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Novel properties of neighbourly edge irregular interval-valued neutrosophic graphs

## M. Ghassemi, A.A. Talebi

In this paper, some types of edge irregular interval-valued neutrosophic graphs such as neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs are introduced. A comparative study between neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs is done. Likewise some properties of them are studied.
(pp. 1-18)
Strongly regular relation and $n$-Bell groups derived from it
Aazam Nokhbeh, Ali Iranmanesh, M.M Zahedi
A new strongly regular relation $\theta_{n}^{*}$ is defined on polygroup $P$ such that the quotient $P / \theta_{n}^{*}$, the set of all equivalence classes, is a Bell group for $n \in\{2,3\}$.
(pp. 19-28)
Applications of extended Hadamard $K$-fractional integral
Muhammad Khurshed Azam, Shan E. Farooq, Farooq Ahmad, Muhammad Iqbal, Khurram Shabbir, Muhammad Aziz Rehman

In this paper, we use the extended Hadamard $k$-fractional integral to obtain some new fractional integral inequalities by introducing the new parameters $s$ and k . These extended fractional integral inequalities also hold true for usual Hadamard fractional integral when we substitute $k$ is equal to one and s is equal to zero.

Convergence of a modified PRP conjugate gradient method with a new formula of step-size

Kh. Bouaziz, T. Bachawat
We present in this paper the global convergence of a modified PRP (Polak-Ribière-Polyak) conjugate gradient method suggested by Min and Jing [11], by using a new formula of step-size that combination by Wu [14], and by Sun and colleagues $[3,12]$. Some numerical results are also presented.
(pp. 45-53)

Nonexistence of global solutions of some nonlinear ultra-parabolic equations on the Heisenberg group

Lamairia Abd Elhakim
This article provides sufficient conditions for non existence Global weak solutions for non-local and non-linear equivalent equations on $\mathbb{H}^{\mathbb{N}} \times(0, \infty) \times(0, \infty)$, where $\mathbb{H}^{\mathbb{N}}$ is the Heisenberg group. Our method of proof relies on a suitable choice of a test function and the weak formulation approach of the sought for solutions.
(pp. 54-70)

Neural dynamic optimization algorithm based on event triggered algorithm and its application
J. Chen, J. Hu, Y. Li, Y. Feng, B.O. Onasanya

Wireless sensor networks consist of microprocessor controlled sensors that communicate with each other over multi-hop communication networks. In WSN, the energy consumption of sensor networks for communication can be obviously bigger than the energy required to operate computation that would bring us unimaginable benefits if communication and computation between each node can be somehow isolated. In this paper, a neurodynamic optimization approach is proposed based on the event-triggered algorithm for handling standard NUM problem in WSN. We first confirm that the equilibrium point set of the designed neural network model based on event-triggered algorithm corresponds to the optimal solution of the NUM problem. Then, it is proved that the proposed neural network model is stable in the sense of Lyapunov and is convergent to the optimal solution. Finally, a numerical example is provided to illustrate the performance of the proposed neural network.
(pp. 71-81)

A note on extended Hurwitz-Lerch Zeta function
M. Ghayasuddin, N. U. Khan, Waseem A. Khan, Moin Ahmad

In the present research note, we introduce another extension of Hurwitz-Lerch Zeta function (HLZF) by using the generalized extended Beta function defined by Parmar [7]. We investigate its integral representations, Mellin transform, generating relations and differential formula. In view of diverse applications of the Hurwitz-Lerch Zeta functions, the results presented here are potentially useful in some other related research areas.
(pp. 82-89)

Global stability analysis and persistence for an ecological food web-model
R. Mudar Hussien, K. Qahtan Al-Jubouri, N.M. G. Al-Saidi, F. Nazarimehr

The ecological food web problems and their impact on the environment play vital role for balancing of some environments in our daily life. In the present work, the analytic results of an ecological food web-model are rigorously examined and analyzed. The model includes interactions and natural variables occur in different organisms of the species that influence by the competition and refuge as two basic conditions. The persistence of variant species for the resources competition is also analyzed. The global asymptotic stability of the positive equilibrium points is investigated numerically based on the Runge-Kutta predictor-corrector algorithm. Finally, the effects of the variation of each parameter on the proposed model are inspected numerically.
(pp. 90-103)
On bi-univalent functions involving Srivastava-Attiya operator
Mazin Sh. Mahmoud, Abdul Rahman S. Juma, Raheam A. Al-Saphory
Two subclasses $L \sum_{\Sigma}^{b, \delta}(\mu, \alpha)$ and $L \sum_{\Sigma}^{b, \delta}(\mu, \beta)$ of the class $\sum$ of Bi-univalent functions have been introduced by making use of the Srivastava-Attiya operator. The estimates of the coefficients $\left|a_{2}\right|$ and $\left|a_{3}\right|$ of functions have been found for these subclasses. The results obtained are quit interesting and new.
(pp. 104-112)
On a complex Matsumoto space
K.S. Venkatesha, S.K. Narasimhamurthy

In this paper we studied complex Matsumoto space. The expressions for fundamental metric tensor, angular metric tensor, Chern-Finsler connection coefficients and the formula of holomorphic curvature are obtained.

