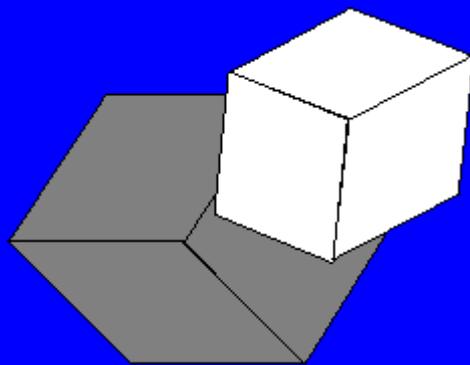


N° 47 – March 2022

# Italian Journal of Pure and Applied Mathematics

*ISSN 2239-0227*

## Papers Abstracts



**FORUM EDITRICE UNIVERSITARIA UDINESE**  
**FARE srl**

## Quadratic residues and squares of the ring of dual numbers modulo $n$

Basem Alkhamaiseh

For an integer  $n \geq 1$ , we denote the ring of dual numbers over the integers modulo  $n$  by  $\mathbb{Z}_n[\alpha]$ . The ring  $\mathbb{Z}_n[\alpha]$  is a commutative extension for the ring  $\mathbb{Z}_n$ . We introduce and study square and quadratic residue elements in  $\mathbb{Z}_n[\alpha]$ . We also find multiplicative functions that count squares and quadratic residues for the ring  $\mathbb{Z}_n[\alpha]$ . (pp. 1–11)

## Predicting Jordanian's GDP based on ARIMA modeling

O. Alsinglawi, S. Al Wadi, M. Aladwan and M. Saleh

Gross Domestic Product (GDP) is the market value of all goods and services that are produced within the country's national borders in a year, Our study aims to estimate and predict Jordanian's GDP using a time series data for the period from 1978 till 2017, the data has been taken from Jordanian's department of statistics, Minitab and Matlab statistical software's are used , we deploy a wavelet transform (WT) model to decomposes the time series data then detecting the fluctuations and outliers values, also an ARIMA (autoregressive integrated moving average) is established, the fitted ARIMA (2, 2, 1) time series model is the best for modeling the Jordanian's GDP according to the recognition rules and stationary test of time series. The results show that the predicted values are within the range of 5%, and the prediction capability of this model is relatively adequate and efficient in modeling the annual GDP for the next 20 years, thus, the prediction accuracy is considered high. It is concluded that Jordanian's GDP is an upward trend for upcoming 20 fiscal years, Furthermore, Jordanian's government has to follow more comprehensive economic policies and should implement key growth-enhancing reforms to strengthen its economy, also it has to stimulate job-creating growth and creates conditions to increase private investment and improve country's competitiveness. (pp. 12–25)

### Oscillation of fractional Emden-Fowler type neutral vector partial differential equations with mixed nonlinearities and deviating arguments

N. Nagajothi and V. Sadhasivam

The main purpose of this paper is to extend and improve existing oscillation criteria of mixed fractional order Emden-Fowler type neutral vector partial differential equations with mixed nonlinearities subject to Robin and Dirichlet boundary conditions. Several sufficient conditions are obtained for oscillation of solutions of such class of equations by using the generalized Riccati substitution and integral average method. We support our results by an illustrative example. (pp. 26–54)

### Numerical solution of nonlinear time-fractional Cable equation by finite volume element method

A. Yazdani and R. Yousefian

In this paper, we study the use of finite volume element method and see how we can employ finite element bases to numerical solution of nonlinear time fractional Cable equation in one dimension. Finite element bases are as linear polynomials which are compactly supported. Caputo in the proposed scheme constitutes the fractional derivative. This scheme will be analyzed and the stability of the method will be discussed. Furthermore, the numerical results are compared with analytical solution to ascertain how accurate the presented method is. (pp. 55–69)

### Coefficient estimates for subclasses $B_{\Sigma}^m(\alpha, \lambda)$ and $B_{\Sigma}^m(\beta, \lambda)$ of analytic and bi-univalent functions defined by a differential operator

A. TA. Yousef, Z. Salleh and Tariq Al-Hawary

In this paper two subclasses  $B_{\Sigma}(\alpha, \lambda)$  and  $B_{\Sigma}(\beta, \lambda)$  are studied and a new estimation for the initial coefficients  $|a_2|$  and  $|a_3|$  in these subclasses are found. (pp. 70–78)

### SDA-COVID-19: Social Distancing App for COVID-19 track and control

E. Abu-Taieh, I. Alhadid, H. Saqer Al-Bdour, A. Zolait, J. Abu-Tayeh et al

COVID-19 is a highly contagious and lethal virus-based disease. Social distancing is the only way to stop the virus from spreading. In this context, the researchers suggest a social distancing application named SDA-COVID-19. The suggested App (SDA-COVID-19) will help individuals maintain social distancing by the exchange of data among phones about potentially infected and/or contaminated people with COVID-19, with whom an individual socialized or came in contact, whereby, an individual will be alerted if a COVID-19 infected person is in close proximity. Two versions are suggested for SDA-COVID-19 one is Service-Oriented and the other is Bluetooth oriented. (pp. 79–90)

### On quaternion algebras that split over specific quadratic number fields

V. Acciario, D. Savin, M. Taous and A. Zekhnini

Let  $d$  and  $m$  be two distinct squarefree integers and  $\mathcal{O}_K$  the ring of integers of the quadratic field  $K = \mathbb{Q}(\sqrt{d})$ . Denote by  $H_K(\alpha, m)$  a quaternion algebra over  $K$ , where  $\alpha \in \mathcal{O}_K$ . In this paper we give necessary and sufficient conditions for  $H_K(\alpha, m)$  to split over  $K$  for some values of  $\alpha$ , and we obtain a complete characterization of division quaternion algebras  $H_K(\alpha, m)$  over  $K$  whenever  $\alpha$  and  $m$  are two distinct positive prime integers. Examples are given involving prime Fibonacci numbers. (pp. 91–107)

### An hybrid block method for direct integration of first, second and third order IVPs

E.O. Adeyefa, J.O. Kuboye, O.A. Olajide and S.A. Osikoya

Development of numerical methods for the solution of initial value problems (IVPs) in ordinary differential equations (ODEs) has been considered overwhelmingly in literature. However, the use of a single numerical method for the integration of ODEs of different order has not been commonly reported.

