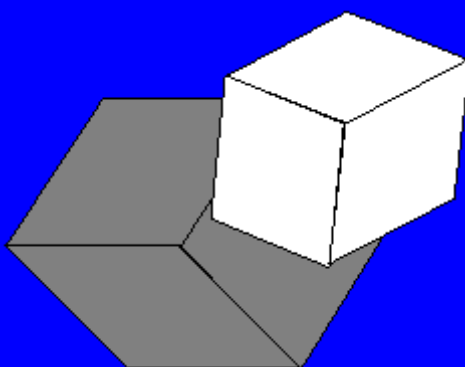


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## Papers Abstracts



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### On the $(\mathcal{W}_{G_b, \varphi}; \alpha)$ -diaphony of the nets of type of Halton-Zaremba constructed over finite groups

Vesna Dimitrievska Ristovska, Vassil Grozdanov

In the present paper, the authors introduce an arithmetic based on finite groups with respect to arbitrary bijections. This algebraic background is used to construct the function system  $\mathcal{W}_{G_b, \varphi_b}$  of the Walsh functions over the set  $G_b$  of groups with respect to the set  $\varphi_b$  of bijections. The developed algebraic base is also used to introduce a wide class of two-dimensional nets  $_{G_b, \varphi_b} Z_{b, \nu}^{\kappa, \mu}$  of type of Halton-Zaremba. Four concrete nets of this class are constructed and graphically illustrated. The so-called  $(\mathcal{W}_{G_b, \varphi}; \alpha)$ -diaphony is applied as a appropriate tool for studying the nets of the introduced class. An exact formula for the  $(\mathcal{W}_{G_b, \varphi}; \alpha)$ -diaphony of the nets of class  $_{G_b, \varphi_b} Z_{b, \nu}^{\kappa, \mu}$  is presented. This formula allows us to show the influence of the vector  $\alpha$  on the exact order of the  $(\mathcal{W}_{G_b, \varphi}; \alpha)$ -diaphony of these nets.

(pp. 1–26)

### Some properties of regular topology on $C(X, Y)$

Mir Aaliya, Sanjay Mishra

The recently introduced regular topology for the function space  $C(X, Y)$  has been explored up to some metrizability and various countability and completeness properties. The main aim of this paper is to explore the regular topology on the function space  $C(X, Y)$  in which we study submetrizability and extend various properties equivalent to the metrizability of the space  $C_r(X, Y)$ . We also study number of maps corresponding to the space  $C_r(X, Y)$  and prove that the regular topology on the space  $C(X, Y)$  is strong when  $X$  is taken discrete. Furthermore, we study various separation axioms on the space  $C_r(X, Y)$ , where we prove that the function space  $C_r(X)$  is normal by taking  $X$  to be countable, compactly generated compact space and prove certain equivalent conditions to various separation axioms on the space  $C_r(X, Y)$ .

(pp. 27–43)

### Petrov-discontinuous Galerkin finite element method for solving diffusion-convection problems

Mohammed Waleed Abdulridha, Hashim A. Kashkool, Ali Hasan Ali

In this paper, we present a new modification of the discontinuous Galerkin Finite element method (DGFEM). The proposed modification is considered when the symmetric interior penalty Galerkin scheme involves only space variables by using the Petrov discontinuous Galerkin Finite element method (PDGFEM), while the time in the linear diffusion-convection problem remains continuous. We prove the properties of the bi-linear form (V-elliptic, continuity and stability), and we show that the error estimate is of second order with respect to the space. We also present some numerical experiments to validate the proposed method, and we simulate these peppermints to illustrate the theoretical results.

(pp. 44–60)

### On structures of rough topological spaces based on neighborhood systems

Radwan Abu-Gdairi

Keeping in view the generalized approximation space, the goal of this paper is to suggest and investigate four different styles for approximating rough sets. The proposed approximations are based on various general topologies. In fact, we first generalize the notion of the initial-neighborhood and thus we construct four different topologies generated from these neighborhoods. The relationships between the new neighborhoods (respectively, topologies) and the previous are studied. Comparisons of the degrees of different accuracy of the presented approximations are investigated. The essential characteristics of these operators are obtained.

