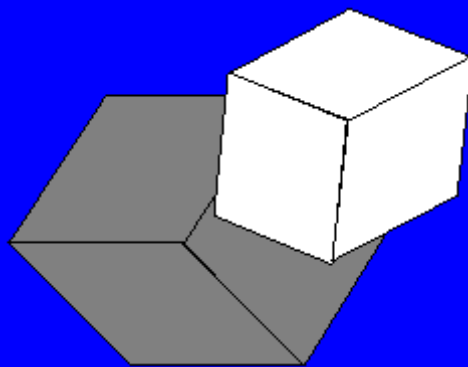


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## Papers Abstracts



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### On primary subgroups and the structure of finite groups

Qiang Zhou, Hong Pan

If  $P$  is a  $p$ -group for some prime  $p$  we shall write  $\mathcal{U}(P)$  to denote the set of all maximal subgroups of  $P$  and  $\mathcal{U}_d(P) = \{P_1, \dots, P_d\}$  to denote any set of maximal subgroups of  $P$  such that  $\bigcap_{i=1}^d P_i = \Phi(P)$  and  $d$  is as small as possible. In this paper, the structure of a finite group  $G$  under some assumptions on the  $c$ -normal or  $ss$ -quasinormal subgroups in  $\mathcal{U}_d(P)$ , for each prime  $p$ , and Sylow  $p$ -subgroups  $P$  of  $G$  is researched. Some known results are generalized.

(pp. 1–7)

### Trapezoidal cubic fuzzy weighted geometric operator and their application to multiple attribute group decision making

A. Fahmi, S. Abdullag, F. Amin, A. Ali

In this paper, the score function and the accuracy function of trapezoidal cubic fuzzy number are developed. We extend cubic fuzzy aggregation operators as the trapezoidal cubic fuzzy weighted geometric operator. Finally, we give an illustrate example to show the multiple attribute group decision-making method with trapezoidal cubic fuzzy weighted geometric operator.

(pp. 8–17)

### On weakly $S$ -permutable subgroups and $P$ -nilpotency of finite groups

Qingfeng Liu

Suppose that  $G$  is a finite group and  $H$  is a subgroup of  $G$ .  $H$  is said to be weakly  $s$ -permutable in  $G$  if there is a subnormal subgroup  $T$  of  $G$  such that  $G = HT$  and  $H \cap T \leq H_{sG}$ , where  $H_{sG}$  is the subgroup of  $H$  generated by all those subgroups of  $H$  which are  $s$ -permutable in  $G$ . We fix in every non-cyclic Sylow subgroup  $P$  of  $G$  some subgroup  $D$  satisfying  $1 < |D| < |P|$  and study the  $p$ -nilpotency of  $G$  under the assumption that every subgroup  $H$  of  $P$  with  $|H| = |D|$  is weakly  $s$ -permutable in  $G$ . Some recent results and the Frobenius' theorem are generalized.

(pp. 18–23)

Two-warehouse inventory model for instantaneous deteriorating items with price and time dependent demand, complete backlogging

G. Santhi, K. Karthikeyan

This paper analyses an inventory model for deteriorating items which is stored in two different warehouses (one is the existing storage known as own warehouse ( $OW$ ) and the other is hired on rental basis known as rented warehouse ( $RW$ ). The  $RW$  is used for storing the excess units over the fixed capacity  $w$  of the  $OW$  and is assumed to offer better preserving facilities than the  $OW$  resulting in a lower rate of deterioration. Also, the  $RW$  is assumed to charge higher holding cost than the  $OW$ . Here the demand is considered as two factors such as selling price and time dependent demand, since, not only the selling price, but also the time is a crucial factor to enhance the demand in the market as well as affecting the overall finance. In this model, shortage is allowed in  $OW$  and the excess demand is backlogged. The purpose of this study is to find the optimal replenishment policies for minimizing the total relevant inventory costs. Furthermore, Numerical examples are provided to illustrate the proposed model.

(pp. 24–33)

On solving matrix games with payoffs presented by dual hesitant fuzzy sets

Y. Song, Z.H. Yang

The solving methodology of matrix games, with payoffs presented by dual hesitant fuzzy sets, is investigated in this paper. Firstly, the notion of dual hesitant fuzzy sets is given. Next, the concept of solutions for the matrix games is defined on the basis of dual hesitant fuzzy sets, which thereby prove the solutions of the matrix games can be obtained through solving a pair of linear programming models. Lastly, a numerical example is given to illustrate the effectiveness of the proposed method.

(pp. 34–52)

On condition  $(P'_E)$

Mahdiyeh Abbasi, Akbar Golchin, Hossein Mohammadzadeh Saany

In (Semigroup and Applications: Proceedings of Conference, world scientific, (1998) 72-77) Golchin and Renshaw introduced Condition  $(P_E)$  and showed that this condition implies weak flatness, but not the converse. In this paper, we introduce a generalization of Conditions  $(P_E)$ , called Condition  $(P'_E)$ , and will show that this condition implies principal weak flatness. Also we give a characterization of monoids by this condition over their right acts.

(pp. 53–70)

### Solution of random ordinary differential equations using Laplace variational iteration method

Fadhel S. Fadhel, Ali A. Abdulsahib, Salah H. Abid

In this article, we shall introduce the Laplace transformation method in connection with the variational iteration method to study and solve random ordinary differential equations. The sequence of approximated closed form iterated solutions is derived based on the general Lagrange multiplier evaluated using the well-known convolution theorem of Laplace transformation method. Two illustrative examples are considered, linear and nonlinear random ordinary differential equations. The obtained results of the closed form solution have very high accuracy in comparison with the exact solution, if exist, or have a very high convergence between the iterated solutions.

(pp. 71–81)

### Closed multiplication modules

Shukur Al-Aeashi, Inaam M.A. Hadi

In this paper, all rings are commutative with identity, all modules are unitary,  $S$  denotes a ring and  $W$  denotes an  $S$ -module. A submodule  $U$  of an  $S$ -module  $W$  is said to be closed in  $W$ , if  $U$  has no proper essential extensions in  $W$ . In this paper we introduce the notion of multiplication module concerning closed submodule as a new generalization of multiplication module namely closed multiplication module. Some basic properties of this notion are given. Some related modules with this concept are investigated and studied too.

(pp. 82–90)

### Certain subclass of Janowski functions associated with symmetrical functions

Fuad Alsarari

The objective of the present paper is to study results that are defined using notions of Janowski classes and  $(j, k)$ -symmetrical functions. A representation theorem, Polya-Schoenberg theorem and some interesting properties are also pointed out.

(pp. 91–100)

### Scale elasticity in the presence of undesirable and nondiscretionary factors: an application to bank branches

H. Amirmoahammadi, A. Amirteimoori, S. Kordrostami, M. Vaez Ghasemi

The concept of returns to scale and scale elasticity has been frequently studied in the framework of data envelopment analysis. Returns to scale is a qualitative characterization to the frontier points of the technology set and its estimation using tools such as data envelopment analysis (DEA) has attracted considerable attention among researchers. In order to quantify this characterization, we can use scale elasticity that is a quantitative measure of response of the outputs to the change of the inputs of the frontier points in a production set. In the current paper, we propose a method to calculate scale elasticity in banking sector when there are non-discretionary and undesirable factors in the process.

