}.5.11^2.13$	2
A_{13}	$2^{9}.3^5.5^2.7.11.13$	2
A_{14}	$2^{10}.3^5.5^2.7^2.11.13$	2
A_{15}	$2^{10}.3^6.5^3.7^2.11.13$	2
$L_6(3)$	$2^{11}.3^{15}.5.7.11^2.13^2$	4
Suz	$2^{13}.3^7.5^2.7.11.13$	2
A_{16}	$2^{14}.3^6.5^3.7^2.11.13$	2
Fi_{22}	$2^{17}.3^9.5^2.7.11.13$	2
$\mathfrak{S}_{17}$	$17 \in \pi(G) \subseteq \{2,3,5,...,17\}$	$\lvert Out(G)\rvert$
$L_2(17)$	$2^4.3^2.17$	2
$L_2(16)$	$2^4.3.5.17$	4
$S_4(4)$	$2^8.3^2.5^2.17$	4
He	$2^{10}.3^3.5^2 7^3.17$	2
$O_8^-(2)$	$2^{12}.3^4.5.7.17$	2
$L_4(4)$	$2^{12}.3^4.5^2.7.17$	4
$S_8(2)$	$2^{16}.3^5.5^2.7.17$	1
$U_4(4)$	$2^{12}.3^2.5^3.13.17$	4
$U_3(17)$	$2^6.3^4.7.13.17^3$	6
$O_{10}^-(2)$	$2^{20}.3^6.5^2.7.11.17$	2
$L_2(13^2)$	$2^3.3.5.7.13^2.17$	4
$S_4(13)$	$2^6.3^2.5.7^2.13^4.17$	2
$L_3(16)$	$2^{12}.3^2.5^2.7.13.17$	24
$S_6(4)$	$2^{18}.3^4.5^3.7.13.17$	2
$O_8^+(4)$	$2^{24}.3^5.5^4.7.13.17^2$	12
$F_4(2)$	$2^{24}.3^6.5^2.7^2.13.17$	2
$\mathbb{A}_{17}$	$2^{14}.3^6.5^3.7^2.11.13.17$	2
$\mathbb{A}_{18}$	$2^{15}.3^8.5^3.7^2.11.13.17$	2
$\mathfrak{S}_{19}$	$19 \in \pi(G) \subseteq \{2,3,5,...,19\}$	$Out(G)$

$L_2(19)$	$2^2.3^2.5.19$	2
$L_3(7)$	$2^5.3^2.5^3.19$	6
$U_3(8)$	$2^9.3^4.7.19$	18
$U_3(19)$	$2^5.3^2.5^2.7^3.19^3$	2
$L_4(7)$	$2^9.3^4.5^2.7.19$	4
J_3	$2^7.3^5.5.17.19$	2
J_4	$2^3.3.5.7.11.19$	1
$L_3(11)$	$2^4.3.5^2.7.11^3.19$	2
HN	$2^{14}.3^6.5^6.7.11.19$	2
$U_4(8)$	$2^{18}.3^7.5.7^2.13.19$	6
$\mathbb{A}_{19}$	$2^{15}.3^8.5^3.7^2.11.13.17.19$	2
$\mathbb{A}_{20}$	$2^{17}.3^8.5^4.7^2.11.13.17.19$	2
$\mathbb{A}_{21}$	$2^{17}.3^9.5^4.7^3.11.13.17.19$	2
$\mathbb{A}_{22}$	$2^{18}.3^9.5^4.7^3.11^2.13.17.19$	2
${}^2E_6(2)$	$2^{36}.3^9.5^2.7^2.11.13.17.19$	6

2. Main result

At first, we consider the alternating group on 27 letters, $\mathbb{A}_{27}$, and will show it is OD-characterizable. It is easy to verify that $|\mathbb{A}_{27}| = 2^{22}.3^{13}.5^6.7^3.11^2.13^2.17.19.23$ and $D(\mathbb{A}_{27}) = (8,8,7,7,5,5,4,4,2)$. Let G be a finite group such that $|G| = |\mathbb{A}_{27}|$, $\pi(G) = \{2,3,5,7,11,13,17,19,23\}$, $D(G) = (8,8,7,7,5,5,4,4,2)$, where $D(G)$ is the degree pattern of G in its prime graph. Since $deg(2) = 8$ we deduce that the prime 2 is joined to all vertices in $\pi(G) - \{2\}$, hence $\Gamma(G)$ is a connected graph. Now it is easy to see that $\Gamma(G) = \Gamma(\mathbb{A}_{27})$ and that

$$\pi_e(G) \supseteq \left\{ \begin{array}{l} 2,3,5,7,11,13,17,19,23,6,10,14,22,26,34,38,46,15,27, \\ 33,39,51,57,69,35,55,65,85,95,77,91,119,133,143 \end{array} \right\}.$$

Lemma 1. *Let K be the maximal normal solvable subgroup of G. Then K is a $\{2,3\}-$group. In particular, G is a non-solvable group.*

