

WEAKLY \mathfrak{b} -I-OPEN SETS AND WEAKLY \mathfrak{b} -I-CONTINUOUS FUNCTIONS

Jamal M. Mustafa

Samer Al Ghour¹

*Department of Mathematics
Al al-Bayt University
P.O. Box: 130095, Mafraq
Jordan
e-mail: jjmmrr971@yahoo.com
algore@just.edu.jo*

Khalid Al Zoubi

*Department of Mathematics
Faculty of Science
Yarmouk University
Jordan
e-mail : khalidz@yu.edu.jo*

Abstract. The notion of weakly \mathfrak{b} -I-open sets is introduced and used to define the notions of weakly \mathfrak{b} -I-continuous functions, weakly \mathfrak{b} -I-open functions, and weakly \mathfrak{b} -I-closed functions. Some characterizations and properties regarding these concepts are discussed.

Keywords and phrases: Ideal topological spaces, weakly \mathfrak{b} -I-open sets, weakly \mathfrak{b} -I-continuous functions, weakly \mathfrak{b} -I-irresolute functions, weakly \mathfrak{b} -I-open functions.

2000 Mathematics Subject Classification: 54C08, 54C10, 54A05.

Introduction

Let (X, τ) be a topological space and $A \subseteq X$. The complement of A in X , the closure of A , the interior of A and the power set of A will be denoted by $X - A = A^c$, $Cl(A)$, $Int(A)$ and $\mathcal{P}(A)$, respectively. The subject of ideals in topological spaces has been studied by Kuratowski [9] and Vaidyanathaswamy [13]. An ideal on a topological space (X, τ) is defined as a non-empty collection I of subsets of X satisfying the following two conditions:

- (1) If $A \in I$ and $B \subseteq A$, then $B \in I$;
- (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

¹Samer Al Ghour did this research during the sabbatical leave which was given from Department of Mathematics and Statistics, Jordan University of Science and Technology, Irbid, Jordan.

An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I) . For a subset $A \subseteq X$, $A^*(I) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau \text{ with } x \in U\}$ is called the local function of A with respect to I and τ [9]. We simply write A^* instead of $A^*(I)$ in case there is no chance of confusion. It is well known that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$. First, we shall recall some definitions used in the sequel.

Definition 1.1. Let A be a subset of a topological space (X, τ) . Then

- (a) A is called *semi-open* [10] if $A \subseteq Cl(Int(A))$.
- (b) A is called *pre-open* [11] if $A \subseteq Int(Cl(A))$.
- (c) A is called *b-open* [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.
- (d) A is called *semi-closed* [2] if it is the complement of a semi-open set.
- (e) The *semi-closure* of A [2], denoted by $sCl(A)$, is the smallest semi-closed set that contains A .

Definition 1.2. A subset A of an ideal topological space (X, τ, I) is said to be

- (a) *I-open* [8] if $A \subseteq Int(A^*)$.
- (b) *semi-I-open* [7] if $A \subseteq Cl^*(Int(A))$.
- (c) *pre-I-open* [3] if $A \subseteq Int(Cl^*(A))$.
- (d) *b-I-open* [4] if $A \subseteq Cl^*(Int(A)) \cup Int(Cl^*(A))$.
- (e) *weakly semi-I-open* [5] if $A \subseteq Cl^*(Int(Cl(A)))$.
- (f) *weakly pre-I-open* [6] if $A \subseteq sCl(Int(Cl^*(A)))$.

2. Weakly b-I-open sets

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is said to be *weakly b-I-open* if $A \subseteq Cl^*(Int(Cl(A))) \cup Cl(Int(Cl^*(A)))$.

The family of all weakly b-I-open sets of the space (X, τ, I) will be denoted by $WBIO(X, \tau)$.

Theorem 2.2. For a subset of an ideal topological space, the following properties hold:

- (a) Every b-I-open set is weakly b-I-open.
- (b) Every weakly semi-I-open set is weakly b-I-open.
- (c) Every weakly pre-I-open set is weakly b-I-open.