(pp. 101–114)

### Note on a relative Hilali conjecture

S. Chouingou, M. Anas Hilali, M. Rachid Hilali, A. Zaim

In this paper, we focus on the relative Hilali conjecture proposed by T. Yamaguchi and S. Yokura, that for a continuous map between two simply connected elliptic spaces  $f : X \rightarrow Y$ ,  $\dim \text{Ker } \Pi_*(f)_{\mathbb{Q}} \leq \dim \text{Ker } H_*(f; \mathbb{Q}) + 1$ . Our aim is to prove this conjecture for fibrations whose fibre has at most two-oddly generators. Also we show it in the case  $H \rightarrow G \rightarrow G/H$ , where  $G$  is a compact connected Lie group and  $H$  is a closed sub-Lie group of  $G$ .

(pp. 115–128)

### Adjoint of some weighted composition operators on the Fock space

Asma Negahdari, Mahsa Fatehi

In this paper, we find the adjoints of weighted composition operators  $C_{\psi, \varphi}$  on the Fock space, when  $\psi = K_c$  for  $c \in \mathbb{C}$ . Then, we show that  $C_{\psi, \varphi}^*$  and  $C_{\psi, \varphi}$  commute, when  $\varphi(z) = az + b$ ,  $a, b \in \mathbb{C}$  and  $|a| = 1$ .

(pp. 129–133)

### Some more algebraic hyperstructures in chemistry and biochemistry

B.O. Onasanya, D. Tu, Y. Feng, O.D. Ogunbiyi, O.A. Ogunmola

In classical group theory, composition of two elements yields one element only. This theory, definitely is not applicable in the study of reactants in the blood system and chlorination where composition of two elements can yield more than one.

Many researchers have studied a lot about finite hyperstructures. But, in this paper, it is shown that chlorination of methane is an infinite hyperstructure. This motivates reason for more studies on algebraic properties of infinite hyperstructures rather than the finite ones.

Besides, it shows that the interaction between the reactants in the blood system and chemical structure of ozone layer satisfy the axioms of some kind of hyperstructures.

(pp. 134–146)

### Eventually constant intertwining linear maps between complete locally convex spaces

C. Bosch, César L. García, T.E. Gilsdorf, C. Gómez-Wulschner, R. Vera

Starting from Sinclair's 1976 work [6] on automatic continuity of linear operators on Banach spaces, we prove that sequences of intertwining continuous linear maps are eventually constant with respect to the separating space of a fixed linear map. Our proof uses a gliding hump argument. We also consider aspects of continuity of linear functions between locally convex spaces and prove that such a linear function  $T$  from the locally convex space  $X$  to the locally convex space  $Y$  is continuous whenever the separating space  $G(T)$  is the zero vector in  $Y$  and for which  $X$  and  $Y$  satisfy conditions for a closed graph theorem.

(pp. 147–163)

### Banach gamma-algebra modules and full stability

Nadia M. J. Ibrahem

In this paper the full stable Banach gamma-algebra modules, fully stable Banach gamma-algebra modules relative to ideal are introduced. Some properties and characterizations of these classes of full stability are studied.

(pp. 164–170)

### The bifurcation analysis of an ecological model involving stage structures in both populations with toxin

Azhar Abbas Majeed

In this paper, the sufficient conditions of the affair the local bifurcation for instance saddle node, transcritical and pitchfork for a mathematical model consisting of stage-structured in prey as well as predator and toxicity is considered to study. Lotka–Volterra type of functional response has been proposed; later the model analysed, it is found that the transcritical bifurcation occurs adjacent the equilibrium point  $E_0$ . While the saddle - node bifurcation occurs at the equilibrium points  $E_1$  and  $E_2$ . It is worth sying; there is no circumstance occurrence of the pitchfork bifurcation to each point. Finally, some arithmetical simulations are used to illustrate the circumstance of local bifurcation of this model.

(pp. 171–183)

### Wrapped shanker distribution

Ayat T. R. Al-Meanazel, Ahmad M. H. Al-Khazaleh

By wrapping Shanker distribution on the real line around a circle, we drive a new circular distribution named wrapped Shanker distribution. The expressions for the densities functions, trigonometric moments, and some related parameters for the new distribution are also obtained.

(pp. 184–194)

### The Lie integral method for differential equations

Zheng Mingliang

The integral theory of differential equations is a very important subject in mathematical science. In this paper, we mainly introduce the Lie integral method for some nonlinear integrable differential equations. The detailed steps and main conclusions of the one-order ordinary differential equations, the two-order ordinary differential equations and the typical partial differential equations are discussed respectively. Three examples are given to illustrate the calculation. The results show that the key of the Lie integral method is the calculation of Lie symmetry generators, which is easy to program. The purpose is to spread the idea of group analysis for differential equations.

(pp. 195–204)

### Approximate methods for solving one-dimensional partial integro-differential equations of fractional order

Osama H. Mohammed, Ahmed K. Mohsin

In this paper, an analytic methods which are so-called ADM and HAM are applied to obtain series solutions to a class of one dimensional partial-intgro differential equations of fractional order(FPIDEs). First, existence of a unique solution under certain condition is proved. Then, the ADM and HAM are performed to get a general solution for one dimensional partial-intgro differential equations. In the next step, the convergence analysis of the proposed methods is investigated. Finally, some examples are included to demonstrate the efficiency of the proposed methods.

(pp. 205–220)

### Generalized-pre-irresolute and $\lambda$ -irresolute functions in closure spaces

Sarhad F. Namiq, Halgwrd M. Darwesh

In this paper, we introduce and study two new types of maps called generalized-pre-irresolute functions and  $\lambda$ -irresolute functions. We discuss the relation among generalized-pre-irresolute functions  $\lambda$ -irresolute functions, generalized-pre-continuous functions and  $\lambda$ -continuous functions. And also by  $T_\lambda$ -spaces,  $T_{p\lambda}$ -spaces and  $T_{gp}$ -spaces,we find some relation between them.

(pp. 221–233)

### Study on value distribution of $L$ -functions sharing a small function

S.H. Naveenkumar, S. Rajeshwari

In this paper, we mainly concentrate on Nevanlinna theory and the properties of  $L$ -functions in the extended selberg class, we mainly study the uniqueness problems on  $L$ -functions related to Brück conjecture. This extend the result due to Q.C. Zhang.

(pp. 234–244)

### Continuity of the farthest point map in vector spaces

H. Ranjbar, A. Niknam

In the paper, using the notion of sesquilinear form, we extend the Banach contraction principle in the vector space, involving rational square terms in the inequality. Furthermore, we discuss the sectional continuity of the farthest point map in vector space.

(pp. 245–250)



### On generalized weakly conharmonically symmetric manifolds

A. Patra, S.K. Hui, A. Patra

The object of the present paper is to study generalized weakly conharmonically symmetric manifold and obtained some of its geometric properties. Existence of such manifold is ensured by a non-trivial example.

(pp. 251–258)

### Some results on uniqueness of homogeneous differential polynomials of meromorphic functions

Dilip Chandra Pramanik, Jayanta Roy

In this paper, we deal with the uniqueness problem of homogeneous differential polynomials of meromorphic functions sharing one small function  $a(z)$  with weight  $l$ , where  $l$  is a non negative integer, with certain essential conditions and prove some results which generalize some results due to Lahiri and Pal [9].

