p-FUZZY HYPERGROUPOIDS ASSOCIATED TO THE PRODUCT OF p-FUZZY HYPERGRAPHS[1]

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Abstract. We construct fuzzy hyperoperations from the product of p-fuzzy hypergraphs and then we prove that the fuzzy hyperstructures determined by these hyperoperations are commutative p^2-fuzzy quasi-hypergroups. Some properties of these fuzzy hyperoperations are also listed.

Keywords: p-fuzzy hypergroupoid; product; p-fuzzy hypergraph.

1. Introduction and preliminaries

The connections between graphs and hypergroups had been looked into by several researchers (see, for instance, [4], [7]). Corsini studied the connections between hypergraphs and hypergroups in [5]. Ali studied the hypergroupoid associated to the product of hypergraphs in [1].

In this paper, we construct fuzzy hyperoperations from the product of p-fuzzy hypergraphs and then we prove that the fuzzy hyperstructures determined by these hyperoperations are commutative p^2-fuzzy quasi-hypergroups. Some properties of these fuzzy hyperoperations are also listed. This paper can be seen as a fuzzy version of [1].

We recall some notations of fuzzy hyperstructure theory. A fuzzy subset of a nonempty set H is a function $M : H \to [0, 1]$. The collection of all fuzzy subsets of H is denoted by $F(H)$. The p-cut of a fuzzy subset M of H is defined by

$$M_p \doteq \{x \in H \mid M(x) \geq p\}.$$

Given a fuzzy hyperoperation $* : H \times H \to F(H)$, for all $a \in H$, $B \in F(H)$, the fuzzy subset $a * B$ of H is defined by

$$(a * B)(x) \doteq \bigvee_{B(b)>0} (a * b)(x).$$

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Given $A, B \in F(H)$, we give the following definitions

$$\begin{aligned} A \subseteq B &\doteq A(x) \leq B(x), & \forall x \in H. \\ A = B &\doteq A(x) = B(x), & \forall x \in H. \\ (A \cup B)(x) &\doteq A(x) \vee B(x), & \forall x \in H. \\ (A \cap B)(x) &\doteq A(x) \wedge B(x), & \forall x \in H. \end{aligned}$$

Proposition 1.1. ([8]) $\forall A, B, C \in F(H)$, *we have the following properties*

(1) $A \cup A = A,\ A \cap A = A$;
(2) $A \cup B = B \cup A,\ A \cap B = B \cap A$;
(3) $(A \cup B) \cup C = A \cup (B \cup C),\ (A \cap B) \cap C = A \cap (B \cap C)$;
(4) $A \cap (A \cup B) = A,\ A \cup (A \cap B) = A$;
(5) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C),\ (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$;
(6) $A \cup \emptyset = A,\ A \cap \emptyset = \emptyset, A \cup H = H, A \cap H = A$.

Definition 1.2. Let A, B be fuzzy subsets of nonempty set H, $A \times B$ is defined to be a fuzzy subset of $H \times H$ such that

$$(A \times B)(a, b) = A(a) \cdot B(b), \quad \forall a, b \in H.$$

Proposition 1.3. *Let A, B, C be fuzzy subsets of nonempty set H, then*

$$A \times (B \cup C) = A \times B \cup A \times C.$$

Proof. For all $a, b \in H$ we have

$$\begin{aligned} (A \times (B \cup C))(a, b) &= A(a) \cdot (B \cup C)(b) \\ &= A(a) \cdot (B(b) \vee C(b)) \\ &= A(a) \cdot B(b) \vee A(a) \cdot C(b) \\ &= (A \times B)(a, b) \vee (A \times C)(a, b) \\ &= ((A \times B) \cup (A \times C))(a, b). \end{aligned}$$

And so $A \times (B \cup C) = A \times B \cup A \times C$. ∎

Proposition 1.4. *Let A, B be fuzzy subsets of a nonempty set H, then for all $p \in [0, 1]$ we have*

$$A_p \times B_p \subseteq (A \times B)_{p^2}.$$

Proof. For all

$$\begin{aligned} x = (a, b) \in A_p \times B_p &\Rightarrow a \in A_p, b \in B_p \\ &\Rightarrow A(a) \geq p, B(b) \geq p \\ &\Rightarrow (A \times B)(a, b) = A(a)B(b) \geq p^2 \\ &\Rightarrow x \in (A \times B)_{p^2}. \end{aligned}$$

■

A fuzzy hypergroupoid $\langle H; *\rangle$ is a nonempty set H endowed with a fuzzy hyperoperation (i.e. a function $*$ from $H \times H$ to $F(H)$). A p-fuzzy quasi-hypergroup is a fuzzy hypergroupoid such that

$$(x * H)_p = H = (H * x)_p, \quad \forall x \in H.$$

The readers can consult [2], [3], [8] to learn more about hyperstructures and fuzzy sets.