The converse of each part in the above theorem need not be true as shown in the following three examples.

Example 2.3. Let $X = \{1, 2, 3\}$, $\tau = \{\phi, X, \{1, 2\}\}$ and $I = \{\phi, \{1\}\}$. Then $A = \{1\}$ is weakly b-I-open but not b-I-open.

Example 2.4. Let $X = \{1, 2, 3\}$, $\tau = \{\phi, X, \{1\}, \{3\}, \{1, 3\}\}$ and $I = \{\phi, \{1\}\}$. Then $A = \{1, 2\}$ is weakly b-I-open but not weakly semi-I-open.

Example 2.5. Let $X = \{1, 2, 3, 4\}$, $\tau = \{\phi, X, \{1, 2\}, \{3, 4\}\}$ and $I = \mathcal{P}(X)$. Then $A = \{2, 3\}$ is weakly semi-I-open and so weakly b-I-open but not weakly pre-I-open since $sCl(Int(Cl^*(A))) = \phi$.

Lemma 2.6. [8] *Let A and B be subsets of an ideal topological space (X, τ, I) .*

- (a) *If $A \subseteq B$, then $A^* \subseteq B^*$.*
- (b) *If $U \in \tau$, then $U \cap A^* \subseteq (U \cap A)^*$.*
- (c) *A^* is closed in (X, τ) .*

Theorem 2.7. *Let (X, τ, I) be an ideal topological space and A, B subsets of X .*

- (1) *If $U_\alpha \in WBIO(X, \tau)$ for each $\alpha \in \Delta$, then $\bigcup\{U_\alpha : \alpha \in \Delta\} \in WBIO(X, \tau)$.*
- (2) *If $A \in WBIO(X, \tau)$ and $B \in \tau$, then $A \cap B \in WBIO(X, \tau)$.*

Proof. (1) Since $U_\alpha \in WBIO(X, \tau)$, we have

$$\begin{aligned} \bigcup_{\alpha \in \Delta} U_\alpha &\subseteq \bigcup_{\alpha \in \Delta} [Cl^*(Int(Cl(U_\alpha))) \cup Cl(Int(Cl^*(U_\alpha)))] \\ &\subseteq \bigcup_{\alpha \in \Delta} \{[(Int(Cl(U_\alpha))) \cup (Int(Cl(U_\alpha)))^*] \cup [Cl(Int(U_\alpha \cup U_\alpha^*))]\} \\ &\subseteq [Int(Cl(\bigcup_{\alpha \in \Delta} U_\alpha)) \cup (Int(Cl(\bigcup_{\alpha \in \Delta} U_\alpha)))^*] \cup [Cl(Int((\bigcup_{\alpha \in \Delta} U_\alpha) \cup (\bigcup_{\alpha \in \Delta} U_\alpha)^*))] \\ &= Cl^*[Int(Cl(\bigcup_{\alpha \in \Delta} U_\alpha))] \cup Cl[Int(Cl^*(\bigcup_{\alpha \in \Delta} U_\alpha))]. \end{aligned}$$

Hence $\bigcup_{\alpha \in \Delta} U_\alpha \in WBIO(X, \tau)$.

(2) Let $A \in WBIO(X, \tau)$ and $B \in \tau$. Then $A \subseteq Cl^*(Int(Cl(A))) \cup Cl(Int(Cl^*(A)))$

