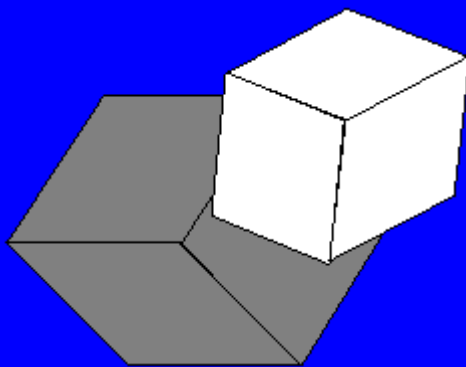


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## Papers Abstracts



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### Recursion formulas for Humbert's matrix functions

Ayed Al E'damat, Jihad Younis, Ashish Verma

Special matrix functions have become a major area of study for mathematicians and physicists over the last two decades. The famous Humbert's matrix functions have received considerable attention by many authors from different points of view [5,16,24]. Inspired by the recent work by Abd-Elmageed *et.al.* [1], who established recursion formulas satisfied by the first Appell matrix function, namely  $F_1$ . In this paper, we find the recursion formulas for Humbert's matrix functions. This enriches the theory of special matrix functions. The obtained results are believed to be newly presented.

(pp. 1–17)

### Extended of generalized power series reversible rings

Eltiyeb Ali, Ayoub Elshokry

Let  $R$  be a ring and  $(S, \leq)$  a strictly ordered monoid. This paper aims to introduce and study generalized power series *nil*-reversible rings. The researchers obtains various necessary or sufficient conditions for a generalized power series *nil*-reversible rings are 2-primal, *nil*-semicommutative and *nil*-symmetric. Examples are given to show that, a generalized power series *nil*-reversible which is neither generalized power series semicommutative nor generalized power series reversible. Also, we proved that a multiplicatively closed subset of  $R$  consisting of central non-zero divisors is generalized power series *nil*-reversible if and only if  $R$  is generalized power series *nil*-reversible. Moreover, other standard ring-theoretic properties are given.

(pp. 18–32)

### Some even-odd mean graphs in the context of arbitrary super subdivision

M. Basher

In this paper, we establish some new results on even vertex odd mean labeling of graph. We prove that the graphs obtained by arbitrary super subdivision of cycle, comb, crown, slanting ladder and planar grid are even-odd mean graphs.

(pp. 33–52)

### The separator of Green's classes of the full transformation semigroup

Janeth G. Canama, Gaudencio C. Petalcorin, JR.

This paper investigates the separator of Green's classes of the full transformation semigroup. The separator of a subset  $A$  of a semigroup  $S$  is the set of all elements  $x \in S$  satisfying the following conditions:  $xA \subseteq A$ ,  $Ax \subseteq A$ ,  $x(S \setminus A) \subseteq S \setminus A$  and  $(S \setminus A)x \subseteq S \setminus A$ . We establish the relationship between the separator of Green's classes and the permutations preserving partition and/or permuting image.

(pp. 53–63)

### Picture fuzzy multisets

Longzhou Cao, Yuming Feng, Taiwo O. Sangodapo

In this paper, the notion of picture fuzzy multiset was introduced. Some standard operations on picture fuzzy multiset such as intersection, union, complement were defined and their properties were investigated. Also, cut set and Cartesian product of picture fuzzy multiset were defined and the connections of Cartesian product with intersection and union were obtained.

(pp. 64–76)

