ON (λ, μ)-FUZZY SUBHYPERLATTICES[1]

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Abstract. We first introduce the concepts of (λ, μ)-fuzzy subhyperlattices and (λ, μ)-fuzzy ideals. Secondly, we list some equivalent conditions of them. Lastly, we prove that the Cartesian product of two (λ, μ)-fuzzy subhyperlattices is still a (λ, μ)-fuzzy subhyperlattice. This paper can be seen as a generalization of [1].

Keywords: Cartesian product; (λ, μ)-fuzzy; subhyperlattice; ideal.

1. Introduction and preliminaries

The concept of fuzzy sets was first introduced by Zadeh [18] in 1965. The theory of fuzzy sets has been developed fast and has many applications in many branches of sciences. In mathematics, the study of fuzzy algebraic structures was first initiated by pioneer paper of Rosenfeld[11]. He first studied the fuzzy subgroup of a group and since then, many researchers have been engaged in extending the concepts and results of abstract algebra based on fuzzy sets.

Hyperstructure theory was first introduced in 1934 by Marty at the 8th Congress of Scandinavian Mathematicians (see [10]). Later on, hyperstructures have been developed in both pure and applied sciences. A comprehensive review of the theory of hyperstructures can be found in [2, 3]. Hyperstructures are generalizations of classic structures. For example, hypergroup [2] is a generalization of group, hyperlattice[9] and superlattice[6] are generalizations of lattice and so on.

Fuzzy hyperstructures have been introduced rather recently. Corsini and Tofan studied fuzzy hypergroups in [4], Hasankhani and Zahedi studied fuzzy hyperrings in [8], Serafimidis, Konstantinidou and Kehagias studied fuzzy hyperlattices in [12] and so on.

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Recently, Yuan [17] introduced the concept of fuzzy subgroup with thresholds. A fuzzy subgroup with thresholds λ and μ is also called a (λ,μ)-fuzzy subgroup. Yao continued to research (λ,μ)-fuzzy normal subgroups, (λ,μ)-fuzzy quotient subgroups and (λ,μ)-fuzzy subrings in [14, 15, 16]. Ali and Ray discussed the product of fuzzy sublattices in [1].

In this paper, we introduced the concept of fuzzy hyperlattice with thresholds.

Let us recall some definitions and notions.

By a fuzzy subset of a nonempty set X we mean a mapping from X to the unit interval $[0,1]$. If A is a fuzzy subset of X, then we denote $A_\alpha = \{x \in X | A(x) \geq \alpha\}$ for all $\alpha \in [0,1]$.

A partial hypergroupoid $< H; * >$ is a nonempty set H with a function from $H \times H$ to the set of subsets of H, i.e.,

$$\begin{array}{rcl} * : H \times H & \to & \mathbf{P}(H) \\ (x,y) & \to & x * y. \end{array}$$

A hypergroupoid is a nonempty set H, endowed with a hyperoperation, that is a function from $H \times H$ to the set of nonempty subsets of H.

If $A, B \in \mathbf{P}(H) - \{\emptyset\}$, then we define $A * B = \cup\{a * b | a \in A, b \in B\}$, $x * B = \{x\} * B$ and $A * y = A * \{y\}$.

Definition 1.1 ([5],[7]) Let H be a nonempty set , $\sqcup : H \times H \to P^*(H)$ be a hyperoperation, where $P(H)$ is the power set of H and $P^*(H) = P(H) - \{\emptyset\}$ and $\wedge : H \times H \to H$ be an operation. Then $(H, \sqcup, \wedge)$ is called a hyperlattice if for all $a, b, c \in H$:

(1) $a \in a \sqcup a$, $a = a \wedge a$;
(2) $a \sqcup b = b \sqcup a$,$a \wedge b = b \wedge a$;
(3) $(a \sqcup b) \sqcup c = a \sqcup (b \sqcup c)$,$(a \wedge b) \wedge c = a \wedge (b \wedge c)$;
(4) $a \in (a \sqcup b) \wedge a$, $a \in (a \wedge b) \sqcup a$;
(5) $b \in a \sqcup b \Leftrightarrow a = a \wedge b \Leftrightarrow a \leq b$.