and so

$$\begin{aligned} A \cap B &\subseteq [Cl^*(Int(Cl(A))) \cup Cl(Int(Cl^*(A)))] \cap B \\ &= [Cl^*(Int(Cl(A))) \cap B] \cup [Cl(Int(Cl^*(A))) \cap B] \\ &= [(Int(Cl(A)) \cup (Int(Cl(A)))^*) \cap B] \cup [Cl(Int(A \cup A^*)) \cap B] \\ &\subseteq [(Int(Cl(A)) \cap B) \cup ((Int(Cl(A))) \cap B)^*] \cup [Cl(Int[(A \cap B) \cup (A^* \cap B)])] \\ &\subseteq [(Int(Cl(A)) \cap B) \cup (Int(Cl(A)) \cap B)^*] \cup [Cl(Int[(A \cap B) \cup (A \cap B)^*])] \\ &\subseteq [(Int(Cl(A \cap B))) \cup (Int(Cl(A \cap B)))^*] \cup [Cl(Int[(A \cap B) \cup (A \cap B)^*])] \\ &= Cl^*[Int(Cl(A \cap B))] \cup Cl[Int(Cl^*(A \cap B))]. \end{aligned}$$

This shows that $A \cap B \in WBIO(X, \tau)$.

The following example shows that the finite intersection of weakly b-I-open sets need not be weakly b-I-open.

Example 2.8. Let $X = \{1, 2, 3, 4\}$, $\tau = \{\phi, X, \{1, 2\}, \{1, 2, 3\}\}$ and $I = \{\phi, \{3\}, \{4\}, \{3, 4\}\}$. Then $A = \{2, 4\}$ and $B = \{1, 4\}$ are weakly b-I-open but $A \cap B = \{4\}$ is not weakly b-I-open.

Definition 2.9. A subset A of an ideal topological space (X, τ, I) is said to be *weakly b-I-closed* if its complement is weakly b-I-open.

Theorem 2.10. *If a subset A of an ideal topological space (X, τ, I) is weakly b-I-closed, then $\text{Int}(\text{Cl}(\text{Int}(A))) \subseteq A$.*

Proof. Since A is weakly b-I-closed, $X - A \in \text{WBIO}(X, \tau)$. Then

$$\begin{aligned} X - A &\subseteq \text{Cl}^*(\text{Int}(\text{Cl}(X - A))) \cup \text{Cl}(\text{Int}(\text{Cl}^*(X - A))) \\ &\subseteq \text{Cl}(\text{Int}(\text{Cl}(X - A))) \cup \text{Cl}(\text{Int}(\text{Cl}(X - A))) \\ &= \text{Cl}(\text{Int}(\text{Cl}(X - A))) \\ &= X - \text{Int}(\text{Cl}(\text{Int}(A))). \end{aligned}$$

Therefore, $\text{Int}(\text{Cl}(\text{Int}(A))) \subseteq A$.

3. Weakly b-I-continuous functions

Definition 3.1.

- (1) A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is called *b-I-continuous* [4] if the inverse image of each open set in Y is a b-I-open set in X .
- (2) A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is called *weakly semi-I-continuous* [5] if the inverse image of each open set in Y is a weakly semi-I-open set in X .
- (3) A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is called *weakly pre-I-continuous* [6] if the inverse image of each open set in Y is a weakly pre-I-open set in X .

Definition 3.2. A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is called *weakly b-I-continuous* if $f^{-1}(V)$ is weakly b-I-open in (X, τ, I) for every open set V in (Y, ρ) .

Remark 3.3.

- (1) Every b-I-continuous function is weakly b-I-continuous.
- (2) Every weakly semi-I-continuous function is weakly b-I-continuous.
- (3) Every weakly pre-I-continuous function is weakly b-I-continuous.

The converse in each part of the above remark need not be true as shown in the following three examples.

Example 3.4. Let (X, τ) be the real line with the indiscrete topology and (Y, ρ) the real line with the usual topology. Then the identity function $f : (X, \tau, \mathcal{P}(X)) \rightarrow (Y, \rho)$ is weakly b-I-continuous but not b-I-continuous.

Example 3.5. Let $X = Y = \{1, 2, 3\}$, $\tau = \{\phi, X, \{1\}, \{3\}, \{1, 3\}\}$, $I = \{\phi, \{1\}\}$ and $\rho = \{\phi, Y, \{1, 3\}\}$. Define a function $f : (X, \tau, I) \rightarrow (Y, \rho)$ as follows: $f(1) = 1$, $f(2) = 3$ and $f(3) = 2$. Then f is weakly b-I-continuous but not weakly semi-I-continuous.