### The matrix inverse based on the EP-nilpotent decomposition of a complex matrix

X.F. Cao, T.Y. Zhao, S.Z. Xu, Q.S. Feng, H.S. Chen

A generalized inverse for matrices is introduced, which is called the MPEPN-inverse. Let  $A$  be a complex matrix, the MPEPN-inverse can be described by using the part  $A_1$  in the EP-nilpotent decomposition of  $A$  and the Moore-Penrose inverse of  $A$ . Let  $A = A_1 + A_2$  be the EP-nilpotent decomposition of  $A$ ,  $A^{E,\ddagger}$  be the MPEPN-inverse of  $A$  and  $A^\dagger$  be the Moore-Penrose inverse of  $A$ , one can show that  $A^{E,\ddagger}AA^{E,\ddagger} = A^{E,\ddagger}$  does not hold in general, moreover, necessary and sufficient conditions to make the MPEPN-inverse to be an outer inverse of  $A$  are given, that is  $A^{E,\ddagger}AA^{E,\ddagger} = A^{E,\ddagger}$  hold if and only if one of the conditions  $(A_1A^\dagger)^2 = A_1A^\dagger$  and  $P_{\mathcal{R}(A_2)}A^\oplus = 0$  holds, where  $A^\oplus$  is the Core-EP inverse of  $A$  and  $P_{\mathcal{R}(A_2)}$  is the projection on  $\mathcal{R}(A_2)$ . If  $A_1A^\dagger$  is an idempotent, then the MPEPN-inverse of  $A$  coincides with the  $(A^\dagger A_1 P_{\mathcal{R}(A^*)}, P_{\mathcal{R}(A)} A_1 A^\dagger)$ -inverse of  $A$ , i.e. coincides the inverse along  $A^\dagger A_1 P_{\mathcal{R}(A^*)}$  and  $P_{\mathcal{R}(A)} A_1 A^\dagger$ .

(pp. 77–90)

### Fantastic (weak) hyper filters in hyper BE-algebras

Xiaoyun Cheng, Xiaolong Xin, Xiaoli Gao

In this paper, fantastic (weak) hyper filters in hyper BE-algebras are introduced and investigated. The relationships between fantastic (weak) hyper filters and (weak) hyper filters are discussed and the related examples are delivered. Then, fantastic (weak) hyper filters are characterized respectively. Moreover, examples are given in which fantastic weak hyper filters and fantastic hyper filters may not be deduced from each other in hyper BE-algebras, meanwhile the conditions are found that fantastic weak hyper filters become fantastic weak hyper filters in hyper BE-algebras.

(pp. 91–98)

### Flow-selfdual curves in a geometric surface

Mircea Crasmareanu

For a natural parametrization of a curve  $\gamma$  in an orientable two-dimensional Riemannian manifold, we compare two differential equations associated to  $\gamma$ . The main tool of our study is the geodesic curvature  $k$  of  $\gamma$  and when these equations coincide we call  $\gamma$  as being flow-selfdual since the second equation corresponds to the flow-curvature  $k_f$  of  $\gamma$  in the same manner as the first equation involves  $k$ . We obtain that these curves have a constant geodesic curvature and then we discuss four examples. Also, we generalize this type of differential equations to vector fields on Riemannian manifolds of arbitrary dimension.

(pp. 99–105)

### On the completion of symmetric metric spaces

E.I. Rodríguez-Juárez, J.E. Macías-Díaz

In this work, we investigate particular properties on the completion of symmetric spaces. Symmetric spaces are metric spaces and, naturally, question arises as to whether their completions are also symmetric. In this work, we provide an affirmative response to this question. More precisely, we prove that every metric space is isometrically a subset of a symmetric space. In addition, we prove that the completion of a symmetric metric space is likewise symmetric. Some additional functorial properties are established along with some other results. Additionally, generic examples of symmetric spaces will be provided in this manuscript.

(pp. 106–114)

### [Tight partitions for packing circles in a circle](#)

Dinesh B. Ekanayake, Douglas J. Lafountain

We develop a new strategy for proving optimal packing densities for  $N$  congruent circles in a circle. Specifically, we introduce tight partitions, which generalize filled rings of circles, and show that for the densest packing, the union of tight partitions forms a connected graph containing the center of every circle, except for possibly rattlers on the container boundary. We then apply this to the case of  $N = 14$  to reduce the list of potentially optimal solutions to one basic shape, which in turn admits a one-parameter family of configurations with two local extrema, one of which is the global optimal.