In this paper, we focus on development of a numerical method capable of obtaining the numerical solution of first, second and third order IVPs. The method is formulated from continuous schemes obtained via collocation and interpolation techniques and applied in a block-by-block manner as numerical integrator for first, second and third order ODEs. The convergence properties of the method are discussed via zero-stability and consistency. Numerical examples are included and comparisons are made with existing methods in the literature. (pp. 108–118)

### Ordinary differential equations of the probability functions of the Benktander kind II distribution with $b = 1$ and its properties

Ahmad M.H. Al-Khazaleh and ALI H. Handam

The idea of convolution is the sum of independent and identically distributed (iid) random variables and the structure of linear combination of random variables. The cases of ordinary differential equation (ODE) of the convolution of probability distributions of mixture of Benktander distribution of the second kind have been studied. Moreover, the ODE of quantile function (QF), survival function (SF), hazard function (HF) and reversed hazard function (RHF) of convoluted probability distributions has been considered. We obtain explicit forms for the densities and distribution functions for Benktander distribution of the second kind, as well as their moments and related parameters. We derive basic properties of these laws and illustrate their modeling possible using a simulated data. (pp. 119–134)

### [Hadamard product of meromorphic multivalent functions with positive coefficient](#)

[Osama Ogilat and Radwan Al-Smadi](#)

In the present paper, we introduced a class  $S(\gamma, \tau, \delta, p)$  of a meromorphic multivalent function and used this class of function Hadamard product (or convolution) to prove some nice and attractive properties such that the coefficient inequalities, distortion theorem, radii of starlikeness and convexity. Moreover, we verified that the class  $S(\gamma, \tau, \delta, p)$  is closed under convex linear combination. (pp. 135–146)

### [A mathematical approach for linking/integrating management plans: Marine Spatial Planning \(MSP\) and Integrated Coastal Zone Management \(ICZM\) as a case study](#)

[Reda Albotoush and Aileen Tan Shau-Hwai](#)

This research has proposed the Management Plan's Linking/Integrating Methodology (MPLIM), that is articulated around the fundamental differences between the management plans to be linked. MPLIM consists of five main steps: identification of similar sectors or actions between the two management plans; evaluating management plans; identifying and classifying the differences between the management plans into themes; analysing the differences and making recommendations; and lastly, linking all desirable sectors or actions. MPLIM approach had been justified with the help of geometrical shapes and mathematical formulae, based on the concept of union and intersection sets. However, it is the preparation for linkage that is the challenge and not the link itself, as the link is taking decisions that are in favour of all parties, supported by the harmony generated from the recommendations step. When implementing MPLIM approach on two-management plans concepts (i.e., Integrated Coastal Zone Management, ICZM and Marine Spatial Planning, MSP), it was noted that the proposed methodology could be used as a promising tool for linking/integrating various management plans. In addition, it turned out that it facilitated understanding of stubborn conceptual issues related to ICZM and MSP and their relations. (pp. 147–172)

### [Share price forecast: using ARIMA model special study on Amman Stock Exchange \(ASE\)](#)

[O. Alhwatmeh](#)

The goal of this research is to explore the events and Stock price prediction of Amman Stock Exchange (ASE) so for their scientific importance in economics for predict financial time series, using the ARIMA model for the active Participation's of stock market as well as macro decision makers to be able to predict

trend of the market. The data examined in the paper had been collected from Amman Stock Exchange (ASE) from 2012 to 2018. 15 well-known banks on the ASE have thus been selected for stock price forecasting. The data set examined information about: opening price, closing price Concerning the results of previous researches, which confirm the relative superiority of linear models in price index prediction, an appropriate model has been offered in this research ARIMA model, in order to predict price index of Amman Stock Exchange. The results show ability of a to predict stock price time series data using ARIMA model, but more variables should be studied to support results and non-impartiality.

(pp. 173–181)

#### [Internal audit practices consistency with international internal auditing standards for entities with government contributions](#)

[Omar Alhwatmeh, Mohammad Aladwan and Amneh Alkurdi](#)

The study aims to investigate the consistency of internal auditing practices with international internal auditing standards for Jordanian companies with government contributions. The study employs both qualitative and quantitative research methods in addressing the problem of the study. To achieve the study objective, the researchers distributed a questionnaire to (460) employees of (115) Jordanian companies with government contribution, (434), (90) percent of the questionnaires were returned. On order to analyze the data, the researchers used the Mean, Standard Deviation, Percentages, and T-test. The general findings of the study revealed that Jordanian companies with government contributions apply internal auditing practices that are enacted by government agencies but unfortunately these practices are not fully consistent with international internal auditing standards. the results showed weak commitment from sampled companies to such necessary standards. In the light of the finding, the researchers gave a number of recommendations that are necessary to achieve sufficient and effective internal auditing standards. the originality of this study is steamed from addressing the importance of adopting internal auditing standards for performance and their role in increasing the control over organizations activities.

(pp. 182–204)

#### [On binary block codes associated to UP-algebras](#)

[Moin A. Ansari, Ali N.A. Koam and Azeem Haider](#)

In this article, we define the notion of a UP-algebra valued function on a set and investigate related properties. We establish the binary block codes generated by UP-algebras valued function. We have also shown that for every binary block-code  $C$ , there exists a UP-algebra and UP-valued function which determines  $C$  whereas UP-algebras associated to a binary block code are not unique up to isomorphism.

(pp. 205–220)

### Behaviour of motion of infinitesimal variable mass oblate body in the generalized perturbed circular restricted three-body problem

A.A. Ansari, Laxmi Narain, Sada Nand Prasad and Mehtab Alam

The main goal of the present study is to reveal the behaviour of motion of the infinitesimal body in case of circular restricted three-body problem where all the participating bodies have oblate shapes and both primaries have the effect of solar radiation pressure. The third infinitesimal body is varying its mass according to Jeans law. We also consider that the system is affected by the small perturbations in Coriolis and centrifugal forces. We evaluate the equations of motion of the infinitesimal oblate body under the generalized sense in the perturbed circular restricted three-body problem by using the Meshcherskii-space time transformations to preserve the dimensions of the position as well as time, and then determine the Jacobi-integral. Further we numerically illustrated the equilibrium points, Poincaré surfaces of section, regions of possible and forbidden motion and then basins of the attracting domain by supposing the effects of all the parameters used. Further more, we examine the stability of these equilibrium points with the help of Meshcherskii space-time inverse transformations and found them unstable. (pp. 221–239)

### Soft ii-mappings in soft topological spaces

Sabih W. Askandar and Amir A. Mohammed

In this paper, we have presented new ideas of soft mappings from a soft topological space into another are called soft i-open, soft inter-open and soft ii-open mappings, (soft *i*-continuous, soft inter-continuous and soft ii-continuous mappings), (soft topological i-homeomorphisms, soft topological inter-homeomorphisms and soft topological ii-homeomorphisms. The relations among these concepts and some different concepts of soft mappings as soft open, soft semi-open and soft  $\alpha$ -open mappings (separately, soft continuous, soft semi-continuous and soft  $\alpha$ -continuous mappings), (separately, soft topological homeomorphisms, soft topological semi-homeomorphisms and soft topological  $\alpha$ -homeomorphisms) are examined by utilizing evidences and guides to clarify and explain it. (pp. 240–257)

### Comparison of two types of rough approximation via grill

A.A. Azzam

Rough set theory and topology are now branching far into applied areas, such as economics, data processing, imaging and chemistry. As a consequence of this importance gained from topology after the advent of a rough set theory which helps to quantify things that were previously difficult to measure. It was

important to work on the extension of the topological space with new concepts such as grill and ideal. In this paper, we present new approximations of rough sets via a grill concept which has helped to extend the topological spaces. In addition, the topology created by the present method is finer than other methods. Finally, grill topological spaces will be obtained in terms of relations and grills aimed at minimizing the boundary regions. (pp. 258–270)