On closedness of rectangular bands and left[right] normal bands
Shabnam Abbas, Wajih Ashraf, Rizwan Alam
In this paper, first we have shown that the variety of rectangular bands is closed in the variety of all left[right] semiregular bands. Further, we have shown that the variety of left[right] normal bands are closed in some containing varieties of semigroups defined by the identities $a x y=a^{n} \operatorname{yax}\left[a x y=x y a y^{n}\right]$ and $a x y=$ $a y a^{n} x\left[a x y=x y^{n} a y\right]$, where $(\mathbf{n} \in \mathbb{N})$.
(pp. 127-137)

Junction interface conditions for asymptotic gradient full-observer in Hilbert space

Raheam A. Al-Saphory, Zinah A. Khalid, Mahmood K. Jasim
The fundamentals concept of boundary asymptotic gradient observer of full order type $\partial \Omega A G F O$-observer via internal case in link with the strategic sensors in different system domains have been presented. The results so obtained for linear dynamical systems which is created by a strongly continuous semi-group ( $S C S$-group) in Hilbert space $H^{1 / 2}(\partial \Omega)$ have been analyzed. Consequently, the existence of sufficient conditions for $\partial \Omega A G F O$-estimator in parabolic infinite dimensional systems have been studied and scrutinized. In addition to that, we have observed at the junction interface that the interior solution is harmonized with the exterior solution for asymptotic gradient full observation.
(pp. 138-153)
Fixed point theorems for monotone mappings on partial $M^{*}$-metric spaces
Alsauodi, Gharib, Malkawi, Rabaiah, Shatanawi
In this paper, we introduce the concept of partial $M^{*}$-metric on a nonempty set $X$, and we give some properties supported by some examples to illustrate our results. Furthermore, we establish some fixed points results for partial $M^{*}$-metric. Also, we extend our result for monotone mappings on partial $M^{*}$-metric spaces.

Certain classes of meromorphic functions by using the linear operator
Sultan Abdullah Alzahrani, Maslina Darus
In this paper, we introduce a new certain differential operator $A_{\lambda}^{n} f(z)$ with subclass $S_{p}^{*}(\alpha, \lambda, n, \beta)$ for functions of the form $f(z)=\frac{1}{z^{p}}+\sum_{k=1}^{\infty} a_{k} z^{k}$. For functions in $S_{p}^{*}(\alpha, \lambda, n, \beta)$, we give coefficient inequalities, distortion theorem, radii of starlikeness and convexity.
(pp. 173-181)

The total graph of a commutative ring with respect to multiplication

## Mohammad Ashraf, J.H. Asalool, Mohit Kumar

Let $R$ be a commutative ring with $1 \neq 0, Z(R)$ be the set of zero-divisors of $R$, and $\operatorname{Reg}(R)$ be the set of regular elements of $R$. In this paper, we introduce and investigate the dot total graph of $R$ and denote by $T_{Z(R)}(\Gamma(R))$. It is the (undirected) simple graph with all elements of $R$ as vertices, and any two distinct vertices $x, y \in R$ are adjacent if and only if $x y \in Z(R)$. The
graph $T_{Z(R)}(\Gamma(R))$ is shown to be connected and has a small diameter of at most two. Furthermore, $T_{Z(R)}(\Gamma(R))$ divides into two distinct subsets of $R$, i.e., $Z(R)$ and $\operatorname{Reg}(R)$. Following that, the connectivity, clique number, and girth of the graph $T_{Z(R)}(\Gamma(R))$ were investigated. Finally, the traversability of the graph $T_{Z(R)}(\Gamma(R))$ is investigated.

On nonsolvability of exponential Diophantine equations via transformation to elliptic curves
R.J.S. Mina, J.B. Bacani

Exponential Diophantine equations of the form $p^{X}+q^{Y}=Z^{2}$, with unknowns $(X, Y, Z)$ in the set of positive integers, are of interest to many number theorists. Many of these equations are solved using congruence techniques and the quadratic reciprocity. The goal of this paper is to show unsolvability of some Diophantine equations of this type using the concept of elliptic curves. Similar types of exponential Diophantine equations are also considered in this study. To illustrate the results, examples are provided.

A nonmonotone damped Gauss-Newton method for nonlinear complementarity problems

Li Dong
The damped Gauss-Newton methods have been successfully applied to solve the nonlinear complementarity problem (NCP). This class of methods is usually designed based on a monotone Armijo line search. In this paper, we propose a damped Gauss-Newton method with a nonmonotone line search to solve the NCP. Without requiring any problem assumptions, we prove that the proposed method is well defined and it is globally convergent. Moreover, under the nonsingularity assumption, we show that the proposed method is locally superlinearly/quadratically convergent. Some numerical results are reported.
(pp. 206-215)
$e$-semicommutative modules
Hanan Abd-Elmalk
$e$-semicommutative modules and their related properties. Also, we investigate some extensions of rings and modules in terms of $e$-semicommutativity.
(pp. 216-224)

The quasi frame and equations of non-lightlike curves in Minkowski $\mathbb{E}_{1}^{3}$ and $\mathbb{E}_{1}^{4}$
H. K. Elsayied, A. M. Tawfiq, A. Elsharkawy

