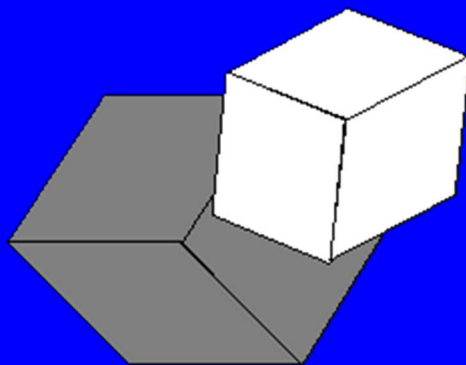


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## Papers Abstracts



**FORUM**

### Approximate approach for solving two points fuzzy boundary value problems

Arslan Hojat Ansari, Kamaleldin Abodayeh

In this paper, we prove the existence and uniqueness of a fixed point of a self mapping on partial  $S$ -metric spaces under the partially  $\alpha$ -contractive condition. (pp. 1–13)

### Characterization of some linear groups by their conjugacy class sizes

Yanheng Chen, Guiyun Chen, Yuming Feng, B.O. Onasanya

Let  $G$  be a group and denote by  $N(G)$  the set of conjugacy class sizes of  $G$ . In this paper, we proved that if  $Z(G) = 1$  and  $N(G) = N(PGL(3, q))$ , where  $q \in \{2, 3, 4, 5, 7, 8, 9, 11\}$ , then  $G \cong PGL(3, q)$ . (pp. 14–24)

### On new class of contra continuity in nano topology

M. Lellis Thivagar, Saeid Jafari, V. Sutha Devi

The purpose of this paper is to introduce and study the stronger form of nano continuity called nano contra continuity. Further the concept of nano kernel and nano Bi-contra continuity is also discussed. These type of mappings can be used in Biotechnology, to study contra or two way contra effects. Here we applied both contra and Bi-contra mapping between a set of viruses to a set of antiviruses as a treatment for disease causing viruses. (pp. 25–36)

### Heptavalent symmetric graphs of order $8p$

Song-Tao Guo, Yong Xu, Guiyun Chen

A graph is symmetric if its automorphism group acts transitively on the set of arcs of the graph. In this paper, we classify connected heptavalent symmetric graphs of order  $8p$  for each prime  $p$ . As a result, a connected heptavalent symmetric graph of order  $8p$  with  $p$  a prime exists if and only if  $p = 2$  or  $3$ , and up to isomorphism, there are only two such graphs: one for each  $p = 2$  and  $3$ . (pp. 37–46)

### On almost generalized pseudo-Ricci symmetric spacetime

Kanak Kanti Baishya

The notion of an almost generalized pseudo-Ricci symmetric space time has been introduced and studied. The beauty of such spacetime is that it has the flavour of Ricci symmetric, Ricci recurrent, generalized Ricci recurrent, pseudo-Ricci symmetric, generalized pseudo-Ricci symmetric and almost pseudo-Ricci symmetric space. Having found, faulty example in [8] the present paper attempts to construct a nontrivial example of an almost pseudo Ricci symmetric spacetime. (pp. 47–54)

### Some remarks on generalizations of prime submodules

Masoud Zolfaghari, Mohammad Hosein Moslemi Koopaei

Let  $R$  be a commutative ring with identity and  $M$  be a unitary  $R$ -module. Let  $S(M)$  be the set of all submodules of  $M$  and  $\phi : S(M) \rightarrow S(M) \cup \{\emptyset\}$  be a function. A proper submodule  $N$  of  $M$  is called  $(n-1, n)$ - $\phi$ -prime, if  $r_1 \dots r_{n-1}x \in N \setminus \phi(N)$  where  $r_1, \dots, r_{n-1} \in R$  and  $x \in M$ , then there exists  $i \in \{1, \dots, n-1\}$  such that  $r_1 \dots r_{i-1}r_{i+1} \dots r_{n-1}x \in N$  or  $r_1 \dots r_{n-1} \in (N : M)$  ( $n \geq 2$ ). In this work,  $(n-1, n)$ - $\phi$ -prime submodules are studied and some results are obtained. Also, the characterization of  $(n-1, n)$ - $\phi$ -prime submodules of a free multiplication module is given. (pp. 55–64)

### A new view of closed-CS-module

Majid Mohammed Abed

This paper give a new fact about the extending module. A module  $M$  is called extending if every closed submodule  $N$  of  $M$  is a direct summand. Study of the concepts complement closed submodule  $((\text{Closed-}N)^c)$  is achieved. Also we expose to a new way to obtain generalization of extending module by complement closed submodule. (pp. 65–72)

### Hesitant fuzzy sets approach to ideal theory in ordered $\Gamma$ -semigroups

A.F. Talee, M.Y. Abbasi, G. Muhiuddin, Sabahat Ali Khan

The aim of this paper is to apply hesitant fuzzy sets theory in ordered  $\Gamma$ -semigroups. The hesitant fuzzy ideals in ordered  $\Gamma$ -semigroups are introduced and some related properties are explored. Using this concept, some characterizations on hesitant fuzzy left (right and bi-) ideals are given. The hesitant fuzzy interior ideal in ordered  $\Gamma$ -semigroup are defined and their related properties are studied. Finally, the characterization of a simple ordered  $\Gamma$ -semigroup in terms of a hesitant fuzzy simple ordered  $\Gamma$ -semigroup is presented.

(pp. 73–85)

### Intuitionistic fuzzy rare $\alpha$ -continuity

R. Dhavaseelan, S. Jafari

In this paper, the concepts of intuitionistic fuzzy rare  $\alpha$ -continuous function, intuitionistic fuzzy rarely continuous function, intuitionistic fuzzy rarely pre-continuous function, intuitionistic fuzzy rarely semi-continuous function are introduced and studied in light of the concept of rare set in intuitionistic fuzzy setting. We present some basic properties.