### Modelling the impact of interventions against malaria-schistosomiasis co-infection dynamics

E.A. Bakare and S. Hoskova-Mayerova

Sub-Saharan Africa is known to possess the greater part of the global burden of malaria and schistosomiasis infection. The co-endemicity of these two tropical diseases has initiated the investigation into the mechanisms of their co-infection due to the competing immunological responses associated with each disease in the recent time. It is known that malaria and schistosomiasis have similar epidemiological dispersal and cause challenges to public health and socio-economic development throughout the sub-Saharan region. There are very few works done on the application of optimal control theory to the dynamics of malaria-schistosomiasis co-infection to the best of our knowledge. Our aim here is to predict the impact of the present control interventions to provide necessary information for the policy makers against future control strategies. In this regard, we proposed a malaria-schistosomiasis co-infection model using a system of compartmental deterministic non-linear ordinary differential equations. Optimal control theory was applied to examine the best control strategies against malaria-schistosomiasis disease using insecticides treated bed nets ( $u_1$ ), prevention by avoiding swimming or wading in freshwater ( $u_2$ ), treatments of malaria with artemisinin combined therapy ( $u_3$ ), treatment of schistosomiasis with praziquantel ( $u_4$ ), treatment of malaria-schistosomiasis ( $u_5$ ), biological control ( $u_6$ ) and insecticide spray, destruction of stagnant water and mosquito breeding sites ( $u_7$ ) as control interventions in line with World Health Organisation (WHO) suggestion. Scenarios on various control strategies were developed using combinations of the seven control interventions either one at a time, combinations of two at a time or more than two at a time to minimize the transmission of malaria-schistosomiasis disease. Finally, the proposed model predicted that there were reductions in the transmission dynamics of malaria-schistosomiasis co-infection in the presence of at least two or more control interventions even in areas where there are limited resources. (pp. 271–302)

### The total game chromatic number of paths, cycles and stars

H. Bharadwaj and T.A. Mangam

The total game coloring is one of the extensions of the game coloring problem in which two players alternatively color both vertices and edges of a given graph.

In this paper, the total game chromatic number is determined for the classes of paths, cycles, and stars. The incidence game chromatic number for large paths is also discussed by relating the incidence and the total graphs of paths.

(pp. 303–311)

$(p, q)$ - $\varphi$  relative Gol'dberg order and  $(p, q)$ - $\varphi$  relative Gol'dberg type oriented certain growth properties of entire functions of several complex variables

Ritam Biswas and Tanmay Biswas

In this paper our concern is to study some growth properties based on different growth indicators such as  $(p, q)$ - $\varphi$  relative Gol'dberg order,  $(p, q)$ - $\varphi$  relative Gol'dberg type etc. of entire functions of several complex variables.

(pp. 312–318)

Generalized relative order  $(\alpha, \beta)$  and generalized relative type  $(\alpha, \beta)$  oriented some growth analysis of composite analytic functions in the unit disc

Chinmay Biswas and Tanmay Biswas

In this paper we introduce the idea of generalized relative order  $(\alpha, \beta)$  and generalized relative type  $(\alpha, \beta)$  of an analytic function with respect to another analytic function in the unit disc where  $\alpha$  and  $\beta$  are continuous non-negative on  $(-\infty, +\infty)$  functions. Hence we study some growth properties relating to the composition of two analytic functions in the unit disc on the basis of generalized relative order  $(\alpha, \beta)$  and generalized relative type  $(\alpha, \beta)$  as compared to the growth of their corresponding left and right factors.

(pp. 319–327)

Generalized relative order  $(\alpha, \beta)$  and generalized relative type  $(\alpha, \beta)$  oriented some growth properties of composite entire and meromorphic functions

Tanmay Biswas and Chinmay Biswas

In this paper we wish to prove some results relating to the growth rates of composite entire and meromorphic functions with their corresponding left and right factors on the basis of generalized relative order  $(\alpha, \beta)$  and generalized relative type  $(\alpha, \beta)$  where  $\alpha$  and  $\beta$  are continuous non-negative functions defined on  $(-\infty, +\infty)$ .

(pp. 328–337)

### On some class of degenerate elliptic equations with $L^1(\Omega)$ coefficients in $P(X)$ -Sobolev spaces

M. Boukhrij, B. Aharrouch and J. Bennouna

This paper deals with a class of nonlinear degenerate elliptic problem with coefficients in  $L^1(\Omega)$  of the form  $-\operatorname{div} a(x, u, \nabla u) = f$ , where  $a(x, u, \nabla u)$  is allowed to be degenerate with the unknown  $u$ . We prove the existence of weak and entropy solutions under some hypothesis on  $f$ . We also study the same problem with a lower order term. (pp. 338–364)

### Time optimizing of fractal image compression using the Scatter Search algorithm

Shaimaa S. Al-Bundi and Nadia M.G. Al-Saidi

Image compression is one of the essential requirements for the efficient use of storage space and bandwidth. A new technique based on fractal theory is proposed for encoding the image; it is known as fractal image compression. In the procedure of encoding, the mechanism of search is considered as one of the main problems of this technique. In this work, an attempt to speed up the encoding process with minimal loss of the compressed image quality is adopted based on the Scatter Search algorithm. It is a sibling of Tabu search based on similar origins. The experimental results show a significant reduction in the computation time, where the mean square error measures between blocks are decreased after comparing them to full search methods. Consequently, the decoding process evinced that the reconstructed images were of high quality. (pp. 365–381)

### Optimum solution of time fractional coupled system of partial differential equations

Samia Bushnaq, Rashid Nawaz and Laiq Zada

In the present article, the Optimal Homotopy Asymptotic Method (OHAM) has been extended for the first time to derive the approximate solution of coupled system of fractional order partial differential equations. The fractional Whitham-Broer-Kaup system has been solved as test example. Numerical results obtained by the proposed method are compared with that of Adomian Decomposition Method (ADM) and Variational Iteration Method (VIM). The fractional derivatives are described in the Caputo sense. Numerical results show that the proposed method is reliable and efficient for solution of fractional order coupled system partial differential equations. The accuracy of the method increases by taking higher order approximations. (pp. 382–401)

### Weakly $\omega$ -continuous functions in bitopological spaces

C. Carpintero, R. Rajalakshmi, N. Rajesh and E. Rosas

In this paper, as a generalization of  $u$ - $\omega$ -continuous functions, we introduce the notion of weakly  $\omega$ -continuous functions in bitopological spaces and obtain several characterizations and some of its properties. (pp. 402–412)

### $\omega^*$ -topology

H.M. Darwesh and H.A. Shareef

For any space  $(X, \tau)$ , we introduced the concept of  $\omega^*$ -open sets, then we found and discussed a new topology on  $X$ , namely  $\tau_{\omega^*}$  which is strictly closed between  $\tau_{\theta}$  ( $\theta$ -topology) and  $\tau_{\omega}$  ( $\omega$ -topology) on  $X$  and it is independent with the original topology  $\tau$ . Furthermore, the relationship of  $\omega^*$ -open sets with some other types of sets are given. (pp. 413–422)

### Analysis of chaos dynamics of emotions in employment relation model

Saureesh Das and Rashmi Bhardwaj

This paper studies the mathematical modelling and analysis of chaos for employment relation system in which the interactions between tendencies of employer, employee and intermediate employee are being studied. In the modelling the social frame work of an urban class office system is being considered where employee and employers have their own set of disappointments and grief in their mutual relation. Stable phase, critical phase and chaotic phase have been observed in the simulation of the dynamics of the office system for different parameter values. Through bifurcation plot for variation in the level of disappointment the critical value of disappointment is observed to be in accordance with the critical value evaluated through stability analysis. It is concluded that the tendencies of both employer and employee remain stable till they have their disappointment level below the critical value. The moment the critical value is crossed the employment relation transits from stable to critical phase where the tendencies of both the partners start oscillating which later becomes chaotic as disappointment level further increases. (pp. 423–437)