(pp. 61–74)

### Some separation axioms via nano $S_\beta$ -open sets in nano topological spaces

Nehmat K. Ahmed, Osama T. Pirbal

In this present study, we shed light on some separation axioms via nano  $S_\beta$ -open sets including nano  $S_\beta$ -regular,  $S_\beta$ -normal,  $S_\beta - S_0$  and  $S_\beta - S_1$  axioms in nano topological spaces where nano  $S_\beta$ -open set is defined and related to nano semi-open and nano  $\beta$ -closed sets. Here, we implement each axiom on the family of all nano  $S_\beta$ -open sets according to upper and lower approximations in which there exist exactly six families of nano  $S_\beta$ -open sets. This research work brings out some interesting results such as it is shown that in which condition a nano topological space is always nano  $S_\beta$ -normal space where upper and lower approximations are leading conditions. In addition, the relationship among those axioms is also considered.

(pp. 75–85)

### On graded weakly classical 2-absorbing submodules of graded modules over graded commutative rings

S. Alghueiri, K. Al-Zoubi

In this paper, we introduce the concept of graded weakly classical 2-absorbing submodule as a generalization of a graded classical 2-absorbing submodule. We give a number of results concerning this class of graded submodules and their homogeneous components.

(pp. 86–97)

### Strong modular product and complete fuzzy graphs

Talal Al-Hawary

In this paper, we provide an improvement of the modular product of fuzzy graphs defined by [?] in 2015, which we call strong modular product. We give sufficient conditions for the strong modular product of two fuzzy graphs to be complete and we show that if the strong modular product of two fuzzy graphs is complete, then at least one factor is a complete fuzzy graph. Moreover, we give necessary and sufficient conditions for the strong modular product of two balanced fuzzy graphs to be balanced.

(pp. 98–104)

### Chain dot product graph of a commutative ring

Basem Alkhamaiseh

In this article, we generalized the concepts of total dot product graph (the chain zero-divisor dot product), which were investigated in 2015 by A. Badawi, to what we call chain total dot product graph  $CTD(R)$  (the chain zero-divisor dot product graph  $CZD(R)$ ). We give some basic graph properties for the graphs  $CTD(R)$  and  $CZD(R)$  such as connectedness, diameter and the girth.

(pp. 105–112)

### Projection graphs of rings and near-rings

Teresa Arockiamary S., Meera C., Santhi V.

Association of graphs with algebraic structures facilitates the process of understanding the properties of algebraic structures through graphs. In this paper, projection graph  $P(R)$  of a ring  $R$  is introduced as an undirected graph, whose vertices are the nonzero elements of  $R$  and any two distinct vertices  $x$  and  $y$  are adjacent if and only if their product is equal to either  $x$  or  $y$ . The projection graph  $P(N)$  of a near-ring  $N$  is also defined in the same way. It is proved that  $P(R)$  is a star graph if and only if  $R$  has no nonzero zero-divisors. A method

of finding adjacent vertices with the help of annihilators is developed. The projection graphs of certain classes of rings are found to be bipartite and  $P(R)$  is proved to be weakly pancyclic when  $R$  is a local ring with ascending chain condition on the annihilator ideals of its elements.  $P(\mathbb{Z}_n)$  are constructed for certain values of  $n$  and their properties are studied. Moreover,  $P(N)$  is shown as a complete graph when  $N$  is either a constant near-ring or an almost trivial near-ring.

(pp. 113–135)

### Characterization of generalized $n$ -semiderivations of 3-prime near rings and their structure

Asma Ali, A. Mamouni, Inzamam Ul Huque

Let  $N$  be a near ring and  $n$  be a fixed positive integer. An  $n$ -additive (additive in each argument) mapping  $F : \underbrace{N \times N \times \dots \times N}_{n\text{-times}} \rightarrow N$  is said to be a permuting

generalized  $n$ -semiderivation on a near ring  $N$  if there exists an  $n$ -semiderivation  $d : \underbrace{N \times N \times \dots \times N}_{n\text{-times}} \rightarrow N$  associated with a map  $g : N \rightarrow N$  such that

the relation  $F(x_1x'_1, x_2, \dots, x_n) = F(x_1, x_2, \dots, x_n)x'_1 + g(x_1)d(x'_1, x_2, \dots, x_n) = d(x_1, x_2, \dots, x_n)g(x'_1) + x_1F(x'_1, x_2, \dots, x_n)$  and  $g(F(x_1, x_2, \dots, x_n)) = F(g(x_1), g(x_2), \dots, g(x_n))$  hold, for all  $x_1, x'_1, x_2, \dots, x_n \in N$ . The purpose of the present

paper is to prove some commutativity theorems in case of a semigroup ideal of a 3-prime near ring admitting a generalized  $n$ -semiderivation, thereby extending some known results of derivations, semiderivations and generalized derivations.