(pp. 86–94)

### En-semi prime subacts over monoids with zero

Shireen O. Dakheel

Let  $S$  be a monoids with zero and  $A_s$  be a right  $S$ -act . In this paper, we introduce the notion of En- semi prime subact of an  $S$ -act  $A_s$  which is define as : A proper subact  $B$  of an  $S$ -act  $A_s$  is said to be En- semi prime subact, if whenever  $f^2(a)S \subseteq B$  for some endomorphism  $f$  of an  $S$ -act  $A_s$ , and  $a \in A_s$ , then  $f(a)S \subseteq B$ . An  $S$ -act  $A_s$  itself is called En-semi prime if the zero subact ( $\theta$ ) of  $A_s$  is En-semi prime subact of  $A_s$ . Also, we study and gives some related concepts of this notion like: semi prime subact, En-prime subact, En- pure subact and En-radical of subact.

(pp. 95–99)

### Join hesitant fuzzy filters of residuated lattices

G. Muhiuddin, Shuaa Aldhafeeri

The notions of join hesitant fuzzy filters and join hesitant fuzzy  $G$ -filters in residuated lattices are introduced, and related properties are investigated. Characterizations of join hesitant fuzzy filters and join hesitant fuzzy  $G$ -filters are discussed. Conditions for a join hesitant fuzzy filter to be a join hesitant fuzzy  $G$ -filter are provided, and a new join hesitant fuzzy filter is construct by the given join hesitant fuzzy filter.

(pp. 100–114)

### Application of differential transformation method for solving prey predator model with holling type I

J. Alebraheem, S. Hussain, F. Ahmad, K. Nimer

Nonlinear differential equations are used for describing many phenomena in the real world as prey predator interactions. Prey predator models are classified as one of the most important applications in applied mathematics. In this paper, modified structure of prey predator model is used, theoretical properties of the model are presented, the boundedness of the model is shown and the dynamical behavior of the model is proved as globally stable. Semi analytical solution by using differential transformation method (DTM) is obtained for non-dimensional prey predator model with Holling type I in the case of persistence dynamics of the model. The results seem to satisfy biological domain of the problem. We conclude that the results of differential transformation method is in good agreement with numerical results from interpolation method (IM) by using MATHEMATICA program. (pp. 115–127)

### Some classes of mappings on generalized quaternion metric spaces II

Ayda Mohammed Ayed Al-Ahmadi

In connection with the theory of  $(m, q)$ -isometries mappings on metric spaces ([3]) and the theory of  $m$ -quaternion-valued  $G$ -isometric mappings ([1]), we introduce the concept of  $(m, \infty)$ -generalized isometric mappings on a generalized real-valued metric space. We present some essential properties of these classes of mappings. (pp. 128–140)

### On two-sided group digraphs and graphs

Farzaneh Nowroozi Larki, Shahram Rayat Pisheh

In this paper, we consider a generalization of Cayley graphs and digraphs (directed graphs) introduced by Iradmusa and Praeger. For non-empty subsets  $L, R$  of group  $G$ , two-sided group digraph  $\vec{2S}(G; L, R)$  has been defined as a digraph having the vertex set  $G$ , and an arc from  $x$  to  $y$  if and only if  $y = l^{-1}xr$  for some  $l \in L$  and  $r \in R$ . This article has strived to answer some open problems posed by Iradmusa and Praeger related to these graphs. Further, we determine sufficient conditions by which two-sided group graphs to be non-planar, and then we consider some specific cases on subsets  $L, R$ . We prove that the number of connected components of  $\vec{2S}(G; L, R)$  is equal to the number of double cosets of the pair  $L, R$  when they are two subgroups of  $G$ . (pp. 141–152)

### A characterization of simple group $S_4(7)$

Shao Changguo, Jiang Qinhui, Li Kefeng

In this paper, we will use use to give a new characterization of simple group  $S_4(7)$ . (pp. 153–157)

### Redefined neutrosophic filters in BE-algebras

Xiaohong Zhang, Peng Yu, F. Smarandache, Choonkil Park

Neutrosophic set theory is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 2015, neutrosophic set theory is applied to BE-algebra, and the notion of neutrosophic filter is introduced. In this paper, some mistakes and deficiencies of original definition of neutrosophic filter are pointed out by some examples. Moreover, a new definition of neutrosophic filter is established, some basic properties are presented, and the relationships between fuzzy filters and neutrosophic filters are discussed. Finally, the concept of implicative neutrosophic filter in BE-algebra is introduced, and some necessary and sufficient conditions for a neutrosophic filter to be implicative are presented. (pp. 158–176)

### More on almost countably compact spaces

Zuhier Altawallbeh

We study a recent general space of countably compact space called almost countably compact. A topological space  $X$  is almost countably compact space if for every countable open cover  $\{U_n : n \in \mathbb{N}\}$  of  $X$ , there is a finite subfamily  $\{U_{n_i}\}_{i=1}^m$ , where  $m \in \mathbb{N}$  such that  $X = \bigcup_{i=1}^m Cl(U_{n_i})$ . In particular, we investigate this new class of spaces and some other properties in the view of regular cover notion and semiregularization topology. (pp. 177–184)

### Fixed point results with $\Omega$ -distance by utilizing simulation functions

Anwar Bataihah, Abdalla Tallafha, Wasfi Shatanawi

In this paper, we utilize the concept of simulation functions in sense of Khojasteh et al [10] and the notion of  $\Omega$ -distance in the sense of Saadati et. al. [1] to introduce the notion of  $(\Omega, \mathcal{Z})$ -contraction and  $(\Omega, \varphi, \mathcal{Z})$ -contraction. We employ our contractions to formulate and prove many fixed point results for  $\Omega$ -distance. Our results unify and improve many fixed point results in literature. Also, we give fixed point results of integral type as well as we support our result by introducing an example. (pp. 185–196)

### On coefficient inequalities for certain subclasses of meromorphic bi-univalent functions

Amol B. Patil, Uday H. Naik

In the present paper, we investigate and define two subclasses of meromorphic bi-univalent function class  $\Sigma'$  which are defined on the domain  $\mathbb{U}^* = \{z \in \mathbb{C} : 1 < |z| < \infty\}$ . Further, by using the well-known coefficients estimates of the Carathéodory functions (i.e functions with positive real part) we obtain the estimates on the coefficients  $|b_0|$ ,  $|b_1|$  and  $|b_2 + b_0^3|$  for functions in these subclasses. (pp.