(pp. 259–271)

### The construction of fraction $N$ -subgroups in near rings

Mahdieh Sadeghi Gougheri, Tahereh Roodbarylor

One of the first constructions of algebra is the quotient module. In this paper, we introduce the concepts of fraction  $N$ -subgroups and pure  $N$ -subgroups in near rings. Meanwhile, some properties of them are investigated. Finally, we show that the relationship between pure  $N$ -subgroups and fraction  $N$ -subgroups.

(pp. 272–282)

### A posteriori error analysis for discontinuous Galerkin methods for time discrete semilinear parabolic problems

Mohammad Sabawi

*A posteriori* error estimates for time discretisations for semilinear parabolic (evolution) problems by the discontinuous Galerkin method DG(r) of arbitrary order  $r \geq 0$  are derived and analysed. The semilinear evolutionary problems of the form  $u' + Au = f(u)$  with  $A$  is either linear or monotone  $\gamma^2$ -angle bounded operator are considered. The main tool in this analysis is the time reconstruction function  $\hat{U}$  of the approximate discrete solution  $U$  of the exact solution  $u$ . Two classes of nonlinearities are addressed: firstly, when the source term is globally Lipschitz continuous and secondly when the source term is locally Lipschitz continuous.

(pp. 283–301)

### Controllability of semilinear boundary control inequality problem in infinite dimensional state space

Sameer Qasim Hasan

This study aims to propose the basic theory for controllability of mild solutions of semilinear boundary control inequality problems in an infinite dimension state space by establishing the necessary and sufficient conditions for this issue.

(pp. 302–313)

### Some parameterized inequalities related to $(s, m)$ -convexity and their applications

Gou Hu, Hui Lei, Tingsong Du

By introducing multi-parameters, the authors establish an integral identity for a mapping whose second derivative is integrable, present several integral inequalities of the Hadamard–Simpson type involving  $(s, m)$ -convexity, and apply these inequalities to the special means.

(pp. 314–329)

### A comment on weakly $\pi g$ -closed sets in topological spaces

J.B. Toranagatti

In this paper, we show that the notions of  $\pi gp$ -closed sets and weakly  $\pi g$ -closed sets are equivalent.

(pp. 330–332)

### Partial dynamical systems and partial geodesic equations

Seyed Ebrahim Akrami, Shervin Farzi

In this paper, we generalize the Kato's approach to solve the Cauchy problem for quasi-linear symmetric hyperbolic systems by permitting the coefficients of first-order space-derivatives of the unknown depend on their first-order time-derivative. As an application, we show the existence and uniqueness of the solutions of *partial dynamical systems* and *partial geodesic equation*. These are modifications of the ordinary dynamical systems and geodesic equation in which we replace the ordinary differential operator with the partial differential operator. The motivation comes from quantum theory.

(pp. 333–347)

### Fixed point theory for study the controllability of control systems in reflexive Banach space

Naseif J. Al-Jawari, Riyam B. Abdulmaged

In this paper, fixed point theory and strongly continuous semigroup theory are introduced to study the sufficient conditions for controllability of nonlinear control systems in reflexive Banach spaces. Example is gave to illustrate the value of the theory.

(pp. 348–360)

### On graded $J_{gr}$ -semiprime submodules

K. Al-Zoubi, S. Alghueiri

Let  $G$  be a group with identity  $e$ . Let  $R$  be a  $G$ -graded commutative ring and  $M$  a graded  $R$ -module. A proper graded submodule  $S$  of  $M$  is called a *graded semiprime submodule* if whenever  $r \in h(R)$ ,  $m \in h(M)$  and  $k \in \mathbb{Z}^+$  with  $r^k m \in S$ , then  $rm \in S$ . In this paper, we introduce the concept of graded  $J_{gr}$ -semiprime submodule as a new generalization of graded semiprime submodule and we give a number of results concerning such graded modules. We say that a proper graded submodule  $N$  of  $M$  is a *graded  $J_{gr}$ -semiprime submodule of  $M$*  if whenever  $r \in h(R)$ ,  $m \in h(M)$  and  $k \in \mathbb{Z}^+$  with  $r^k m \in N$ , then  $rm \in N + J_{gr}(M)$ , where  $J_{gr}(M)$  is the graded Jacobson radical of  $M$ .

(pp. 361–369)

### An improved transformation-based kernel estimator for population abundance with shoulder condition

B. Albadareen, N. Ismail, J.J. Jaber

The estimation of parameter  $f_X(0)$ , which is the probability density function at the perpendicular distance  $x = 0$ , is a common target in the line transect sampling to estimate the population abundance,  $D$ . The key assumption of the density shape in the line transect sampling is known as the shoulder condition ( $f'_X(0) = 0$ ). In this paper, we propose a log-transformation application as an adaptation of the classical kernel method to estimate the population abundance in the line transect sampling. The proposed transformation produces a simple and interpretable estimator as the usual kernel estimator while holding theoretical and practical advantages. The asymptotic properties of the proposed transformation are derived. A simulation study using the half-normal detection function is also investigated and applied using various sample sizes. Theoretical and practical results show the superior potential properties of the proposed transform estimator over the usual kernel estimator.

(pp. 370–381)

## Multi-fuzzy hypergroups

A. Al-Husban

In this paper, the concept of multi-fuzzy hyperstructure and multi-fuzzy hyperoperation are presented and developed. This concept generalized the concept of fuzzy hypergroup based on fuzzy space to the context of multi-fuzzy hypergroup based on multi-fuzzy space.

(pp. 382–390)

## Some results on Wiener indices for a connected graph $G$

Ahmed Mohammed Ali

Wiener, hyper-Wiener and partial-Wiener indices are single number and defined by  $W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u, v|G)$ ,  $WW(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} [d^2(u, v|G) + d(u, v|G)]$  and  $W_i(G) = \sum_{\{u,v\} \subseteq V(G) d(u,v|G) \geq i} d(u, v|G)$ ,  $1 \leq i \leq \delta$ ; respectively, where  $d(u, v|G)$  is the distance between two vertices  $u$  and  $v$  in a graph  $G$ ,  $V(G)$  is the vertex set of  $G$  and  $\delta$  is the diameter of  $G$ . In this paper, we find a relationship between some Wiener indices and boundary of hyper-Wiener index depending on the diameter and Wiener index.