(pp. 115–136)

### [On the characterization of regular ring lattices and their relation with the Dirichlet kernel](#)

Marco Fabris

Regular ring lattices (RRLs) are defined as peculiar undirected circulant graphs constructed from a cycle graph, wherein each node is connected to pairs of neighbors that are spaced progressively in terms of vertex degree. This kind of network topology is extensively adopted in several graph-based distributed scalable protocols and their spectral properties often play a central role in the determination of convergence rates for such algorithms. In this work, basic properties of RRL graphs and the eigenvalues of the corresponding Laplacian and Randić matrices are investigated. A deep characterization for the spectra of these matrices is given and their relation with the Dirichlet kernel is illustrated. Consequently, the Fiedler value of such a network topology is found analytically. With regard to RRLs, properties on the bounds for the spectral radius of the Laplacian matrix and the essential spectral radius of the Randić matrix are also provided, proposing interesting conjectures on the latter quantities.

(pp. 137–160)

### [On pseudo picture fuzzy cosets](#)

Yuming Feng, Taiwo O. Sangodapo

In this paper, the concepts of pseudo picture fuzzy cosets, pseudo picture fuzzy double cosets and pseudo picture fuzzy middle cosets were introduced and some of their characteristics were established. In addition, we investigated the connections between pseudo picture fuzzy double cosets and picture fuzzy normal subgroup, also between pseudo picture fuzzy middle cosets and picture fuzzy normal subgroup.

(pp. 161–176)

### Area integral characterizations and $\Phi$ -Carleson measures for harmonic Bergman-Orlicz spaces

Xi Fu, Meina Gao, Xiaoqiang Xie

Let  $\Phi$  be a growth function. In this paper, we define a harmonic Bergman-Orlicz space  $\mathcal{B}_\alpha^\Phi$  and characterize it in terms of area integral functions. Furthermore, we define  $\Phi$ -Carleson measures and then discuss  $\Phi$ -Carleson measures for harmonic Bergman-Orlicz spaces.

(pp. 177–193)

### Mycielskian of signed graphs

Albin Mathew, Germina K.A.

In this paper, we define the Mycielskian of a signed graph and discuss the properties of balance and switching in the Mycielskian of a given signed graph. We provide a condition for ensuring the Mycielskian of a balanced signed graph remains balanced, leading to the construction of a balanced Mycielskian. We establish a relation between the chromatic numbers of a signed graph and its Mycielskian. We also study the structure of different matrices related to the Mycielskian of a signed graph.

(pp. 194–206)

### Topological factor groups relative to normal soft int-groups

Essam H. Hamouda, Howida Adel Alfran

Given a group  $\mathcal{G}$ , let  $\alpha_{\mathcal{G}}$  be a normal soft int-group in  $\mathcal{G}$ . We construct the factor group  $\mathcal{G}/\alpha$  relative to  $\alpha_{\mathcal{G}}$  by defining a congruence relation on  $\mathcal{G}$ . Using this construction, we establish soft Isomorphism Theorems which generalize the classical group Isomorphism Theorems. Finally, we give some topological structures on  $\mathcal{G}$  and  $\mathcal{G}/\alpha$ .

(pp. 207–217)

### $\mathcal{J}$ - $\omega^*$ -open sets and $\mathcal{J}$ - $\omega^*$ -topology in ideal topological spaces

Nawroz O. Hessean, Halgwrđ M. Darwesh, Sarhad F. Namiq

The aim of this study is to introduce  $\mathcal{J}$ - $\omega^*$ -open sets as a new set in ideal space which form topology on  $\mathcal{X}$  known as  $\mathcal{T}_{\mathcal{J}\omega^*}$  (or  $\mathcal{J}$ - $\omega^*$ -topology) which is strictly placed between  $\mathcal{T}_{\omega^*}$  and  $\mathcal{T}_{\omega}$ . Additionally, we investigate the relationships of  $\mathcal{J}$ - $\omega^*$ -open sets with some other classes of sets.