(pp. 136–158)

### Subspace diskcyclic tuples of operators on Banach spaces

Nareen Bamerni

In this paper, we study subspace diskcyclic and subspace-disk transitive tuples of operators. We give some characterizations of these tuples. Also, we give a set of sufficient conditions for a tuple to be subspace-diskcyclic. We find a relation between the subspace-diskcyclicity of a tuple of operators and the tuple of the direct sum of those operators. Finally, we show that if a tuple of operators is subspace-diskcyclic, then not every operator in the tuple has to be subspace-diskcyclic.

(pp. 159–167)

### Nonlinear mappings preserving the kernel or range of skew product of operators

H. Benbouziane, M. Ech-ChÉRif El Kettani, A. M. Vadel

Let  $\mathcal{H}$  be a Hilbert space and let  $\mathcal{B}(\mathcal{H})$  be the algebra of all bounded linear operator on  $H$ . We characterise surjective maps  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H})$ , such that  $F(\phi(A) \diamond \phi(B)) = F(A \diamond B)$ , for all  $A, B \in \mathcal{B}(\mathcal{H})$ , where  $F(A)$  denotes any of  $R(A)$  or  $N(A)$  and  $A \diamond B$  denotes any binary operations  $A^*B$ ,  $AB^*A$  for all  $A, B \in \mathcal{B}(\mathcal{H})$ .

(pp. 168–177)

### On the application of $M$ -projective curvature tensor in general relativity

K. Chattopadhyay, A. Bhattacharyya, D. Debnath

In this paper the application of the  $M$ -projective curvature tensor in the general theory of relativity has been studied. Firstly, we have proved that an  $M$ -projectively flat quasi-Einstein spacetime is of a special class with respect to an associated symmetric tensor field, followed by the theorem that a spacetime with vanishing  $M$ -projective curvature tensor is a spacetime of quasi-constant curvature. Then we have proved that an  $M$ -projectively flat quasi-Einstein spacetime is infinitesimally spatially isotropic relative to the unit timelike vector field  $\xi$ . In the next section we have proved that an  $M$ -projectively flat Ricci semi-symmetric quasi-Einstein spacetime satisfying a definite condition is an  $N(\frac{2l-m}{6})$ -quasi Einstein spacetime. In the last section, we have firstly proved that an  $M$ -projectively flat perfect fluid spacetime with torse-forming vector field  $\xi$  satisfying Einstein field equation with cosmological constant represents an inflation, then we have found out the curvature of such spacetime, followed by proving the theorem that the spacetime also becomes semi-symmetric under these conditions. Lastly, we have found out the square of the length of the Ricci tensor in this type of spacetime and also proved that if an  $M$ -projectively flat perfect fluid spacetime satisfying Einstein field equation with cosmological constant, with torse-forming vector field  $\xi$  admits a symmetric  $(0, 2)$  tensor  $\alpha$  parallel to  $\nabla$  then either  $\lambda = \frac{k}{2}(p - \sigma)$  or  $\alpha$  is a constant multiple of  $g$ .

(pp. 178–190)

### Torsion section of elliptic curves over quadratic extensions of $\mathbb{Q}$

Z. Cheddour, A. Chillali, A. Mouhib

In this paper, we will study and determine all possible torsion sections of elliptic curves that can appear on quadratic extensions of the set of rational numbers endowed by the usual addition and a non-standard way of multiplication.

(pp. 191–200)

### On $k$ -perfect polynomials over $\mathbb{F}_2$

H. Chehade, Y. Alkhezi, W. Zeid

A polynomial  $A$  is called  $k$ -perfect over the finite field  $\mathbb{F}_2$  if the sum of the  $k^{\text{th}}$  powers of all distinct divisors of  $A$  equals  $A^k$ , where  $k$  is a positive integer. We show that a  $k$ -perfect polynomial  $A$  over  $\mathbb{F}_2$  must be even when  $k = 2^n$ ,  $n$  is a non-negative integer, and we characterize all  $2^n$ -perfect polynomials over  $\mathbb{F}_2$  that are of the form  $x^a(x+1)^b \prod_{i=1}^r P_i^{h_i}$ , where each  $P_i$  is a Mersenne prime and  $a, b$  and  $h_i$  are positive integers.