Proof. Let p be a prime divisor of the order of K and S_p be a Sylow $p-$subgroup of K. Since $K \trianglelefteq G$, by the Frattini argument we obtain $G = KN_G(S_p)$. Next we distinguish several cases:

Case (1) $p = 23$. In this case, S_{23} is a cyclic group of order 23, hence $\overline{N} = \frac{N_G(S_{23})}{C_G(S_{23})}$ is isomorphic to a subgroup of $\mathbb{Z}_{22}$. Therefore, $|\overline{N}|$ is a divisor of 22. But $deg(23)$ in $\Gamma(G)$ is 2 and 23 is joined to 2 and 3 only, hence $C_G(S_{23})$ is a $\{2,3,23\}-$group. Thus, $|N_G(S_{23})| = 2^\alpha.3^\beta.11^\gamma.23$, where $0 \le \alpha \le 22$, $0 \le \beta \le 13$, $0 \le \gamma \le 2$ and from $G = KN_G(S_{23})$ we deduce that $19 \mid |K|$. Since K is assumed to be solvable, we can consider a $\{19-23\}-$Hall subgroup of K which has order 19.23

and should be cyclic. This implies $19 \sim 23$ which is a contradiction because $\mathbb{A}_{27}$ has no element of order 19.23. We conclude that $23 \nmid |K|$.

Case (2) $p = 19$ or 17. This case is treated with the same technique as in case (1) and a contradiction is obtained.

Case (3) $p = 13$ or 11. First, we consider $p = 13$, the case $p = 11$ is treated similarly. If a Sylow 13−subgroup of K has order 13, then with the same techniques as in case (1) we obtain a contradiction, therefore we may assume that a Sylow 13−subgroup of K has order $|S_{13}| = 13^2$. We have $\overline{N} = \frac{N_G(S_{23})}{C_G(S_{23})} \leq Aut(S_{13})$. If S_{13} is a cyclic group, then $|Aut(S_{13})| = 13.12$ and if S_{13} is elementary abelian, then $|Aut(S_{13})| = |GL_2(13)| = 2^5.3^2.7.13$. Therefore, in any case we deduce that $\overline{N}$ is a $\{2,3,7,13\}-$group. Since $deg(13) = 5$ and $13 \sim 2$, $13 \sim 3$, $13 \sim 5$, $13 \sim 7$, $13 \sim 11$, we obtain that $N_G(S_{23})$ is a $\{2,3,5,7,11,13\}-$group, hence $19 \mid |K|$. Again considering a $\{19,13\}-$Hall subgroup in K results in a contradiction. Therefore, $|K|$ cannot be divisible by 13 or 11.

Case (4) $p = 7$. In this case, a Sylow 7−subgroup of K has order 7, 7^2 or 7^3. If $|S_7| = 7$ or 7^2, then using the same techniques as in case (1) and case (3) we will obtain a contradiction. Hence, suppose $|S_7| = 7^3$. From $G = KN_G(S_7)$ and the fact that $23 \nmid |K|$, we deduce that $23 \mid |N_G(S_7)|$, so S_7 is normalized by an element σ of G with order 23. Since G has no element of order 23.7, $\langle\sigma\rangle$ should act fixed-point-freely on S_7, implying $23 \mid 7^3 - 1$, a contradiction.

Case (5) $p = 5$. In this case, $|S_5| = 5^\alpha$, $1 \leq \alpha \leq 6$. By case (1), we have $23 \nmid |K|$, and from $G = KN_G(S_5)$ we deduce that S_5 is normalized by an element of order 23, hence $23 \mid 5^\alpha - 1$. But considering all the numbers $1 \leq \alpha \leq 6$ impossibility of the last divisibility is proved.

Therefore, we have proved that $|K|$ can only be divisible by 2 and 3 and part of the lemma is proved. If G is a solvable group, then it has a $\{23,19\}-$Hall subgroup of order 23.19 implying that $23 \sim 19$, a contradiction. ∎

In the following lemma, we keep fixed the notation used in Lemma1.

Lemma 2. *$\frac{G}{K}$ is an almost simple group, i.e. $S \leq \frac{G}{K} \leq Aut(S)$ where S is a simple group isomorphic to $\mathbb{A}_{27}$.*

Proof. Let us put $\overline{G} = \frac{G}{K}$ and $\overline{S} = Soc(\overline{G})$ where $Soc(\overline{G})$ denotes the socle of the group $\overline{G}$, i.e., the subgroup of $\overline{G}$ generated by the set of all the minimal normal subgroups of $\overline{G}$. Then, $\overline{S} \cong S_1 \times S_2 \times ... \times S_n$ where $S_i^{,}s$ are finite non-abelian simple groups. We assume $n \geq 2$ and derive a contradiction.

If $23 \mid |\overline{S}|$, then the order of only one S_i is divisible by 23. We assume $23 \mid |S_1|$. Because only $23 \sim 2$ and $23 \sim 3$ hold in the prime graph of G, S_i, $i \geq 2$, must be $\{2,3\}-$groups contradicting simplicity of S_i. Hence, $23 \nmid |\overline{S}|$.