(pp. 218–231)

### A study on co-intersection graphs of rings

S. Jaber Hoseini, Yahya Talebi

Let  $R$  be a ring and  $\mathcal{I}^*(R)$  be the set of all nontrivial left ideals of  $R$ . The Co-intersection graph of ideals of  $R$ , denoted by  $\Omega(R)$ , is an undirected simple graph with the vertex set  $\mathcal{I}^*(R)$ , and two distinct vertices  $I$  and  $J$  are adjacent if and only if  $I + J \neq R$ .

This paper derives a sufficient and necessary condition for  $\Omega(R)$  to be a connected graph. We characterize the values of  $n$  for which the graph  $\Omega(\mathbb{Z}_n)$  is Eulerian and Hamiltonian. Furthermore, the bad (and nice) decision number of  $\Omega(\mathbb{Z}_n)$  are studied in the paper.

(pp. 232–242)

### Refinements of unitary invariant norm inequalities for matrices

Xingkai Hu, Yuan Yi, Wushuang Liu

In this paper, we first establish an arithmetic-geometric mean inequality of unitary invariant norm for matrices, which is an improvement of the result proposed by Zou and He [Linear Algebra Appl., 436(2012), 3354-3361]. Then, we use it to refine the existing inequality. Moreover, we derive two unitarily invariant norm inequalities for matrices, which refine the result of Cao and Wu.

(pp. 243–253)

### Nondestructive evaluation of interface defects in layered media

Gabriele Inglese, Roberto Olmi

In a layered thermal conductor, the inaccessible interface could be damaged by mechanical solicitation, chemical infiltration, aging. In this case, the original thermal properties of the specimen are modified. The defect occurs typically in form of *delamination*. The present paper deals with nondestructive evaluation of interface thermal conductance  $h$  from the knowledge of the surface temperature when the specimen is heated in some controlled way. The goal is achieved by expanding  $h$  in powers of the thickness of the upper layer. The mathematical analysis of the model produces exact formulas for the first coefficients of  $h$  which are tested on simulated and real data. The evaluation of interface flaws comes from reliable approximation of  $h$ .

(pp. 254–274)

### Trend modeling and multi-step taxi demand prediction

Shan Jiang, Yuming Feng, Xiaofeng Liao, B.O. Onasanya

At present, there is a serious mismatch between the supply and demand of taxis, and reasonable demand forecasting can effectively reduce the supply-demand gap, which is an important foundation for taxi scheduling. This article proposes three modeling methods for taxi demand cycle trends, namely the Fourier series based method, the principal component analysis trend based method, and the average trend based method. Finally, based on a weighted combination of three periodic features, a multi-step prediction model for taxi demand was established. On actual data, the method proposed in this paper achieved an MAE error of 1.91, indicating that it can effectively predict taxi multi-step demand. Furthermore, after comparison, the method proposed in this paper outperforms other comparative methods in predicting taxi demand.

(pp. 275–294)

### Strong edge-coloring of planar graphs with girth at least seven

Jiixin Yuan, Mingfang Huang

A *strong edge-coloring* of a graph  $G$  is that two edges  $e_1$  and  $e_2$  that are adjacent to each other or adjacent to the same edge must be colored with distinct colors. In this paper we prove that every planar graph  $G$  with girth  $g \geq 7$  and maximum degree  $\Delta \geq 5$  has a strong edge-coloring using at most  $3\Delta - 1$  colors. In addition, we prove that every planar graph  $G$  without adjacent  $7 - cycles$ , with girth  $g \geq 7$  and the maximum degree  $\Delta \geq 4$  has a strong edge-coloring using at most  $3\Delta - 1$  colors.