(pp. 391–399)

## Optimality analysis of fuzzy critical path for projects (an applied study)

H. Ali Chachan, H.M. A. Razzaq Al-Badri, W. Mohammed Elaibi

The Critical Path Method (CPM) is a crucial agent for the control of complex let alone the planning projects. The prominent achievement of CPM for each activity needs the existence of a valid identified time duration. However, in applicable situations this requirement is usually difficult to fulfill however a great number of activities will be fulfilled for the first time. Thus, there is always a doubt in the network planning about the time durations of activities, a matter that leads to the advancement and development of fuzzy (CPMs.). Rather than employing objective probability measures only A rational approach toward decision making should take into account human subjectivity. We used a method to schedule and control project of extending railway (Basra - Faw) using a fuzzy critical path method. Time is considered to be the major element and the real cost in every project. Thus, estimating activity time methods affect the calculations of total project time and finally the total cost. The total time of the project was calculated with the crispy time defined by  $\alpha - Cut$ . The researchers determined the fuzzy critical path, fuzzy critical activities, and fuzzy float for the rest of the activities. The total expected time to complete the project will be between (54.4, 68, 84) weeks, which was calculated by using the fuzzy critical

path method, which is identical to the length of the first fuzzy critical path when calculating the lengths of all paths, and the estimate of expected time with this method. It enables the decision-maker to follow two policies in the completion of the project that accommodate all the conditions that can be encountered without making mistakes, and it also allows him to obtain a clear value in the amount determined by the level of the cut( $\alpha$ ).

(pp. 400–411)

### Injectivity versus dense injectivity in topological spaces

H. Barzegar

An injective space is a topological space with a strong extension property for continuous functions with values on it. The topological characterizations by injectivity turns out to follow from the algebraic characterizations and general category theory (Escardo 1998). In this paper we study injectivity in the context of topological spaces. We show that, in the category  $\mathcal{Top}_*$ , dense injective spaces are precisely the injective spaces, and in Tychonoff spaces dense injective spaces are precisely compact spaces. This way we obtain enriched versions of known results about injective topological spaces and obtain new proofs and new characterizations of topological spaces by injectivity.

(pp. 412–416)

### $LG$ -fuzzy tangent spaces of $C^\infty$ $L$ -fuzzy manifolds with $L$ -gradation of openness

M. Sadeghi, A. Behzadi, M. Mostafavi

In this paper, we introduce the concept of the  $L$ -fuzzy tangent spaces of an  $C^\infty$   $L$ -fuzzy manifold with  $L$ -gradation of openness  $(X, \mathfrak{T})$ , which  $X$  itself is an  $L$ -fuzzy subset of a crisp set  $M$  and  $\mathfrak{T}$  is an  $L$ -gradation of openness of fuzzy subsets of  $M$  which are less or equal to  $X$  and  $L = \langle L, \leq, \wedge, \vee, ' \rangle$  denotes a complete distributive lattice with at least two elements. Then, we define  $C^\infty$   $LG$ -fuzzy vector fields and  $C^\infty$   $LG$ -fuzzy one-parameter and  $LG$ -fuzzy local one-parameter group acting on an  $C^\infty$   $L$ -fuzzy manifold with  $L$ -gradation of openness. We prove some theorems and give some examples about them.

(pp. 417–437)

### Variations of diagonal cyclicity of Latin squares formed by permutation polynomials

Vadiraja, Shankar, Prasanna, Manasa, Vishnu

Cryptographic applications of Latin squares require to study them in various aspects. The Latin squares, which are due to bivariate polynomials, show some interesting patterns of entries. In this paper, we discussed the diagonally cyclic nature of Latin squares over some small finite rings with the help of the bivariate permutation polynomials, which formed them.

(pp. 438–452)

### $P$ systems on iso-triangular arrays

K. Bhuvaneswari, T. Kalyani, D.G. Thomas, D. Lalitha

Membrane computing is a branch of natural computing in theoretical computer science. This theoretical model was initiated by G.H. Paun in 1988 and it is called  $P$  system, inspired by the structure and functioning of the living cells.  $P$  system has become an appropriate platform for solving many computational problems arising in different fields. For certain DNA molecules, the behavior of the molecules are recombined by an operation called splicing. Splicing was introduced by Tom Head. Splicing operation on arrays inspired Helan Chandra et.al. introduced H-array splicing system, which generated 2D arrays. This Paper introduced iso-triangular domino array splicing system and iso-triangular domino array splicing  $P$  system. The computational complexity of introduced  $P$  systems is examined with few existing models.

(pp. 453–472)

### Prime-valent one-regular graphs of order $12p$

Qiao-Yu Chen, Song-Tao Guo

A graph is *one-regular* and *arc-transitive* if its full automorphism group acts on its arcs regularly and transitively, respectively. In this paper, we classify connected one-regular graphs of prime valency and order  $12p$  for each prime  $p$ . By analyzing the structure of the full automorphism group of such graphs and using the classification of arc-transitive graphs of order  $2p$ , we prove that there is only one such graph, that is, the cycle  $C_{12p}$  with valency two and order  $12p$ .

(pp. 473–479)

### A note on alternative axiomatic results and sports applications of the precore

Chien-Yuan Cheng, Geng-Jie Zhang, Yu-Hsien Liao, En-Cheng Chi

Different from related results of the precore due to Hwang et al. (2015), we characterize the precore by means of a revised reduction. Furthermore, some sports applications are also proposed.

(pp. 480–487)

### Logical entropy of partitions in hyperproduct MV-algebras

R. Chinram, T. Kaewnoi, A. Iampan

In this paper, we introduce the algebraic structure hyperproduct MV-algebras which is a generalization of product MV-algebras. In addition, we study the logical entropy and the logical conditional entropy of partitions in a hyperproduct MV-algebra and provide their properties.

(pp. 488–498)

### On some properties of divisible and pure fuzzy multigroups

P.A. Ejegwa, Y. Feng, W. Zhang, J. Zhou, X. Ming

Many group's theoretic notions have been established in fuzzy algebra although with some modifications. The theory of fuzzy multigroups is the study of group theory in the context of fuzzy multisets. In this paper we propose and characterize the ideas of divisible and pure fuzzy multigroups with a number of results which are duly proved. It is shown that the homomorphic image and preimage of divisible and pure fuzzy multigroups are as well divisible and pure fuzzy multigroups. The relationship between divisible fuzzy multigroups and divisible groups as well as between pure fuzzy multigroups and pure groups are instituted using the idea of alpha-cuts. Finally, it is established that a fuzzy multigroup of a divisible cyclic group is constant.

(pp. 499–512)

### [A new multi-step pseudo-spectral method for the approximate solution of inverse reaction-diffusion equation](#)

I. Gholampoor, M. Tavassoli Kajani

In this paper, we propose a novel multi-step pseudo-spectral method for numerical solution of an inverse reaction-diffusion equation. This method is based on the approximation of the original problem with a sequence of sub-problems. We utilize the collocation method based on a new set of nonclassical basis functions using Chebyshev-Gauss-Radau quadrature points. By using the Pseudo-spectral method, inverse reaction diffusion equation with initial and boundary conditions, would reduce to a system of nonlinear algebraic equations. Due to the ill-posed system of nonlinear algebraic equations, the Tikhonov regularization scheme is employed to obtain a numerical stable solution. The proof the stability and convergence of the method is presented. Some numerical examples are given to show the accuracy of the proposed method and results are compared with those found in literature.

(pp. 513–529)

### [On an analytical and numerical solutions within the conformable fractional derivative for Fitzhugh-Nagumo fractional equation](#)

Meriem Guechi, Abdelouahab Kadem

In this article, a fractional Ricatti expansion method in order to solve a nonlinear fractional Fitzhugh-Nagumo equation in the frame of conformable derivatives, we also apply it to the nonlinear fractional Newell-Whitehead-Segel and Zeldovich equations which are a particular case of fractional Fitzhugh-Nagumo equation. In order to illustrate the accuracy and validity of this method, some numerical solution are given.