### CANOPY, COBWEB, DBSCAN clustering data mining techniques: study and analysis

Varsha Duhoon and Rashmi Bhardwaj

Clustering is a process of grouping objects belonging to similar class or kind of objects which are collected and put together in same cluster or else grouped

in other cluster based on similarity. The study focuses on the CANOPY, COBWEB and DBSCAN methods of clustering to cluster weather parameters in order to gain insight of the pattern being followed. Clustering technique as an unsupervised form of learning in Data mining helps in providing insight into the distribution of data to visualise and analyze characteristic of each cluster. The data considered in the study is of daily weather parameters for Delhi region from 1<sup>st</sup> January, 2017 to 31<sup>st</sup> October, 2018. The clustering of data is carried out to study the nature using different methods of clustering and the efficiency of these methods are then compared to assess the best suited method based on time taken to form clusters. (pp. 438–448)

### Coupled fixed point theorem in quasi metric space

Manjusha P. Gandhi and Anushree A. Aserkar

In the present paper a unique common coupled fixed point theorem has been proven for quasi metric space with modified -  $\omega$  distance function. This result is improvement, modification and extension in the study of quasi metric space. An example has been given to illustrate the work. (pp. 449–457)

### Hybrid of shifted Legendre and rational Legendre spectral methods on a semi-infinite interval

M. Ghanbarpoor and M. Tavassoli Kajani

In this study, spectral and pseudospectral methods on the half-line are studied using orthogonal systems of shifted Legendre polynomials on a subinterval  $[0, S]$  and rational Legendre functions on  $[S, \infty)$ . Based on this idea a hybrid orthogonal system is introduced. We establish primary results on hybrid approximations of interpolations and some orthogonal projections. The obtained results organize the theory of developing spectral and pseudospectral methods for solving differential equations on a semi-infinite interval. Error analysis for a model problem are established. Numerical results are included to support the theoretical results and shows the effectiveness and performance of this method. (pp. 458–485)

### A study on the fuzzy dynamical systems by using of dynamical relations

A. Gorouhi, M. Ebrahimi and U. Mohammadi

In this paper, some dynamical relations on fuzzy dynamical systems are introduced and their properties are studied. This relations are special subsets of  $X \times X$ , where  $X$  denotes the base space of the fuzzy dynamical systems  $(X, f, M, *)$ . By using this relations (i.e.  $O_F f, R_F f, N_F f$ ) a new method for studying fuzzy dynamical systems is introduced. (pp. 486–494)

### Dynamically defined topological entropy of co-compact open covers

A. Gorouhi, M. Ebrahimi and U. Mohammadi

In this paper, the concept of dynamically defined topological entropy of co-compact open sets for iteration of a continuous system on non-compact Hausdorff space is introduced. Dynamically defined topological entropy is introduced as an invariant of topological conjugation for perfect mappings defined on any Hausdorff space (compactness and metrizability are not necessarily required). This is achieved through the consideration of a topological dynamic system and co-compact covers of the space. An example in the genetic space is presented.

(pp. 495–501)

### CLR property in Menger spaces and related common fixed point theorems

Vishal Gupta, Balbir Singh and Pawan Kumar

In this paper, we have proved some common fixed point theorems for weakly compatible mappings in Menger space using the notion of CLR and JCLR property and control functions. Some illustrative examples are also given to show the usability of the presented results.

(pp. 502–520)

### Fractional Bessel differential equation and fractional Bessel functions

M. Abu Hammad, Sh. Alsharif, R. Khalil and A. Shmasneh

Using a new simple and well-behaved definition of the fractional derivative which is different from the Caputo and Riemann-Liouville fractional derivative and recently introduced by Khalil and others, we reformulate the second order Bessel differential equation in this new setting. In this article by the use of power series, one of the solution of the fractional differential equation is obtained. Moreover, we find the generating function and use it to prove some nice standard results and recurrence relations. Finally, we present some application and integral representations of Bessel functions of fractional type including sines and cosines.

(pp. 521–531)

### On a Diophantine inequality with different powers of primes

Huafeng Liu

Let  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$  be non-zero real numbers, not all negative. Assume that  $\frac{\lambda_1}{\lambda_2}$  is irrational and algebraic. Let  $\mathcal{V}$  be a well-spaced sequence, and  $\delta > 0$ . In this paper, we proved that, for any  $\varepsilon > 0$ , the number of  $v \in \mathcal{V}$  with  $1 \leq v \leq X$  such that the inequality

$$|\lambda_1 p_1^2 + \lambda_2 p_2^2 + \lambda_3 p_3^3 + \lambda_4 p_4^3 + \lambda_5 p_5^4 + \lambda_6 p_6^4 - v| < v^{-\delta}$$

has no solution in primes  $p_1, p_2, p_3, p_4, p_5, p_6$  does not exceed  $O(X^{1-\frac{1}{8}+2\delta+\varepsilon})$ .

(pp. 532-549)

### On the edge version of topological indices for certain networks

Mohamad Nazri Husin and Anamila Ariffin

In QSPR/QSAR study, topological indices such as Shultz index, generalized Randić index, Zagreb index, general sum-connectivity index, atom-bond connectivity (ABC) index and geometric-arithmetic (GA) index are utilized to guess the physico-chemical and bioactivity of chemical compounds. They can be classified based on the structural properties of graphs used for calculation. There is numerous applications of graph theory in this field of research. The aims of this paper is to investigate the generalize on the edge version Randić, GA, ABC, multiplicative ABC, Zagreb and inverse sum indices for certain graph networks by using the concept of line graphs. (pp. 550–564)

### Some results on uniqueness of certain types of difference polynomials

V. Husna, Veena and S. Rajeshwari

In this article we investigate the uniqueness of certain type of difference polynomials that share a small function and obtain some results which improve and extend some recent results of Sujoy Majumder [12]. (pp. 565–577)

### Solving oscillation problems using optimized integrator method

Kasim Abbas Hussain

In this paper, an explicit optimized integrator method of order four is developed for solving second order ordinary differential equations with oscillatory solutions. The new optimized integrator method (NOIM) depend on the existing hybrid methods with dissipative of order infinity. The constant coefficients of new method is found after using the phase-lag, the amplification error (dissipative error) and the first derivative of the phase-lag. Numerical results are presented to illustrate the robustness and competency of the proposed integrator method compared with the existing methods in the scientific literature for solving oscillatory problems. (pp. 578–587)

### On supra $\hat{\omega}_p$ -Lindelöf spaces

Kasim Abbas Hussain, Laheeb Muhsen Noman and Haider Jebur Ali

We submit in this research a new kind of supra compact and supra Lindelöf spaces via new kind of open sets in supra topological space, also, to introduce new type of supra continuous functions. Our results supported by examples, remarks and some facts. (pp. 588–595)

### Inverse one-dimensional wave equation problem under upper-base as additional information

S.O. Hussein

Final displacement measurement 'upper-base' is used in this research for finding force in non-homogeneous hyperbolic equation. Same as [7,8] finite difference method (FDM) with separation variable are applied, but different additional condition is investigated. That means additional data were given by the left end displacement measurement in [7], and the right end displacement measurement in [8].