(pp. 295–304)

### Predator and prey dynamics with Beddington-DeAngelis functional response with in kinesis model

Tahani Al-Karkhi, Nurdan Çabukoğlu

In mathematical ecology, the study of interactions that are reactive-diffusive in nature between different species and their relevant systems has been researched extensively. However, there is still room for contribution on this rich topic. Therefore, we study a spatial-temporal prey-predator model which includes kinesis terms representing plankton dynamics under info-chemical mediated trophic interactions. The Beddington-DeAngelis functional response is coupled with a simplified two species approach within the model to describe the grazing pressure of zooplankton (M) on phytoplankton (P). This pressure is controlled through an external info-chemical (C). The mutual interference by predators within the ecosystem is implemented through the Beddington-DeAngelis



functional response, a distinctive feature of this response type. This feature is utilized in this study to indicate the effect of changes in prey density in relation to predator density. In our model, a stability analysis is performed between the two aforementioned species to provide a system dynamics comparison. The critical conditions for kinesis are derived on the basis that increases in the reproduction coefficient decrease the diffusion. This means that species prefer to stay in good conditions to facilitate the reproduction process, but are likely to escape in bad conditions. The kinesis terms within our Phytoplankton-Zooplankton model impact factors such as survival and traveling wave behavior. Numerical experiments are performed in this work to examine the traveling waves and the monotonic dependence of the reproduction coefficient in the species population. Moreover, the possible benefits of purposeful kinesis are demonstrated.

(pp. 305–329)

### On the localization of a type $B$ semigroup

Chunhua Li, Lingxiang Meng, Jieying Fang

This paper mainly investigates the localization of a type  $B$  semigroup. Firstly, the unique localization of a type  $B$  semigroup on its idempotent semilattice is given, and some properties of the localization of a type  $B$  semigroup are studied. It is proved that the localization of a type  $B$  semigroup on its idempotent semilattice is the maximum cancellative monoid homomorphic image. Finally, the relationships between localizations and the minimum cancellative congruence of a type  $B$  semigroup are discussed.

(pp. 330–338)

### On completeness of fuzzy metric spaces

Changqing Li, Yanlan Zhang

Recently,  $p$ -convergence in fuzzy metric spaces, in George and Veeramani's sense, has been explored by Gregori et al. [6]. In this paper, we study consistency of Cauchyness (completeness, respectively) and  $p$ -Cauchyness ( $p$ -completeness, respectively) in fuzzy metric spaces.

(pp. 339–345)

### Generalization of fuzzy Ostrowski like inequalities for $(m_1, m_2, \alpha, \beta, \gamma, \mu)$ -convex functions

Faraz Mehmood, Akhmadjon Soleev

In the present paper, we present the very 1st time the generalized notion of  $(m_1, m_2, \alpha, \beta, \gamma, \mu)$ -convex function in mixed kind, which is the generalization of 22 functions, which are presented in sequel manner. Our aim is to establish generalized Ostrowski like inequalities for  $(m_1, m_2, \alpha, \beta, \gamma, \mu)$ -convex functions via

Fuzzy Riemann Integrals by applying several techniques in which power mean inequality and Hölder's inequality are included. Moreover, we would obtain various results with respect to the convexity of function as special cases and also recapture several established results of different authors of different papers.

(pp. 346–363)

On a class of half-discrete Hilbert-type inequalities in the whole plane involving some classical special constants

Minghui You

In this paper, we first define a new half-discrete kernel function in the whole plane, which involves some exponent functions and unifies some homogeneous and non-homogeneous kernels. By employing some techniques of real analysis, a new half-discrete Hilbert-type inequality with the newly defined kernel function, as well as its equivalent forms are established. Furthermore, the constant factors of the newly obtained inequalities are proved to be optimal. At last, assigning special values to the parameters, we get some interesting Hilbert-type inequalities involving hyperbolic functions, and with the constant factors related to Euler numbers, Bernoulli numbers, and Catalan constant.