(pp. 530–539)

### [A note on restriction semigroups](#)

Haijun Liu, Xiaojiang Guo

It is proved that if  $S$  is a weakly reductive semigroup, then  $S$  is a (proper; Clifford; E-reflexive) restriction semigroup if and only if  $\Omega(S)$  is a (proper; Clifford; E-reflexive) restriction semigroup and  $\Pi(S)$  is a  $(2, 1, 1)$ -subsemigroup of  $\Omega(S)$ .

(pp. 540–547)



### The convergent sequence to the weighted operator

Ehsan M. Hameed, Ahmed K. Al-Jaberi

This paper focuses on the periodic weighted operator and explains its spectrum. The resolvent of the operator sequence of the weighted operator is discussed, and proof is given to indicate that this operator sequence is restricted to the space of all real valued Lebesgue measurable functions that are square integrable on real numbers. The sequence of this operator's convergence to the resolvent of the second derivative operator with the individual boundary conditions is then demonstrated. Consequently, this convergence is used to analyze the results of weighted operator model in the image processing.

(pp. 548–556)

### Pathological diagnosis and clustering of single-valued neutrosophic sets based on the new similarity measure

Han-Liang Huang, Xue-Hong Cai, Jiong-Mei Mo

Similarity measure is an effective tool for decision-making and clustering analysis. Most of the existing similarity measures only to consider the similarity between the corresponding positions of the data. On this basis, we not only consider the relationship between the corresponding positions of the data, but also consider the influence of the overall difference between the data onto the similarity in single-valued neutrosophic environment. Furthermore, similarity and distance measures with weights are given and their properties are investigated. Three examples, which include pathologic diagnosis, clustering and decision making are given to demonstrate the effectiveness of methods. It can be seen that compared with the existing methods, our methods have the characteristics of high accuracy and wide application range.

(pp. 557–576)

### A modified Newton's method for solving functions of one variable

Basim A. Hassan, Ekhllass S. Al-Rawi

In this research, we developed the Newton method to diminish the number of iterations to arrive at the optimal solution for the problem, depending on the Taylor series to approximate the derivatives applying the minimum value of the problem. We compare the method presented with the traditional Newton method in requisites of the number of iteration and the time of execution. Shown numerical results effectiveness for the proposed algorithm after applying it to a group of unimodal functions.

(pp. 577–582)

[New step sizes of the gradient methods for unconstrained optimization problem](#)

Basim A. Hassan, Fadhil Alfarag, Snezana Djordjevic

In this work, we derive a new formula for step-length in the frame of gradient descent methods. The idea of method is based on the quadratic model for using a new approximation of the Hessian of the minimizing function. The rate of convergence is linear and it belongs to the same class of gradient descent methods. Numerical experiment shows that new method is very promising compared to the Barzilai-Borwein approach.

(pp. 583–590)

[Analytical investigation of transversely vibrating nonlinear beams using the iterative method](#)

Md. Alal Hosen

An iterative method is presented to obtain approximate frequencies and corresponding periodic solutions of transversely vibrating nonlinear beams, namely, the cubic-quintic Duffing oscillator. The main purpose of this study is to determine analytical results with high accuracy. In this paper, the truncated terms of the Fourier series have been used and utilized in all steps of iteration. To evaluate the correctness of the present method, the approximated results are compared with the existing results and corresponding numerical results that are considered to be exact. It is shown that the obtained approximate solutions are valid for whole ranges of amplitude of the oscillations and can meet the exact solutions with a high level of accuracy. The error analysis has also carried out and shown acceptable results for the presented iterative method. Effectiveness of the mentioned method found from comparison with the other existing methods and has revealed this method's novelty, reliability and wider applicability to other types of nonlinear engineering problem.

(pp. 591–613)

[The theta-complete graph Ramsey number  \$r\(\theta\_k, K\_5\); k = 7, 8, 9\$](#) 

A.M.M. Jaradat, A. Baniabedlruhman, M.S. Bataineh, M.M.M. Jaradat

Finding the Ramsey number is an important problem of the well-known family of the combinatorial problems in Ramsey theory. In this work, we investigate the Ramsey number  $r(\theta_s, K_5)$  for  $s = 7, 8, 9$  where  $\theta_s$  is the set of theta graphs of order  $s$  and  $K_5$  is a complete graph of order 5. Our result closed the problem of finding  $R(\theta_s, K_5)$  for each  $s \geq 6$ .

(pp. 614–620)

## A note on $G_\delta$ -diagonals in semitopological groups

Jing Zhang

In this note, we prove that:

(1) if  $G$  is a sof-countable semitopological group, then the diagonal  $\Delta$  of  $G$  can be represented as the intersection of a countable family of sequentially open neighborhoods of  $\Delta$  in  $G \times G$ , which generalizes the corresponding result in [?];

(2) if  $G$  is a sof-countable paratopological group, then  $G$  has a  $G_{s\delta}$ -diagonal of infinite rank; (3) let  $G$  be a regular semitopological group and  $H$  a closed invariant subgroup of  $G$  with a quasi- $G_\delta$ -diagonal, if  $G/H$  has a quasi- $G_\delta$ -diagonal, then  $G$  has a quasi- $G_\delta$ -diagonal.

(pp. 621–627)

## On solvability of finite groups with some nonnormal nonabelian subgroups

Wangqiang Kang, Jiakuan Lu, Wei Meng

For a finite group  $G$ , the symbol  $\mathcal{M}(G)$  denotes the number of conjugacy classes of maximal subgroups of  $G$ , and  $\mathcal{T}(G)$  denotes the number of conjugacy classes of non-normal non-abelian subgroups of  $G$ . In this paper, we prove the finite groups with  $\mathcal{T}(G) < \mathcal{M}(G)$  are solvable.

(pp. 628–631)

## Rough structure on groups

Rabah Kellil

In this paper, we introduce the notion of rough mapping between two approximation spaces, then rough structure on a group. Many properties have been investigated and examples have been given to illustrate our study. In accordance with our definitions, we show that left and right translations are rough isomorphisms. As a second topic treated is the behavior of the family of lower and upper approximations of a set and of a subgroup with respect to a family of increasing equivalence relations defined on the approximation space.

(pp. 632–644)

### Rings whose elements are a sum of a unit regular and nilpotent

Hani A. Khashan

The concept of nil clean rings has been extended in several ways. This paper continues investigation new generalizations of such rings. We define a  $U$ -nil clean ring as the one in which every element is a sum of a nilpotent and a unit regular. A  $U$ -nil clean ring is actually an important example of the class of  $NR$ -clean rings introduced in [?]. We introduce many examples and fundamental properties of these rings and investigate the behavior of such properties under various ring extensions. For a ring  $R$  and a group  $G$ , we determine some necessary conditions on  $R$  and  $G$  such that the group ring  $RG$  is  $U$ -nil clean.