Furthermore, the wave equation with force split in two parts, part one is direct homogenous problem and numerically using FDM and second one is inverse non-homogeneous ill-posed problem, solved separation variable has been applied. Sequentially, for solving inverse force problem require initial and the boundary condition with additional data, in this study extra data is upper-base. Moreover, second part is unstable due to ill-posedness of the problem where a small error in addition data causes major error in out force data [6]. A stable solution for direct problem and reasonable for inverse problem has been obtained as shown on figures and presented numerical result. Compare with the [7,8] different shapes of figures and different condition numbers are provided.

(pp. 596–608)

### Properties of weakly 2-absorbing primal ideals

Ameer Jaber

Let  $R$  be a commutative ring with unity ( $1 \neq 0$ ). In this paper we introduce the concept of weakly 2-absorbing primal ideal which is a generalization of weakly primal ideal. Let  $I$  be a proper ideal of  $R$ . An element  $a \in R$  is defined to be a weakly 2-absorbing prime to  $I$  if for any  $r, s, t \in R$  with  $0 \neq rsta \in I$ , then  $rs \in I$  or  $rt \in I$  or  $st \in I$ . An element  $a \in R$  is not a weakly 2-absorbing prime to  $I$  if there exist  $r, s, t \in R$ , with  $0 \neq rsta \in I$ , such that  $rs, rt, st \in R \setminus I$ . We denote by  $\nu_0(I)$  the set of all elements in  $R$  that are not weakly 2-absorbing prime to  $I$ . We define a proper ideal  $I$  of  $R$  to be a weakly 2-absorbing primal if the set  $\nu_0(I) \cup \{0\}$  forms an ideal of  $R$ . Many results concerning weakly 2-absorbing primal ideals and examples of weakly 2-absorbing primal ideals are given.

(pp. 609–619)

### Relative relation matrix based approaches for updating approximations in neighborhood multigranulation rough sets

Jianxin Huang and Peiqiu Yu

With the revolution of computing and biology technology, data sets containing information could be huge and complex that sometimes are difficult to handle. Dynamic computing is an efficient approach to solve problems. Since neighborhood multigranulation rough sets (NMGRS) have been proposed, few papers focused on how to calculate approximations in NMGRS and how to update approximations in NMGRS dynamically. The purpose of this study is try to propose relative relation matrix based approaches for computing approximations in NMGRS and updating them dynamically. First, static approaches for computing approximations in NMGRS were proposed. Second, relative relation matrix based approaches for updating approximations in NMGRS while decreasing and increasing neighborhood classes were proposed. Third, incremental algorithms for updating approximations in NMGRS while decreasing and increasing neighborhood classes were designed. Finally, the efficiency and the validity of the designed algorithms were verified by experiments.

(pp. 620–648)

### Symmetry classification and solutions for Shigesada-Kawasaki-Teramoto system

Jina Li and Jingfu Zhao

In this paper, the symmetry classification of the Shigesada-Kawasaki-Teramoto system is presented, then the symmetry reduction and solutions of the classified system are constructed by the generalized conditional symmetry method.

(pp. 649–663)

### A unified inequality of differential polynomials related to small functions

J.F. Xu and S.C. Ye

In this paper, we consider the value distribution of the differential polynomials  $\varphi f^n f' - 1$  ( $n \geq 2$ ), where  $f$  is a transcendental meromorphic function and  $\varphi$  is a small function, and obtain a precise unified inequality by the reduced counting function.

(pp. 664–676)

### On refined Young's inequality

G. Karamali, P. Harikrishnan and H.R. Moradi

In this paper, we obtain new refinements of the classical Young's inequality for positive real numbers, and by using these results, we establish the corresponding inequalities for Hilbert space operators.

(pp. 677–683)

### On classes of Janowski functions associated with a conic domain

K.R. Karthikeyan, K. Amarender Reddy and G. Murugusundaramoorthy

The purpose of this paper is to define a new class of close-to-convex function which are analytic, normalized functions in the open unit disk subordinating with a conic region and to derive initial coefficient estimates  $a_2$ ,  $a_3$  for the function class. We also investigate interesting characteristic properties like sufficient conditions, inclusion relationship, and radius of convexity. Further by appropriate choice of parameters, we provided some new and well known results of our main result. (pp. 684–698)

### 2D MA parameters identification using higher-order spatial cumulants

Soumia Kharfouchi and Hanane Beghriche

In this paper, the almost-sure convergence properties of sample estimates of higher-order spatial statistics are derived. As a practical framework, we address the problem of identification of 2D moving average (MA) models with non-Gaussian errors based on cumulants alone under a nonminimum phase assumption first and on a generalized method of moments approach after. A simulation study verifies the performance of the proposed methods. (pp. 699–720)

### Study on Kenmotsu manifolds admitting generalized Tanaka-Webster connection

Kiran Kumar D.L., H.G. Nagaraja, U. Manjulamma and S. Shashidhar

In this study, we use the generalized Tanaka-Webster connection to observe curvature properties and Ricci soliton in Kenmotsu manifold. Here we look at projective curvature tensor, conharmonic curvature tensor, Weyl projective curvature tensor and recurrent conditions of Kenmotsu manifold with generalized Tanaka-Webster connection. Likewise, we demonstrate identical conditions for a Ricci soliton in a Kenmotsu manifold with generalized Tanaka-Webster connection. (pp. 721–733)

### The basis of knowledge space via Boolean matrix

Yidong Lin, Jinjin Li, Xun Ge and Wen Sun

Knowledge space theory as a mathematical theory in learning evaluation is a crucial component in mathematical psychology. In real world, basis of knowledge space can be significantly much smaller than the knowledge space, which may be essential for some purposes. In this paper, a new mechanism

drawn lessons from Boolean matrix is formulated to search for the basis. In this process, according to the link between knowledge space and its projections, the judgement theorems of atoms (or irreducible elements) are put forward to suit for different situations. Furthermore, an algorithm for basis is presented. And numerical experiments are conducted to evaluate the effectiveness of the proposed approach. (pp. 734–750)

#### Coincidence and fixed point results for $(\Psi, L)$ - $M$ -weak contraction mapping on $Mb$ -metric spaces

Abed Al-Rahman M. Malkawi, Abdallah Talafhah and Wasfi Shatanawi

In this paper, we introduce the  $(\Psi, L) - M - weak$  contraction and we prove some common fixed point results for self-mappings  $T$  and  $S$  and some fixed point results for a single mapping  $T$  by using a  $(c) - comparison$  function and a comparison function in the sense of  $Mb - metric$  space. (pp. 751–768)

#### Ricci soliton on Sasakian manifolds admitting Zamkovoy connection

A. Mandal, A. Das and A.H. Sarkar

Object of this paper is to study Ricci soliton on concircularly flat,  $W_2$ -flat,  $W_3$ -flat,  $W_4$ -flat Sasakian manifolds with respect to Zamkovoy connection. Besides these, we discuss Ricci soliton on a Sasakian manifold satisfying  $W_2^*(\xi, Y) \cdot R^* = 0$ , where  $R^*$  denotes Riemannian curvature tensor with respect to Zamkovoy connection and  $W_2^*$ -denotes the  $W_2$ -curvature tensor with respect to Zamkovoy connection. (pp. 769–779)