The quasi frame is an alternate frame to the Frenet-Serret frame but it is defined when the second derivative of the curve vanishes. It has the same behavior as a parallel transport frame but is easier in computation and has the same accuracy. In this paper, we investigate the quasi frame and equations of non-lightlike curves in 3-dimensional Minkowski space $\mathbb{E}_{1}^{3}$ and in 4-dimensional Minkowski space-time $\mathbb{E}_{1}^{4}$. Furthermore, we show the quasi frame can be considered as a generalization of Bishop frame in $\mathbb{E}_{1}^{3}$ and $\mathbb{E}_{1}^{4}$.
(pp. 225-239)
Upper and lower estimates for products of two hyperbolic $p$-convex functions
Nashat Faried, Mohamed S. S. Ali, Zeinab M. Yehia
In this study, we obtain upper and lower estimates for product of two hyperbolic $p$-convex functions, which is analogous to Hermite-Hadamard type inequalities for product of two hyperbolic $p$-convex functions.
(pp. 240-248)
Chaotic dynamics of the Duffing-Holms model with external excitation
Guo Feng, Yu Hong, Ding Yin
The dynamics of the Duffing-Holms model are researched, and the critical conditions for chaos of the model with external excitation are obtained using Melnikov method. The expression of Melnikov function is given. The results show that the criteria obtained for chaos motion in the sense of the Smale horseshoe is consistent with that obtained by the numerical simulation. Research shows the Melnikov function is an effective analytical method to judge the occurrence of chaotic motion.
(pp. 249-260)
Strongly $m$-system and strongly primary ideals in posets

## J. Catherine Grace John, B. Elavarasan

In this paper, we study and establish some interesting results of strongly prime ideal and strongly $m$-system in posets. Also, we study the notion of strongly primary ideals in posets and show some properties of the set $\sqrt{I}=\left\{x: L(x)^{*} \cap\right.$ $I \neq \phi\}$ for ideal $I$ of $P$.
(pp. 261-270)

Bivariate extension of $\lambda$-hybrid type operators
Md. Heshamuddin, A. Kumar, N. Rao, M. Shadab, A. Srivastava

In this manuscript, we develop a bi-variate extension of hybrid type operators. We discuss the order of approximation via modulus of continuity, Peetre's K-functional,the rate of convergence, Lipschitz maximal functions and Voronovskaja type result. In addition to this, we investigate global approximation results. In the last section, we study the approximation properties of the operators in Bögel-spaces in terms of mixed-modulus of continuity.
(pp. 271-292)
Topological approaches for generalized multi-granulation rough sets with applications
S. Hussein, A.S. Salama, A.K. Salah

Methods of data classifications are considered as a major preprocessing step for pattern recognition, machine learning, and data mining. In this paper, we give two topological approaches to generalize multi-granular rough sets using families of binary relations. In the first approach, we define a family of topological spaces using families of relations to maximize the interiors and minimize the closures. In the second approach we define minimal neighborhoods to classify multi-data of information systems and generate a multi-granular knowledge base. Moreover, we present some important algorithms to reduce all topological reductions of the information system using topological bases. We round off by studying real life applications of this work using medical data.
(pp. 293-311)
On the oscillatory behavior of a class of fourth order nonlinear damped delay differential equations with distributed deviating arguments

## S. Janaki, V. Ganesan

The present study concerns the oscillation of a class of fourth-order nonlinear damped delay differential equations with distributed deviating arguments. We offer a new description of oscillation of the fourth-order equations in terms of oscillation of a related well studied second-order linear differential equation without damping. Some new oscillatory criteria are obtained by using the generalized Riccati transformation, integral averaging technique and comparison principles. The effectiveness of the obtained criteria is illustrated via example.
(pp. 312-323)

The shortest path problem on chola period built temples with Dijkstra's algorithm in intuitionistic triangular neutrosophic fuzzy graph
K. Kalaiarasi, R. Divya, Vishnu Narayan Mishra

In this article, Intuitionistic Triangular Neutrosophic Fuzzy Graph of Shortest Path Problem was Inaugurated, which is drew on triangular numbers and Intuitionistic Neutrosophic Fuzzy Graph. Real-world application is given as an illustrative model for Intuitionistic Triangular Neutrosophic Fuzzy Graph. Here we introduced famous chola period temples. These types of temples builted in various king of cholas. Here we assume only seven types of temples as vertices of Intuitionistic Triangular Neutrosophic Fuzzy Graph. Use of fuzzification method, edge weights of this Graph was calculated. Score function of Intuitionistic Triangular Neutrosophic Fuzzy Graph is inaugurated, with the help of this score function in the proposed algorithm, shortest way is determined.. This present Chola period temples Shortest Path Problem. Obtained shortest path is verified through Dijkstra's Algorithm with the help of Python Jupyter Notebook (adaptation) programming.
(pp. 324-344)
Invariant approximation property under group passes to extensions with a finite quotient

## Kankeyanathan Kannan

Analytic properties of invariant approximation property, studies analytic techniques from operator theory that encapsulate geometric properties of a group. also we show that the invariant approximation property passes to finite extensions.