(pp. 645–658)

### On generalized fuzzy soft compact spaces

F.H. Khedr, M.A. Abd Allah, S.A. Abd El-Baki, M.S. Malfi

In the present paper, we continue studying generalized fuzzy soft topological spaces. We first introduce generalized fuzzy soft  $p$ -cover and utilize it to define a new type of generalized fuzzy soft compact topological spaces so-called a generalized fuzzy soft  $p^*$ -compact topological spaces which is a generalization of compactness in fuzzy soft topological spaces in [?]. In fact, a generalized fuzzy soft  $p^*$ -compact topological space is more general than generalized fuzzy soft compact spaces in [?]. In general, we investigate some basic results, relations and properties of generalized fuzzy soft  $p^*$ -compact space and provide some illustrative examples.

(pp. 659–671)

### Ricci solitons on ( $LCS$ )-manifolds under $D$ -homothetic deformation

Prabhavati G. Angadi, G.S. Shivaprasanna, G. Somashekhara, P.S.K. Reddy

The object of the present paper is to study Ricci solitons in  $LCS$ -manifolds under  $D$ -homothetic deformation. We examine the behavior of Ricci solitons when potential vector field is orthogonal to Reeb vector field and pointwise collinear with the Reeb vector field. Also, we have show the Ricci solitons in  $D$ -homothetically transformed  $LCS$ -manifolds are expanding, shrinking and steady.

(pp. 672–683)

Ricci solitons and generalized weak symmetries under  $D$ -homothetically deformed  $LP$ -Sasakian manifolds

G. Somashekhar, S. Girish Babu, P.S.K. Reddy

The object of the present paper is to investigate the nature of Ricci solitons and generalized weak symmetries under  $D$ -homothetically deformed  $LP$ -Sasakian manifolds.

(pp. 684–695)

Relative study on means in linear path and two positive arguments in a curved path

K. Sridevi, P.S.K. Reddy, K.M. Nagaraja

Inspired by the Ky Fan type inequalities established for a symmetric triangular wave form over the interval  $[0, 1]$ , a similar symmetric function is introduced over  $[0, 1]$ . A relative study of arithmetic, geometric and power means between symmetric linear path and this curved path is carried out. Also some important inequality chains among these means are established.

(pp. 696–705)

On certain Somos's theta function identities of level 14 and their application to colored partition

Shruthi, B.R. Srivatsa Kumar

M. Somos discovered around 6200 theta function identities using PARI/GP scripts without offering the proof. In this paper, we give a new proof of Somos's theta function identities of level 14 using modular equation of degree 7 and further we extract some interesting combinatorial interpretations of colored partitions.

(pp. 706–713)

Updating reducts in fuzzy  $\beta$ -covering via matrix approaches while adding and deleting a covering element

Liting Xu, Jinjin Li, Peiqiu Yu

Since many real data sets may vary dynamically, reduct changes dynamically as attribute set vary over time. How to update reduct based on previous information is an important task that can help to improve the efficiency of knowledge discovery. This paper focuses on the change of reduct when an element adding

into or deleting from a fuzzy  $\beta$ -covering. In order to derive the reduct algorithms for computing reduct in a fuzzy  $\beta$ -covering, the relation character matrix is defined. Then, by the properties of relation character matrix, the corresponding algorithms for computing new reduct are derived when an element is added into or deleted from the fuzzy  $\beta$ -covering, respectively. Finally, the effectiveness of the proposed algorithms are validated by numerical experiments.

(pp. 714–735)

[A note on unification of the arithmetic-geometric mean and Hölder inequalities for unitarily invariant norms](#)

Limin Zou, Ji Li

Let  $A, X, B \in M_n$ ,  $\frac{1}{p} + \frac{1}{q} = 1, p, q > 1, \alpha \in [0, 1]$ . In this note, we show that if  $r < \max\left\{\frac{1}{p}, \frac{1}{q}\right\}$ , then the following inequality

$$\left\| |A^*XB|^{2r} \right\| \leq \left\| |T_X(\alpha)|^{rp} \right\|^{1/p} \left\| |T_X(1-\alpha)|^{rq} \right\|^{1/q}$$

is note always true, where

$$T_X(\alpha) = \alpha AA^*X + (1-\alpha)XBB^*.$$

(pp. 736–739)

[More generalized forms of fuzzy subgroups of groups](#)

T. Manikantan, S. Vijay Peter

In this paper, we generalize the notion of belongs to relation ( $\in$ ) by  $\in_\gamma^h$  relation of a fuzzy point with a fuzzy subset. We introduce the concepts of  $(\alpha, \beta)$ -fuzzy subgroups of a group by using  $\in_\gamma^h$  relation and  $q_\delta^k$  relation between a fuzzy point and a fuzzy subset. We also introduce the notion of  $(\in_\gamma^h, \in_\gamma^h \vee q_\delta^k)$ -fuzzy subgroup of a group which is a generalization of twelve different types of fuzzy subgroups. Further, we introduce the notions of  $q_\delta^k$ -level subset,  $\in_\gamma^h \vee q_\delta^k$ -level subset,  $(\delta, k)$ -lower and upper parts,  $(\gamma, h)$ -lower and upper parts of fuzzy subsets and investigate some related properties. Finally, we discuss the homomorphic image and pre-image of  $(\in_\gamma^h, \in_\gamma^h \vee q_\delta^k)$ -fuzzy subgroups.

(pp. 740–767)

### Epidemia di Coronavirus Covid19 causata da SARS-COV-2 in Cina

Salvatore Mazzullo, Guglielmo Paganetto

A logistical mathematical model allowed analyzing daily data on the evolution of the Covid19 coronavirus epidemic in China. The model made it possible to anticipate some epidemiological data, not easily identifiable by other means. Three forecasts are particularly relevant: the final cumulative value of the infected people equal to 81750, the time when the daily infection reaches the peak, estimated at 06/02/2020, the practical duration of the epidemic estimated at 100 days from the beginning of the infection. The trend of daily contagion data is compatible with a time decreasing average life span of the virus.

(pp. 768–780)

### On fixed point results for family of multivalued mapping with application

M. Mehmood, A. Shahzad, Q. Mahmood, A. Shoaib, T. Rasham

The purpose of this article is to find out some fixed point results for the family of multifunctions on proximinal sets in complete dislocated metric space. We also presented an example which shows the novelty of our results. An application to the system of nonlinear integral equations has been presented. Our results combine, extend and infer several comparable results in the literature.

(pp. 781–794)

### Characterizations of FI-lattices associated to implicative derivations

Mei Wang, Nana Ma, Nan Jiang, Juntao Wang

In this paper, we discuss related properties of implicative derivations and give some characterizations of them in FI-lattices. Then we show that every implicative derivation on regular FI-lattice is principle. Finally we prove that every Boolean algebra is isomorphic to the algebra of all implicative derivations.

(pp. 795–804)

### On purity and divisibility of $S$ -acts

Gh. Moghaddasi, M. Haddadi, S. Delavari

In this paper we study purity of  $S$ -acts where  $S$  is a special type of monoid of the form  $S = G \dot{\cup} I$ , in which  $G$  is a group and  $I$  is an ideal of  $S$ . Here every  $S$ -act can be naturally considered as an  $I^1$ -act. So the question is that

what is the relationship between purity and divisibility of  $I^1$ -acts and purity and divisibility of  $S$ -acts?