2. Fuzzy hyperoperation $*$

We borrow some definitions from [6].

Definition 2.1. H is a nonempty set, $\{A_i\}_i$ are fuzzy subsets of H, if there exists a $p \in (0,1]$ such that

$$\bigcup_i (A_i)_p = H,$$

then $\langle H; \{A_i\}_i\rangle$ is called a p-fuzzy hypergraph.

Definition 2.2. Let $\Gamma = \langle H; \{A_i\}_i\rangle$ be a p-fuzzy hypergraph, set $E_p(x) = \bigcup\limits_{A_i(x)\geq p} A_i$. The fuzzy hypergroupoid $H_\Gamma = \langle H; *\rangle$ where the fuzzy hyperoperation $*$ is defined by

$$x * y \doteq E_p(x) \cup E_p(y), \quad \forall x, y \in H$$

is called a p-fuzzy hypergraph hypergroupoid or a p-f.h.g. hypergroupoid.

Proposition 2.3. ([6]) *The p-f.h.g. hypergroupoid H_Γ has the following properties for any $x, y \in H$:*

(1) $x * y = x * x \cup y * y$;

(2) $x \in (x * x)_p$;

(3) $y \in (x * x)_p \Leftrightarrow x \in (y * y)_p$;

(4) $\{x, y\} \subseteq (x * y)_p$;

(5) $x * y = y * x$;

(6) $(x * H)_p = H$;

(7) $\langle H; \{x * x\}_{x\in H}\rangle$ *is a p-fuzzy hypergraph;*

(8) $x * x * x = \bigcup\limits_{(x*x)(z)>0} z * z$;

(9) $(x * x) * (x * x) = x * x * x$.

Proof. (1) $x*y = E_p(x)\cup E_p(y) = (E_p(x)\cup E_p(x))\cup(E_p(y)\cup E_p(y)) = x*x\cup y*y$.

(2) It is a special case of (4).

(3) Since $\bigcup_i (A_i)_p = H$, then for any $x \in H$ there exists some $A_i \in F(H)$ such that $A_i(x) \geq p$.

We only prove the implication "$\Rightarrow$" . Since

$$\begin{aligned}(x*x)(y) &= (E_p(x)\cup E_p(x))(y) = (E_p(x))(y)\\ &= \left(\bigcup_{A_i(x)\geq p} A_i\right)(y) = \bigvee_{A_i(x)\geq p} A_i(y) \geq p,\end{aligned}$$

then there exists $A_i \in F(H)$ such that $A_i(x) \geq p$ and $A_i(y) \geq p$. So

$$(y*y)(x) = \bigvee_{A_j(y)\geq p} A_j(x) \geq p.$$

Thus $x \in (y*y)_p$.

(4) $(x*y)(x) = (E_p(x)\cup E_p(y))(x) = (E_p(x))(x)\vee(E_p(y))(x) \geq (E_p(x))(x) = \bigvee_{A_i(x)\geq p} A_i(x) \geq p$. So $x \in (x*y)_p$. Similarly we can prove $y \in (x*y)_p$.

(5) $x*y = E_p(x)\cup E_p(y) = E_p(y)\cup E_p(x) = y*x$.

(6) for any $y \in H$,

$$\begin{aligned}(x*H)(y) &= \left(\bigcup_{t\in H} x*t\right)(y) = \left(\bigcup_{t\in H}(E_p(x)\cup E_p(t))(y)\right)\\ &\geq \left(\bigcup_{t\in H} E_p(t)\right)(y) \geq (E_p(y))(y) = \left(\bigcup_{A_i(y)\geq p} A_i\right)(y)\\ &= \bigvee_{A_i(y)\geq p} A_i(y) \geq p.\end{aligned}$$

So $H \subseteq (x*H)_p$ and thus $(x*H)_p = H$.

