THE CATEGORY OF HYPER S-ACTS

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Abstract. The actions of a semigroup or a monoid S on sets have been studied and applied in many branches of mathematics. In this paper, we generalize this notion, and introduce the category of hyper S-acts with the homomorphisms between them. Then, using the usual notion of congruences defined for hyper S-acts, quotients are defined and isomorphism theorems are proved. Finally, limits and colimits in the category of hyper S-acts are studied.

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1. Introduction and preliminaries

The study of hyperstructures started in [7] by introducing hypergroups. Since then other classic hyperstructures have been studied in [2], [9], [10], [11], and the notion has been generalized to universal hyperalgebras and studied in [1], [3], [4], [8]. In this paper we introduce a special type of hyperstructure, namely, hyper S-acts, and study some notions such as congruences, quotients, isomorphism theorems, limits, and colimits in the category they form.

In the rest of this section we recall the definition of the category of S-acts. Let S be a semigroup. Recall that a (right) S-*act* or S-*system* is a set A together with a function $\lambda : A \times S \to A$, called the *action* of S (or the S-action) on A, such that for $a \in A$ and $s, t \in S$ (denoting $\lambda(a, s)$ by as) $a(st) = (as)t$. If S is a monoid with an identity e, we add the condition $ae = a$.

A morphism $f : A \to B$ between S-acts A, B is called an S-*map* if, for each $a \in A$, $s \in S$, $f(as) = f(a)s$.

Since id_A and the composite of two S-maps are S-maps, we have the category **Act-**S of all S-acts and S-maps between them (for more information about acts see [5] and [6]).

2. The category of hyper S-acts

In this section, first the notion of a hyper S-act over a monoid S is defined and then defining the proper homomorphisms between them, the category of hyper S-acts is introduced.

Definition 2.1 Let S be a monoid and A be a set. If we have a mapping

$$\begin{array}{rccc} \mu: & A \times S & \longrightarrow & \mathcal{P}(A) \\ & (a,s) & \mapsto & \mu(a,s) = a \circ s \in \mathcal{P}(A) \end{array}$$

called the *hyper action* of S (or the hyper S-action) on A, such that for $a \in A$ and $s, t \in S$

(i) $a \in a \circ e$,

(ii) $a \circ (st) = (a \circ s) \circ t$, where

$$B \circ s = \bigcup_{b \in B} b \circ s, \forall B \subseteq A.$$

Then we call A a *right hyper S-act* or a *right hyper act* over S and write $A_{\mathcal{H}}$.

Analogously, we define a *left hyper S-act* A and write ${}_{\mathcal{H}}A$.

Remark 2.2 Every S-act A_S is naturally a hyper S-act, by defining

$$\begin{array}{rccc} \mu: & A \times S & \longrightarrow & \mathcal{P}(A) \\ & (a,s) & \longmapsto & \mu(a,s) = \{as\}. \end{array}$$

But there are hyper S-acts which are not ordinary as above. Take $S = \{1, s\}$ where $s^2 = s$ and $A = \{a, b\}$ with the action $a \circ 1 = \{a\}$, $a \circ s = \{a, b\}$, $b \circ 1 = \{b\}$, $b \circ s = \{a, b\}$. Then $A_{\mathcal{H}}$ is a right hyper S-act which is not a right S-act.

Definition 2.3 A function $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$, where A and B are hyper S-acts, is called a homomorphism if $f(a \circ s) \subseteq f(a) \circ s$ for all $a \in A, s \in S$.
f is called a strong homomorphism if $f(a \circ s) = f(a) \circ s$ for all $a \in A, s \in S$.

Definition 2.4 A homomorphism $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$, where A and B are hyper S-acts, is called an isomorphism if it is bijective.

One can easily see that the hyper S-acts with their homomorphism form a category, denoted by $\mathcal{H}Act - S$.

3. Congruences and quotients

This section is devoted to the study of congruences and quotients of hyper S-acts.