(pp. 201–214)

### Prime-valent one-regular graphs of order $18p$

Qiao-Yu Chen, Song-Tao Guo

A graph is *one-regular* and *arc-transitive* if its full automorphism group acts on its arcs regularly and transitively, respectively. In this paper, we classify connected one-regular graphs of prime valency and order  $18p$  for each prime  $p$ . As a result there are two infinite families of such graphs, one is the cycle  $C_{18p}$  with valency two and the other is the normal Cayley graph on the generalized dihedral group  $(\mathbb{Z}_{3p} \times \mathbb{Z}_3) \rtimes \mathbb{Z}_2$  with valency three and  $p \equiv 1 \pmod{6}$ .

(pp. 215–221)

### The $\omega$ -continuity of group operation in the first (second) variable

Halgwrđ M. Darwesh, Adil K. Jabbar, Diyar M. Mohammed

The present paper aims to introduce and study the  $\omega$ -continuity of the group operation in the first (resp., second) variable and some basic properties and relationships concerning left and right translation functions are obtained. Also, we have shown that the group operation is  $\omega$ -continuous at the first (resp., second) variable if and only if it is  $\omega$ -irresolute at the first (resp., second) variable.

(pp. 222–234)

### The group of integer solutions of the Diophantine equation $x^2 + mxy + ny^2 = 1$

Ghader Ghasemi

Let  $m$  and  $n$  be two integers. It is shown that the set of all integer solutions of the Diophantine equation  $x^2 + mxy + ny^2 = 1$  has an Abelian group structure. Furthermore, it is shown that this Abelian group is isomorphic to one of the groups  $\mathbb{Z}_2$ ,  $\mathbb{Z}_4$ ,  $\mathbb{Z}_6$  and  $\mathbb{Z}_2 \times \mathbb{Z}$ .

(pp. 235–253)

### On open question of prominent interior GE-filters in GE-algebras

Yingmin Guo, Wei Wang, Hui Wu

In an interior GE-algebra, the concept of prominent interior GE-filter of type 1 was introduced to serve as a generalization of prominent interior GE-filters. However, there are some work need to be done for this goal. For example, the extension property for prominent interior GE-filter of type 1 still remains unproved so there is an open question on the extension property of such GE-filters need to be proved that 'Let  $(X, f)$  be an interior GE-algebra. Let  $F$  and  $G$  be interior GE-filters in  $(X, f)$ . If  $F \subseteq G$  and  $F$  is a prominent interior GE-filter of type 1 in  $(X, f)$ , then is  $G$  also a prominent interior GE-filter of type 1 in  $(X, f)$ ?' In this paper, we propose the condition for an interior GE-filter to be a prominent interior GE-filter of type 1, then we prove the extension property for prominent interior GE-filter of type 1 in an interior GE-algebra, and thus the open question is solved.

(pp. 254–260)

### On soft $p_c$ -regular and soft $p_c$ -normal spaces

Q.H. Hamko, N.K. Ahmed, A.B. Khalaf

The aim of this paper is to introduce two new types of soft separation axioms called soft  $p_c$ -regular and soft  $p_c$ -normal spaces by using the concept of soft  $p_c$ -open sets in soft topological spaces. We explore several properties and relations of such spaces. Also, we investigate hereditary and soft invariance properties by considering certain soft mappings.

(pp. 261–279)

### Schatten class weighted composition operators on weighted Hilbert Bergman spaces of bounded strongly pseudoconvex domain

Cheng-Shi Huang, Zhi-Jie Jiang

Let  $D$  be a bounded strongly pseudoconvex domain in  $\mathbb{C}^n$ ,  $\delta(z) = d(z, \partial D)$  the Euclidean distance from the point  $z$  to the boundary  $\partial D$  and  $H(D)$  the set of all holomorphic functions on  $D$ . For given  $\beta \in \mathbb{R}$ , the weighted Hilbert Bergman space on  $D$ , denoted by  $A^2(D, \beta)$ , consists of all  $f \in H(D)$  such that

$$\|f\|_{2,\beta} = \left[ \int_D |f(z)|^2 \delta(z)^\beta dv(z) \right]^{\frac{1}{2}} < +\infty,$$

where  $dv$  is the Lebesgue measure on  $D$ . The aim of the paper is to completely characterize the Schatten class of weighted composition operators on  $A^2(D, \beta)$  when  $\delta(z)$  satisfies certain integrable condition.