(7) From $x \in (x*x)_p$ we know $\bigcup_{x\in H}(x*x)_p = H$. And then $\langle H; \{x*x\}_{x\in H}\rangle$ is a p-fuzzy hypergraph.

(8) $x*x*x = \bigcup_{(x*x)(z)>0} z*x = \bigcup_{(x*x)(z)>0} (z*z)\cup(x*x) = \bigcup_{(x*x)(z)>0} z*z$.

(9) $(x*x)*(x*x) = \bigcup_{(x*x)(a)>0,(x*x)(b)>0} a*b = \bigcup_{(x*x)(a)>0,(x*x)(b)>0} (a*a \cup b*b) = \bigcup_{(x*x)(a)>0} a*a = x*x*x$. ∎

Remark 2.4. From (5), (6) of the previous proposition, we know that H_Γ is a commutative p-fuzzy quasi-hypergroup.

3. Fuzzy hyperoperations $\otimes$ and $\oplus$

Proposition 3.1. *Let $\Gamma_1 = \langle H; \{A_i\}_i\rangle$, $\Gamma_2 = \langle H; \{B_j\}_j\rangle$ be two p-fuzzy hypergraphs, then*

$$\Gamma_1 \times \Gamma_2 \doteq \langle H \times H; \{A_i\}_i \times \{B_j\}_j\rangle$$

is a p^2-fuzzy hypergraph and called the product of Γ_1 and Γ_2.

Proof. From Proposition 1.4 we know

$$\bigcup_{i,j}(A_i \times B_j)_{p^2} \supseteq \bigcup_{i,j}((A_i)_p \times (B_j)_p) = \left(\bigcup_i (A_i)_p\right) \times \left(\bigcup_j (B_j)_p\right) = H \times H.$$

And then $\bigcup_{i,j}(A_i \times B_j)_{p^2} = H \times H.$ ∎

Definition 3.2. Let $H_{\Gamma_1} = \langle H; *\rangle$, $H_{\Gamma_2} = \langle H; \circ\rangle$ be two p-f.h.g. hypergroupoids associated to Γ_1, Γ_2 respectively. The product p^2-f.h.g. hypergroupoid of $\Gamma_1 \times \Gamma_2$ is defined by $H_{\Gamma_1\times\Gamma_2} = \langle H \times H; \otimes\rangle$ where the fuzzy hyperoperation $\otimes$ is defined by

$$(x_1, x_2) \otimes (y_1, y_2) \doteq (x_1 * y_1) \times (x_2 \circ y_2), \quad \forall x_1, x_2, y_1, y_2 \in H.$$

Proposition 3.3. *The p^2-f.h.g. hypergroupoid $H_{\Gamma_1\times\Gamma_2}$ has the following properties for any $X, Y \in H \times H$:*

(1) $X \otimes Y \supseteq X \otimes X \cup Y \otimes Y$;

(2) $X \in (X \otimes X)_{p^2}$;

(3) $Y \in (X \otimes X)_{p^2} \nRightarrow X \in (Y \otimes Y)_{p^2}$;

(4) $\{X, Y\} \subseteq (X \otimes Y)_{p^2}$;

(5) $X \otimes Y = Y \otimes X$;

(6) $(X \otimes (H \times H))_{p^2} = H \times H$;

(7) $\langle H; \{X \otimes X\}_{X\in H\times H}\rangle$ *is a p^2-fuzzy hypergraph;*

(8) $X \otimes X \otimes X \supseteq \bigcup_{(X\otimes X)(Z)>0} Z \otimes Z$;

(9) $(X \otimes X) \otimes (X \otimes X) \supseteq \bigcup_{(X\otimes X)(Z)>0} Z \otimes Z.$

Proof. Set $X = (x_1, x_2), Y = (y_1, y_2)$, then

(1) $X \otimes Y = (x_1, x_2) \otimes (y_1, y_2) = (x_1 * y_1) \times (x_2 \circ y_2) = ((x_1 * x_1) \cup (y_1 * y_1)) \times ((x_2 \circ x_2) \cup (y_2 \circ y_2)) \supseteq ((x_1 * x_1) \times (x_2 \circ x_2)) \cup ((y_1 * y_1) \times (y_2 \circ y_2)) = ((x_1, x_2) \otimes (x_1, x_2)) \cup ((y_1, y_2) \otimes (y_1, y_2)) = X \otimes X \cup Y \otimes Y.$

(2) It is a special case of (4).