Example 3.6. Let $X=Y=\{1, 2, 3, 4\}$, $\tau=\{\phi, X, \{1\}, \{3\}, \{1, 3\}\}$, $\rho = \{\phi, Y, \{1\}, \{3\}, \{1, 3\}\}$, and $I = \{\phi, \{3\}\}$. Define a function $f : (X, \tau, I) \rightarrow (Y, \rho)$ as follows: $f(1) = f(3) = 4$ and $f(2) = f(4) = 2$. Then f is weakly b-I-continuous but not weakly pre-I-continuous.

Theorem 3.7. For a function $f : (X, \tau, I) \rightarrow (Y, \rho)$ the following are equivalent:

- (1) f is weakly b-I-continuous.
- (2) For each $x \in X$ and each $V \in \rho$ with $f(x) \in V$, there exists $U \in WBIO(X, \tau)$ with $x \in U$ such that $f(U) \subseteq V$.
- (3) The inverse image of each closed set in Y is weakly b-I-closed in X .

Proof. Straightforward.

Definition 3.8. Let A be a subset of a space (X, τ, I) and let $x \in X$. Then A is called a *weakly b-I-neighborhood* of x , if there exists a weakly b-I-open set U containing x such that $U \subseteq A$.

Theorem 3.9. For a function $f : (X, \tau, I) \rightarrow (Y, \rho)$, the following statements are equivalent:

- (1) f is weakly b-I-continuous.
- (2) For each $x \in X$ and each open set V in Y with $f(x) \in V$, $f^{-1}(V)$ is weakly b-I-neighborhood of x .

Proof. (1) \Rightarrow (2). Let $x \in X$ and let V be an open set in Y such that $f(x) \in V$. By Theorem 3.7, there exists a weakly b-I-open set U in X with $x \in U$ such that $f(U) \subseteq V$. So $x \in U \subseteq f^{-1}(V)$. Hence $f^{-1}(V)$ is a weakly b-I-neighborhood of x .

(2) \Rightarrow (1). Let V be an open set in Y and let $f(x) \in V$. Then by assumption, $f^{-1}(V)$ is a weakly b-I-neighborhood of x . Thus for each $x \in f^{-1}(V)$ there exists a weakly b-I-open set U_x containing x such that $x \in U_x \subseteq f^{-1}(V)$. Hence $f^{-1}(V) = \cup\{U_x : x \in f^{-1}(V)\}$ and so $f^{-1}(V) \in WBOI(X, \tau)$.

Definition 3.10. A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is called *weakly b-I-irresolute* if $f^{-1}(V)$ is weakly b-I-open in (X, τ, I) for every weakly b-I-open set V in (Y, ρ) .

Theorem 3.11. Let $f : (X, \tau, I) \rightarrow (Y, \rho, J)$ and $g : (Y, \rho, J) \rightarrow (Z, \sigma, K)$ then

- (1) gof is weakly b - I -continuous if f is weakly b - I -continuous and g is continuous.
- (2) gof is weakly b - I -continuous if f is weakly b - I -irresolute and g is weakly b - I -continuous.

We note that Theorem 3.11(1) is not true if g is assumed to be only b - I -continuous as it is shown in the next example.

Example 3.12. Let $X = Y = Z = \{1, 2, 3, 4\}$, $\tau = \{\phi, X, \{1\}, \{3\}, \{1, 3\}\}$, $\rho = \{\phi, Y, \{1\}, \{3\}, \{1, 3\}\}$, $\sigma = \{\phi, Z, \{1, 2\}\}$, $I = \{\phi, \{3\}\}$ and $J = \{\phi, \{2\}\}$. Define a function $f : (X, \tau, I) \rightarrow (Y, \rho, J)$ by $f(1) = f(3) = 4$ and $f(2) = f(4) = 2$. Also let $g : (Y, \rho, J) \rightarrow (Z, \sigma)$ be the identity function. Then f is weakly b - I -continuous and g is b - I -continuous but gof is not weakly b - I -continuous since $(gof)^{-1}(\{1, 2\}) = \{2, 4\}$ is not weakly b - I -open.