Error estimates of two-grid method for second-order nonlinear hyperbolic equation

Keyan Wang, Qisheng Wang
In this paper, the full discrete scheme of mixed finite element approximation is introduced for second-order nonlinear hyperbolic equation. In order to deal with the nonlinear mixed-method equations efficiently, a two-grid algorithm is considered. Numerical stability and error estimate are proved on both the coarse grid and fine grid. It is shown that the two-grid method can achieve asymptotically optimal approximation as long as the mesh sizes satisfy $h=\mathcal{O}\left(H^{(2 k+1) /(k+1)}\right)$. Some numerical results are provided to confirm the theoretical analysis.
(pp. 362-381)

Approximation properties of $(p, q)$ bivariate Szász Beta type operators
Shuzaat Ali Khan, Nadeem Rao, Taqseer Khan
In the present research article, we construct a new sequence of bivariate $(p, q)$ hybrid type operators using $(p, q)$ - beta functions via Dunkl analogue. In the subsection sequence, we investigate the rate of convergence and the order of approximation for these sequences positive linear operators. Further, we study local approximation results in various class of functions. In the last section, we give the global approximation results using weight function.
(pp. 382-399)

Common fixed point theorems for four self maps satisfying generalized $(\psi, \phi)$-weak contraction in metric space

Manoj Kumar, Preeti, Poonam
In this manuscript, we shall prove a common fixed point theorem for four weakly compatible self-maps $P, Q, R$ and $S$ on a metric space $\left(M, d^{*}\right)$ satisfying the following generalized $(\psi, \phi)$-weak contraction:

$$
\psi\left(d^{*}(R u, S v)\right) \leq \psi(\Delta(u, v))-\phi(\Delta(u, v))
$$

where

$$
\begin{aligned}
\Delta(u, v) & =\max \left\{d^{*}(R u, S v), d^{*}(R u, P u), d^{*}(S v, Q v)\right. \\
& \frac{1}{2}\left[d^{*}(P u, S v)+d^{*}(Q v, R u)\right] \\
& \frac{d^{*}(P u, R u) d^{*}(Q v, S v)}{1+d^{*}(R u, S v)}, \frac{d^{*}(P u, S v) d^{*}(Q v, R u)}{1+d^{*}(R u, S v)}, \\
& \left.d^{*}(R u, P u)\left[\frac{1+d^{*}(R u, Q v)+d^{*}(S v, P u)}{1+d^{*}(R u, P u)+d^{*}(S v, Q v)}\right]\right\}
\end{aligned}
$$

Also, we have proved common fixed point theorems for the above mentioned contraction using weakly compatible self-maps along with E.A. property and (CLR) property. An illustrative example is also provided to support our results.

Non-cancellation group of a direct product
Rugare Kwashira, Jules Clement MBA
The non-cancellation set of a group $G$, denoted by $\chi(G)$, is defined to be the set of all isomorphism classes of groups $H$ such that $G \times \mathbb{Z} \cong H \times \mathbb{Z}$. While investigating when $\mathbb{Z}$ can be cancelled in this direct product, $\chi(G)$ has become
the focus of many studies. For the semidirect product $G_{i}=\mathbb{Z}_{n_{i}} \rtimes_{\omega_{i}} \mathbb{Z}, i=1,2$, methods for computation of the non-cancellation groups $\chi\left(G_{1} \times G_{2}\right), \chi\left(G_{i}^{k}\right)$, $k \in \mathbb{N}$ and $\chi\left(G_{i}, h_{i}\right)$ have been developed. We present in this study, a general method of computing $\chi\left(G_{1} \times G_{2}, h\right)$, where $h: F \hookrightarrow G_{1} \subseteq G_{1} \times G_{2}$ and $F$ a finite group.
(pp. 417-425)
On the semiring variety generated by $B^{0},\left(B^{0}\right)^{*}, N_{2}, T_{2}, Z_{2}, W_{2}$
Aifa Wang, Lili Wang
In this paper, we study the semiring variety generated by $B^{0},\left(B^{0}\right)^{*}, N_{2}, T_{2}, Z_{2}, W_{2}$. We prove that this variety is finitely based and prove that the lattice of subvarieties of this variety is a distributive lattice of order 1014. Moreover, we deduce this variety is hereditarily finite based.

Fixed point theorem for $(\phi, F)$-contraction on $C^{*}$-algebra valued partial metric spaces

## Hafida Massit, Mohamed Rossafi

Recently, a new type of mapping called ( $\phi, F$ )- contraction was introduced in the literature as a generalization of the concepts of contractive mappings. This present article extends the new notion in $C^{*}$-algebra valued partial metric spaces and establishing the existence and uniqueness of fixed point for them. Nontrivial examples are further provided to support the hypotheses of our results.
(pp. 439-453)
The structure of $\left(\theta_{1}, \theta_{2}\right)$-isoclinism classes of groups
A.M.Z. Mehrjerdi, M.R.R. Moghaddam, M.A. Rostamyari

In 1940, Philip Hall introduced the concept of isoclinism among all groups, and it is generalized to a more general notion called isologism. This concept is isoclinism with respect to a given variety of groups. The equivalence relation of isologism partitions the class of all groups into families.

In this article, we introduce a kind of isoclinism with respect to $\theta$-centre, $Z^{\theta}(G)$, and right $\theta$-commutator subgroup $K^{\theta}(G)$, for some automorphism $\theta$ of the group $G$, and we investigate some of its properties.