The readers can consult [2],[13] to learn more about hyperstructures and fuzzy sets.

Throughout this paper, we will always assume that $0 \leq \lambda < \mu \leq 1$.

2. (λ, μ)-fuzzy subhyperlattices

Throughout this section H always denotes a hyperlattice. The meet, hyperjoin and partial order of H, will be denoted as $\wedge$, $\sqcup$, and $\leq$, respectively.

Definition 2.1 A fuzzy subset A of a hyperlattice H is said to be a (λ,μ)-fuzzy subhyperlattice of H if $\forall a, b \in H$,

$$A(a \wedge b) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu$$

and

$$\inf_{t \in a \sqcup b} A(t) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

Remark 2.2 From the previous definition, we know that a fuzzy subhyperlattice is a $(0, 1)$-fuzzy subhyperlattice.

Theorem 2.3 *Let A be a fuzzy subset of H. Then the following are equivalent:*

(1) *A is a (λ, μ)-fuzzy subhyperlattice of H;*
(2) *A_α is a subhyperlattice of H, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.*

Proof. 1. (1) $\Rightarrow$ (2). Let A be a (λ, μ)-fuzzy subhyperlattice of H. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$ and $x \sqcup y \subseteq A_\alpha$, for all $x, y \in A_\alpha$.

From $x \in A_\alpha$ we know that $A(x) \geq \alpha$. And similarly we obtain that $A(y) \geq \alpha$. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Note that $\lambda < \alpha$ and so $x \wedge y \in A_\alpha$.

From $A(x) \geq \alpha$ and $A(y) \geq \alpha$ we know that $\inf\limits_{t \in x \sqcup y} A(t) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu \geq \alpha \wedge \mu = \alpha$ and $\lambda < \alpha$, we conclude that $\inf\limits_{t \in x \sqcup y} A(t) \geq \alpha$. So $A(t) \geq \alpha$ for any $t \in x \sqcup y$. Thus $x \sqcup y \subseteq A_\alpha$.

2. (2) $\Rightarrow$ (1). If there exist $x_0, y_0 \in H$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \wedge A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \wedge A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ and $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is a subhyperlattice of H. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu$ holds for all $x, y \in H$.

Again, if there exist $x_0, y_0 \in H$ such that $\inf\limits_{t \in x_0 \sqcup y_0} A(t) \vee \lambda < \alpha = (A(x_0) \wedge A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \wedge A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ and $y_0 \in A_\alpha$. But $\inf\limits_{t \in x_0 \sqcup y_0} A(t) < \alpha$, that is $A(t) < \alpha$ for some $t \in x_0 \sqcup y_0$. So $x_0 \sqcup y_0 \not\subseteq A_\alpha$. This is a contradiction with that A_α is a subhyperlattice of H. Thus $\inf\limits_{t \in x \sqcup y} A(t) \vee \lambda \geq (A(x) \wedge A(y)) \wedge \mu$ holds for all $x, y \in H$. ∎

3. (λ, μ)-fuzzy ideals

Definition 3.1 Let $(H, \sqcup, \wedge)$ be a hyperlattice. A nonempty subset I of H is called an ideal of H if for all $a, b \in H$,

$$a, b \in I \Rightarrow a \sqcup b \subseteq I$$

and

$$a \in H, b \in I \Rightarrow a \wedge b \in I.$$

Proposition 3.2 *Suppose I is a subset of a hyperlattice H, then the following are equivalent for all $a, b \in H$,*

(1) *$a \in H, b \in I \Rightarrow a \wedge b \in I$;*
(2) *$a \in I$ and $b \leq a \Rightarrow b \in I$.*

Proof. (1) $\Rightarrow$ (2). If $b \leq a$, then $b = a \wedge b$. From (1) we know that $a \wedge b \in I$. And so $b \in I$.