197–205)

### A note on unitarily invariant norm inequalities for accretive-dissipative operator matrices

Junjian Yang

In this paper, we present a unitarily invariant norm inequality for accretive-dissipative operator matrices, which is similar to an inequality obtained by Zhang in [J. Math. Anal. Appl. 412 (2014) 564-569]. Examples are provided to show that neither Zhang's inequality nor our inequality is uniformly better than the other. (pp. 206–212)

### Limiting direction of Julia sets and infinite radial order of solutions to complex linear differential equations

Guowei Zhang

In this paper we find that for infinite order entire functions, the ray where it takes infinite radial order is a common limiting directions of Julia sets of their derivatives and their primitives. Applying this result to the solutions of some complex differential equations, we obtain the lower bound of the measure of sets of common limiting directions of Julia sets of the derivatives and integral primitives of any non-trivial solution of these equations, which give alternative proofs of previous results. (pp. 213–223)

### Intuitionistic fuzzy ideals on approximation systems

H. Jude Immaculate, Saeid Jafari, I. Arockiarani

In this paper, we initiate the concept of intuitionistic fuzzy ideals on rough sets. Using a new relation we discuss some of the algebraic nature of intuitionistic fuzzy ideals of a ring.

(pp. 224–236)

### On modular flats and pushouts of matroids

Talal Al-Hawary

In this paper, a sufficient condition for a submatroid of a loopless matroid to be a modular flat is given. Moreover, it is shown that if the injective pushout of two loopless matroids relative to a common submatroid exists, then the join of the given matroids exists and is isomorphic to the indicated pushout.

(pp. 237–241)

### Chebyshev wavelet method (CWM) for the numerical solutions of fractional boundary value problems

S.T. Mohyud-Din, H. Khan, M. Arif, S. Bushnaq

This research article, is concerned with the numerical solutions of fractional boundary value problems by using Chebyshev Wavelet Method (CWM). Simulations based on (CWM), are better in terms of the numerical solutions of higher order boundary value problems. The results obtained by the proposed method are compared with the results of Optimal Homotopy Asymptotic Method (OHAM), Modified Optimal Homotopy Asymptotic Method (MOHAM), Variation Iteration Method (VIM) and exact solutions of the problems. By comparison, it is obvious that the current method improved the accuracy and is easy to implement. The numerical solutions of some examples are discussed to show the suitability of (CWM).

(pp. 242–255)

### Fuzzy zero suffix algorithm to solve fully fuzzy transportation problems by using element-wise operations

S. Dhanasekar, S. Hariharan, David Maxim Gururaj

In this paper zero suffix method with element-wise operations of fuzzy numbers is proposed to solve fully fuzzy transportation problem. The proposed method assures the optimality, feasibility and positivity conditions of the fuzzy solution. The proposed method is easy to understand since it follows zero suffix algorithm and easy to compute since it considers the fuzzy numbers as ordered pairs as it uses the element-wise operations.

(pp. 256–267)



Pricing European call options with default risk under a jump-diffusion model via FFT transform

Sang Wu, Chao Xu, Yinghui Dong

This paper considers the pricing of European call options with default risk within the framework of reduced-form model. We model the stock price and the default intensity by two dependent jump-diffusion models with common jumps. By using a Girsanov theorem, we give the explicit expression for the Fourier transform of the price of call options with default risk. (pp. 268–278)

Estimation for the parameter of a class of diffusion processes

Chao Wei

This paper is concerned with the parameter estimation problem for a stationary ergodic diffusion process with drift coefficient  $a(X_t, \theta)$  and diffusion coefficient  $b(X_t)$  under the case of continuous-time observations. Firstly, we find a closed interval on which the likelihood function is continuous and does not attain the maximum at two endpoints of this interval. Secondly, we prove that the maximum likelihood estimator exists in the interval when the sample size is large enough. Finally, the strong consistency of the estimator and the asymptotic normality of the error of estimation are proved. All of the results are obtained by applying the maximal inequality for martingales, Borel-Cantelli lemma and uniform ergodic theorem. (pp. 279–290)

Relationships between tropical eigenvectors and tropical fixed points of the group  $GL(2, \mathbb{R})$

Umar Hayat, Ghulam Farid, Erdal Karapinar

The eigenvalues, eigenvectors and fixed points of matrices have many applications in various branches of science and many mathematical disciplines. In this paper first we introduce the concept of tropical fixed points, then we calculate the tropical eigenvalues and tropical eigenvectors of  $GL(2, \mathbb{R})$ . Furthermore we give relationships between tropical eigenvectors and tropical fixed points of  $GL(2, \mathbb{R})$ . (pp. 291–300)

### [L<sup>∞</sup>-asymptotic behavior for a finite element approximation to optimal control problems](#)

[S. Boughaba, M. El Amine Bencheikh Le Hocine, M. Haiour](#)

In this paper, a system of parabolic quasi-variational inequalities relevant to the management of energy production is considered where a quasi-optimal of error estimate on uniform norm is proved, by using semi-implicit scheme combined with Galerkin method. Furthermore, an asymptotic behavior result in the same norm is given, taking into consideration the discrete stability properties.

(pp. 301–312)

### [Rings with strongly algebraically closed lattices](#)

[A. Molkhasi](#)

In this article, we prove that if the central idempotents lattice of a Baer ring and the projection lattice of a \*-Baer ring center and the set of all saturated subsets of a Noetherian regular ring are  $q'$ -compact, then they are strongly algebraically closed lattice. Also, for a commutative ring  $R$ , it is shown that if the set of idempotents of a Specker  $R$ -algebra is  $q'$ -compact, then it is a strongly algebraically closed lattice.