Hyper BCK-hashing algorithm: employing encoding system based on logical algebra in enhancing the secure hash algorithms

Hussein A. Jad, Samy M. Mostafa, Mokhtar A. Abdel Naby, Bayumy A. B. Youssef, Mona S. Kashkoush, Ashraf A. Taha

Cryptographic algorithms perform essential functions to generate data from digital form to comprehensible patterns such that the permitted user is the only one who can understand the message. In this study, we propose Hyper BCK-Hashing (HBCK-HASHING) Algorithm based on a hyper BCK-valued function and hash function (SHA-2). It targets to enhance the Secure Hash algorithms (SHA-2) with an algorithm of hyper BCK-valued function which based on the redundant encoding to maximize the security level of the cryptographic process of n-ary block codes ( U ) through maximize the quantity of information with the fewest number of visible characteristics. The redundant encoding based on making a unique - identified HBCK-algebra (H) for $n$-ary block codes ( U ) with applying the hyper BCK-valued function on $(H)$ to generate n-ary block $\operatorname{codes}\left(U_{H}\right)$. In addition, we perform the computational Secure Hash algorithms on $\left(U_{H}\right)$ to map the size of $n$-ary block codes $\left(U_{H}\right)$ into a fixed size. The proposed algorithm was evaluated by using the avalanche effect parameter in comparison with the Secure Hash algorithm (512 and 256). Experimental outcomes indicate that the HBCK-HASHING algorithm shows a significant-high.

A note on $b$-generalized derivations in rings with involution
M.R. Mozumder, A. Abbasi, N.A. Dar, A. Fosner, M.S. Khan

Let $R$ be a ring with involution $*$. The purpose of this paper is to investigate the special type of mappings defined on $(R, *)$. In fact it is shown that these mappings are actually the $b$-generalized derivation defined on $R$.
(pp. 482-489)
Quasi-metric hyper dynamical systems
SH. Niknezhad, M.R. Molaei
We start this paper by introducing the concept of quasi-metric hypergroups. We show that the product of two quasi-metric hypergroups is a quasi-metric hypergroup. Quasi-metric hyperdynamical systems are defined, and a method for constructing quasi-metric hyperdynamical systems via two given quasi-metric hyperdynamical systems, is deduced. Attracting sets for quasi-metric hyperdynamical systems are considered. A method for constructing quasi-metric hyperdynamical systems with attracting sets via two given quasi-metric hyperdynamical systems with attracting sets, is presented.
(pp. 490-495)

Efficient block approach for the numerical integration of higher-order ordinary differential equations with initial values
E. Oluseye Adeyefa, O. Olayemi Olanegan

This paper considered an innovative procedure to numerically approximate higher-order Initial Value Problems (IVPs) of Ordinary Differential Equations (ODEs). The proposed method is a one-step, self-starting Block integrator method employed to approximate higherorder (Third, Fourth, and Fifth-order) IVPs without reduction to lower order. The method was developed through collocation and interpolation approach. The basic properties of the method such as convergence, consistency, zero stability, order and error constant are well investigated. The accuracy of the method over existing methods are validated by numeral experiments. The method produces more interesting and superior results when compared to some existing numerical methods in terms of accuracy and absolute errors.

> (pp. 496-513)

On a class of Lorentzian paracontact metric manifolds
K. L. Sai Prasad, S. Sunitha Devi, G.V.S.R. Deekshitulu

In this present paper, we consider a class of Lorentzian almost paracontact metric manifolds namely Lorentzian para-Kenmotsu (briefly $L P$-Kenmotsu) manifolds admitting a pseudo-projective curvature tensor $\bar{W}(X, Y)$. We study and have shown that the scalar curvature of Lorentzian para-Kenmotsu manifold is constant if and only if the time like vector field $\xi$ is harmonic, whenever the $L P$-Kenmotsu manifold satisfying $R(X, Y) \cdot \bar{W}=0$ is not an Einstein manifold. Further we have shown that Lorentzian para-Kenmotsu manifolds admitting an irrotational pseudo-projective curvature tensor and a conservative pseudoprojective curvature tensor are an Einstein manifolds of constant scalar curvature. At the end, we construct an example of a 3 -dimensional $L P$-Kenmotsu manifold admitting a pseudo-projective curvature tensor which verifies the results discussed in the present work.

Roughness of soft sets over a semigroup
A. A. Ramadan, E. H. Hamouda, S. Amira

In this study we discuss the concept of rough soft sets over a semigroup. Basic results of the lower and upper approximations of soft semigroups, soft ideals, soft bi-ideals and soft interior ideals over a semigroup with a congruence relation are introduced. Finally, topological structures of rough soft sets are presented.

On the primary-like dimension of modules
Fatemeh Rashedi
Let $R$ be a ring and let $M$ be a left $R$-module. In this article, we introduce and study the primary-like dimension of $M$ was defined to be the supremum of the lengths of all strong-like chains of primary-like submodules of M and denoted by P.L. $\operatorname{dim}(M)$.

A characterization of $\operatorname{PSL}\left(4, p^{2}\right)$ by some character degree
Younes Rezayi, Ali Iranmanesh
Let $G$ be a finite group and $\operatorname{cd}(G)$ be the set of irreducible character degree of $G$. In this paper we prove that if $p$ is a prime number, then the simple group $\operatorname{PSL}\left(4, p^{2}\right)$ are uniquely determined by its order and some its character degrees.

Separation coordinates in a Hamiltonian quartic system
Nicola Sottocornola
The separability of Hamiltonian integrable systems has been the object of a considerable amount of attention in the last decades. Over the years several techniques have been proposed to deal with this difficult problem. In this paper we make use of the method of the Kowalewski's Conditions. To illustrate the effectiveness of the method we consider the Hénon-Heiles system known as HH4 $1: 6: 8$. This system is integrable in two cases. For one of them, separated only in some particular cases, we provide the separation coordinates in the generic form. The other case remains unsolved.