(3) For example, let $\Gamma = \langle \{a, b, c, d\}; \{A_i\}_{i=1}^{3} \rangle$ where $A_1 = \frac{0.5}{a} + \frac{0}{b} + \frac{0.4}{c} + \frac{0}{d}$, $A_2 = \frac{0}{a} + \frac{0.5}{b} + \frac{0}{c} + \frac{0.8}{d}$, $A_3 = \frac{0}{a} + \frac{0}{b} + \frac{0.5}{c} + \frac{0}{d}$, then

$$\bigcup_{i=1}^{3} (A_i)_{0.5} = (A_1)_{0.5} \cup (A_2)_{0.5} \cup (A_3)_{0.5} = \{a, b, c, d\}.$$

Hence, Γ is a 0.5-fuzzy hypergraph.

Set $U = (a, b), V = (c, d)$, then

$$\begin{aligned}
(U \otimes U)(V) &= ((a, b) \otimes (a, b))(c, d) \\
&= ((a * a) \times (b \circ b))(c, d) \\
&= (a * a)(c) \cdot (b \circ b)(d) \\
&= \left(\bigvee_{A_i(a) \geq 0.5} A_i(c) \right) \cdot \left(\bigvee_{A_j(b) \geq 0.5} A_j(d) \right) \\
&= A_1(c) \cdot A_2(d) \\
&= 0.4 \times 0.8 \\
&= 0.32 \\
&\geq 0.25 \\
&= (0.5)^2.
\end{aligned}$$

But

$$\begin{aligned}
(V \otimes V)(U) &= ((c, d) \otimes (c, d))(a, b) \\
&= ((c * c) \times (d \circ d))(a, b) \\
&= (c * c)(a) \cdot (d \circ d)(b) \\
&= \left(\bigvee_{A_i(c) \geq 0.5} A_i(a) \right) \cdot \left(\bigvee_{A_j(d) \geq 0.5} A_j(b) \right) \\
&= A_3(a) \cdot A_2(b) \\
&= 0 \times 0.5 \\
&= 0 \\
&< 0.25 \\
&= (0.5)^2.
\end{aligned}$$

So $V \in (U \otimes U)_{0.5^2}$ but $U \overline{\in} (V \otimes V)_{0.5^2}$.

(4) $(X\otimes Y)(X) = ((x_1,x_2)\otimes(y_1,y_2))(x_1,x_2) = ((x_1*y_1)\times(x_2\circ y_2))(x_1,x_2) = (x_1*y_1)(x_1)\cdot(x_2\circ y_2)(x_2) \geq p\cdot p = p^2$. Thus $X \in (X\otimes Y)_{p^2}$. Similarly we can prove $Y \in (X\otimes Y)_{p^2}$.

(5) $X\otimes Y = (x_1,x_2)\otimes(y_1,y_2) = (x_1*y_1)\times(x_2\circ y_2) = (y_1*x_1)\times(y_2\circ x_2) = (y_1,y_2)\otimes(x_1,x_2) = Y\otimes X$.

(6) for any $T \in H\times H$,

$$\begin{aligned}(X\otimes(H\times H))(T) &= \left(\bigcup_{Y\in H\times H} X\otimes Y\right)(T)\\ &\geq (X\otimes T)(T)\\ &\geq ((X\otimes X)\cup(T\otimes T))(T)\\ &\geq (T\otimes T)(T)\\ &\geq p^2.\end{aligned}$$

So $H\times H \subseteq (X\otimes(H\times H))_{p^2}$ and thus $(X\otimes(H\times H))_{p^2} = H\times H$.

(7) From $\bigcup_{X\in H\times H}(X\otimes X)_{p^2} \supseteq \bigcup_{X\in H\times H}\{X\} = H\times H$ we know

$$\bigcup_{X\in H\times H}(X\otimes X)_{p^2} = H\times H.$$

Then $\langle H; \{X\otimes X\}_{X\in H\times H}\rangle$ is a p^2-fuzzy hypergraph.