(pp. 280–293)



### [Hermite-Hadamard inequality for preinvex functions](#)

[Akhilad Iqbal, Khairul Saleh, Izhar Ahmad](#)

We derive integral inequalities of Hermite-Hadamard type for the functions that have preinvex absolute values of third order derivatives. Moreover, we also discuss applications to several special means.

(pp. 294–302)

### [Constructions of indecomposable representations of algebras via reflection functors](#)

[Wanwan Jia, Fang Li](#)

The main aim of this paper is to reform the Bernstein-Gelfand-Ponomarev theory in order to characterize representations of some (non-basic) artinian algebras. All non-isomorphic indecomposable projective and injective representations are constructed via Coxeter functors for a generalized path algebra of acyclic quiver and then for an artinian hereditary algebra of Gabriel-type with an admissible ideal. The methods given via natural quivers and reformed modulations are helpful for one to study some properties which are *not Morita-invariant* in representation theory.

(pp. 303–335)

### [Semigroup of transformations with restricted partial range: regularity, abundance and some combinatorial results](#)

[Jiulin Jin, Taijie You](#)

Suppose that  $X$  be a nonempty set. Denote by  $\mathcal{T}(X)$  the full transformation semigroup on  $X$ . For  $\emptyset \neq Z \subseteq Y \subseteq X$ , let  $\mathcal{T}(X, Y, Z) = \{\alpha \in \mathcal{T}(X) : Y\alpha \subseteq Z\}$ . Then,  $\mathcal{T}(X, Y, Z)$  is a subsemigroup of  $\mathcal{T}(X)$ . In this paper, we characterize the regular elements of the semigroup  $\mathcal{T}(X, Y, Z)$ , and present a necessary and sufficient condition under which  $\mathcal{T}(X, Y, Z)$  is regular. Furthermore, we investigate the abundance of the semigroup  $\mathcal{T}(X, Y, Z)$  for the case  $Z \subsetneq Y \subsetneq X$ . In addition, we compute the cardinalities of  $\mathcal{T}(X, Y, Z)$ ,  $\text{Reg}(\mathcal{T}(X, Y, Z))$  and  $E(\mathcal{T}(X, Y, Z))$  when  $X$  is finite, respectively.

(pp. 336–351)

### On some properties of Nörlund ideal convergence of sequence in neutrosophic normed spaces

Vakeel A. Khan, Mohammad Arshad

The purpose of this paper is to introduce the Nörlund ideal convergent sequence spaces with respect to these spaces  $\mathcal{N}_{I_0(S)}^f$ ,  $\mathcal{N}_{I(S)}^f$  and  $\mathcal{N}_{I_\infty(S)}^f$ . Also, we studied the Nörlund ideal Cauchy criterion in neutrosophic normed space and its properties. Also, we define an open ball  $B(x, \epsilon, \gamma)$  and closed ball  $B[x, \epsilon, \gamma]$  in neutrosophic norm space. Furthermore, we also look at some of these convergent sequence spaces' topological and algebraic properties.

(pp. 352–373)

### Some aspects of the vertex-order graph

Kiruthika G., Kalamani D.

The vertex-order graph of the finite cyclic group  $G$  is based on its components  $C_d$  of the vertex-order graph  $\mathfrak{S}(G)$ , whose vertices are of order ' $d$ ' as the divisors of the order of the group  $G$ . The important properties of the vertex-order graph and its complements namely girth, radius, diameter, clique number, independence number and rank are derived. Further, the complement  $\overline{\mathfrak{S}(G)}$  of the vertex-order graph is proved as a complete  $t$ -partite graph and shown with an example. Later, we compute the first, second and third Zagreb indices of the graph  $\mathfrak{S}(G)$ ,  $\overline{\mathfrak{S}(Z_p)}$  and  $\overline{\mathfrak{S}(Z_{pq})}$ .

(pp. 374–389)

### Lebesgue's theorem and Egoroff's theorem for complex uncertain sequences

Ö. Kişi, M. Gürdal

In this paper, within framework uncertain theory, we investigate Lebesgue's theorem, Egoroff's theorem and Riesz's theorem for complex uncertain sequences.