A mathematical model of COVID-19 transmission dynamics with treatment and quarantine

## H. Tessema Alemneh, I. Haruna, G. Teshome Tilahun

Corona-virus disease (COVID-19) is caused by the novel-virus (SARS-COV2). This disease mainly targets human respiratory system. COVID-19 (Coronavirus) has affected day to day life and is slowing down the global economy. This pandemic has affected thousands of peoples, who are either sick or are being killed due to the spread of this disease. In this paper we developed an eight compartmental model with quarantine and treatment of COVID-19. After proposing the model, we analysed the qualitative behaviors of the model, like the disease free and endemic equilibrium points and their stability analysis.

Moreover, we obtained the basic reproduction number using next-generation matrix method and we performed the sensitivity analysis to identify the most affecting parameters in terms of disease control and spreed. To investigate the detail effect of each major parameters, we performed numerical simulation. We obtained that using both quarantine and treatment is best way to combating COVID-19 in the community. Therefore, stakeholders and policy makers should work both quarantine and treatment simultaneously in combating the pandemic from the population.
(pp. 567-583)
Ruled hypersurfaces in nonflat complex space forms satisfying Fischer-Marsden equation

## Wenjie Wang

In this paper, we prove that there exist no ruled hypersurfaces in a nonflat complex space form satisfying the Fischer-Marsden equation. This answers partially an open question posed by Venkatesha et al. in (Ann. Univ. Ferrara, 67 (2021), 203-216).
(pp. 584-590)
A unified generalization and refinement of Hermite-Hadamard-type and Simpson-type inequalities via $s$-convex functions

Yiting Wu, Qiuyue Li
In this paper, by introducing the incomplete beta function, we establish a multiparameter integral inequality via $s$-convex functions, which provides a unified generalization and refinement of Hermite-Hadamard-type and Simpson-type inequalities. As applications, we illustrate that a number of Hermite-Hadamardtype and Simpson-type inequalities can be derived from the special cases of the main result.
(pp. 591-605)

Oscillation criteria of fractional damped differential equations
Yunhui Zeng, Zhihong Wang, Anning Wang
In this paper, we prove some properties of oscillation for a class of fractional damped differential equations using generalized Riccati transformation and inequality technique, we prove some new oscillatory criteria. Recent results in the literature are generalized and significant improved. Example is shown to illustrate our main results.
(pp. 606-618)

Hedges in quasi-pseudo- $M V$ algebras
Zhaoying Chen, Wenjuan Chen
In this paper, we introduce the notions of multiplicative interior operators (mioperators, for short), additive closure operators (ac-operators, for short) and hedges in quasi-pseudo-MV algebras which will generalize the related contents in pseudo-MV algebras. First we discuss the relationship between mi-operators and ac-operators in a quasi-pseudo-MV algebra and investigate the properties of mioperators in quasi-pseudo-MV algebras. Second we define and study hedges in quasi-pseudo-MV algebras. We also show that mi-operators are hedges. Finally, the properties of filters and weak filters in a quasi-pseudo-MV algebra with hedge are discussed.

Congruence-free restriction semigroups
Zhiqin Zhang, Junying Guo, Xiaojiang Guo
Restriction semigroups are common generalizations of ample semigroups and inverse semigroups. The main aim of this paper is to probe restriction semigroups with certain congruence properties. In this paper we give some characterizations of restriction semigroups each of whose proper ( $2,1,1$ )-congruences are reduced, so called H-reduced restriction semigroups. In particular, the classification of congruence-free restriction semigroups is obtained; that is, it is proved that a restriction semigroup is congruence-free if and only if it is either a simple group or an H-reduced restriction semigroup without nontrivial reduced restriction monoid ( $2,1,1$ )-congruences. These results extend and enrich the related results of inverse semigroups.

A new single step hybrid block algorithm for solving fourth order ordinary differential equations directly

Raft Abdelrahim, Walid Hdidi
In this paper, a one-step hybrid block method with generalized three off-step points for solving general fourth order ordinary differential equations is developed using power series of order eight as a basis function. The technique employed for the derivation of this method are to interpolate the power series at $x_{n}$ and all off-step points and to collocate the fourth derivative of the basis function at all points in the selected interval. The method derived is proven to be zero stable, consistent and then convergent. The performance of the method is tested by solving linear and non-linear fourth order initial value problems.
(pp. 651-668)

Acceptance sampling plans for truncated lifetime tests under two-parameter Pranav distribution

## A.I. Al-Omari, N. Alotaibi, G.A. Alomani, A.S. Al-Moisheer

The single acceptance sampling plans (SASP) are one of the main statistical tools in industry and production fields. Both of the customers and producers are interesting in the product, where the customers want a product of good quality with long life time and the producers want to keep the quality of the products with minimum cost and variation. In this study, it is supposed that the lifetime of the products follows the two parameters Pranav distribution (TPPD) and the mean is taken as a quality parameter. The necessary tables of the minimum sample size, operating characteristic (OC) function and the producer's risk values are obtained for various model parameters. Also, for applicability investigation of the suggested SASP based on TPPD, a real data set of failure times of 20 identical components is analyzed and used. It turns out that the new ASP gives minimum sample sizes and it is recommended for practitioners.
(pp. 669-686)

Inequalities of DVT-type-the one-dimensional case continued

## Barbora Batíková, Tomáš Kepka, Petr Němec

In this note, the investigation of particular inequalities of DVT-type in integer numbers is continued.