We respond to this question and show that divisibility of an  $S$ -act  $A$  is extendable from  $I^1$ -acts to  $S$ -acts. Also, we show that every consistent system of the third type of equations has a solution in  $A$  whenever every consistent system of the third type of equations over  $I^1$ -act  $A$  has a solution in  $A$ .

(pp. 805–814)

#### A note on extended beta, Gauss and confluent hypergeometric functions

Musharraf Ali, Mohd Ghayasuddin

The main object of this article is to present a new extension of beta function by making use of the generalized Mittag-Leffler function. Some integral representations and summation formulae for this function are also given in a systematic way. Next, we define a new type of beta distribution as an application of our extended beta function. Moreover, we consider a further extension of Gauss and confluent hypergeometric functions by introducing our new beta function.

(pp. 815–826)

#### Coupled fixed point theorems for mappings satisfying Geraghty type contractive conditions

E. Prajisha, P. Shaini

In this paper we establish coupled common fixed point theorems for a pair of weakly increasing mappings satisfying Geraghty type contractive conditions involving a control function, in partially ordered metric spaces. An example is given to illustrate the theorem.

(pp. 827–835)

#### Equitable coloring on subdivision-vertex join and subdivision-edge join of graphs

K. Praveena, M. Venkatachalam, A. Rohini, V.N. Mishra

Adding a new vertex to any edge of a graph  $G$  gives a subdivision of  $G$  denoted by  $S(G)$ . Let  $G_1$  and  $G_2$  be two disjoint graphs. The subdivision-vertex join of  $G_1$  and  $G_2$ , denoted by  $G_1\dot{V}G_2$ , is the graph obtained from  $S(G_1)$  and  $G_2$  by joining every vertex of  $V(G_1)$  with every vertex of  $V(G_2)$ . The subdivision-edge join of  $G_1$  and  $G_2$ , denoted by  $G_1\underline{V}G_2$ , is the graph obtained from  $S(G_1)$  and  $G_2$  by joining every vertex of  $I(G_1)$  with every vertex of  $V(G_2)$ ,



where  $I(G_1)$  is the set of inserted vertices of  $S(G_1)$ . In this paper we determine the equitable chromatic number of subdivision-vertex join and subdivision-edge join of path graph with path graph, complete graph and star graph.

(pp. 836–849)

### Entire solution of certain type of delay-differential equations

S. Rajeshwari, V. Husna, Sheeba Kousar Buzurg

In this article, we shall study the conditions regarding the existence of entire solutions of certain type of delay-differential equations. Our results are supplements to some results obtained recently by W. Lu et al., and references therein.

(pp. 850–856)

### Categorical representation of generalized topologies, generalized neighbourhood systems and generalized interior and closure operators

Rajni Bala

We define a new operator namely strong generalized closure operator on a generalized topological space and investigate some of its properties. Further we give a categorical representation of concepts namely generalized topology, strong generalized interior operator, strong generalized closure operator and complete generalized neighbourhood system and study the relationship between these concepts by means of natural transformations and adjunctions. Also we establish an adjunction between the functors  $\Gamma$  and  $GT$  to prove that the functor  $\Gamma$  is left adjoint of the functor  $GT$ .

(pp. 857–873)

### Soft interior ideals over semigroups

A.A. Ramadan, Essam.H. Hamouda, S. Amira

The concept of a soft interior ideal over a semigroup  $S$  is presented in this paper. Relations between soft left (right) ideals and soft interior ideals are investigated and illustrative examples are provided. Regular and intra-regular semigroups are characterized in terms of soft interior ideals. It is shown that soft interior ideals over a semi-simple semigroup  $S$  is soft ideals. Furthermore, semi-simple semigroups are characterized by means of soft interior ideals. Finally, we introduced a characterization of fuzzy interior ideals of  $S$  by certain soft interior ideals over  $S$ .

(pp. 874–884)

### [z-clean and strongly z-clean rings](#)

Maryam Raofi, Yahya Talebi

In this article we introduce the concept of  $z$ -clean and strongly  $z$ -clean rings. The ring  $R$  is said to be a  $z$ -clean ring if every element of  $R$  is sum of a zero divisor of  $R$  and an idempotent element of  $R$ .

We present the necessary and sufficient condition when the clean rings, and  $z$ -clean rings become equivalent. We study various properties of the  $z$ -clean and strongly  $z$ -clean rings.

(pp. 885–893)

### [Some types of multiplication \$N\$ -group in near rings](#)

Elahe Khodadadpour, Tahereh Roodbarylor

In this paper, the authors have defined multiplication and cyclic  $N$ -groups in near rings. In addition, their relationships have been found. Also, the symbol weak multiplication  $N$ -groups and  $F^*$ - weak multiplication  $N$ -groups in near rings have been introduced. Finally, it has been shown through examples and demonstration of some theorems that these definitions are not equivalent.

(pp. 894–902)

### [On almost weak flatness of acts](#)

A. Sabzbani, A. Golchin, H. Mohammadzadeh Saany

Laan in (Commun. Algebra 29(2), 829-850 (2001)) introduced Condition  $(E')$ , a generalization of Condition  $(E)$ . Golchin and Mohammadzadeh in (Semigroup Forum, 86(2), 413-430 (2012)) introduced Condition  $(P')$ , a generalization of Condition  $(P)$ . Sedaghatjoo in (Semigroup Forum, 87, 653-662 (2013)) introduced a generalization of weak flatness property, called *almost weak flatness*. In this paper, we give a characterization of monoids by this property of their right acts. We also will give a characterization of almost weakly flat coherent monoids in general and monoids coming from some special classes.

(pp. 903–921)

### [Econometric analysis of Jordanian phosphate industry](#)

M. Saleh, A. Rawashdeh

The aim of this study is to assess the competitiveness of the Jordanian extractive phosphate industry using the econometrics analysis, i.e., a common integration approach and error correction model, as well as to appraise some of the competitiveness indicators of the Jordan Phosphate Mines Company (JPMC) for the period (1990-2018), on the basis of Porter's index of comparative advantage, technical efficiency, as well as allocative efficiency. The results pointed out that the Jordanian extractive phosphate industry has a comparative advantage, moderate technical efficiency and decreasing returns to scale. However, the results of the econometrics analysis showed that there is no significant long-term statistical relationship between the company's performance and the variables affecting it. Moreover, the results revealed that the profitability of the company fluctuates during the studied period. On that account, the study presented a set of recommendations, including raising the technical efficiency and efficiency of scale through the optimal use of resources, opening new export markets, using modern technologies, in addition to expanding exploration and extraction areas in order to boost productivity, profitability and competitiveness.

(pp. 922–940)

### [A novel artificial variable-free simplex algorithm for solving neutrosophic linear programming problems](#)

Essam El Seedy, Amani Elrayes, Elsayed Badr, Aya Rabie

In this article, we propose a novel fuzzy artificial variable-free version of simplex algorithm (FAVFSA) for solving fuzzy linear programming problems. Also, we develop this algorithm as a simplified neutrosophic artificial variable-free version of simplex algorithm (NAVFSA) for solving neutrosophic linear programming problems. A computational comparison between the proposed algorithm and the traditional two-phase algorithm is introduced. The computational results show that the size and the running time of the new algorithm is less than the size and the running time of the traditional two-phase algorithm. We also use numerical examples to compare between the fuzzy approach and the neutrosophic approach. The results show that the neutrosophic approach is more accurate than the fuzzy approach.