(pp. 313–319)

### [On the \$n^\*\$ - and \$\gamma\_n^\*\$ - complete fuzzy hypergroups](#)

[N. Fahimi, T. Nozari, R. Mahjoob](#)

We extend the fuzzy approach of algebraic hyperstructures to the context of complete fuzzy hypergroups. In this paper we introduce the classes of  $n^*$ -complete fuzzy hypergroups and  $\gamma_n^*$ -complete fuzzy hypergroups which they are generalizations of two important classes of hypergroups, also we find some properties of them. Finally, we study  $2^*$ -complete fuzzy hypergroups and give some properties and examples in this regard.

(pp. 320–330)

### [Bifurcations of Liouville tori of generalized two-fixed center problem](#)

[F.M. El-Sabaa, M. Hosny, S.K. Zakria](#)

We study the topological type of the level sets of generalized two-fixed center problem. Furthermore, all generic bifurcation of the level sets are presented. We determine the families of periodic solutions by giving the solution in terms of Jacobi's elliptic functions. Finally, the phase portrait is studied, and the singular points are classified.

(pp. 331–352)

### Optimization technique for solving fuzzy partial differential equations under strongly generalized differentiability

Ghaleb Gumah

In this article, we develop and analyze the use of the combined Laplace transform-homotopy perturbation method C(LT-HPM) to find the exact and approximate solutions for fuzzy partial differential equations under strongly generalized differentiability. The C(LT-HPM) allows the solution of the fuzzy partial differential equation to be calculated in the form of an infinite series in which the components can be easily computed. The method is tested on some linear and nonlinear fuzzy partial differential equations with fuzzy initial conditions to show the effectiveness and accuracy of this method.

(pp. 353–379)

### On some properties of certain subclasses of univalent functions

A.H. El-Qadeem, D.A. Mohan

This study interested in two subclasses of analytic functions defined on the open unit disc of the complex plain, we discuss some neighborhood properties, integral means inequalities and some results concerning the partial sums of the functions belonging to these subclasses.

(pp. 380–390)

### On uniformly primary hyperideals and uniformly 2-absorbing primary hyperideals

M. Anbarloei

Let  $R$  be a multiplicative hyperring. In this paper, we introduce the concepts of uniformly primary hyperideal and uniformly 2-absorbing primary hyperideal of  $R$ , which impose a certain boundedness condition on the usual notions of primary hyperideal and 2-absorbing primary hyperideal, respectively. We will show some properties of them.

(pp. 391–401)

### Classes of weighted tent function spaces and mixed norms with some applications

A. El-Sayed Ahmed, M.Y. Youssif

In this paper, some new definitions for weighted classes of analytic functions are introduced. Moreover, certain properties are presented for functions belonging to the defined classes in the unit disk. Besides, a class of weighted tent functions is also considered. Furthermore, some properties for identity operator are studied for the new tent function spaces.

(pp. 402–415)

### Toward strictly singular fractional operator restricted by Fredholm-Volterra in Sobolev space

Shatha Hasan, Mona Sakkijha

In this paper, a reliable numerical technique is proposed for solving a class of singular fractional differential equations involving Fredholm and Volterra operators subjected to suitable three-point boundary conditions. The solution methodology is presented based on reproducing-kernel method (RKM), which is used directly without employing linearization and perturbation. However, a favorable Hilbert spaces are constructed, and then the orthonormal function system is generated by using Gram-Schmidt orthogonalization process. Error analysis is given in Sobolev space. Numerical example is tested to multipoint singular fractional differential problems with Fredholm and Volterra operators to show the theoretical statements of the RKHS method. The results obtained indicate that the RKHS method is easy to implement, reliability and capability with a great potential of such singular problems. (pp. 416–427)

### Nonlinear left $*$ -Lie triple mappings of standard operator algebras

Lin Chen, Jun Li, Jianhua Zhang

Let  $\mathcal{H}$  be an infinite dimensional complex Hilbert space and  $\mathcal{A}$  be a standard operator algebra on  $\mathcal{H}$  which is closed under the adjoint operation. For  $A, B \in \mathcal{A}$ , define by  $*[A, B] = AB - B^*A$  the left  $*$ -Lie product of  $A$  and  $B$ . In this paper, we prove that a mapping  $\phi : \mathcal{A} \rightarrow \mathcal{B}(\mathcal{H})$  satisfies  $\phi(*[A, *[B, C]]) = *[\phi(A), *[B, C]] + *[A, *[\phi(B), C]] + *[A, *[B, \phi(C)]]$ , for all  $A, B, C \in \mathcal{A}$  is automatically linear. Moreover,  $\phi$  is an inner  $*$ -derivation. (pp. 428–438)

### A common fixed point theorem without continuity under weak compatible mappings in uniform convex Banach spaces

Deepti Thakur

In this paper, we prove a common fixed point theorem for four mappings under the condition of weak compatibility on a closed subset of a uniformly convex Banach space without taking under consideration the continuity of mappings. We provide an example in support of our result. (pp. 439–446)

### A further study on the hyperideals of ordered semihypergroups

Jian Tang, Ze Gu, Xiangyun Xie

In this paper, we first introduce the concepts of prime, weakly prime and semiprime hyperideals in ordered semihypergroups, and give some characterizations of them. Furthermore, we consider the extensions of hyperideals in commutative ordered semihypergroups. As a generalization of the concept of prime hyperideals of ordered semihypergroups, the concept of  $n$ -prime hyperideals of ordered semihypergroups is introduced, and related properties are discussed. In particular, we prove that every  $(n - 1)$ -prime hyperideal of ordered semihypergroups is  $n$ -prime for any positive integer  $n \geq 3$ . Moreover, we investigate the relationship between  $n$ -prime hyperideals and extensions of hyperideals, and prove that a hyperideal  $I$  of a commutative ordered semihypergroup is  $n$ -prime if and only if any extension of  $I$  is  $(n - 1)$ -prime ( $n \geq 3$ ). Finally, we prove that if  $I$  is a semiprime hyperideal of a commutative ordered semihypergroup  $S$ , then  $I$  is the intersection of all extensions of  $I$ . Especially, if  $I$  is also  $n$ -prime ( $n \geq 3$ ), then  $I$  can be expressed as the intersection of all  $(n - 1)$ -prime hyperideals of  $S$  containing it. (pp. 447–470)