Prime-valent one-regular graphs of order $20 p$
Qiao-Yu Chen, Song-Tao Guo
A graph is one-regular and arc-transitive if its full automorphism group acts on its arcs regularly and transitively, respectively. In this paper, we classify connected one-regular graphs of prime valency and order $20 p$ for each prime $p$. As a result there is only one infinite family of such graphs, that is, the cycle $C_{20 p}$ with valency two.

Units of a class of finite rings of characteristic $p^{3}$

## Chitengá John Chikunji

Let $R$ be a commutative completely primary finite ring with Jacobson radical $\mathcal{J}$ such that $\mathcal{J}^{3}=(0), \mathcal{J}^{2} \neq(0)$ and $R / \mathcal{J} \cong G F\left(p^{r}\right)$, the finite field with $p^{r}$ elements, for any prime $p$ and any positive integer $r$. Then, characteristic of $R$
is either $p, p^{2}$ or $p^{3}$. In this paper, we determine the structure and generators of the group of units of the ring $R$ in the special case when the characteristic of $R$ is $p^{3}$. We treat the problem by considering fixed dimensions and bases for the vector spaces $\mathcal{J}^{i} / \mathcal{J}^{i+1}(i=1,2)$ over the residue field $R / \mathcal{J}$ and by fixing the order of the ideal $\mathcal{J}^{2}$. This complements the author's earlier solution to the problem in the case when the characteristic of $R$ is $p$ or $p^{2}$ and $\mathcal{J}^{2} \subseteq \operatorname{ann}(\mathcal{J})$, the annihilator of $\mathcal{J}$.
(pp. 703-712)
Solvability in fuzzy multigroup context

## Paul Augustine Ejegwa, Yuming Feng, Wei Zhang

The adventure of fuzzy sets has witnessed myriad of group's theoretical concepts being studied as fuzzy algebraic structures. In this present work, the notion of solvable fuzzy multigroups is considered for the first time as an algebraic structure in fuzzy multigroup context. Solvable series for a fuzzy multigroup is defined in such a way that the family of the fuzzy submultigroups of the considered fuzzy multigroup has the same support. Some precursory results in normality and quotient of fuzzy multigroups are considered. It is established that there exists an if and only if condition between the solvability of a fuzzy multigroup and its support. Finally, certain results on solvable fuzzy multigroups are obtained.

Total edge irregularity strength and edge irregular reflexive labeling for calendula graph
M. R. Zeen El Deen, G. Elmahdy

The calendula graph $C l_{m, n}$ is a graph constructed from a cycle on $m$ vertices $C_{m}$ and $m$ copies of $C_{n}$ which are $C_{n_{1}}, C_{n_{2}}, \ldots, C_{n_{m}}$ and pasting the $i$-th edge of $C_{m}$ to an edge of $C_{n_{i}}$ for each $i \in\{1,2, \ldots, m\}$. For a simple graph $G(V, E)$ a labeling of vertices and edges by a mapping $\Phi: V(G) \cup E(G) \rightarrow\{1,2, \ldots, k\}$ providing that the weights of any two pair of edges are distinct is called an edge irregular total $k$-labeling, where the weight of an edge is the sum of the label of the edge itself and the labels of its two end vertices. If $k$ is minimum and $G$ admits an edge irregular total $k$-labeling, then $k$ is called the total edge irregularity strength, $\operatorname{tes}(G)$. The total $k$ - labeling is called the reflexive edge strength of $G$ if the edge labeling $\Phi_{e}: E(G) \rightarrow\left\{1,2, \ldots, k_{e}\right\}$ and a vertex labeling $\Phi_{v}: V(G) \rightarrow\left\{0,2,4, \ldots, 2 k_{v}\right\}$, where $k=\max \left\{k_{e},, 2 k_{v}\right\}$. In the current paper, we investigate the existence of edge irregular total $k$ - labeling for the calendula graphs $C l_{m, n}$ and precise the exact value of total edge irregularity strength of calendula graphs $C l_{m, n}$. Besides, we explore the presence of edge reflexive irregular $r$ - labeling for calendula graphs and determine the perfect value of reflexive edge strength.

An efficient optimal fourth-order iterative method for scalar equations
Faisal Ali, Waqas Aslam, Ghulam Akbar Nadeem
In the present paper, using linear combination technique, we introduce an optimal three-step iterative scheme for solving nonlinear equations. We prove the convergence of the proposed method. In order to demonstrate the performance of newly developed method, we consider some commonly used nonlinear equations for numerical as well as graphical comparisons. We also explore polynomiographs in the context of some complex polynomials.

> (pp. 754-770)

A study on Riesz I-convergence in intuitionistic fuzzy normed spaces
Vakeel A. Khan, Ekrem Savas, Izhar Ali Khan, Zahid Rahman
The primary objective of this study is to introduce the concept of ideal convergent sequences as a domain of regular Riesz triangular matrix in the settings of intuitionistic fuzzy normed spaces(IFNS). Some properties of this notion with respect to the intuitionistic fuzzy norm are also presented in this study. We demonstrate the Riesz ideal Cauchy criterion in intuitionistic fuzzy normed spaces and later on, we show that in an arbitrary IFNS, a sequence is Riesz ideal convergent if and only if it satisfies Riesz ideal Cauchy criterion. We also present major counterexamples for the converse part of some results. Lastly, we define the notion of Riesz $I^{*}$-convergence in intuitionistic fuzzy normed spaces and establish the relationship with Riesz I-convergence in IFNS with a certain counterexample.
(pp. 771-789)