Since K is a $\{2,3\}-$group, we have $23 | |\overline{G}|$. From $\frac{N_{\bar{G}}(\overline{S})}{C_{\bar{G}}(\overline{S})} = \frac{\overline{G}}{C_{\bar{G}}(\overline{S})} \leq Aut(\overline{S})$, we deduce that $23 \in \pi(\overline{G}) \subseteq \pi(Aut(\overline{S}))$. But $|Aut(\overline{S})| = |\overline{S}| . |Out(\overline{S})|$, hence $23 \mid |Out(\overline{S})|$. Let $P_1, P_2, ..., P_k$ be non-isomorphic simple groups among $S_1, ..., S_n$

such that $\overline{S} \cong P_1^{t_1} \times P_2^{t_2} \times ... \times P_k^{t_k}$, where $t_1 + t_2 + ... + t_k = n$. We have $Out(S) = Out(P_1^{t_1}) \times Out(P_2^{t_2}) \times ... \times Out(P_k^{t_k})$, and from $23 \nmid |\overline{S}|$ we have $23 \nmid |P_i|$, for all $1 \le i \le k$, so P_is are simple $\{2, 3, 5, 7, 11, 13, 17, 19\}-$group and by Table 1 we obtain $23 \nmid |Out(P_i)|$, $1 \le i \le k$. But $Aut(P_i^{t_i}) \cong P_i \,\mathrm{Wr} S_{t_i}$, and from $23 \mid |Aut(P_1^{t_1})| = |P_1|^{t_1}.t_1!$ we obtain $23 \mid t_1!$ which implies $t_1 \ge 23$. Therefore, $(23!)_2 \times 2^{23} = 2^{42}$ must divide the order of G, a contradiction. Therefore, $n = 1$ and $\overline{S}$ is a simple group. ∎

Next, we consider the fact that $|\overline{G}| \mid |Aut(\overline{S})|$, $|\overline{G}| = 2^{22}.3^{13}.5^6.7^3.11^2.13^2.17.19.23$, and $\overline{S}$ an S_{19}-group. Now, by Table 1, we obtain $\overline{S} \cong \mathbb{A}_{27}$ and $\overline{S} \le \frac{G}{K} \le Aut(\overline{S})$.

Theorem 1. *G is isomorphic to $\mathbb{A}_{27}$.*

Proof. By Lemma 2, $\mathbb{A}_{27} \le \frac{G}{K} \le Aut(\mathbb{A}_{27}) = \mathbb{S}_{27}$, therefore $\frac{G}{K} \cong \mathbb{A}_{27}$ or $\mathbb{S}_{27}$. If $\frac{G}{K} \cong \mathbb{A}_{27}$, then $|K| = 1$, because $|G| = |\mathbb{A}_{27}|$, so $G \cong \mathbb{A}_{27}$. If $\frac{G}{K} \cong \mathbb{S}_{27}$, then $2\,|K| = 1$, which is impossible. ∎

Next, we consider OD-characterizability of the symmetric group $\mathbb{S}_{27}$. The degree pattern of $\mathbb{S}_{27}$ is the same as the degree pattern of $\mathbb{A}_{27}$, i.e., $D(\mathbb{S}_{27}) = (8, 8, 7, 7, 5, 5, 4, 4, 2)$, but $|\mathbb{S}_{27}| = 2^{23}.3^{13}.5^6.7^3.11^2.13^2.17.19.23$. We assume that G is a finite group with $|G| = |\mathbb{S}_{27}|$ and $D(G) = D(\mathbb{S}_{27})$. As in the case of $\mathbb{A}_{27}$, if K is the maximal normal solvable subgroup of G, then we can prove that K is a $\{2, 3\}-$group. Also, if we set $\overline{G} = \frac{G}{K}$ and $\overline{S} = Soc(\overline{G})$, then similar techniques yields $\overline{S} \cong \mathbb{A}_{27}$ and $\overline{S} \le \overline{G} \le Aut(\overline{S})$.

Theorem 2. *The symmetric group $\mathbb{S}_{27}$ is $3-$fold OD-characterizable.*

Proof. Since $\mathbb{A}_{27} \le \frac{G}{K} \le Aut(\mathbb{A}_{27}) = \mathbb{S}_{27}$ we will obtain $\frac{G}{K} = \mathbb{A}_{27}$ or $\mathbb{S}_{27}$. If $\frac{G}{K} = \mathbb{S}_{27}$, then we obtain $|K| = 1$ and $G = \mathbb{S}_{27}$. If $\frac{G}{K} = \mathbb{A}_{27}$, then $|K| = 2$, hence $K \le Z(G)$. Since K is the maximal normal solvable subgroup of G we obtain $K = Z(G)$ and G is the central extension of $\mathbb{Z}_2$ by $\mathbb{A}_{27}$. Therefore, we have two possibilities for G , i.e., $G = \mathbb{Z}_2 \times \mathbb{A}_{27}$ or $G = \mathbb{Z}_2.\mathbb{A}_{27}$ the covering group of $\mathbb{A}_{27}$. Both groups have the same degree pattern and orders as $\mathbb{S}_{27}$ and theorem is proved. ∎

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