### Finite group with coincide automizer and central automorphism of subgroups

Lü Gong, Libo Zhao

In this paper, we characterize the finite group  $G$  such that  $Aut_G(H) = Aut_c(H)$  for every (abelian, non-abelian) subgroup  $H$  of  $G$ , where  $Aut_G(H)$  and  $Aut_c(H)$  are automizer and central automorphism of  $H$  in  $G$ . (pp. 471–476)

### Ricci semi-symmetric normal complex contact metric manifolds

Aysel Turgut Vanli, Inan Unal

In this paper, we obtain the necessary and sufficient conditions a complex almost contact metric manifold to be normal. In addition, we give some new identities for the Riemann curvature and the Ricci curvatures of normal complex contact metric manifolds. Furthermore, we show that a Ricci semi-symmetric normal complex contact metric manifold is Einstein. (pp. 477–491)

### The path graph of the amalgamated graph of $C_3$ and $C_n$ at an edge or at a vertex

Eman Hussein, Hasan Al-Ezeh, Omar Abu Ghneim

Path graphs were proposed as a generalization of line graphs. The 2-path graph denoted by  $P_2(G)$ , of a graph  $G$  has vertex set the set of all paths of length two. Two such vertices are adjacent in the new graph if their union is a path of length three or a cycle of length three. In this paper we will introduce the path graph of the amalgamated graph of  $C_3$  and  $C_n$  at an edge and at a vertex. Also, some new properties of these graphs will be given such as the independence number, domination number and matching number.

(pp. 492–502)

### A characterization of some alternating group by its order and special conjugacy class sizes

Shitian Liu, Xianhua Li

Let  $G$  be a group and  $N(G)$  be the set of the sizes of conjugacy class of  $G$ . Let  $m_p(G)$  be the number from  $N(G)$  which is not divisible by  $p$  and let  $A_n$  be the alternating group of degree  $n$ . The alternating groups  $A_5$ ,  $A_6$ ,  $A_7$ ,  $A_8$ , and  $A_9$  are characterized by their orders and special conjugacy class sizes. So in generality, are the alternating groups characterized by their orders and some special conjugacy class size(s)? In this paper, we show that  $G$  is a finite group such that  $m_p(G) = m_p(A_n)$  and  $m_2(G) = m_2(A_n)$  where  $n \in \{p, p+1, p+2\}$ , then  $G$  is isomorphic to  $A_n$ .

(pp. 503–516)

### Invo-clean rings associated with central polynomials

N.R. Abed Alhaleem, A.H. Handam

Let  $R$  be an associative ring with identity and let  $C(R)$  be the center of a ring  $R$  and let  $g(x)$  be a fixed polynomial in  $C(R)[x]$ . We defined  $R$  to be  $g(x)$ -invo clean if every element in  $R$  can be written as a sum of an involution and a root of  $g(x)$ . In this paper, we investigate conditions on a ring to be  $g(x)$ -invo clean ring. Some properties and several examples are given.

(pp. 517–522)

### On developing an optimal Jarratt-like class for solving nonlinear equations

Maryam Attary, Praveen Agarwal

It is attempted to derive an optimal class of methods without memory from Ozban's method [A. Y. Ozban, Some New Variants of Newton's Method, Appl. Math. Lett. 17 (2004) 677-682]. To this end, we try to introduce a weight function in the second step of the method and to find some suitable conditions, so that the modified method is optimal in the sense of Kung and Traub's conjecture. Also, convergence analysis along with numerical implementations are included to verify both theoretical and practical aspects of the proposed optimal class of methods without memory. (pp. 523–530)

### Some spectral inclusion for strongly continuous semigroups operators

A. Tajmouati, M. Amouch, M.R.F. Alhomidi Zakariya

Let  $(T(t))_{t \geq 0}$  be a  $C_0$ -semigroup on a Banach space  $X$ . In this paper, we show that if there exists  $t_0 > 0$  such that  $T(t_0)$  is a pseudo B-Fredholm operator, then  $T(t)$  is pseudo B-Fredholm for all  $t \geq 0$ , which is equivalent that  $T(t)$  is generalized Drazin invertible for all  $t \geq 0$ . Also we prove that the spectral inclusion of strongly continuous semigroup hold for pseudo Fredholm, generalized Drazin and pseudo B-Fredholm spectra. (pp. 531–543)

### A reliable non-standard finite difference scheme for solving nonlinear biochemical reaction model

Khaled Moaddy, A. Alhashash

In this paper, we present an efficient and accurate numerical scheme for the solution of a model biochemical reaction. The non-standard finite difference scheme based on Adomian decomposition method does not need to linearized or non-locally linearized for the nonlinear term of differential equation. The decomposition method is adopted to construct the numerical solutions. The results demonstrate reliability and efficiency of the algorithm developed. (pp. 544–551)

### Numerical simulations for reactive nitrogen compounds pollution measurements in a stream using Saulyeve method

Areerat Vongkok, Nopparat Pochai

Nutrient pollution is one of most harmful environmental problems, and is caused by surplus nitrogen in water. This nitrogen concentration occurring in water can take several forms, such as organic nitrogen, ammonia, nitrite, nitrate, and dissolved nitrogen gas. Pollution levels can be measured via data collection; however, this is a rather difficult and complex process, and the results obtained widely deviate in term of measurement. A mathematical model can be used in complicated water-quality measurement. The advection-dispersion-reaction model provides a pollutant concentration field. In this research, there are five numerical models for nitrogen pollutant concentration measurement in a stream proposed: a total nitrogen dispersion model, an organic nitrogen dispersion model, an ammonia dispersion model, a nitrite dispersion model, and a nitrate dispersion model. The traditional Forward Time Central Space finite difference technique and the unconditionally explicit Saulyeve technique are employed to obtain five approximated types of organic and inorganic nitrogen pollutant concentrations in each time and place. This paper proposes five forms of nitrogen pollutant measurement model for the unconditionally stable Saulyeve method, so as to make it more accurate without incurring any significant loss of computational efficiency. The five approximated forms of pollutant concentrations obtained indicate that all models improve the nutrient pollution measurement process. (pp. 552–582)