Corollary 3.13. If $f : (X, \tau, I) \rightarrow \prod_{\alpha \in \Delta} X_\alpha$ is a weakly b - I -continuous function from a space (X, τ, I) into a product space $\prod_{\alpha \in \Delta} X_\alpha$ then $p_\alpha \circ f$ is weakly b - I -continuous for each $\alpha \in \Delta$, where p_α is the projection function from the product space $\prod_{\alpha \in \Delta} X_\alpha$ onto the space X_α for each $\alpha \in \Delta$.

If (X, τ, I) is an ideal topological space and A is a subset of X , we denote by $\tau|_A$ the relative topology on A and $I|_A = \{A \cap I : I \in I\}$ is obviously an ideal on A .

Lemma 3.14. [8] Let (X, τ, I) be an ideal topological space and A, B be subsets of X such that $B \subseteq A$. Then $B^*(\tau|_A, I|_A) = B^*(\tau, I) \cap A$.

Lemma 3.15. [5] Let (X, τ, I) be an ideal topological space, $A \subseteq X$ and $U \in \tau$. Then $Cl^*(A) \cap U = Cl_U^*(A \cap U)$.

Theorem 3.16. Let (X, τ, I) be an ideal topological space, $A \subseteq U \in \tau$. If $A \in WBIO(X, \tau)$ then $A \in WBIO(U, \tau|_U, I|_U)$.

Proof. Since $U \in \tau$ and $A \in WBIO(X, \tau)$, we have

$$\begin{aligned} A &= U \cap A \subseteq U \cap [Cl^*(Int(Cl(A))) \cup Cl(Int(Cl^*(A)))] \\ &= [U \cap (Cl^*(Int(Cl(A))))] \cup [U \cap (Cl(Int(Cl^*(A))))] \\ &\subseteq Cl^*(U \cap Int(Cl(A))) \cup Cl(U \cap Int(Cl^*(A))) \\ &= Cl^*(Int(U \cap Cl(A))) \cup Cl(Int(U \cap Cl^*(A))) \\ &= Cl^*(Int_U(U \cap Cl(A))) \cup Cl(Int_U(U \cap Cl^*(A))) \end{aligned}$$

Since $U \in \tau \subseteq \tau^*$, we obtain

$$\begin{aligned} A &= U \cap A \subseteq U \cap [Cl^*(Int_U(U \cap Cl(A))) \cup Cl(Int_U(U \cap Cl^*(A)))] \\ &= [U \cap (Cl^*(Int_U(U \cap Cl(A))))] \cup [U \cap (Cl(Int_U(U \cap Cl^*(A))))] \\ &= Cl_U^*(Int_U(U \cap Cl(A))) \cup Cl_U(Int_U(U \cap Cl^*(A))) \\ &= Cl_U^*(Int_U(Cl_U(A))) \cup Cl_U(Int_U(Cl_U^*(A))) \end{aligned}$$

Then $A \in WBIO(U, \tau|_A, I|_A)$.

Corollary 3.17. *Let (X, τ, I) be an ideal topological space. $U \in \tau$ and $A \in WBIO(X, \tau)$, then $U \cap A \in WBIO(U, \tau|_U, I|_U)$.*

Proof. Since $U \in \tau$ and $A \in WBIO(X, \tau)$, by Theorem 2.7, $U \cap A \in WBIO(X, \tau)$. Since $U \in \tau$, by Theorem 3.16, $U \cap A \in WBIO(U, \tau|_U, I|_U)$.