(2) $\Rightarrow$ (1). From $a \wedge b \leq b \in I$ and (2) we know that $a \wedge b \in I$. ∎

Definition 3.3 A fuzzy subset A of a hyperlattice H is a (λ, μ)-fuzzy ideal of H if for all $a, b \in H$,

$$A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$$

and

$$\inf_{t \in a \sqcup b} A(t) \vee \lambda \geq (A(a) \wedge A(b)) \wedge \mu.$$

Proposition 3.4 *Suppose A is a fuzzy subset of a hyperlattice H, then the following are equivalent for all $a, b \in H$,*

(1) $A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu$;
(2) $a \leq b \Rightarrow A(a) \vee \lambda \geq A(b) \wedge \mu$.

Proof. (1) $\Rightarrow$ (2). If $a \leq b$, then $a \wedge b = a$. Thus $A(a) \vee \lambda = A(a \wedge b) \vee \lambda \geq (A(a) \vee A(b)) \wedge \mu \geq A(b) \wedge \mu$.

(2) $\Rightarrow$ (1). From $a \wedge b \leq a$ we know that $A(a \wedge b) \vee \lambda \geq A(a) \wedge \mu$ and from $a \wedge b \leq b$ we conclude that $A(a \wedge b) \vee \lambda \geq A(b) \wedge \mu$. Thus $A(a \wedge b) \vee \lambda \geq (A(a) \wedge \mu) \vee (A(b) \wedge \mu) = (A(a) \vee A(b)) \wedge \mu$.

Hence, we complete the proof. ∎

Theorem 3.5 *Let A be a (λ, μ)-fuzzy subhyperlattice of H. Then the following are equivalent:*

(1) *A is a (λ, μ)-fuzzy ideal of H;*
(2) *A_α is an ideal of H, for any $\alpha \in (\lambda, \mu]$, where $A_\alpha \neq \emptyset$.*

Proof. (1) $\Rightarrow$ (2). Let A be a (λ, μ)-fuzzy ideal of H. For any $\alpha \in (\lambda, \mu]$, such that $A_\alpha \neq \emptyset$, we need to show that $x \wedge y \in A_\alpha$, for all $x \in A_\alpha$ and $y \in H$.

From $A(x) \geq \alpha$ we obtain that $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu \geq \alpha$. Note that $\lambda < \alpha$, we conclude that $A(x \wedge y) \geq \alpha$. So $x \wedge y \in A_\alpha$.

(2) $\Rightarrow$ (1). If there exist $x_0, y_0 \in H$ such that $A(x_0 \wedge y_0) \vee \lambda < \alpha = (A(x_0) \vee A(y_0)) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0) \vee A(y_0) \geq \alpha$. So $x_0 \in A_\alpha$ or $y_0 \in A_\alpha$. But $A(x_0 \wedge y_0) < \alpha$, that is $x_0 \wedge y_0 \notin A_\alpha$. This is a contradiction with that A_α is an ideal of H. Thus $A(x \wedge y) \vee \lambda \geq (A(x) \vee A(y)) \wedge \mu$ holds for all $x, y \in H$.

The proof is ended. ∎

Example 3.6 Let $H = \{0, a, b, 1\}$ and define $\sqcup$ and $\wedge$ as following

$\wedge$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

$\sqcup$	0	a	b	1
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{1\}$
a	$\{a\}$	$\{0, a\}$	$\{1\}$	$\{b, 1\}$
b	$\{b\}$	$\{1\}$	$\{0, b\}$	$\{a, 1\}$
1	$\{1\}$	$\{b, 1\}$	$\{a, 1\}$	H

Then $(H, \sqcup, \wedge, 0, 1)$ is a bounded distributive hyperlattice (see Example 1.8 of [9]).