(pp. 390–397)

### Novel concepts in fuzzy graphs

Kishore Kumar P.K, Alimohammad Fallah Andevvari

Today, fuzzy graphs have a variety of applications in other fields of study, including medicine, engineering, and psychology, and for this reason many researchers around the world are trying to identify their properties and use them in computer science as well as finding the shortest problem in a network. So, in this paper, some new fuzzy graphs are introduced and some properties of them are investigated. As a consequence of our results, some well-known assertions in the graph theory are obtained.

(pp. 398–413)

### On the sub- $\eta$ - $n$ -polynomial convexity and its applications

Lei Xu, Tingsong Du

This study addresses a new family of functions, to be named as the sub- $\eta$ - $n$ -polynomial convex functions, which is defined as a general form of the  $n$ -polynomial convex functions and the sub- $\eta$ -convex functions, and some of their significant properties are presented as well. In addition, by means of the sub- $\eta$ - $n$ -polynomial convexity, certain Hermite–Hadamard-type inequalities are established here. The sufficient conditions regarding optimality for sub- $\eta$ - $n$ -polynomial convex programming are discussed as applications.

(pp. 414–439)

### A weighted power distribution mechanism under transferable-utility systems: axiomatic results and dynamic processes

Yu-Hsien Liao

By applying the notion of the efficient Banzhaf value, any additional fixed utility should be distributed equally among the players who are concerned. However, in several applications, this notion seems unrealistic for the situation being modeled. Therefore, we adopt weights to introduce a modification of the efficient Banzhaf value, which we name the weighted Banzhaf value. To present the rationality, we adopt some reasonable properties to characterize this weighted value. Based on different viewpoints, we further define excess functions to propose alternative formulations and related dynamic processes for this weighted value.

(pp. 440–454)

### Recognition of decomposable posets by using the poset matrix

S. U. Mohammad, M. R. Talukder, S. N. Begum

We introduce the notion of a composition of square matrices. We recall the notion of poset matrix, a square  $(0, 1)$ -matrix, to represent posets. We show that this composition of poset matrices gives generalizations of the ordinal product as well as the direct sum and ordinal sum of poset matrices. We give an interpretation of the composition of poset matrices in posets. We show that the composition of poset matrices is also a poset matrix, and it represents a decomposable poset. This result gives, consequently, a matrix recognition of the decomposable posets.

(pp. 455–466)

### Generalized hesitant fuzzy $N$ -soft sets and their applications

A. Nazra, Jenizon, A.K. Chan, G. C. Wicaksono, Y. S. Sari, Zulvera

The  $N$ -soft Set as a generalization of the Soft Sets was introduced in 2018 by Fatimah et al. The concept of the  $N$ -soft Sets combined with the hesitant fuzzy sets is called hesitant fuzzy  $N$ -soft sets. On the other hand, the concept of fuzzy soft sets as a combination of soft sets and fuzzy sets was generalized by Majumdar and Samanta in 2010, called Generalized fuzzy soft sets, where many scholars have studied their properties and characteristics. This paper aims to extend the hesitant fuzzy  $N$ -soft set to a generalized hesitant fuzzy  $N$ -soft set that incorporates some characteristics of generalized fuzzy soft sets. Definition of the generalized hesitant fuzzy  $N$ -soft set, complements, and some of their operations are defined. Moreover, some of their properties, such as associative and distributive related to binary operations, are studied. Finally, we propose two algorithms for decision-making problems by extending the TOPSIS method to apply under generalized hesitant fuzzy  $N$ -soft set information.

(pp. 467–494)

### A note on $k$ -zero-divisor hypergraphs of some commutative rings

Elham Mehdi-Nezhad, Amir M. Rahimi

The main object of this paper is to study and characterize the connectedness, diameter, dominating sets and domination number of the  $k$ -zero-divisor hypergraph  $H_k(R)$  of a finite direct product of integral domains and a class of commutative Artinian rings  $R$ , respectively. We will show that the  $k$ -zero-divisor hypergraph associated to the direct product of  $k \geq 3$  integral domains (resp., commutative Artinian rings which are the direct product of  $k \geq 3$  local rings) are connected with diameter at most 3 and domination number at most 2 (resp., connected with diameter at most 4 and domination number at most  $2k$ ). We will also provide some examples related to these results.