Theorem 3.18. *Let $f : (X, \tau, I) \rightarrow (Y, \rho)$ be a weakly b-I-continuous function and $U \in \tau$. Then the restriction $f|_U : (U, \tau|_U, I|_U) \rightarrow (Y, \rho)$ is weakly b-I-continuous.*

Proof. Let V be any open set in (Y, ρ) . Since f is weakly b-I-continuous, we have $f^{-1}(V) \in WBIO(X, \tau)$. Since $U \in \tau$, by Theorem 3.16, we have $U \cap f^{-1}(V) \in WBIO(U, \tau|_U, I|_U)$. On the other hand, $(f|_U)^{-1}(V) = U \cap f^{-1}(V)$ and $(f|_U)^{-1}(V) \in WBIO(U, \tau|_U, I|_U)$. This shows that $f|_U : (U, \tau|_U, I|_U) \rightarrow (Y, \rho)$ is weakly b-I-continuous.

Theorem 3.19. *A function $f : (X, \tau, I) \rightarrow (Y, \rho)$ is weakly b-I-continuous if and only if the function $g : X \rightarrow X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly b-I-continuous.*

Proof. \Rightarrow) Let f be weakly b-I-continuous. Let $x \in X$ and W be an open set in $X \times Y$ with $g(x) \in W$. Then there exists a basic open set $U \times V \subseteq W$ such that $g(x) = (x, f(x)) \in U \times V \subseteq W$. Since f is weakly b-I-continuous, there exists a weakly b-I-open set U_0 in X with $x \in U_0$ such that $f(U_0) \subseteq V$. By Theorem 2.7, $U_0 \cap U$ is weakly b-I-open and $g(U_0 \cap U) \subseteq U \cap V \subseteq W$. This shows that g is weakly b-I-continuous.

\Leftarrow) Suppose that g is weakly b-I-continuous. Let $x \in X$ and V be any open set in Y with $f(x) \in V$. Then $X \times V$ is open in $X \times Y$. Since g is weakly b-I-continuous, there exists a weakly b-I-open set U in X with $x \in U$ such that $g(U) \subseteq X \times V$. Therefore, we obtain $f(U) \subseteq V$. This shows that f is weakly b-I-continuous.

Definition 3.20. An ideal topological space (X, τ, I) is said to be *weakly b-I-normal* if for each pair of non-empty disjoint closed subsets A and B of X , there exist two weakly b-I-open subsets U and V of X such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \phi$.

Theorem 3.21. *If $f : (X, \tau, I) \rightarrow (Y, \rho)$ is weakly b-I-continuous, closed injection and Y is normal, then X is weakly b-I-normal.*

Proof. Let A and B be two disjoint closed subsets of X . Since f is closed and injective, $f(A)$ and $f(B)$ are disjoint closed subsets of Y . Since Y is normal, there exist two open subsets U and V of Y such that $f(A) \subseteq U$, $f(B) \subseteq V$ and $U \cap V = \phi$. Now $f^{-1}(U)$ and $f^{-1}(V)$ are weakly b-I-open in X with $A \subseteq f^{-1}(U)$, $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is weakly b-I-normal.

Definition 3.22. An ideal topological space (X, τ, I) is said to be *weakly b-I-connected* if X can't be written as a union of two disjoint weakly b-I-open subsets of X .

Theorem 3.23. *The weakly b-I-continuous image of a weakly b-I-connected space is connected.*

Proof. Let $f : (X, \tau, I) \rightarrow (Y, \rho)$ be a weakly b-I-continuous function of a weakly b-I-connected space X onto a topological space Y . Assume that Y is disconnected, then $Y = A \cup B$ where A and B are non-empty clopen with $A \cap B = \phi$. Since f is weakly b-I-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty weakly b-I-open in X . Also, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not weakly b-I-connected which is a contradiction. Therefore, Y is connected.

Lemma 3.24. [12] *For any function $f : (X, \tau, I) \rightarrow (Y, \rho)$, $f(I)$ is an ideal on Y .*

Definition 3.25. [12] An ideal topological space (X, τ, I) is said to be *I-compact* if for every open cover $\{W_\alpha : \alpha \in \Delta\}$ of X , there exists a finite subset Δ_0 of Δ such that $(X - \cup\{W_\alpha : \alpha \in \Delta_0\}) \in I$.