On lacunary statistical convergence of difference sequences
Ö. Kişi, M. Gürdal
In this paper, the $S_{\theta}(\Delta)$ and $N_{\theta}(\Delta)$ summabilities are used along with the notion of weakly unconditionally Cauchy series (in brief $w u C$ series) to characterize a Banach space. We examine these two kinds of summabilities which are regular methods and we recall some features. Furthermore, we investigate the spaces $S_{N_{\theta}}\left(\sum_{p} \Delta w_{p}\right)$ and $S_{S_{\theta}}\left(\sum_{p} \Delta w_{p}\right)$ which will be thought to characterize the completeness of a space.
(pp. 790-804)

On finite groups with $s$-weakly normal subgroups
Lijun Huo, Weidong Cheng
A subgroup $H$ of a group $G$ is weakly normal in $G$ if $H^{g} \leq N_{G}(H)$ implies that $g \in N_{G}(H)$ for any element $g \in G$. A subgroup $H$ of a group $G$ is $s$-weakly normal in $G$ if there exists a normal subgroup $T$ such that $G=H T$ and $H \cap T$ is weakly normal in $G$. Clearly a weakly normal subgroup of $G$ is an $s$-weakly normal subgroup of $G$. In this paper, we investigate the influence of $s$-weakly normal subgroups on the structure of a finite group, especially some criteria for supersolvability, nilpotency, formation and hypercenter of a finite group are proved. Based on our results, some recent results can be generalized easily.
(pp. 805-815)
On a maximal subgroup $\bar{G}=5^{4}:\left(\left(3 \times 2 L_{2}(25)\right): 2_{2}\right)$ of the Monster $\mathbb{M}$
D.M. Musyoka, L.N. Njuguna, A.L. Prins, L. Chikamai

The split extension $\bar{G}=5^{4}:\left(\left(3 \times 2 L_{2}(25)\right): 2_{2}\right)$ is a maximal subgroup of the sporadic Monster group $\mathbb{M}$ of order $58500000=2^{5} .3^{2} .5^{6} .13$. The technique of Fischer-Clifford matrices has been applied to numerous examples of split and non-split extensions where the kernels are either elementary abelian 2 or 3 groups but very few examples exist where the kernel is an elementary abelian 5 -group. In this paper, the Fischer-Clifford matrices technique is applied to the group $\bar{G}=5^{4}:\left(\left(3 \times 2 L_{2}(25)\right): 2_{2}\right)$, where the kernel $5^{4}$ of the extension is an elementary abelian 5 -group.

A note on $(I, J)$-e-continuous and $(I, J)$-e*-continuous functions
E. Rosas, C. Carpintero, J. Sanabria. M. Salas, L. VÁsquez

In this paper the notions of $I$-e-open set and $I-e^{*}$-open set are introduced and used to define a large number of modifications of the concept of continuous function, such as $(I, J)$-e-continuous functions, $(I, J)-e^{*}$-continuous functions, contra $(I, J)$-e-continuous functions, contra $(I, J)-e^{*}$-continuous functions, almost weakly $(I, J)$-e-continuous functions, almost weakly $(I, J)$ - $e^{*}$-continuous functions, almost $(I, J)$-e-continuous functions, almost $(I, J)$ - $e^{*}$-continuous functions, almost contra $(I, J)$-e-continuous functions and almost contra $(I, J)-e^{*}$ continuous functions. Also, several characterizations of these new classes of functions are given and finally relations between them are investigated.
(pp. 834-846)

Extended GE-filters in weak eGE-algebras
M.K. Shaik, R.K. Bandaru, Y.B. Jun

A broader concept than eGE algebra, called weak eGE algebra, is introduced, and related properties are studied. The concept of transitive and tightly (weak) eGE algebra is also considered and some properties are discussed. A weak eGEalgebra with additional conditions is used to give a way to create a GE-algebra. Extended GE filters are described in the last section. The concept of eGEfilters and upper sets is introduced and associated properties are investigated. Conditions for a superset of $E$ in a weak eGE-algebra $(X, *, E)$ to be an eGEfilter are provided. Also, conditions for the upper set to become an eGE-filter are discussed. The characterization of the eGE-filter is established.
(pp. 847-862)
A study on intuitionistic fuzzy topological operators

## Sinem Tarsuslu (Yilmaz)

In this paper, new intuitionistic fuzzy topological operators are introduced by considering Marinov and Atanassov's last operators. We show that these operators are also pair of conjugate preinterior-preclosure operators. In addition, some properties of these operators are examined.

Normal structure and the modulus of weak uniform rotundity in Banach spaces
Z. Zuo, Y. Huang

In this paper, we present some sufficient conditions for which a Banach space $X$ has normal structure in term of the modulus of weak uniform rotundity $\delta_{X}(\epsilon, f)$, the Domínguez-Benavides coefficient $R(1, X)$ and the coefficient of weak orthogonality $\omega(X)$. Some known results are improved and strengthened.
(pp. 876-884)
A note on the $p$-length of a $p$-soluble group
Huaquan Wei, Jiao Li, Huilong Gu, Yangming Li, Liying Yang
Suppose that the finite group $G=A B$ is a mutually permutable product of two $p$-soluble subgroups $A$ and $B$. By use of several invariant parameters of $A$ and $B$, we present some bounds of the $p$-length of $G$. Some known results are improved.

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