(pp. 364–385)

Some algebraic identities on prime near rings with generalized derivations

Phool Miyan, Seleshi Demie, Adnew Markos, Leta Hailu

The purpose of the present paper is to investigate the commutativity of a prime near ring  $N$  with a generalized derivation  $F$  associated with a nonzero derivation  $d$  satisfying one of the conditions: For some nonnegative integers  $p$  and  $q$ :

- (i)  $[F(x), y] = \pm y^p(x \circ y)y^q$ ;
- (ii)  $[x, F(y)] = \pm x^p(x \circ y)x^q$ ;
- (iii)  $F(x) \circ y = \pm y^p[x, y]y^q$ ;
- (iv)  $x \circ F(y) = \pm x^p[x, y]x^q$ ;
- (v)  $F(x) \circ y = \pm y^p(x \circ y)y^q$ ;
- (vi)  $[x, F(y)] = \pm x^p[x, y]x^q$ ;
- (vii)  $[F(x), y] = \pm y^p[x, y]y^q$ ;
- (viii)  $x \circ F(y) = \pm x^p(x \circ y)x^q$ ,

for all  $x, y \in N$ . Moreover, we give an example which shows the necessity of primness hypothesis in the theorems.

(pp. 386–397)

### Binary soft simply\* alpha open sets and continuous function

C.R. Parvathy, A. Sofia

A topological rough approximation space is defined over two different universes using binary soft relations. A new class of binary soft set and its corresponding soft topology is obtained. Continuity functions for the newly defined set are introduced. The characteristics of continuity functions between two binary soft topological spaces and that between binary soft topological space and topological rough approximation space are examined. The proposed definitions and properties are demonstrated with suitable examples.

(pp. 398–410)

### Structural invariants of the product maximal graph

G. Ramya, D. Kalamani

In this paper, some structural properties of the product maximal graph like matching, vertex covering, edge covering and cordial labeling are studied. Furthermore, the number of triangles of  $\Gamma_{pm}(R)$  are calculated. The isomorphism between the product maximal graph of cartesian product of two commutative rings and cartesian product of two product maximal graphs of commutative rings and its relations is interpreted with an example.

(pp. 411–426)

### Degree sum exponent distance energy of non-commuting graph for dihedral groups

M.U. Romdhini, A. Nawawi

The non-commuting graph is defined on a finite group  $G$ , denoted by  $\Gamma_G$ , with  $G \setminus Z(G)$  is the vertex set of  $\Gamma_G$  and  $v_p \neq v_q \in G \setminus Z(G)$  are adjacent whenever they do not commute in  $G$ . In this paper, we focus on  $\Gamma_G$  for dihedral groups of order  $2n$ ,  $D_{2n}$ , where  $n \geq 3$ . We show the spectrum, spectral radius and energy of the graph corresponding to the degree sum exponent distance matrix and analyze the hyperenergetic property. Moreover, we then present the correlation between the obtained energy and the adjacency energy.

(pp. 427–442)

### On divisor labeling of co-prime order graphs of finite groups

M. Saini, G. Singh, A. Sehgal, D. Singh

The co-prime order graph of a finite group  $G$  is an undirected graph whose vertex set is  $G$  and two distinct vertices  $u, v \in G$  are adjacent if  $\gcd(o(u), o(v)) = 1$  or a prime number. Labeling a graph is the process of assigning integers to its vertices and/or edges subject to certain conditions. In other words, vertex (edge) labeling is a function of the set of vertices (edges) to a set of labels (generally integers). A graph  $\Gamma$  is a divisor graph if all its vertices can be labeled with positive integers such that two distinct vertices  $x$  and  $y$  are adjacent if and only if  $x|y$  or  $y|x$ . This paper focuses on some conditions under which the co-prime order graphs of finite groups, especially abelian groups and permutation groups, are divisor graphs.

(pp. 443–451)

### Common fixed point for compatible self-maps in an orbitally complete $b$ -metric space

P. Swapna, T. Phaneendra, M.N. Raja Shekhar

A common fixed point theorem is obtained for three self-maps on a  $b$ -metric space, satisfying a rational type condition, through the notions of orbital completeness, orbital continuity and the compatibility.

(pp. 452–460)

### Simultaneous approximation of translation operators

N. Tsirivas

Let  $(m_n)_{n \in \mathbb{N}}$  be an unbounded sequence of complex numbers and  $(a_v)_{v \in \mathbb{N}}$  be a sequence of numbers in the unit circle

$$C(0, 1) = \{z \in \mathbb{C} \mid |z| = 1\},$$

where  $\mathbb{N}$  is the set of natural numbers.