Consider the following fuzzy subset of H, defined by

H	0	a	b	1
A	1.0	0.2	0.3	0.2

Then A is a $(0.2, 1.0)$-fuzzy ideal of H. Also A is a $(\lambda, 1.0)$-fuzzy ideal of H, where $0 \leq \lambda \leq 0.2$.

We give the definition of (λ, μ)-fuzzy prime ideal as following.

Definition 3.7 A proper (λ, μ)-fuzzy ideal of H is called a (λ, μ)-fuzzy prime ideal, if for all $a, b \in H$,

$$(A(a) \vee A(b)) \vee \lambda \geq A(a \wedge b) \wedge \mu.$$

Example 3.8 Consider the bounded distributive hyperlattice H of the previous example, and consider the following fuzzy subsets of H defined by

H	0	a	b	1
A	0.25	0.16	0.25	0.16

Then A is a $(0.16, 0.25)$-fuzzy prime ideal of H.

Theorem 3.9 *Let A be a (λ, μ)-fuzzy subhyperlattice of H. Then the following are equivalent:*

(1) *A is a (λ, μ)-fuzzy prime ideal of H;*

(2) *A_α is a prime ideal of H, for any $\alpha \in (\lambda, \mu]$, where A_α is a proper ideal of H.*

Proof. (1) $\Rightarrow$ (2). Let A be a (λ, μ)-fuzzy prime ideal of H. For any $\alpha \in (\lambda, \mu]$, such that A_α is a proper ideal of H, we need to show that $x \wedge y \in A_\alpha \Rightarrow x \in A_\alpha$ or $y \in A_\alpha$. From $x \wedge y \in A_\alpha$ we obtain that $A(x \wedge y) \geq \alpha$. So $(A(x) \vee A(y)) \vee \lambda \geq A(x \wedge y) \wedge \mu \geq \alpha \wedge \mu = \alpha$. Note that $\lambda < \alpha$, we conclude that $A(x) \vee A(y) \geq \alpha$. Thus $x \in A_\alpha$ or $y \in A_\alpha$.

(2) $\Rightarrow$ (1). If there exist $x_0, y_0 \in H$ such that $(A(x_0) \vee A(y_0)) \vee \lambda < \alpha = A(x_0 \wedge y_0) \wedge \mu$, then $\alpha \in (\lambda, \mu]$, $A(x_0 \wedge y_0) \geq \alpha$. So $x_0 \wedge y_0 \in A_\alpha$. But $A(x_0) \vee A(y_0) < \alpha$, that is $x_0 \notin A_\alpha$ and $y_0 \notin A_\alpha$. This is a contradiction with that A_α is a prime ideal of H. Thus $(A(x) \vee A(y)) \vee \lambda \geq A(x \wedge y) \wedge \mu$ holds for all $x, y \in H$.

The proof is ended. ∎

4. Cartesian product of (λ, μ)-fuzzy subhyperlattices

Let H_1 and H_2 be two hyperlattices. The Cartesian product of H_1 and H_2 is defined by $H_1 \times H_2 \doteq \{(x, y) | x \in H_1, y \in H_2\}$.

For $(a, b), (c, d) \in H_1 \times H_2$, we define

$$(a, b) \leq (c, d) \doteq \begin{cases} a \leq c \\ b \leq d \end{cases},$$

$$(a, b) \wedge (c, d) \doteq (a \wedge c, b \wedge d)$$

and

$$(a, b) \sqcup (c, d) \doteq \cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d} (t_1, t_2)$$

Proposition 4.1 *Let H_1 and H_2 be two hyperlattices. Then $(H_1 \times H_2, \sqcup, \wedge)$ is a hyperlattice.*

Proof. For all $(a,b),(c,d),(e,f) \in H_1 \times H_2$, we have

(1) From $(a,b) \sqcup (a,b) = \cup_{t_1 \in a \sqcup a, t_2 \in b \sqcup b}(t_1,t_2)$, $a \in a \sqcup a$ and $b \in b \sqcup b$ we know that $(a,b) \in (a,b) \sqcup (a,b)$.