(8) $X\otimes X\otimes X = \bigcup_{(X\otimes X)(Z)>0} Z\otimes X \supseteq \bigcup_{(X\otimes X)(Z)>0}((Z\otimes Z)\cup(X\otimes X)) = \bigcup_{(X\otimes X)(Z)>0} Z\otimes Z$.

(9) $(X\otimes X)\otimes(X\otimes X) = \bigcup_{(X\otimes X)(S)>0,(X\otimes X)(Z)>0} S\otimes Z \supseteq \bigcup_{(X\otimes X)(S)>0,(X\otimes X)(Z)>0}(S\otimes S\cup Z\otimes Z) = \bigcup_{(X\otimes X)(Z)>0} Z\otimes Z$. ∎

Remark 3.4. From (5), (6) of the above Proposition we know that $H_{\Gamma_1\times\Gamma_2}$ is a commutative p^2-fuzzy quasi-hypergroup.

Definition 3.5. The fuzzy hyperoperation $\oplus$ on $H_{\Gamma_1\times\Gamma_2}$ is defined by

$$(x_1,x_2)\oplus(y_1,y_2) \doteq (x_1*x_1)\times(x_2\circ x_2)\cup(y_1*y_1)\times(y_2\circ y_2),\quad \forall x_1,x_2,y_1,y_2\in H.$$

Proposition 3.6. *The p^2-f.h.g. hypergroupoid $H^{\oplus}_{\Gamma_1\times\Gamma_2} = \langle H\times H;\oplus\rangle$ has the following properties for any $X,Y\in H\times H$:*

(1) $X\oplus Y = X\oplus X\cup Y\oplus Y$;

(2) $X \in (X \oplus X)_{p^2}$;

(3) $Y \in (X \oplus X)_{p^2} \nRightarrow X \in (Y \oplus Y)_{p^2}$;

(4) $\{X, Y\} \subseteq (X \oplus Y)_{p^2}$;

(5) $X \oplus Y = Y \oplus X$;

(6) $(X \oplus (H \times H))_{p^2} = H \times H$;

(7) $\langle H; \{X \oplus X\}_{X \in H \times H}\rangle$ *is a* p^2*-fuzzy hypergraph;*

(8) $X \oplus X \oplus X = \bigcup_{(X \oplus X)(Z) > 0} Z \oplus Z$;

(9) $(X \oplus X) \oplus (X \oplus X) = X \oplus X \oplus X$.

Proof. Set $X = (x_1, x_2), Y = (y_1, y_2)$, then

(1) Since $X \oplus Y = (x_1, x_2) \oplus (y_1, y_2) = (x_1 * x_1) \times (x_2 \circ x_2) \cup (y_1 * y_1) \times (y_2 \circ y_2)$ and $X \oplus X \cup Y \oplus Y = (x_1, x_2) \oplus (x_1, x_2) \cup (y_1, y_2) \oplus (y_1, y_2) = (x_1 * x_1) \times (x_2 \circ x_2) \cup (y_1 * y_1) \times (y_2 \circ y_2)$. Then $X \oplus Y = X \oplus X \cup Y \oplus Y$.

(2) It is a special case of (4).

(3) For example, set $\Gamma = \langle \{a, b, c, d\}; \{A_i\}_{i=1}^3 \rangle$ where $A_1 = \frac{0.5}{a} + \frac{0}{b} + \frac{0.4}{c} + \frac{0}{d}$, $A_2 = \frac{0}{a} + \frac{0.5}{b} + \frac{0}{c} + \frac{0.8}{d}$, $A_3 = \frac{0}{a} + \frac{0}{b} + \frac{0.5}{c} + \frac{0}{d}$, then from Proposition 3.3 we know that Γ is a 0.5-fuzzy hypergraph.

