

A NOTE ON TESTING OF HYPOTHESIS

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Abstract. In this paper problem of testing of hypothesis is discussed when the samples have been drawn from normal distribution. The study of hypothesis testing is also extended to Bayes set up.

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Let the random variable (r.v.) X have a normal distribution $N(\theta, \sigma_2)$, where σ_2 is assumed to be known.

The hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$ is to be tested.

Let X_1, X_2, \dots, X_n be a random sample from $N(\theta, \sigma^2)$ population.

Let $\bar{X} \left(= \frac{1}{n} \sum_{i=1}^n X_i \right)$ be the sample mean.

By Neyman–Pearson lemma, the most powerful test rejects H_0 at α % level of significance,

if $\frac{\sqrt{n}(\bar{X} - \theta_0)}{\sigma} \geq d_\alpha$, where d_α is such that

$$\int_{d_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dZ = \alpha.$$

If the sample is such that H_0 is rejected, then will it imply that H_1 will be accepted?

In general, this will not be true for all values of θ_1 , but will be true for some specific value of θ_1 , i.e., when θ_1 is at a specific distance from θ_0 .

H_0 is rejected

$$(1) \quad \text{if } \frac{\sqrt{n}(\bar{X} - \theta_0)}{\sigma} \geq d_\alpha, \text{ i.e., } \bar{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Similarly, the Most Powerful Test will accept H_1 against H_0

$$(2) \quad \text{if } \frac{\sqrt{n}(\bar{X} - \theta_0)}{\sigma} \geq d_\alpha, \text{ i.e., } \bar{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Rejecting H_0 will mean accepting H_1

if (1) \implies (2)

$$(3) \quad \text{i.e., } \bar{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \implies \bar{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}$$

$$\text{i.e., } \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Similarly, accepting H_1 will mean rejecting H_0

if (2) \implies (1)

$$(4) \quad \text{i.e., } \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}.$$

From (3) and (4) we have

$$(5) \quad \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} = \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \text{ i.e., } \theta_1 - \theta_0 = 2d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Thus,

$$d_\alpha \frac{\sigma}{\sqrt{n}} = \frac{\theta_1 - \theta_0}{2} \quad \text{and} \quad \theta_1 = \theta_0 + 2d_\alpha \frac{\sigma}{\sqrt{n}}.$$

From (1),

$$\text{Reject } H_0 \text{ if } \bar{X} > \theta_0 + \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2}$$

and from (2),

$$\text{Accept } H_1 \text{ if } \bar{X} > \theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2}.$$

Thus, rejecting H_0 will mean accepting H_1 when

$$\bar{X} > \frac{\theta_0 + \theta_1}{2}.$$

From (5), this will be true only when

$$\theta_1 = \theta_0 + 2d_\alpha \frac{\sigma}{\sqrt{n}}.$$

For other values of $\theta_1 \neq \theta_0 + 2d_\alpha \frac{\sigma}{\sqrt{n}}$ rejecting H_0 will not mean accepting H_1 .

Therefore, it is recommended that, instead of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1, \theta_1 > \theta_0$, it is more appropriate to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_0$.

In this situation, rejecting H_0 will mean $\theta > \theta_0$ and is not equal to some given value $= \theta_1$.

But in Baye's setup, rejecting H_0 means accepting H_1 whatever may be H_0 and H_1 .

In this set up, the level of significance is not a preassigned constant, but depends on H_0, H_1, σ_2 and n .

Consider $(0, 1)$ loss function and equal prior probabilities $1/2$ for θ_0 and θ_1 . The Baye's test rejects H_0 (accept H_1)

$$\text{if } \bar{X} > \frac{\theta_0 + \theta_1}{2}$$

and accepts H_0 (rejects H_1)

$$\text{if } \bar{X} < \frac{\theta_0 + \theta_1}{2}.$$

[See Rohatagi, p.463, Example 2].

The level of significance is given by

$$P_{H_0} \left[\bar{X} > \frac{\theta_0 + \theta_1}{2} \right] = P_{H_0} \left[\frac{(\bar{X} - \theta_0)\sqrt{n}}{\sigma} > \frac{(\theta_1 - \theta_0)\sqrt{n}}{2\sigma} \right] = 2 - \Phi \left(\frac{\sqrt{n}(\theta_1 - \theta_0)}{2\sigma} \right),$$

where

$$\Phi(t) = \int_{-\infty}^t \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dZ.$$

Thus, the level of significance depends on $\theta_0, \theta_1, \sigma^2$ and n .

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