Definition 3.26. An ideal topological space (X, τ, I) is said to be *weakly b-I-compact* if for every weakly b-I-open cover $\{W_\alpha : \alpha \in \Delta\}$ of X , there exists a finite subset Δ_0 of Δ such that $(X - \cup\{W_\alpha : \alpha \in \Delta_0\}) \in I$.

Theorem 3.27. *The image of a weakly b-I-compact space under a weakly b-I-continuous surjective function is $f(I)$ -compact.*

Proof. Let (X, τ, I) be a weakly b-I-compact space and $f : (X, \tau, I) \rightarrow (Y, \rho)$ be a weakly b-I-continuous surjection. Let $\{V_\alpha : \alpha \in \Delta\}$ be an open cover of Y . Then $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a weakly b-I-open cover of X . Since X is weakly b-I-compact, there exists a finite subset Δ_0 of Δ such that $(X - \cup\{f^{-1}(V_\alpha) : \alpha \in \Delta_0\}) \in I$. Therefore $(Y - \cup\{V_\alpha : \alpha \in \Delta_0\}) \in f(I)$. This shows that $(Y, \rho, f(I))$ is $f(I)$ -compact.

4. Weakly b-I-open functions

Recall that a subset F of a space (X, τ, I) is said to be *weakly semi-I-closed* [5] if its complement is weakly semi-I-open.

Definition 4.1. [5] A function $f : (X, \tau) \rightarrow (Y, \rho, J)$ is called *weakly semi-I-open* (resp., *weakly semi-I-closed*) if the image of every open (resp., closed) set in (X, τ) is weakly semi-I-open (resp., weakly semi-I-closed) in (Y, ρ, J) .

Definition 4.2. A function $f : (X, \tau) \rightarrow (Y, \rho, J)$ is called *weakly b-I-open* (resp., *weakly b-I-closed*) if the image of every open (resp., closed) set in (X, τ) is weakly b-I-open (resp., weakly b-I-closed) in (Y, ρ, J) .

Remark 4.3. Every weakly semi-I-open (resp., weakly semi-I-closed) function is weakly b-I-open (resp., weakly b-I-closed).

The converse of the above remark need not be true as shown in the following example.

Example 4.4. Let $X = \{1, 2, 3\}$ and $\tau = \{\phi, X, \{1, 3\}\}$. Also let $Y = X$, $\rho = \{\phi, Y, \{1\}, \{3\}, \{1, 3\}\}$ and $J = \{\phi, \{1\}\}$. Define a function $f : (X, \tau) \rightarrow (Y, \rho, J)$ as follows: $f(1) = 1$, $f(2) = 3$ and $f(3) = 2$. Then f is weakly b-I-open but not weakly semi-I-open.

Theorem 4.5. *A function $f : (X, \tau) \rightarrow (Y, \rho, J)$ is weakly b-I-open if and only if for each $x \in X$ and each neighborhood U of x there exists $V \in WBJO(Y, \rho)$ containing $f(x)$ such that $V \subseteq f(U)$.*

Proof. \Rightarrow Suppose that f is a weakly b-I-open function. For each $x \in X$ and each neighborhood U of x , there exists $U_x \in \tau$ such that $x \in U_x \subseteq U$. Let $V = f(U_x)$. Since f is weakly b-I-open, $V \in WBJO(Y, \rho)$ and $f(x) \in V \subseteq f(U)$.

\Leftarrow Let U be an open set in (X, τ) . For each $x \in U$, there exists $V_x \in WBJO(X, \tau)$ such that $f(x) \in V_x \subseteq f(U)$. Now $f(U) = \cup\{V_x : x \in U\}$ and so $f(U) \in WBJO(Y, \rho)$. This shows that f is weakly b-I-open.