Definition 3.5 The equivalence relation θ on a hyper S-act $A_{\mathcal{H}}$ is called a *congruence* if for every $a, b \in A$ and $s \in S$,

$$a\theta b \Longrightarrow \frac{a \circ s}{\theta} = \frac{b \circ s}{\theta}$$

where, for $X \subseteq A$, $\frac{X}{\theta} = \{\frac{x}{\theta} = [x]_\theta : x \in X\}$, and $[x]_\theta$ is the equivalence class of x with respect to θ.

Notice that for every $X, Y \subseteq A$, $\frac{X}{\theta} = \frac{Y}{\theta}$ if and only if $X\overline{\theta}Y$, where $X\overline{\theta}Y$ means that for every $x \in X$ there exists $y \in Y$ such that $x\theta y$ and for every $y \in Y$ there exists $x \in X$ such that $x\theta y$.

The set of all equivalence relations on a hyper S-act $A_{\mathcal{H}}$ is denoted by $Eq(A_{\mathcal{H}})$, and the set of all congruences on $A_{\mathcal{H}}$ is denoted by $Con(A_{\mathcal{H}})$.

Remark 3.6 If A_S is an S-act then an equivalence relation θ on A_S is a congruence on A_S if and only if it is a congruence on A_S as a hyper S-act.

Definition 3.7 Let $A_{\mathcal{H}}$ be a hyper S-act and $\theta \in Eq(A_{\mathcal{H}})$. We define a hyper operation $\circ_{\frac{A_{\mathcal{H}}}{\theta}}$ on $\dfrac{A_{\mathcal{H}}}{\theta}$ as follows:

$$\begin{array}{rccc} \circ_{\frac{A_{\mathcal{H}}}{\theta}} : & \dfrac{A_{\mathcal{H}}}{\theta} \times S & \longrightarrow & \mathcal{P}\left(\dfrac{A_{\mathcal{H}}}{\theta}\right) \\ & \left(\dfrac{a}{\theta}, s\right) & \longmapsto & \displaystyle\bigcup_{x \in \frac{a}{\theta}} \frac{x \circ s}{\theta} \end{array}$$

for all $a \in A$ and $s \in S$.

We call $\dfrac{A_{\mathcal{H}}}{\theta}$ with this hyper operation, the *quotient hyper S-act* of $A_{\mathcal{H}}$ with respect to a congruence θ. Notice that if θ is a congruence on $A_{\mathcal{H}}$ then

$$\frac{a}{\theta} \circ s = \frac{a \circ s}{\theta}.$$

Theorem 3.8 *Let $A_{\mathcal{H}}$ be a hyper S-act and $\theta \in Eq(A_{\mathcal{H}})$. Then we have the following:*

(i) *The natural map $\pi : A_{\mathcal{H}} \to \dfrac{A_{\mathcal{H}}}{\theta}$ given by $\pi(a) = \dfrac{a}{\theta}$ is a homomorphism.*

(ii) *The natural map $\pi : A_{\mathcal{H}} \to \dfrac{A_{\mathcal{H}}}{\theta}$ is a strong homomorphism if and only if θ is a congruence and it is called a canonical epimorphism.*

Proof. (i) Let $a \in A$ and $s \in S$. Then we have $\pi(a \circ s) = \dfrac{a \circ s}{\theta} \subseteq \displaystyle\bigcup_{x \in \frac{a}{\theta}} \frac{x \circ s}{\theta} =$ $\pi(a) \circ s$. Hence π is a homomorphism.

(ii) Let $\theta \in Con(A_{\mathcal{H}})$. Then for $a \in A, x \in \dfrac{a}{\theta}$, and $s \in S$, $\dfrac{a \circ s}{\theta} = \dfrac{x \circ s}{\theta}$ and so $\pi(a \circ s) = \dfrac{a \circ s}{\theta} = \displaystyle\bigcup_{x \in \frac{a}{\theta}} \frac{x \circ s}{\theta} = \pi(a) \circ s$. Thus π is a strong homomorphism. Conversely, let π be a strong homomorphism, $a, b \in A, a\theta b$, and $s \in S$. Then

$$\frac{a \circ s}{\theta} \subseteq \bigcup_{x \in \frac{b}{\theta}} \frac{x \circ s}{\theta} = \frac{b}{\theta} \circ s = \pi(b) \circ s = \pi(b \circ s) = \frac{b \circ s}{\theta}.$$