It is obvious that $(a,b) \wedge (a,b) = (a \wedge a, b \wedge b) = (a,b)$.

(2) $(a,b) \sqcup (c,d) = \cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d}(t_1,t_2) = \cup_{t_1 \in c \sqcup a, t_2 \in d \sqcup b}(t_1,t_2) = (c,d) \sqcup (a,b)$.

$(a,b) \wedge (c,d) = (a \wedge c, b \wedge d) = (c \wedge a, d \wedge b) = (c,d) \wedge (a,b)$.

(3) $((a,b) \sqcup (c,d)) \sqcup (e,f) = \cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d}(t_1,t_2) \sqcup (e,f) = \cup_{t_1 \in (a \sqcup c) \sqcup e, t_2 \in (b \sqcup d) \sqcup f}(t_1,t_2)$ $= \cup_{t_1 \in a \sqcup (c \sqcup e), t_2 \in b \sqcup (d \sqcup f)}(t_1,t_2) = (a,b) \sqcup ((c,d) \sqcup (e,f))$.

$((a,b) \wedge (c,d)) \wedge (e,f) = (a \wedge c, b \wedge d) \wedge (e,f) = ((a \wedge c) \wedge e, (b \wedge d) \wedge f) = (a \wedge (c \wedge e), b \wedge (d \wedge f)) = (a,b) \wedge ((c,d) \wedge (e,f))$.

(4) $((a,b) \sqcup (c,d)) \wedge (a,b) = (\cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d}(t_1,t_2)) \wedge (a,b) = \cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d}(t_1 \wedge a, t_2 \wedge b) = \cup_{t_1 \in (a \sqcup c) \wedge a, t_2 \in (b \sqcup d) \wedge b}(t_1,t_2) \ni (a,b)$.

$((a,b) \wedge (c,d)) \sqcup (a,b) = (a \wedge c, b \wedge d) \sqcup (a,b) = \cup_{t_1 \in (a \wedge c) \sqcup a, t_2 \in (b \wedge d) \sqcup b}(t_1,t_2)$ $\ni (a,b)$.

(5) $(a,b) \in (a,b) \sqcup (c,d) \Leftrightarrow (a,b) \in \cup_{t_1 \in a \sqcup c, t_2 \in b \sqcup d}(t_1,t_2) \Leftrightarrow \left\{ \begin{array}{c} a \in a \sqcup c \\ b \in b \sqcup d \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} c = a \wedge c \\ d = b \wedge d \end{array} \right\} \Leftrightarrow (c,d) = (a,b) \wedge (c,d)$.

$(c,d) = (a,b) \wedge (c,d) \Leftrightarrow \left\{ \begin{array}{c} c = a \wedge c \\ d = b \wedge d \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} c \leq a \\ d \leq b \end{array} \right\} \Leftrightarrow (c,d) \leq (a,b)$. ∎

Theorem 4.2 *Let A be a (λ,μ)-fuzzy subhyperlattice of the hyperlattice H_1 and B be a (λ,μ)-fuzzy subhyperlattice of the hyperlattice H_2. Then $A \times B$ is a (λ,μ)-fuzzy subhyperlattice of the hyperlattice $H_1 \times H_2$, where*

$$(A \times B)(x,y) \doteq A(x) \wedge B(y), \quad \forall (x,y) \in H_1 \times H_2.$$