(pp. 495–502)

### Relative averaging operators and trialgebras

Li Qiao, Jun Pei

In this paper, the relative averaging operator is introduced as a relative generalization of the averaging operator. We explicitly determine all averaging operators on the 2-dimensional complex associative algebra. The results show that not every dialgebra can be derived from an averaging algebra. We then generalize the construction of dialgebras and trialgebras from averaging operators to a construction from relative averaging operators. It is shown that this construction from relative averaging operators gives all dialgebras and trialgebras.

(pp. 503–513)

### New sequences of processing times for Johnson's algorithm in PFSP

Shahriar Farahmand Rad

There are many researches about converting  $n$  job  $m$  machine problem to a  $n$  job 2 machine one, and finally using Johnson's rule for minimizing makespan. In one case, this converting leads to the inner product of processing times by Pascal numbers. In this paper, it is shown that there are other suitable numerical sequences with a triangle pattern or without it, producing better makespans in several cases. The quality of results is checked by the benchmark of Taillard in permutation flow shop scheduling problem.

(pp. 514–523)

### Approximate solution of Fredholm type fractional integro-differential equations using Bernstein polynomials

A.H. Sallo, A.B. Khalaf, S.S. Ahmed

The main goal of this paper is to find an approximate solution for a certain type of Fredholm fractional integro-differential equation by using Bernstein polynomials. In the last section, some examples have been presented to compare their approximate and exact solutions.

(pp. 524–539)

### Applications of $\beta$ -open sets

Shallu Sharma, Tsering Landol, Sahil Billawria

In this paper, we establish the validity of the  $\beta$ -open sets. We introduce and study topological properties of  $\beta$ -limit point,  $\beta$ -derived set,  $\beta$ -interior points,  $\beta$ -border,  $\beta$ -frontier and  $\beta$ -exterior. The existence of their relation is also investigated with examples and counter examples.

(pp. 540–553)

### A unified generalization of some refinements of Jensen's inequality

Shou-Hua Shen, Shan-He Wu

In this paper, we establish a unified generalization of three refinements of Jensen's inequality by introducing several parameters. As applications, we illustrate that the improved Jensen's inequality can generate some new inequalities for special means such as arithmetic mean, geometric mean and logarithmic mean.

(pp. 554–560)

### Improvements of Hölder's inequality via Schur convexity of functions

Huan-Nan Shi, Shan-He Wu, Dong-Sheng Wang, Bing Liu

In this paper, we study the Schur convexity of some functions associated with Hölder's inequality, the results obtained are then used to establish the refined versions of Hölder's inequality under certain specified conditions. At the end of the paper, applications to inequalities for special means are given.

(pp. 561–576)

### On nodal filter theory of EQ-algebras

Jie Qiong Shi, Xiao Long Xin

In this article, we mainly focus on a new kind of filter on EQ-algebras. At first, we introduce some new concepts of seminodes, nodes and nodal filters ( $n$ -filters, for short) on EQ-algebras and investigate the relationships among them and some other elements. Also, we investigate their lattice structures and obtain that the set  $\mathcal{SN}(E)$  of all seminodes on an EQ-algebra is a Hertz-algebra and a Heyting-algebra under some conditions. Then, we discuss the properties of  $n$ -filters and show that there is a one-to-one correspondence between nodal principle filter and node element in an idempotent EQ-algebra. Furthermore, the relationships among it and other filters are presented. It is proved that each obstinate filter or each (positive) implicative filter is an  $n$ -filter under some conditions. At last, we introduce the algebraic structures and topological structures of the set of all  $n$ -filters on EQ-algebras and prove that  $(NP(E), \tau)$  is a compact  $T_0$  space. Moreover, we set up the connections from the set  $NF(E)$  of all  $n$ -filters on an EQ-algebra to other algebraic structures, like BCK-algebras, Hertz algebras and so on.

(pp. 577–601)

### Multiset group and its generalization to $(A, B)$ -multiset group

Suma P, Sunil Jacob John

Multiset groups are multisets with its elements taken from a group and the characteristic function of the multiset satisfying certain conditions. Apart from the definition and examples of multiset groups, we try to explain some properties, that a multiset should satisfy in order to become a multiset group. From this point, we broaden the concept of multiset group to a new scenario,  $(A, B)$ -multiset group, where  $A$  and  $B$  are non negative real numbers. The multiplicity of the identity element  $e$  has its own importance in an  $(A, B)$ - multiset group. The count value of the elements depends largely on the values of  $A$  and  $B$ . We have also delved upon the peculiarities of an  $(A, B)$ - multiset group drawn from a cyclic group and defined and explored an  $(A, B)$ - multiset normal group and cosets of  $(A, B)$ - multiset group.