Similarly, $\frac{b \circ s}{\theta} \subseteq \frac{a \circ s}{\theta}$, and hence θ is a congruence on $A_{\mathcal{H}}$. ■

Theorem 3.9 *Let $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$ be a strong homomorphism of hyper S-acts and $\theta \in Con(B_{\mathcal{H}})$. Then $(f \times f)^{-1}(\theta) = \{(x, y) : f(x)\theta f(y)\}$ is a congruence on $A_{\mathcal{H}}$. In particular, $Kerf = \{(x, y) : f(x) = f(y)\}$ is a congruence on $A_{\mathcal{H}}$.*

Proof. Let $a, b \in A, s \in S$ and $(a, b) \in (f \times f)^{-1}(\theta)$. Then $f(a)\theta f(b)$. So $f(a \circ s) = (f(a) \circ s)\overline{\theta}(f(b) \circ s) = f(b \circ s)$. Thus for every $x \in a \circ s$ there exists $y \in b \circ s$ such that $f(x)\theta f(y)$, or equivalently $(x, y) \in (f \times f)^{-1}(\theta)$. Similarly, for every $y \in b \circ s$ there exists $x \in a \circ s$ such that $(x, y) \in (f \times f)^{-1}(\theta)$. Hence $(f \times f)^{-1}(\theta)$ is a congruence on $A_{\mathcal{H}}$. The second part follows from the first part using the fact that $Kerf = (f \times f)^{-1}(\Delta_B)$ where $\Delta_B = \{(b, b) : b \in B\}$ is the identity congruence on $B_{\mathcal{H}}$. ■

Corollary 3.10 *For a hyper S-act $A_{\mathcal{H}}$, the following are equivalent:*

(i) $\theta \in Con(A_{\mathcal{H}})$,

(ii) *There exists a strong homomorphism $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$ such that $\theta = Kerf$.*

Proof. (i) ⇒ (ii) Take f to be the canonical epimorphism $\pi : A_{\mathcal{H}} \to \frac{A_{\mathcal{H}}}{\theta}$.

(ii) ⇒ (i) It is clear by the above Theorem. ■

Remark 3.11 The ordered set $(Eq(A_{\mathcal{H}}), \subseteq)$ is a complete lattice with $\cap$ as infimum and supremum, $\bigvee$, as follows

$$\bigvee_{i \in I} \theta_i = \{(a, b) : \exists x_0, ..., x_n \in A, \exists i_1, ..., i_n \in I \ s.t. \ a = x_0 \ \theta_{i_1} \ x_1 ... \theta_{i_n} \ x_n = b\}$$

for $\theta_i \in Eq(A_{\mathcal{H}})$.

Theorem 3.12 *For every hyper S-act $A_{\mathcal{H}}$, $(Con(A_{\mathcal{H}}), \subseteq)$ is a complete lattice.*

Proof. We show that for $\{\theta_i\}_{i \in I} \in Con(A_{\mathcal{H}}), \bigvee_{i \in I} \theta_i = \theta$ is a congruence. Let $a, b \in A, s \in S$ and $a\theta b$. Then, by the definition of $\bigvee$ which is given in the above Remark, there exist $x_0, ..., x_n \in A, i_1, \ldots, i_n \in I$ such that $a = x_0\theta_{i_1}x_1...\theta_{i_n}x_n = b$. Since each θ_i is a congruence, we have $(x_{t-1} \circ s) \ \overline{\theta_{i_t}} \ (x_t \circ s)$ for $t = 1, ..., n$. Thus $(a \circ s)\overline{\theta}(b \circ s)$ and so θ is a congruence. Therefore, arbitrary supremums exist in $Con(A_{\mathcal{H}})$ and hence it is a complete lattice. ■

4. The isomorphism theorems

In this section, using the usual notion of a congruence defined for hyper S-acts, we prove the decomposition theorem and the generalized version of the second isomorphism theorem from S-act to hyper S-acts.

Theorem 4.13 (Decomposition Theorem) *Let* $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$ *and* $g : A_{\mathcal{H}} \to C_{\mathcal{H}}$ *be strong homomorphisms,* g *be onto and* $Kerg \subseteq Kerf$*. Then there exists a unique strong homomorphism* $h : C_{\mathcal{H}} \to B_{\mathcal{H}}$ *such that* $hg = f$.