Theorem 4.6. *Let $f : (X, \tau) \rightarrow (Y, \rho, J)$ be a weakly b-I-open function. If G is any subset of Y and C is a closed subset of X with $f^{-1}(G) \subseteq C$, then there exists a weakly b-I-closed subset H of Y with $G \subseteq H$ such that $f^{-1}(H) \subseteq C$.*

Proof. Suppose that f is a weakly b-I-open function. Let G be any subset of Y and C a closed subset of X with $f^{-1}(G) \subseteq C$. Then $X - C$ is open. Since f is weakly b-I-open, $f(X - C)$ is weakly b-I-open in Y . Let $H = Y - f(X - C)$. Then H is weakly b-I-closed in Y . Since $f^{-1}(G) \subseteq C$, $G \subseteq H$. Also, we obtain $f^{-1}(H) \subseteq C$.

Theorem 4.7. *Let $f : (X, \tau) \rightarrow (Y, \rho, J)$ be weakly b-I-closed. If G is any subset of Y and U is an open subset of X with $f^{-1}(G) \subseteq U$, then there exists a weakly b-I-open subset H of Y with $G \subseteq H$ such that $f^{-1}(H) \subseteq U$.*

Proof. Similar to that used in Theorem 4.6.

Theorem 4.8. *For any bijective function $f : (X, \tau) \rightarrow (Y, \rho, J)$, the following are equivalent:*

- (1) $f^{-1} : (Y, \rho, J) \rightarrow (X, \tau)$ is weakly b-I-continuous.
- (2) f is weakly b-I-open.
- (3) f is weakly b-I-closed.

Proof. It is straightforward.

Theorem 4.9. *Let $f : (X, \tau) \rightarrow (Y, \rho, J)$ and $g : (Y, \rho, J) \rightarrow (Z, \sigma, K)$.*

- (1) *$g \circ f$ is weakly b - I -open if f is open and g is a weakly b - I -open.*
- (2) *f is weakly b - I -open if $g \circ f$ is open and g is a weakly b - I -continuous injection.*

References

- [1] ANDRIJEVIĆ, D., *On b -open sets*, Mat. Vesnik., 48 (1996), 59-64.
- [2] CROSSLEY, S.G. and HILDEBRAND, S.K., *Semi-closure*, Texas J. Sci., 22 (1971), 99-112.
- [3] DONTCHEV, J., *On pre- I -open sets and a decomposition of I -continuity*, Banyan Math. J., 2 (1996).
- [4] CAKSU GULER, A. and ASLIM, G., *b - I -open sets and decomposition of continuity via idealization*, Proceedings of Institute of Mathematics and Mechanics. National Academy of Sciences of Azerbaijan, 22 (2005), 27-32.
- [5] HATIR, E. and JAFARI, S., *On weakly semi- I -open sets and another decomposition of continuity via ideals*, Sarajevo J. Math., 2 (14) (2006), 107-114.
- [6] HATIR, E. and NOIRI, T., *Weakly pre- I -open sets and decomposition of continuity*, Acta Math. Hungar., 106 (3) (2005), 227-238.
- [7] HATIR, E. and NOIRI, T., *On decomposition of continuity via idealization*, Acta Math. Hungar., 96 (2002), 341-349.
- [8] JANKOVIĆ, D.S. and HAMLETT, T.R., *New topologies from old via ideals*, Amer. Math. Monthly, 97 (4) (1990), 295-310.
- [9] KURATOWSKI, K., *Topology*, Vol. I, Academic Press, New York, 1966.
- [10] LEVINE, N., *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36-41.
- [11] MASHHOUR, A.S., ABD EL-MONSSEF, M.E. and EL-DEEB, S.N., *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47-53.
- [12] NEWCOMB, R.L., *Topologies which are compact modulo an ideal*, Ph.D. dissertation, University of California, Santa Barbara, California, Usa, 1967.
- [13] VAIDYANATHASWAMY, R., *The localization theory in set topology*, Proc. Indian Acad. Sci., 20 (1945), 51-61.