We shall prove that there is an entire function  $f$  so that, for every entire function  $g$  there is a subsequence  $(\lambda_n)$ ,  $n \in \mathbb{N}$  of  $(m_n)_{n \in \mathbb{N}}$  such that, for every compact subset  $L \subseteq \mathbb{C}$  and for every  $v \in \mathbb{N}$ ,

$$\sup_{z \in L} |f(z + \lambda_n a_v) - g(z)| \rightarrow 0 \text{ as } n \rightarrow \infty.$$

In relation with other results about hypercyclic operators, the new element in this paper is that we achieve the approximation with the same sequence  $(\lambda_n)$ , for all numbers  $a_v$  ( $v = 1, 2, \dots$ ).

(pp. 461–471)

### [L-fuzzy ideal theory on bounded semihoops](#)

Zhen Yan Wang, Xiao Long Xin, Xiao Fei Yang

This article mainly focuses on the  $L$ -fuzzy ideal theory on bounded semihoops. Firstly, we propose two classes of  $L$ -fuzzy ideals on bounded semihoop and prove that each  $L$ -fuzzy strong ideal is an  $L$ -fuzzy ideal but an  $L$ -fuzzy ideal may not be an  $L$ -fuzzy strong ideal. We also get some properties and equivalent descriptions of  $L$ -fuzzy strong ideal. Secondly, we introduce the notion of  $L$ -fuzzy prime ideal and the second type of  $L$ -fuzzy prime ideal on bounded semihoops. Moreover, we discuss the relationship between these two types of  $L$ -fuzzy prime ideals. Finally, we present the concept of  $L$ -fuzzy maximal ideal on bounded semihoops and obtain some properties.

(pp. 472–495)

### [Population dynamics of a modified predator-prey model with economic harvesting](#)

Wei Liu

The dynamical behaviors of a predator-prey model with commercial harvesting are studied in the present work. The model is developed from the Leslie predator-prey model with harvesting on predator, which is established by differential-algebra equations. The harvesting is considered from an economic perspective, and the impacts of the harvesting profit on the dynamics of our model are investigated. Firstly, basing on the parameterisation approach of differential-algebra system, the local stability of positive equilibrium point is studied. Further, by treating the harvesting profit as a bifurcation parameter, the Hopf bifurcation occurring at the equilibrium point is analyzed, and we find a qualitative change in the dynamics. Besides, the stability of centre is also considered. Some computer simulations using Matlab software are presented to support the analytical results. Lastly, we relate the results on mathematics and dynamics with the biology, and interpret these results in terms of ecosystem stability and destruction.

(pp. 496–518)

### [On one-sided MPCEP-inverse for matrices of an arbitrary index](#)

S.Z. Xu, X.F. Cao, X. Hua, B.L. Yu

One-sided MPCEP-inverse for matrices was introduced in this paper. The MPCEP-inverse can be described by using the core part  $A_1$  in Core-EP decomposition of  $A$  and the Moore-Penrose inverse of  $A$ . The MPCEP-inverse of  $A$  coincides with the  $(A^\dagger A^k, (A^k)^*)$ -inverse of  $A$ . In addition, the CE matrix was introduced, a necessary and sufficient condition such that a matrix  $A$  to be a CE matrix is the MPCEP-inverse of  $A$  commutes with  $A$ .

(pp. 519–537)

### Inequalities for the generalized inverse trigonometric and hyperbolic functions with one parameter

X. L. Wang, L. Yin

In this paper, we show some inequalities for the generalized inverse trigonometric and hyperbolic functions with one parameter of  $(2, q)$ . Especially, we also present several Shafer-Fink, Wilker and Huygens type inequalities of these functions. These results are consistent with previously known results.

(pp. 538–550)

### Extensions of singular value inequalities for sector matrices

Xuesha Wu

In this paper, we present singular value inequalities for matrices. As a consequence, we prove singular value inequalities for sector matrices. Moreover, we give singular value inequalities involving operator concave function, which are generalizations of some existing results.