Proof. Define h by $h(c) = f(a)$ where $c = g(a)$. Then, h is well-defined since $Kerg \subseteq Kerf$. It is enough to show that h is a strong homomorphism. Let $c \in C, g(a) = c$ for some $a \in A$. Then $h(c \circ s) = h(g(a) \circ s) = h(g(a \circ s)) = f(a \circ s) = f(a) \circ s = hg(a) \circ s = h(c) \circ s$ and hence h is a strong homomorphism. The uniqueness of h follows from its definition. ∎

Corollary 4.14 (First Isomorphism Theorem) *Let* $f : A_{\mathcal{H}} \to B_{\mathcal{H}}$ *be an onto strong homomorphism. Then* $A_{\mathcal{H}}/Kerf \cong B_{\mathcal{H}}$.

Proof. Apply the above Theorem to $\pi : A_{\mathcal{H}} \to A_{\mathcal{H}}/Kerf$ instead of g. Since π is onto and $Ker\pi = Kerf$, there exists $h : A_{\mathcal{H}}/Kerf \to B_{\mathcal{H}}$ such that $h\pi = f$. Since f, π are onto, so is h. Also, if $h(c) = h(c')$ then $f(a) = f(a')$, where $\pi(a) = c, \pi(a') = c'$. But, since $Ker\pi = Kerf$, we get $\pi(a) = \pi(a')$, that is $c = c'$. Hence h is one-one and thus an isomorphism. ∎

Notation 4.15 *Let* A *be a set, and* $\theta, \psi \in Eq(A)$ *and* $\theta \subseteq \psi$*. We denote the set* $\{(x/\theta, y/\theta) \in (A/\theta)^2 : (x, y) \in \psi\}$ *by* ψ/θ.

Theorem 4.16 (Second Isomorphism Theorem) *Let* $A_{\mathcal{H}}$ *be a hyper* S*-act and* $\theta \in Eq(A_{\mathcal{H}})$*. Then*

(i) *For* $\psi \in Con(A_{\mathcal{H}})$ *with* $\theta \subseteq \psi, \psi/\theta$ *is a congruence on* $A_{\mathcal{H}}/\theta$ *and* $(A_{\mathcal{H}}/\theta)/(\psi/\theta) \cong A_{\mathcal{H}}/\psi$.

(ii) *If* θ *is a congruence on* $A_{\mathcal{H}}$*, then all congruences on* $A_{\mathcal{H}}/\theta$ *are of the form* ψ/θ *for some* $\psi \in Con(A_{\mathcal{H}})$ *with* $\theta \subseteq \psi$.

Proof. (i) First we show that the map $f : A_{\mathcal{H}}/\theta \to A_{\mathcal{H}}/\psi$ given by $f(a/\theta) = a/\psi$ is a strong homomorphism. So, let $a \in A$ and $s \in S$. Then

$$\begin{aligned} f\left(\frac{a}{\theta} \circ s\right) &= f\left(\bigcup_{x \in \frac{a}{\theta}} \frac{x \circ s}{\theta}\right) = \bigcup_{x \in \frac{a}{\theta}} f\left(\frac{x \circ s}{\theta}\right) = \bigcup_{x \in \frac{a}{\theta}} \frac{x \circ s}{\psi} = \bigcup_{x \in \frac{a}{\theta}} \frac{a \circ s}{\psi} \\ &= \frac{a \circ s}{\psi} = \frac{a}{\psi} \circ s = f\left(\frac{a}{\theta}\right) \circ s. \end{aligned}$$

Thus f is a strong homomorphism and $\psi/\theta = Kerf \in Con(A_{\mathcal{H}}/\theta)$ and so by Corollary 4.14, $(A_{\mathcal{H}}/\theta)/(\psi/\theta) \cong A_{\mathcal{H}}/\psi$.