#### Comparative numerical study of time fractional coupled Korteweg-de Vries equation

Y. Massoun and A.K. Alomari

In this paper, we construct an analytical solution for a system of the Fractional coupled Korteweg-de Vries differential equations in the sense of Caputo definition. We give the basic properties of the fractional differential equation and a coupled Korteweg-de Vries equation. We applied the homotopy analysis transform method to obtain the analytic solution for this equation, and we compared the result with other obtained solutions by different numerical methods. A comparison study observed the efficacy and accuracy of the present algorithm. (pp. 780–795)

### On properties of composites and monoid domains

Lukasz Matysiak

In this paper we consider all possible properties from commutative algebra for polynomial composites and monoid domains. The aim is full characterization of these structures. We start with the examination of group, ring, modules properties, graded, but also study of invertible elements, irreducible elements, ideals, etc. in these structures. In the second part of this paper we give examples of the use of composites and monoid domains in cryptology. Each such polynomial is the sum of the products of the variable and the coefficient. And what if subsequent coefficient sets are appropriate cryptographic systems? Similarly, monoid domains can be a very good tool between encrypting and decrypting messages. (pp.796–811)

### Some new Hermite-Hadamard Fejér type inequalities for functions whose second-order mixed derivatives are coordinated preinvex

Sikander Mehmood, Artion Kashuri, Fiza Zafar and Nusrat Yasmin

In this paper, we present some new Hermite-Hadamard-Fejér type inequalities for those functions whose second order derivative is coordinated preinvex. We have given some new error bounds for the weighted trapezoidal and weighted mid-point rules for coordinated preinvex functions. The results presented here are extensions of earlier works. (pp. 812–840)

### Cocycles over top spaces

Razieh Moayedi and Mohammad Reza Molaei

Evolution of a potential in a physical system can be determine by a cocycle. This essay contains the notion of cocycles on top spaces. We can associate a semigroup with an identity to each element of a manifold via a cocycle over a top space. This semigroup is a subsemigroup of a Lie group. We prove that these kind of semigroups are invariant under diffeomorphisms of manifolds. We study the concept of forward invariant set, global pullback absorbing set and global forward absorbing set for cocycles. We show that global attractors are persistence by a kind of topological equivalent relation on cocycles. We also define and study the concept of topological entropy for a sequence of cocycles. (pp. 841–851)

### The inverse exponential Rayleigh distribution and related concepts

A.T. Mohammed, M.J. Mohammed, M.D. Salman and R.W. Ibrahim

The main aim of this paper is to introduce a new two-parameter distribution called the inverse exponential Rayleigh distribution (IERD) with the investigated of its statistical properties. We introduce some mathematical complexity of statistical properties such as the moment function, moment generating function, factorial moments generating function, quantity function, skewness, and Kurtosis. We utilize Taylor series expansion to obtain these statistical properties. The process establishes the new distribution relied mainly on the survival functions of both the exponential and Rayleigh distributions. For the application, real data were applied to compare the new distribution with some other distributions using the Akaike information criterion (AIC). It has been indicated that the suggested method is more efficient and high accuracy by the convergence of the real data.

(pp. 852–861)

### Resolving sets in graphs

G.B. Monsanto and H.M. Rara

Let  $G$  be a connected graph. A vertex  $x$  of a connected graph  $G$  is said to *resolve two vertices*  $u$  and  $v$  of  $G$  if  $d_G(x, u) \neq d_G(x, v)$ . For an ordered set  $W = \{x_1, \dots, x_k\} \subseteq V(G)$  and a vertex  $v$  in  $G$ , the  $k$ -vector  $r_G(v/W) = (d_G(v, x_1), d_G(v, x_2), \dots, d_G(v, x_k))$  is called the *representation* of  $v$  with respect to  $W$ . The set  $W$  is a *resolving set* for  $G$  if and only if no two vertices of  $G$  have the same representation with respect to  $W$ . The metric dimension of  $G$ , denoted by  $\dim(G)$ , is the minimum cardinality over all resolving sets of  $G$ . A resolving set of cardinality  $\dim(G)$  is called a *basis*. In this paper, we characterize the resolving sets in the join, corona and lexicographic product of two graphs and determine the resolving number of these graphs.

(pp. 862–871)

### Biharmonic classification of infinite graphs: Bi-networks and Poisson networks

N. Nathiya

In an infinite graph, by using potential-theoretic methods based on the discrete Laplace operator, the properties of biharmonic functions are studied; the notion of bipotentials leads to a classification theory of the infinite graphs. Depending on the solvability of the discrete Poisson equation, we introduce the concept of Poisson networks and investigate some special properties of such infinite graphs.

(pp. 872–883)

### Bach tensor on $N(\kappa)$ -paracontact metric 3-manifolds

K.K. Mirji, D.G. Prakasha and H. Harish

In this paper, we characterize the Bach tensor on  $N(\kappa)$ -paracontact metric 3-manifold. It is proved that a  $N(\kappa)$ -paracontact metric 3-manifold with purely transversal Bach tensor is of constant scalar curvature  $6\kappa$ . (pp. 884–893)

### A note on Einstein-like $\epsilon$ -LP-Sasakian manifolds

R. Prasad, S. K. Verma and S. Kumar

The present paper deals with the study on Einstein-like  $\epsilon$ -Lorentzian para-Sasakian manifolds. A necessary and sufficient condition for an  $\epsilon$ -Lorentzian para-Sasakian manifold to be Einstein-like is obtained in terms of its curvature tensor. We also obtain the scalar curvature of an Einstein-like  $\epsilon$ -Lorentzian para-Sasakian manifold. A necessary and sufficient condition for an  $\epsilon$ -Lorentzian almost para-contact metric hypersurface of an indefinite locally Riemannian product manifold to be  $\epsilon$ -Lorentzian para-Sasakian is derived as well as it is shown that the  $\epsilon$ -Lorentzian para-Sasakian hypersurface of an indefinite locally Riemannian product manifold of almost constant curvature is always Einstein-like.

(pp. 894–904)

### Magic circles cryptosystem

Shatha A. Salman

Magic Squares have been the subject of interest among mathematicians for several centuries because of its magical properties. In this paper a type of magic square was constructed and employed it as a cryptosystem. This method was formulated depending on a set of magic circles that are computed using the proposition of the arithmetic modular together with the operations of addition, rotation and reflection. Due to the importance of the magic circles and the existence of many applications in the practical life, we find a link between the magic circles and the encryption processes. For each character in the plaintext, the ASCII codes is given and then write any plaintext (start from any word) as a linear combination of the elements from the arrangement of magic circles to get the cipher text. An algorithm for generating the magic circles and its application in cryptography was written in MATLAB language. The obtained results within an example on magic circles of order 8 are given together with the inverse of modular magic square. (pp. 905–913)

### Finite groups with $2pqr^2$ elements of maximal order

Sanbiao Tan, Min Zhou, Guiyun Chen and Yanxiong Yan

It is an interesting and difficult topic to determine the structure of a finite group with a given number of elements of maximal order. This topic is related to Thompson's conjecture, that is, if two finite groups have the same order type and one of them is solvable, then the other is solvable. In this article, we continue to this investigation and show that finite groups with  $2pqr^2$  elements of maximal order are solvable, where  $p, q, r$  are primes and  $5 < p < q < r$ .