### Some results on $K$ -frames

Sithara Ramesan, K.T. Ravindran

In this paper we present some results on  $K$ -frames when  $K \in B(H)$  is an injective closed range operator. Also we give a condition on  $K$ -frames  $\{f_n\}_{n \in N}$  and  $\{g_n\}_{n \in N}$  so that  $\{f_n + g_n\}_{n \in N}$  is again a  $K$ -frame for  $H$ . Finally, Schatten class operators are also discussed in terms of  $K$ -frames. (pp. 583–589)

### Analysis and implementation of kidney stones detection by applying segmentation techniques on computerized tomography scans

Mua'ad M Abu-Faraj, Mohammad Zubi

Kidney stone disease is one of the risks for life throughout the world and majority of people with stone formation in kidney at the initial stage do not notice it as disease and it damages the organ slowly. Current estimation is that



there are 30 million people suffering by this disease. There are different imaging techniques for diagnosing kidney diseases, such as CT images, X-rays, and Ultrasound imaging. In this study we explored the deployment of three segmentation techniques using matlab to examine the kidney area, and to enhance kidney stone detection. The segmentation techniques under investigation are: threshold based segmentation, watershed based segmentation, and edge based segmentation. (pp. 590–602)

#### On conjugate trigonometrically $\rho$ -convex functions

Nashat Faried, Mohamed S.S. Ali, Asmaa A. Badr

The aim of this article is to introduce a definition of conjugate trigonometrically  $\rho$ -convex functions by using Young's inequality which plays an important role in linking the concept of duality between trigonometrically  $\rho$ -convex functions, rather the definition given by Fenchel. Furthermore, we show that the integration of any increasing function is trigonometrically  $\rho$ -convex. (pp. 603–616)

#### On decomposable $MS$ -algebras

Ahmed Gaber, Abd El-Mohsen Badawy, Salah El-Din S. Hussein

In this paper we give some results on the direct product, subalgebras and homomorphisms of decomposable  $MS$ -algebras. We Show how direct products and canonical projections are related. Also, we study homomorphic images of subalgebras of decomposable  $MS$ -algebras. (pp. 617–626)

#### Common fuzzy fixed points of $\alpha$ -fuzzy mappings

A. Eyal Al-Mazrooei, J. Ahmad, A. Farrag Sayed

The aim of this paper is to obtain the common fuzzy fixed points of  $\alpha$ -fuzzy mappings satisfying generalized almost  $\Theta$ -contraction in the setting of complete metric space. In this way, we generalize several well known recent and classical results. Finally, we provide an example to show the significance of the investigation of this paper. (pp. 627–641)

### Characterizations of almost $PP$ -ring for three important classes of rings

Huda Odetallah, Hasan Al-Ezeh, Emad Abu Osba

A ring is called an almost  $pp$ -ring if the annihilator of each element of  $R$  is generated by its idempotents. We prove that for a ring  $R$  and an Abelian group  $G$ , if the group ring  $RG$  is an almost  $pp$ -ring then so is  $R$ . Moreover, if  $G$  is a finite Abelian group then  $|G|^{-1} \in R$ . Then we give a counter example to the converse of this. Also, we prove that  $RG$  is an almost  $pp$ -ring if and only if  $RH$  is an almost  $pp$ -ring for every subgroup  $H$  of  $G$ . It is proved that the polynomial ring  $R[x]$  is an almost  $pp$ -ring if and only if  $R$  is an almost  $pp$ -ring. Finally, we prove that the power series ring  $R[[x]]$  is an almost  $pp$ -ring if and only if for any two countable subsets  $S$  and  $T$  of  $R$  such that  $S \subseteq \text{Ann}_R(T)$ , there exists an idempotent  $e \in \text{Ann}_R(T)$  such that  $b = be$  for all  $b \in S$ .

(pp. 642–652)

### Stability of fixed point sets of generalized multivalued $\alpha$ - $\psi$ contraction of Ciric-Berinde type

Anju Panwar Anita

The purpose of our paper is to study the existence of fixed point theorems for generalized multivalued  $\alpha$ - $\psi$  contraction of Ciric-Berinde type by using Hausdorff distance in metric spaces and obtain stability of fixed point sets for such multivalued contraction. Examples are providing to indicate the usefulness of our main result. Moreover, an application to single value mapping is also given.

(pp. 653–670)

### Modules closed full large extensions of cyclic submodules are summands

M.S. Mehany, M.H. Elbaroudy, M.A. Kamal

This paper introduced a new generalization of extending modules, namely modules in which every closed full large extension of a cyclic submodule is a direct summand, introduced a new generalization of the concept of injective modules. In fact, we give and study the properties of the concept of full-LE-Cy-injective modules. Although full-LE-Cy-injective modules are far from injective modules, they are exactly the same on some kind of rings. Then we make use of relatively full-LE-Cy-injectivity on modules to study direct sums of two  $(C_1\text{-LE-Cy})$ -modules. We show that a direct sum of two relatively full-LE-Cy-injective modules is a  $(C_1\text{-LE-Cy})$ -module if and only if each one of them is a  $(C_1\text{-LE-Cy})$ -module. Examples are provided to illustrate and delimit the theory.