(ii) Let φ be a congruence on $A_{\mathcal{H}}/\theta$. Take $\psi = \{(a, b) : (a/\theta, b/\theta) \in \varphi\}$. Then, $\theta \subseteq \psi$ and $\varphi = \psi/\theta$. Further ψ is a congruence on $A_{\mathcal{H}}$. Since θ and ψ/θ are congruences on $A_{\mathcal{H}}$, $\gamma_\theta : A_{\mathcal{H}} \to A_{\mathcal{H}}/\theta$ and $\gamma : A_{\mathcal{H}}/\theta \to \dfrac{A_{\mathcal{H}}/\theta}{\psi/\theta}$ are strong homomorphisms. Thus, $f = \gamma\gamma_\theta$ is also a strong homomorphism. But

$$\begin{aligned} Kerf &= \{(x, y) : f(x) = f(y)\} = \{(x, y) : (x/\theta)/\psi = (y/\theta)/\psi\} \\ &= \{(x, y) : (x/\theta)(\psi/\theta)(y/\theta)\} = \psi \end{aligned}$$

Thus ψ is a congruence by Corollary 3.10. ∎

5. Limits and colimits in the category $\mathcal{H}ACT - S$

In this section the limits and colimits of hyper S-acts are studied.

Remark 5.17 For a semigroup S, the set of all hyper S-actions on a fixed set X is denoted by $H = H(X)$. Let $\circ_i, \circ_j$ be two elements of $H(X)$. Define $\circ_i \leq \circ_j$ if for every $x \in X$ and $s \in S$, $x \circ_i s \subseteq x \circ_j s$. Then $H(X)$ with the relation $\leq$ is a complete Boolean algebra, with $\bigwedge H, \bigvee H$ given by

$$x\left(\bigwedge H\right)s = \bigcap_{\circ \in H}(x \circ s),$$
$$x\left(\bigvee H\right)s = \bigcup_{\circ \in H}(x \circ s),$$

for $x \in X$ and $s \in S$. Specially, $\mathbf{0}, \mathbf{1}$ in $H(X)$ are given by $x\mathbf{0}s = \emptyset, x\mathbf{1}s = X$. Also, the complement $\circ'$ of an element $\circ \in H(X)$ is defined as $x \circ' s = X - (x \circ s)$.

Lemma 5.18 *Let S be a semigroup and X be a set, $F = \{f_i : X \to A_i \mid i \in I\}$, $G = \{g_i : B_i \to X \mid i \in I\}$ be families of functions, where A_i, B_i are hyper S-acts, for all $i \in I$. Then, the greatest (smallest) hyper S-action on the set X, for which $f_i \circ g_i$ are homomorphisms, exists. This hyper S-action on X is called the hyper S-action induced by $F(G)$ and is denoted by $\circ^{\rightarrow}(F)(\circ^{\leftarrow}(G))$, or simply by $\circ^{\rightarrow}(\circ^{\leftarrow})$.*

Proof. Let H be the set of all hyper S-actions on a set X which makes each f_i a homomorphism. Take $\circ^{\rightarrow} = \bigvee H$. It is enough to show that $\circ^{\rightarrow} \in H$. Let $i \in I$ be fixed. We prove that each f_i is a homomorphism from hyper S-act $(X, \circ^{\rightarrow})$ to $(A_i, \circ_i)$. Let $x \in X, s \in S$. For every $\circ \in H, f_i(x \circ s) \subseteq f_i(x) \circ_i s$ where $\circ_i$ is the hyper S-action on A_i. Then

$$f_i(x \circ^{\rightarrow} s) = f_i\left[\bigcup_{\circ \in H}(x \circ s)\right] = \bigcup_{\circ \in H} f_i(x \circ s) \subseteq f_i(x) \circ_i s.$$

Thus f_i is a homomorphism. Dually, taking K to be the set of all hyper S-actions on X which makes each g_i a homomorphism, and $\circ^{\leftarrow} = \bigwedge K$ it can be shown that $\circ^{\leftarrow} \in K$. ∎

Theorem 5.19 *Let $D : I \to \mathcal{H}Act - S$ be a diagram and $U : \mathcal{H}Act - S \to$ **Set** be the forgetful functor. If $\{f_i : A \to UA_i\}_{i \in I}$ is a limit of $U \circ D : I \to$ **Set**, then $\{f_i : A \to A_i\}_{i \in I}$ is a limit of D, where the hyper S-action on A is induced by $\{f_i : A \to A_i\}_{i \in I}$.*