(pp. 602–617)

### Sensitivity analysis of interest rate derivatives in a normal inverse Gaussian Lévy market

A.M. Udoye, G.O.S. Ekhaguere

Abrupt happenings in financial markets contribute to jumps of different magnitudes that invariably affect interest rate derivatives. Many of the existing interest rate models do not capture jumps, leading to inaccurate prediction of option prices and sensitivity analysis in the markets. To incorporate jumps in interest rate derivatives, we extend the Vasicek model with a Brownian motion as an underlying process to a model driven by a normal inverse Gaussian process, which is a subordinated Lévy process, use the extended model to obtain an expression for the price of an interest rate derivative called a zero-coupon bond. We employ Malliavin calculus to compute the greeks *delta* and *vega* of the derived price, which are important risk quantifiers in the interest rate derivative markets driven by a normal inverse Gaussian process.

(pp. 618–638)

### Real hypersurfaces in nonflat complex space forms with Lie derivative of structure tensor fields

Wenjie Wang

In this paper, we obtain some non-existence theorems for real hypersurfaces in nonflat complex space forms such that the structure tensor fields are of Lie Codazzi, Lie Killing or Lie recurrent type.

(pp. 639–648)

### On improved Heinz inequalities for matrices

Yaoqun Wang, Xingkai Hu, Yunxian Dai

In this paper, we improve some Heinz inequalities for matrices by using the convexity of function. Theoretical analysis shows that new inequalities are refinement of the result in the related literature.

(pp. 649–654)

### Schur convexity of a function whose fourth-order derivative is non-negative and related inequalities

Yiting Wu, Qing Meng

In this paper, we study the Schur convexity of a function containing variable upper and lower limit of integration, we prove that the function is Schur-convex if its fourth-order derivative is non-negative. Finally, we use the obtained result to derive an inequality of Hermite-Hadamard type.

(pp. 655–662)

Finite groups of order  $p^3qr$  in which the number of elements of maximal order is  $p^4q$

Qingliang Zhang, Zhilin Qin

Suppose that  $G$  is a finite group. As is known to all, the order of  $G$  and the number of elements of maximal order in  $G$  are closely related to the structure of  $G$ . This topic involves Thompson's problem. In this paper we classify the finite groups of order  $p^3qr$  in which the number of elements of maximal order is  $p^4q$ , where  $p < q < r$  are different primes.

(pp. 663–671)

Derivative-based trapezoid rule for a special kind of Riemann-Stieltjes integral

Weijing Zhao, Zhaoning Zhang

This paper adopts the concept of algebraic precision to construct the derivative-based trapezoid rule for a special kind of Riemann-Stieltjes integral, which uses two derivative values at the endpoints. This kind of quadrature rule obtains an increase of two orders of precision over the trapezoid rule for the Riemann-Stieltjes integral and the error term is investigated. Finally, some numerical examples indicate the numerical superiority of the proposed approach with respect to closed Newton-Cotes formulas.

(pp. 672–681)

A Mehrotra-type algorithm with logarithmic updating technique for  $P_*(\kappa)$  linear complementarity problems

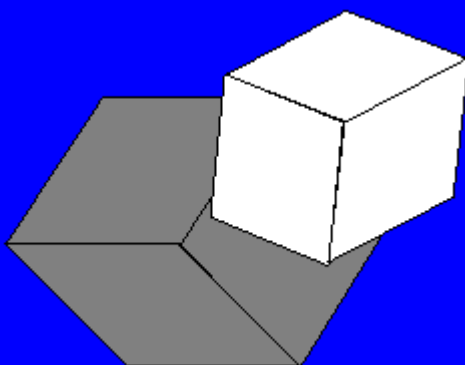
Yiyuan Zhou, Mingwang Zhang, Fangyan Huang

A Mehrotra-type predictor-corrector algorithm for  $P_*(\kappa)$  linear complementarity problems is presented. In this algorithm, the corrector step takes a new direction, and the barrier parameter is the smaller positive root of a logarithmic equation. The iteration complexity of the new algorithm matches the currently best-known results. Numerical results show that the algorithm is efficient.

(pp. 682–695)



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