(pp. 671–679)

### Some generalized forms of soft compactness and soft Lindelöfness via soft $\alpha$ -open sets

T.M. Al-Shami, M.A. Al-Shumrani, B.A. Asaad

By using a notion of soft  $\alpha$ -open sets, we generalize the concepts of soft compact and soft Lindelöf spaces. We define the concepts of soft  $\alpha$ -compact, soft  $\alpha$ -Lindelöf, almost (approximately, mildly) soft  $\alpha$ -compact and almost (approximately, mildly) soft  $\alpha$ -Lindelöf spaces. We present two new kinds of the finite intersection property and utilize them to characterize almost soft  $\alpha$ -compact and approximately soft  $\alpha$ -compact spaces. To elucidate the relationships among the introduced spaces and to illustrate our main results, we supply several interesting examples. Also, we point out that the initiated spaces are preserved under soft  $\alpha$ -irresolute mappings and we investigate certain of results which associate an extended soft topology with the introduced soft spaces. In the end, we conclude some findings which associate the introduced spaces with some soft topological notions such as soft  $\alpha$ -connectedness, soft  $\alpha$ - $T_2$ -spaces, soft  $\alpha$ -partition and soft subspaces. (pp. 680–704)

### Trust-based Testbed for P2P digital library

Rami S. Alkhawaldeh, Moatsum Alawida, Issam Alhadid

Retrieving relevant information from trust-based Peer-to-Peer (P2P) networks is a challenging research task. The users on P2P networks endeavor to ensure that the peers have to provide guaranteed relevant information (or documents) for their information needs. Recent researches in evaluating the trustworthiness or even the reputation of peers lack a well-formulated testbed. As such, building a robust testbed for evaluating trustworthy P2P networks motivates to propose a ground truth testbed as a contribution in P2P systems. In this paper, a trust-based testbed is developed especially in P2P digital libraries using TREC WT10g collection which has been used for evaluation in many P2P networks. The testbed contains a set of peers of trustworthy values for each peer that is estimated using peer rank approach. The statistical factors such as the distribution of peer trustworthiness, distribution of relevant documents and the location of relevant documents within each setting, and the importance of proposed testbed to reach 100% Recall in information retrieval are analyzed. The results are compared with two well-known testbeds. (pp. 705–716)

### A new contraction and existence theorems on fuzzy metric space with a graph

Vishal Gupta, Manu Verma, Jatinderdeep Kaur

In the present paper our aim is to develop coupled fixed point theorems in fuzzy metric space with graph. We introduce the concept of  $\mathcal{J}$ - $\gamma$ -contraction mapping using the control function developed by Wardowski [16]. In current paper, we show the existence of coupled coincidence fixed point in fuzzy metric space with respect to graph. We also give the result having particular value of control function such that the  $\mathcal{J}$ - $\gamma$ -contraction change to  $\mathcal{J}$ -fuzzy contraction.

(pp. 717–729)

### Reliability bounds of dependent linear consecutive k-out-of- n:G systems

Megraoui Fatima Zohra, Soheir Belaloui

Most of researches in the reliability theory dealt to study the independence between components in a system. But, in many real systems, dependence between the components is one of the intractable realistic assumptions that need to be carefully considered. Then, the main purpose of this paper is to provide sharp upper and lower bounds for the reliability of linear consecutive k-out-of-n:G systems consisting of dependent components with identical or arbitrary distribution functions. Some comparisons are done and many examples are treated to prove the performance of the proposed method.

(pp. 730–743)

### Predator-prey model of Holling-type II with harvesting and predator in disease

A. Al Themairi, Manar A. Alqudah

A modified predator-prey model is introduced with Holling -type II. Including a constant rate of harvesting in both infected predators by prey, and predators who are prone to disease. An existence of positive biological equilibrium and uniformly boundedness of the present system are obtained as well. Furthermore, the local stability conditions are defined based on Routh-Hurwitz criteria. Finally, an effective Lyapunov function was performed to check the global asymptotic stability of the interior equilibrium point.

(pp. 744–753)

### Bipolar complex fuzzy sets and their properties

Abdallah Al-Husbana, Ala Amourah, Jamil J. Jaber

The primary motivation behind this paper is to present a brief overview of the bipolar complex fuzzy sets (in short BCFS) which is an extension of bipolar fuzzy set theory. New operations defined over the bipolar complex fuzzy sets some properties of these operations are discussed. (pp. 754–761)

### The impact of organizational and human factors on the successful implementation of ERP system in water authority of Jordan

Issam Alhadid, Suha Afaneh, Heba Almalahmeh

The implementation process of the Enterprise resource planning (ERP) systems is complex, costly and influenced by several critical factors. Organizational and human factors are the most critical success or failure factor for Enterprise Resource Planning (ERP) systems implementation. This paper aims to study the impact of these factors on the successful implementation of ERP system in Water Authority of Jordan; which is a governmental institute responsible of distributing water to participants in Jordan. The methodology of this paper is based on deductive and quantitative method; a questionnaire is designed with (43) questions. The population consists of (77) employees working on the ERP system in the Water Authority of Jordan. Intentional sample of (58) employees working at the main center in Amman was taken. Fifty-seven questionnaires were distributed and (51) were returned. Statistical Package for the Social Sciences (SPSS) program version (12) is used to analyze every item in the questionnaire. While the statistical analysis consists of Cronbach's Alpha, its value is (0. 897), Mean and Standard Deviation, Pearson Correlation, and Linear Regression. The results show that there is statistically significant impact of Organizational and Human factors on the successful implementation of the ERP system in the Water Authority of Jordan. (pp. 762–778)

### Second type nabla Hukuhara differentiability for fuzzy functions on time scales

R. Leelavathi, G. Suresh Kumar, M.S.N. Murty

In this paper, we introduce a new class of derivative called second type nabla Hukuhara derivative for fuzzy functions on time scales under Hukuhara difference. We prove existence and uniqueness of this derivative and obtain its fundamental properties. (pp. 779–801)

### Characterization of generalized projective and injective soft modules

A. Razzaque, I. Rehman, M. Iftikhar Faraz, K. Ping Shum

We first introduce the concepts of projective soft LA-modules, free soft LA-modules, split sequence in soft LA-modules and establish various results on projective soft LA-modules. Then, we consider the injective soft LA-modules and give some relevant results by using free soft LA-modules and split sequences in soft LA-modules. (pp. 802–817)