(pp. 914–921)

### Towards the supercharacter theory of the dicyclic group

H. Saydi

The dicyclic group of order  $4n$  has a presentation

$$T_{4n} = \langle a, b \mid a^{4n} = 1, a^{2n} = b^2, b^{-1}ab = a^{-1} \rangle$$

and is a non-split extension of a cyclic group of order  $2n$  by a cyclic group of order 2. In this paper we investigate a few supercharacter theory for  $T_{4n}$ .

(pp. 922–928)

### Some random iteration processes in modular function space

M. Shahsavari, A. Razani and Gh. Abbasi

In this paper, some random iteration processes for some random operators in modular function space are studied. Then the sequences generated by these iterations are strongly convergent to a random fixed point of the  $\rho$ -generalized Lipschitzian mapping. Our results extend and improve some recent results in the literature. Finally, some numerical examples are presented to indicate the validity of the results.

(pp. 929–949)

### Power gamma extending modules

E.A. Shallal, A.T. Hussein and A.K. Lelo

A submodule  $N$  of Gamma module  $M$  is  $PE\Gamma$ -submodule, if for each  $m$  in  $M$ , ideal  $I$  in  $R$  if  $Im \neq 0$ , then  $I\Gamma(N :_{R\Gamma} m)\Gamma m \neq 0$ .  $PE\Gamma$ -submodule is a proper generalization of essential  $R_\Gamma$ -submodule. An  $R_\Gamma$ -module  $M$  is called power Gamma extending module ( $PE\Gamma$ -module), if every  $R_\Gamma$ -submodule is  $PE\Gamma$ -submodule in a direct summand.

(pp. 950–957)

### Grading and filtrations of gamma rings

S. Shaqaqha and A. Dagher

The aim of this paper is to introduce and study graded and filtered gamma rings and gamma modules. We prove that the filtered  $\Gamma$ -ring (module) is a generalization of the notion of graded ring (module). Also, we construct a graded  $\Gamma$ -ring from a filtered  $\Gamma$ -ring. We investigate some properties of graded and filtered Gamma rings and Gamma modules. Finally we define and study the strongly graded gamma rings. (pp. 958–970)

### Study of different degrees based on various operations on fuzzy soft graphs

S. Shashikala and P.N. Anil

In this paper, two operations, namely conjunction and disjunction on fuzzy soft graphs are defined. The degree and total degree of the vertices of the resultant fuzzy soft graph that are obtained from two given fuzzy soft graphs using these operations are determined. Further, we study properties of degree and total degree of a vertex in fuzzy soft graph based on these operations. (pp. 971–983)

### On $K\alpha c$ -spaces

Alaa Malik Soady, Laheeb Muhsen Noman and Haider Jebur Ali

That research is submitted to introduce new type of supra  $Kc$ -spaces, it is supra  $\alpha K$  ( $\alpha c$ )-space, supra  $L$  ( $\alpha c$ )-space, supra  $(\alpha L)c$ -space and supra  $\alpha L(\alpha c)$ -space. We showed the relation between these types, also we provided some theorems, propositions and examples about the subjects. (pp. 984–990)

### A generalized BL-ring

S.A. Tsafack, A.F. Tchinda, Y. Feng, S. Ndjeya and C. Liao

The purpose of this work is to extend the commutative rings whose lattice of ideals can be equipped with a structure of BL-algebra as carried out by Heubo-Kwegna et al in 2018, to non-commutative ones which are called pseudo BL-rings in this paper. We study and characterize rings whose ideals form a pseudo BL-algebra, we describe them in terms of their subdirectly irreducible factors. We obtain that every unitary pseudo BL-ring with left and right unit is isomorphic to a subring of a direct product of special primary ring and is also isomorphic to a subring of a direct product of discrete valuation rings. (pp. 991–1008)

### Fredholm criteria for a class of regular hypoelliptic operators in multianisotropic spaces in $\mathbb{R}^n$

Ani Tumanyan

We study the Fredholm property of regular hypoelliptic operators, which are the special subclass of Hyormander's hypoelliptic operators. In this paper necessary and sufficient conditions are obtained for the Fredholm property of regular hypoelliptic operators with special variable coefficients in multianisotropic weighted spaces in  $\mathbb{R}^n$ .

(pp. 1009–1028)

### Strong block-block domination of a graph

Sayinath Udupa, R.S. Bhat and Prathviraj N.

Let  $B(G)$  denotes the set of all blocks of a graph  $G$ . Two blocks in  $G$  are adjacent if there is a common cutvertex incident on them. Two blocks  $b_1, b_2 \in B(G)$  are said to *bb-dominate* each other if there is a common vertex incident with  $b_1$  and  $b_2$ . A set  $L \subseteq B(G)$  is said to be a *bb-dominating set (BBD set)* if every block in  $G$  is bb-dominated by some block in  $L$ . *The bb-domination number  $\gamma_{bb} = \gamma_{bb}(G)$*  is the cardinality of a minimum bb-dominating set of  $G$ .

In this paper we define strong (weak) bb-dominating set and strong (weak) bb-full set and obtained relationship between them. We also obtain the relation with existing graph parameters.

(pp. 1029–1036)

### Composition of Abel-Grassmann's strong root of band

Xiaogang An and Xiaohong Zhang

As the role of band in the study of semigroups, AG-band plays an important role in the study of AG-groupoids. In this paper, the concept of strong root of band which is an extension of AG-band, is proposed and its structure is studied in depth. We investigate decomposition of strong root of band induced by an equivalence relation and prove that every strong root of band is the disjoint union of its sub-AG-groups. Furthermore, two different ways how to make a strong root of band are obtained which illuminate the structure of strong root of band.

(pp. 1037–1047)

### [On periodic solutions of Mackey-Glass hematopoiesis model via concave and increasing operator](#)

Zhijian Yao, Jehad Alzabut and Saleem Obaidat

The paper is concerned with a Mackey-Glass hematopoiesis model. By employing the fixed point theorem of  $u_0$  concave and increasing operator, we provide easily verifiable sufficient conditions for the existence of a unique positive periodic solution for the considered model. We provide numerical examples along with illustrative figures to demonstrate the theory. Our approach is new and it is different from previously considered methods. (pp. 1048-1058)

### [Two equations in unequal powers of primes and powers of 2](#)

Yong Cai and Li Yang

In this paper, we obtained when  $k = 176$ , every pair of large even integers satisfying some necessary conditions can be represented in the form of a pair of two prime squares, two prime cubes, two prime fourth powers and  $k$  powers of 2. (pp. 1059-1069)

### [A new characterization of sporadic groups](#)

Zhongbi Wang, Heng Lv, Yanxiong Yan and Guiyun Chen

Let  $G$  be a finite group,  $n$  a positive integer.  $\pi(n)$  denotes the set of all prime divisors of  $n$  and  $\pi(G) = \pi(|G|)$ . The prime graph  $\Gamma(G)$  of  $G$ , defined by Grenberg and Kegel, is a graph whose vertex set is  $\pi(G)$ , two vertices  $p, q$  in  $\pi(G)$  joined by an edge if and only if  $G$  contains an element of order  $pq$ . In this article, a new characterization of sporadic simple groups is obtained, that is, if  $G$  is a finite group and  $S$  a sporadic simple group. Then  $G \cong S$  if and only if  $|G| = |S|$  and  $\Gamma(G)$  is disconnected. This characterization unifies the several characterizations that can conclude the group has disconnected prime graphs, hence several known characterizations of sporadic simple groups become the corollaries of this new characterization. (pp. 1070-1084)