(pp. 551–559)

### A remark on relative Hilali conjectures

Shoji Yokura

S. Chouingou, M. A. Hilali, M. R. Hilali and A. Zaim have recently proved, in certain cases, a relative Hilali conjecture. This is an inequality about the dimensions of the kernel of homomorphisms of rational homotopy groups and rational homology groups, hence shall be called a Kernel-relative Hilali conjecture. In this paper we add another relative Hilali conjecture with respect to the cokernel of such homomorphisms, which shall be called a Cokernel-relative Hilali conjecture. We consider some examples for these conjectures and discuss conditions under which these conjectures hold and also conditions under which they are equivalent to each other. As byproducts of these computations, we show that  $\dim(\pi_*(X) \otimes \mathbb{Q})$  and the formal dimension  $n_X$  of  $X$  have the same parity and that the Hilali conjecture holds when  $\dim(\pi_*(X) \otimes \mathbb{Q}) \leq 4$ .

(pp. 560–582)

### A method for solving quadratic equations in real quaternion algebra by using Scilab software

Geanina Zaharia, Diana-Rodica Munteanu

In this paper, we present some numerical applications for the equation  $x^2 + ax + b = 0$ , where  $a, b$  are two quaternionic elements in  $\mathbb{H}(\alpha, \beta)$ .  $\mathbb{H}(\alpha, \beta)$  represents the algebra of real quaternions with parameterized coefficients by  $\alpha$

and  $\beta$ . The algebra of real quaternions is an extension of complex numbers and is represented by algebraic objects called quaternions. These quaternions are composed of four components: a real part and three imaginary components. In general,  $\mathbb{H}(\alpha, \beta)$  indicates a family of parameterized quaternion algebras, in which the specific values of  $\alpha$  and  $\beta$  determine the specific properties and structure of the quaternion algebra. Based on well-known solving methods, we have developed a new numerical algorithm that solves the equation for any quaternions  $a$  and  $b$  in any algebra  $\mathbb{H}(\alpha, \beta)$ .

(pp. 583–603)

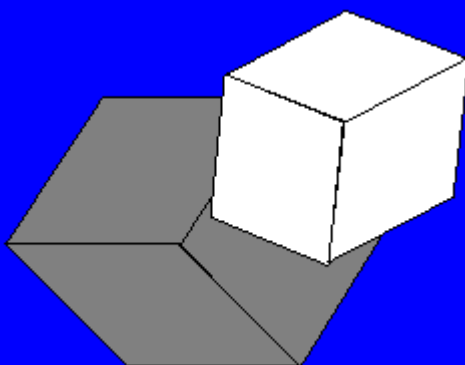
[On nearly  \$CAP\$ -embedded second maximal subgroups of Sylow  \$p\$ -subgroups of finite groups](#)

[Mingmei Zhang, Yuxi Huang, Yong Xu](#)

A subgroup  $D$  of a group  $G$  is called a  $CAP$ -embedded subgroup of  $G$ , if for each prime  $p$  dividing the order of  $D$ , there exists a  $CAP$ -subgroup  $K$  of  $G$  such that a Sylow  $p$ -subgroup of  $D$  is also a Sylow  $p$ -subgroup of  $K$ . Later, we have generalized  $CAP$ -embedded subgroup to nearly  $CAP$ -embedded subgroup. A subgroup  $H$  of a group  $G$  is said to be nearly  $CAP$ -embedded in  $G$  if there is a subnormal subgroup  $T$  of  $G$  and a  $CAP$ -embedded subgroup  $H_{ce}$  of  $G$  contained in  $H$  such that  $G$  is equal to  $HT$  and the intersection of  $H$  and  $T$  is contained in  $H_{ce}$ . The main purpose of this paper is to study the  $p$ -nilpotency of a group which every second maximal subgroup of its Sylow  $p$ -subgroups is nearly  $CAP$ -embedded and some new results are obtained.

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