Proof. Let $\{h_i : C \to A_i\}_{i \in I}$ be a source of D in $\mathcal{H}Act - S$. Consider $\{Uh_i : UC \to UA_i\}_{i \in I}$ in **Set**. Since $\{f_i : A \to UA_i\}_{i \in I}$ is a limit of $U \circ D$, there exists $h : UC \to A$ such that $h_i = f_i h$, for all $i \in I$. Now, it is enough to show that h is a homomorphism, where the hyper S-action on A, say $\circ^{\rightarrow}$, is induced by $\{f_i : A \to A_i\}_{i \in I}$. Define another hyper S-action $\circ_A$ on A as follows:

$$h(x) \circ_A s = \bigcup_{h(y) = h(x)} h(y \circ_C s)$$

for $x \in C, s \in S$, and for other elements of A, $a \circ_A s = a \circ^{\rightarrow} s$. Then $\circ_A$ is a hyper S-action on A which makes each f_i a homomorphism. Indeed, for $i \in I, x \in C, s \in S$,

$$\begin{aligned} f_i[h(x) \circ_A s] &= f_i[\textstyle\bigcup_{h(y)=h(x)} h(y \circ_C s)] \\ &= \bigcup_{h(y)=h(x)} f_i h(y \circ_C s) \\ &\subseteq \bigcup_{h(y)=h(x)} h_i(y \circ_C s) \\ &\subseteq \bigcup_{h(y)=h(x)} h_i(y) \circ_i s \\ &= \textstyle\bigcup_{h(y)=h(x)} f_i h(y) \circ_i s \\ &= f_i h(x) \circ_i s. \end{aligned}$$

The result for the other elements of A follows from the same property of $\circ^{\rightarrow}$. So, $\circ_A \leq \circ^{\rightarrow}$, and then for every $x \in C, s \in S$,

$$h(x \circ_C s) \subseteq h(x) \circ_A s \subseteq h(x) \circ^{\rightarrow} s.$$

Thus, h is a homomorphism, as required. ■

Theorem 5.20 *Let $D : I \rightarrow \mathcal{H}Act - S$ be a diagram and $U : \mathcal{H}Act - S \rightarrow$ **Set** be the forgetful functor. If $\{g_i : UA_i \rightarrow A\}_{i \in I}$ is a colimit of $U \circ D : I \rightarrow$ **Set**, then $\{g_i : A_i \rightarrow A\}_{i \in I}$ is a colimit of D, where the hyper S-action on A is induced by $\{g_i : A_i \rightarrow A\}_{i \in I}$.*

Proof. Similar to the proof of the above theorem, let $\{k_i : A_i \rightarrow C\}_{i \in I}$ be a sink of D in $\mathcal{H}Act - S$. Consider $\{Uk_i : UA_i \rightarrow UC\}_{i \in I}$ in **Set**. Since $\{g_i : UA_i \rightarrow A\}_{i \in I}$ is a colimit of $U \circ D$, there exists $k : A \rightarrow UC$ such that $kg_i = k_i$, for all $i \in I$. Now, we show that k is a homomorphism, where the hyper S-action on A, say $\circ^{\leftarrow}$, is induced by $\{g_i : A_i \rightarrow A\}_{i \in I}$. Define another hyper S-action $\circ_A$ on A as follows:

$$a \circ_A s = k^{-1}(k(a) \circ_C s)$$

for $a \in A, s \in S$. Then $\circ_A$ is a hyper S-action on A which makes each g_i a homomorphism. So, $\circ^{\leftarrow} \leq \circ_A$, and then for every $a \in A, s \in S$,

$$a \circ^{\leftarrow} s \subseteq a \circ_A s = k^{-1}(k(a) \circ_C s)$$

and hence $k(a \circ^{\leftarrow} s) \subseteq k(a) \circ_C s$. Thus, k is a homomorphism. ■

As a corollary of the two preceding theorems we have the following.

Corollary 5.21 *The category $\mathcal{H}Act - S$ has all limits and colimits and $U : \mathcal{H}Act - S \rightarrow$ **Set** preserves limits and colimits.*

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