### [A new secant type method for solving one variable functions](#)

Ekhlass S. Al-Rawi, Basim A. Hassan and Barah M. Sulaiman

For solving one variable functions, Newton method is an important and basic method which convergences quadratically. In this paper, we deriving a new secant type methods based on the approximating the second derivative information for solving one minimization problem. The new secant type iterative method has convergence of order two. Application examples are given for demonstrated that the proposed method has better numerical characteristics compared to Newton method. (pp. 1085-1090)

### An open problem on Tarski associative NET-groupoids and GTA-NET-groupoids

Xiaogang An, Mingming Chen and Xiaohong Zhang

Tarski associative groupoid (TA-groupoid) is a kind of non-associative groupoid satisfying Tarski associative law. Tarski associative NET-groupoid (TA-NET-groupoid) is a neutrosophic extended triplet set with Tarski associative law. In this paper, we prove that every TA-NET-groupoid is a semigroup, thus successfully solving an open problem. Moreover, we propose the new notions of generalized TA-NET-groupoid (GTA-NET-groupoid) and weak commutative GTA-NET-groupoid, and investigate their properties and structural characteristics. We prove that GTA-NET-groupoid is equivalent to quasi strong regular TA-groupoid, and give the necessary and sufficient conditions for TA-groupoid to be a weak commutative GTA-NET-groupoid. (pp. 1091–1112)

### On the solutions of the Diophantine equation $M^x + (M - 1)^y = z^2$

W.S. Gayo Jr. and J.B. Bacani

A Mersenne prime  $M$  is a prime number of the form  $2^p - 1$ , where  $p$  is also a prime number. In this study, we consider the Mersenne prime-involved exponential Diophantine equation  $M^x + (M - 1)^y = z^2$ . The main goal is to find the complete set of solutions of the given Diophantine equation in non-negative integers  $x, y$  and  $z$ . Though it is not yet known whether there are infinitely many Mersenne primes, we have proven that the equation under consideration has only finitely many solutions. The proof uses techniques on quadratic congruence, factorization and modular arithmetic. (pp. 1113–1117)

### Numerical solution of electromagnetic problem in horizontal porous medium

Alaa A. Hammodat and Hamsa D. Saleem

A model of heat transfer in a channel of porous walls under the influence of a vertical electromotive force (EMF) with natural convection and thermal radiation of a dissipative fluid is discussed in the channel plane. Using the finite difference method, the partial differential equations that govern the issue have been formulated and solved. Where, using successive iterations of the above method on the equations that are in the time-dependent form, we were able to reach the required solution to the problem. Besides, the temperature distribution behaviour inside the channel was noticed, and the effect of the thermal behaviour of Rayleigh number, Brinkman number, Prandtl number, Eckert number and Hartmann number was also studied. By creating a computer program using MATLAB, we solved the system. (pp. 1118–1135)

### Finslerian hypersurfaces of a Finsler space with special $(\alpha, \beta)$ -metric

Brijesh Kumar Tripathi and V.K. Chaubey

In the present paper we have studied the Finslerian hypersurfaces of a Finsler space with the special  $(\alpha, \beta)$  metric. We have examined the hypersurfaces of this special metric as a hyperplane of first, second and third kinds. Further, we the condition under which this hypersurface will be C2-like Finsler space also obtained the value of Main Scalar  $I$  in two-Dimensional frame of Finsler Space. (pp. 1136–1147)

### A note on conjugacy degrees of a finite group

Y. Xu, C.J. Wang and Y.R. Ran

Let  $G$  be a group,  $g_1, g_2, \dots, g_k$  are a complete set of representatives for the conjugacy classes of  $G$ . Then

$$k(G) = \frac{1}{|G|^2} \sum_{i=1}^k |(g_i)^G|^2 = \sum_{i=1}^k \frac{1}{|C_G(g_i)|^2}$$

is called the conjugate degree of  $G$  (see [6]). In this short paper, we investigate the influence of the conjugacy degree of  $G$  on the structure of finite groups. The authors get the formulas of the conjugacy degree of the dihedral group and the generalized quaternion group and classify those groups  $G$  such that  $k(G) \geq \frac{1}{3}$ . (pp. 1148–1153)

### Stability aspects for linear Hamiltonian systems and its applications in the three body problem

Abdalla Mansur, Hedia Fgaier

This paper concerns stability and strong stability of the Lagrange equilateral solutions in the three body problem. For given three masses, there exists a family of periodic solutions for which each mass is at the vertex of an equilateral triangle and traveling along an elliptic Kepler orbit. By means of symmetry, we reduce the dimensions of our problem from 12 to 6 dimensions. After making a clever change of coordinates a 4 dimensional system is obtained. We show that this system is G-Hamiltonian where  $G$  can be found by the restriction of an invariant subspace to the linear system. We study the bifurcation of this system as parameter within the system are changed. In particular, we prove that the strong stability of this system at  $e > 0$  is lost in two ways, through period doubling bifurcation, and through two +1 eigenvalues. We also prove that stability and instability regions occur in the circular case ( $e = 0$ ) as the determinant of the characteristic polynomial of the system changes. (pp. 1154–1165)

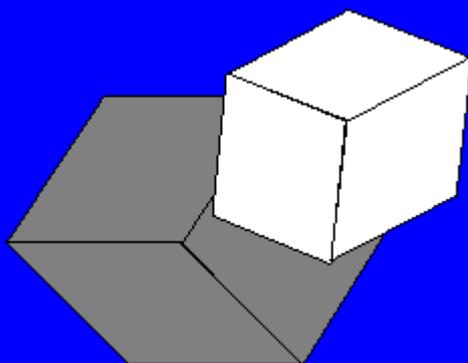
## The Gel'fand spaces of discrete Beurling algebras on $\mathbb{Z}_+^2$ and $\mathbb{Z}^2$

H.V. Dedania, V.N. Goswami

It is well known that the Gel'fand spaces of the Beurling algebras  $l^1(\mathbb{Z}_+, \omega)$  and  $l^1(\mathbb{Z}, \omega)$  can be identified, respectively, with some closed disc and closed annulus in the complex plane  $\mathbb{C}$  (see [?, P.104]). Therefore, it is natural to investigate the Gel'fand spaces of the Beurling algebras  $l^1(\mathbb{Z}_+^2, \omega)$  and  $l^1(\mathbb{Z}^2, \omega)$ . Surprisingly, their Gel'fand spaces are isomorphic to the union (possibly, uncountable) of product of closed discs and closed annuli in  $\mathbb{C}^2$ , respectively.

(pp. 1166–1182)

**IJPAM – Italian Journal of Pure and Applied Mathematics**  
Papers abstracts of Issue n° 47-2022



**FORUM EDITRICE UNIVERSITARIA UDINESE**

**FARE srl**

Via Larga 38 - 33100 Udine

Tel: +39-0432-26001, Fax: +39-0432-296756

[forum@forumeditrice.it](mailto:forum@forumeditrice.it)

Rivista semestrale: Autorizzazione Tribunale di Udine n. 8/98 del 19.3.98

Direttore responsabile: Piergiulio Corsini

**ISSN 2239-0227**