(pp. 941–958)

### On a class of semi-hypergeneralized quasi-Einstein manifolds

Matilal Sen

The object of the present paper is to introduce semi-hypergeneralized quasi-Einstein manifolds and the manifolds with semi-hypergeneralized quasi-constant curvature. Ricci recurrent semi-hypergeneralized quasi-Einstein manifolds have also been studied.

(pp. 959–969)

### Molecular structure investigation of anticancer drug by topological indices

M.C. Shanmukha, N.S. Basavarajappa, K.N. Anil Kumar, A. Usha

The various medical observations represent that the properties of the drugs have an adjacent relation to its molecular structures. Drug properties will be determined by analysing and investigating the corresponding molecular structures of the drug. In chemical graph theory, topological index is one of the significant tools which is applied to understand the properties of a chemical compound. Topological indices defined on these chemical molecular structures can help researchers to better understand the physical features, chemical re-activities and biological activities. This work is to demonstrate various topological indices on anticancer drug, Hyaluronic acid (HA) used in the manufacturing of anticancer drugs.

(pp. 970–983)

### Fuzzy stability of sextic functional equation in normed spaces (direct method)

Shaymaa Alshybbani

Instituted stability result with regard to the sextic functional equation

$$f(nx + y) + f(nx - y) + f(x + ny) + f(x - ny) = (n^4 + n^2)[f(x + y) + f(x - y)] \\ + 2(n^6 - n^4 - n^2 + 1)[f(x) + f(y)]$$

in Fuzzy normed space.

(pp. 984–988)

[Analysis for fractional integro-differential equation with time delay](#)

A.A. Soliman, K.R. Raslan, A.M. Abdallah

The main purpose of this paper is to solve some linear and nonlinear Caputo fractional Volterra-Fredholm integro-differential equations with delay. In addition, we investigate the convergence analysis for these equations. A semi-analytical method is described and illustrated with examples.

(pp. 989–1007)

[Conformal  \$\eta\$ -Ricci solitons in Lorentzian para-Sasakian manifold admitting semi-symmetric metric connection](#)

G. Somashekhar, S. Girish Babu, P.S.K. Reddy

The object of the present research is to study conformal  $\eta$ -Ricci solitons on a semi-symmetric metric connection in a  $LP$ -Sasakian manifold of some curvature tensor like conharmonic curvature,  $W_2$  curvature and projective curvature.

(pp. 1008–1019)

[The small intersection graph of ideals of a lattice](#)

Yahya Talebi, Maryam Eslami

In this paper, we introduce the small intersection graph of ideals of a lattice  $\mathfrak{L}$ , denoted by  $SG(\mathfrak{L})$ , in which the vertex set is being the set of all non-trivial ideals of  $\mathfrak{L}$ , and two distinct vertices  $I$  and  $J$  are adjacent if  $I \wedge J \ll \mathfrak{L}$ . We study basic properties of  $SG(\mathfrak{L})$ . Also, we investigate the properties of  $SG(\mathfrak{L})$  as connectivity, diameter, girth and orthogonal vertices.

(pp. 1020–1028)

[A study on weak pseudoorders in hyper  \$S\$ -posets](#)

Jian Tang, Bijan Davvaz

In this paper, the concept of weak pseudoorders in a hyper  $S$ -poset is introduced, and several related properties are given. Especially, we characterize the structure of factor hyper  $S$ -posets by weak pseudoorders. Furthermore, we discuss the relationship between order-congruences and weak pseudoorders on a hyper  $S$ -poset, and give some characterizations of order-congruences by the properties of weak pseudoorders. In the sequel, we give some homomorphism theorems of hyper  $S$ -posets by weak pseudoorders, which are generalizations of similar results in  $S$ -posets and ordered semigroups.

(pp. 1029–1046)

## Vague soft separation axioms and vague soft continuous functions in vague soft topological spaces

C. Wang, V. Inthumathi, M. Pavithra

In this present study, we introduce the notions of vague soft points, vague soft neighborhoods and vague soft functions which pave a way to define vague soft separation axioms in vague soft topological spaces. This research work brings out the following interesting results in vague soft set theory, which are contrary to their respective basic results in set theory and the interrelation between separation axioms in topological spaces.

1. The intersection of a vague soft set and its complement need not be a null vague soft set.
2. The union of a vague soft set and its complement need not be an absolute (universal) vague soft set.
3. Every vague soft normal space need not be a vague soft regular space.
4. Every vague soft regular space need not be a vague soft Hausdorff space.

Counter examples are given to illustrate the above results. Also, we provided an application in the field of medicine using the concept of vague soft functions.  
(pp. 1047–1067)

## States on hyper equality algebras

Xiaoyun Cheng, Xiaolong Xin

In this paper, states on hyper equality algebras are introduced and studied. Two types of states, namely, Bosbach states and Riečan states are considered and some related properties of them are obtained. Also, the existence of two types of states are given by some examples and the relationships between them are discussed. One of important results is that Bosbach states and Riečan states coincide on bounded involutive hyper equality algebras. Moreover, Bosbach states on quotient hyper equality algebras are investigated. By use of  $\theta$ -invariant Bosbach states, it follows that the induced function  $\bar{s}$  by a Bosbach state  $s$  on  $H$  is also a Bosbach state on  $H/\theta$  and the corresponding example is constructed.  
(pp. 1068–1080)

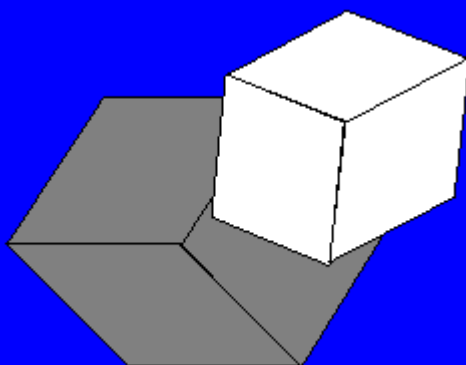
## Granular reduction based matrix in crisp-fuzzy variable threshold concept lattices

Chengling Zhang, Yidong Lin

Attribute reduction is a crucial issue in fuzzy formal concept analysis (FFCA for short). Essentially, the volume of the large database can be decreased by the representation of knowledge as far as possible. With respect to reduction, the discernibility matrix and the discernibility function have played a meaningful affect. Nevertheless, the common approach needs to consider the construction of fuzzy concept lattice before searching reducts, and this invariably leads to the large amount of computation during constructing lattice. Considering adequately this problem, granular concepts are taken into account in this paper. First, the intensions of object set, the extensions of attribute set, and object granular matrix based on matrix operations are investigated in detail. Subsequently, the properties of fuzzy sub-contexts and join-irreducible elements derived from a given crisp-fuzzy variable threshold concept lattice are systematically determined, respectively. Furthermore, the notations of granular consistent set, granular reduct, and discernibility attribute matrix, with a given threshold level, are proposed. In the process of reduction, the construction of a variable threshold concept lattice is averted. Hence, the computational complexity is largely reduced, and our discussion is employed by an algorithm of granular reduction on the basis of matrix theory.

(pp. 1081–1103)

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