Set $U = (a, b), V = (c, d)$, then

$$
\begin{aligned}
(U \oplus U)(V) &= ((a, b) \oplus (a, b))(c, d) \\
&= ((a * a) \times (b \circ b))(c, d) \\
&= (a * a)(c) \cdot (b \circ b)(d) \\
&= \left(\bigvee_{A_i(a) \geq 0.5} A_i(c) \right) \cdot \left(\bigvee_{A_j(b) \geq 0.5} A_j(d) \right) \\
&= A_1(c) \cdot A_2(d) \\
&= 0.4 \times 0.8 \\
&= 0.32 \\
&\geq 0.25 \\
&= (0.5)^2.
\end{aligned}
$$

But

$$\begin{aligned}(V \oplus V)(U) &= ((c,d) \oplus (c,d))(a,b) \\ &= ((c * c) \times (d \circ d))(a,b) \\ &= (c * c)(a) \cdot (d \circ d)(b) \\ &= \left(\bigvee_{A_i(c) \geq 0.5} A_i(a) \right) \cdot \left(\bigvee_{A_j(d) \geq 0.5} A_j(b) \right) \\ &= A_3(a) \cdot A_2(b) \\ &= 0 \times 0.5 \\ &= 0 \\ &< 0.25 \\ &= (0.5)^2.\end{aligned}$$

So $V \in (U \oplus U)_{0.5^2}$ but $U \overline{\in} (V \oplus V)_{0.5^2}$.

(4) $(X \oplus Y)(X) = ((x_1, x_2) \oplus (y_1, y_2))(x_1, x_2) = ((x_1 * x_1) \times (x_2 \circ x_2) \cup (y_1 * y_1) \times (y_2 \circ y_2))(x_1, x_2) \geq ((x_1 * x_1) \times (x_2 \circ x_2))(x_1, x_2) = (x_1 * x_1)(x_1) \cdot (x_2 \circ x_2)(x_2) \geq p \cdot p = p^2$. Thus $X \in (X \oplus Y)_{p^2}$. Similarly we can prove $Y \in (X \oplus Y)_{p^2}$.

(5) $X \oplus Y = (x_1, x_2) \oplus (y_1, y_2) = ((x_1 * x_1) \times (x_2 \circ x_2)) \cup ((y_1 * y_1) \times (y_2 \circ y_2)) = ((y_1 * y_1) \times (y_2 \circ y_2)) \cup ((x_1 * x_1) \times (x_2 \circ x_2)) = (y_1, y_2) \oplus (x_1, x_2) = Y \oplus X$.

(6) for any $T \in H \times H$,

$$\begin{aligned}(X \oplus (H \times H))(T) &= \left(\bigcup_{Y \in H \times H} X \oplus Y \right)(T) \\ &\geq (X \oplus T)(T) \\ &= ((X \oplus X) \cup (T \oplus T))(T) \\ &\geq (T \oplus T)(T) \\ &\geq p^2.\end{aligned}$$

So $H \times H \subseteq (X \oplus (H \times H))_{p^2}$ and thus $(X \oplus (H \times H))_{p^2} = H \times H$.

(7) From $\bigcup_{X \in H \times H} (X \oplus X)_{p^2} \supseteq \bigcup_{X \in H \times H} \{X\} = H \times H$ we know

$$\bigcup_{X \in H \times H} (X \oplus X)_{p^2} = H \times H.$$

Then $\langle H; \{X \oplus X\}_{X \in H \times H} \rangle$ is a p^2-fuzzy hypergraph.

(8) $X \oplus X \oplus X = \bigcup_{(X \oplus X)(Z) > 0} T \oplus X = \bigcup_{(X \otimes X)(Z) > 0} ((Z \otimes Z) \cup (X \otimes X))$ $= \bigcup_{(X \otimes X)(Z) > 0} Z \otimes Z$.

(9) $(X \oplus X) \oplus (X \oplus X) = \bigcup_{(X\oplus X)(S)>0,(X\oplus X)(Z)>0} S \oplus Z$

$$= \bigcup_{(X\oplus X)(S)>0,(X\oplus X)(Z)>0} (S \oplus S \cup Z \oplus Z) = \bigcup_{(X\oplus X)(Z)>0} Z \oplus Z = X \oplus X \oplus X. \quad ■$$

Remark 3.7. From (5), (6) of the previous Proposition we know that $H^{\oplus}_{\Gamma_1\times\Gamma_2} = \langle H \times H; \oplus\rangle$ is a commutative p^2-fuzzy quasi-hypergroup.

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