Proof. Let $(a,b),(c,d) \in H_1 \times H_2$. Then

$$\begin{aligned} (A \times B)\{(a,b) \wedge (c,d)\} \vee \lambda &= (A \times B)(a \wedge c, b \wedge d) \vee \lambda \\ &= \{A(a \wedge c) \wedge B(b \wedge d)\} \vee \lambda \\ &= \{A(a \wedge c) \vee \lambda\} \wedge \{B(b \wedge d) \vee \lambda\} \\ &\geq \{(A(a) \wedge A(c)) \wedge \mu\} \wedge \{(B(b) \wedge B(d)) \wedge \mu\} \\ &= \{A(a) \wedge A(c) \wedge B(b) \wedge B(d)\} \wedge \mu \\ &= \{A(a) \wedge B(b) \wedge A(c) \wedge B(d)\} \wedge \mu \\ &= \{(A \times B)(a,b) \wedge (A \times B)(c,d)\} \wedge \mu \end{aligned}$$

and

$$\begin{aligned}
\left(\inf_{t\in(a,b)\sqcup(c,d)}(A\times B)(t)\right)\vee\lambda &= \left(\inf_{t_1\in a\sqcup c,t_2\in b\sqcup d}(A\times B)(t_1,t_2)\right)\vee\lambda\\
&= \left(\inf_{t_1\in a\sqcup c,t_2\in b\sqcup d}\{A(t_1)\wedge B(t_2)\}\right)\vee\lambda\\
&= \inf_{t_1\in a\sqcup c,t_2\in b\sqcup d}\{(A(t_1)\vee\lambda)\wedge(B(t_2)\vee\lambda)\}\\
&\geq \{(A(a)\wedge A(c))\wedge\mu\}\wedge\{(B(b)\wedge B(d))\wedge\mu\}\\
&= \{A(a)\wedge A(c)\wedge B(b)\wedge B(d)\}\wedge\mu\\
&= \{A(a)\wedge B(b)\wedge A(c)\wedge B(d)\}\wedge\mu\\
&= \{(A\times B)(a,b)\wedge(A\times B)(c,d)\}\wedge\mu
\end{aligned}$$

Hence $A\times B$ is a (λ,μ)-fuzzy subhyperlattice of the hyperlattice $H_1\times H_2$. ■

The following example shows that the product of two (λ,μ)-fuzzy ideals is not necessarily a (λ,μ)-fuzzy ideal.

Example 4.3 Let $H=\{a,b,c,d,e\}$. $\sqcup$ and $\wedge$ are given by the following tables

$\wedge$	a	b	c	d	e
a	a	b	c	d	e
b	b	b	d	d	b
c	c	d	c	d	c
d	d	d	d	d	d
e	e	b	c	d	e

$\sqcup$	a	b	c	d	e
a	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$
b	$\{a,e\}$	b	$\{a,e\}$	b	$\{a,e\}$
c	$\{a,e\}$	$\{a,e\}$	c	c	$\{a,e\}$
d	$\{a,e\}$	b	c	d	$\{a,e\}$
e	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$

It is easy to verify that H is a hyperlattice.
Consider the following fuzzy set A and B of H, respectively.

H	a	b	c	d	e
A	0.2	0.2	0.9	0.4	0.2

H	a	b	c	d	e
B	0.4	0.4	0.8	0.8	0.4

Clearly, A and B are $(0,1)$-fuzzy ideals of H. For $(b,c),(c,d)\in H\times H$, we have

$$(A\times B)(b,c)=A(b)\wedge B(c)=0.2$$

and

$$(A\times B)(c,d)=A(c)\wedge B(d)=0.8.$$

Therefore

$$\{(A\times B)(b,c)\vee(A\times B)(c,d)\}\wedge 1=0.8.$$

On the other hand, we have

$$(A\times B)\{(b,c)\wedge(c,d)\}\vee 0=(A\times B)(b\wedge c,c\wedge d)(A\times B)(d,d)\vee 0=0.4.$$

Thus $(A\times B)\{(b,c)\wedge(c,d)\}\vee 0\ngeq\{(A\times B)(b,c)\vee(A\times B)(c,d)\}\wedge 1$. Hence $A\times B$ is not a $(0,1)$-fuzzy ideal of $H\times H$.

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