### Fixed point theorem for contraction mappings in probabilistic normed spaces

P.K. Harikrishnan, B. Lafuerza-Guillén, Y. Je Cho, K.T. Ravindran

In this paper, the concept of contractive mappings and  $\phi$ - contraction mappings on Menger's probabilistic normed spaces are defined with suitable examples. The unique fixed point theorem for contractive mappings and  $\phi$ - contraction mappings are established in Menger's probabilistic normed spaces. (pp. 818–827)

### Criteria and geometric properties for bounded univalent functions in the unit disk

Oqlah Al-Refai

In this article, we establish new univalence criteria for normalized analytic functions  $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$  with  $f(z)/z \neq 0$  in the unit disk  $\mathbb{U} = \{z : |z| < 1\}$ . Indeed, we prove for any  $n \geq 2$  that the condition  $|(f(z)/z)^{(n)}| \leq (n!/(n+1))(1 - \sum_{k=2}^n k|a_k|)$  is sufficient and sharp for  $f$  to be univalent in  $\mathbb{U}$ . The equality attained for the functions  $f(z) = z + \sum_{k=2}^n a_k z^k$ , where  $\sum_{k=2}^n k|a_k| = 1$ . We investigate interesting geometric properties for such classes of functions. Namely, subordinations, inclusions, distortion and growth theorems, area estimate, starlikeness and convexity. (pp. 828–841)

### Studying the solutions of the delay Sturm Liouville problems

Anmar Hashim Jasim

In this paper, the delay Sturm Liouville problems are introduced with the sufficient and necessary conditions, where the solutions of these problems are studied. Besides, definitions, remarks, examples, theorems, and corollaries are submitted to illustrate the delay Sturm Liouville problems properties. In addition, the inverse of the DSLP are shown in section two. Furthermore, the applications of the results in the second section are given in the third section. (pp. 842–849)

### Common fixed point of faintly compatible in fuzzy metric space

Swati Agnihotri, K.K. Dubey, V.K. Gupta, Arihant Jain

Aim of this paper is to establish a common fixed point theorem for faintly compatible and subsequentially continuous self maps of fuzzy metric space and generalizing the result of Jain et al. [6]. (pp. 850–862)

### Operator inequalities involving improved Young inequality

Jianming Xue

The main purpose of this paper is to present some operator inequalities and matrices inequalities for the Hilbert-Schmidt norm. We first give the refined Young inequality for scalars. After that, based on this inequality, we establish operator inequalities and matrices inequalities for the Hilbert-Schmidt norm. (pp. 863–867)

### On minimal $\lambda_{rc}$ -open sets

Ennis Rosas, Sarhad F. Namiq

We introduce and discuss the notions of minimal  $\lambda_{rc}$ -open sets in topological spaces. We investigate some its fundamental properties. We show that the notions of minimal open sets and minimal  $\lambda_{rc}$ -open sets are independent and finally we obtain some applications of a minimal  $\lambda_{rc}$ -open sets. (pp. 868–877)

### Pseudo analytic approach to estimate drug transport and release in the annular section of human limbs

Saqib Mubarak, M. A. Khanday, Ahsan Ul Haq

A mathematical model has been developed to estimate the concentration of transdermal drug transport in an annular section of the human forearm. The formulation of the model is based on the radial and angular diffusion equation together with appropriate boundary conditions. An analytic method has been employed to determine the steady-state concentration of the drug in the annular region of dermal system and the unsteady-state concentration of drug release and transport has been computed using finite difference explicit method. The proposed model may be useful for drug transport in human subjects especially for the application of drug through transdermal drug delivery system. The model has applications in biomedical sciences especially while dealing with the patients having oral and intravenous drug issues. (pp. 878–889)

### Some cryptographic properties of near bent functions over finite fields

Prasanna Poojary, P.K. Harikrishnan, Vadiraja Bhatta G.R.

We present a method for construction of near bent function with the help of Gold power functions. We have also investigated the cryptographical properties, i.e., non-linearity, correlation immunity, algebraic immunity and algebraic degree of these functions. (pp. 890–898)

### On $(m, n)$ -fully stable Banach algebra modules

Manal Ali Sagban, Muna Jasim Mohammed Ali, Samira Naji Kadhim

In this paper the concept of fully- $(m, n)$  stable Banach Algebra-module ( $F - (m, n) - S - B - A$ -module), we study some properties of  $F - (m, n) - S - B - A$ -module and another characterization have been given. (pp. 899–904)

### Vertex $(n, k)$ -choosability of graphs

Germina K. Augusthy, P. Soorya

Let  $G = (V, E)$ , connected, simple graph of order  $n$  and size  $m$  and let  $V(G) = \{1, 2, \dots, n\}$ . A graph  $G = (V, E)$  is said to be vertex  $(n, k)$ -choosable, if there exists a collection of subsets of the vertex set,  $\{S_k(v) : v \in V\}$  of cardinality  $k$ , such that  $S_k(u) \cap S_k(v) = \emptyset$  for all  $uv \in E(G)$ . This paper initiates a study on vertex  $(n, k)$ -choosable graphs and finds the different integer values of  $k$ , for which the given graph is vertex  $(n, k)$ -choosable. (pp. 905–911)

### Rough approximate operators based on fuzzy soft relation

Anju S. Mattam, Sasi Gopalan

Fuzzy soft set is a mapping from a parameter set to the collection of fuzzy subset of universal set. In this paper fuzzy soft relation is presented based on the cartesian product of fuzzy soft sets and the notion of fuzzy soft equivalence relation is introduced. We prove that every fuzzy soft equivalence relation on an arbitrary fuzzy soft set partition the given fuzzy soft set into equivalence classes and thus induces a new relation on the parameter set. Basic properties of the induced relation are studied. A pair of rough approximate operators are investigated and their related properties are given. Relationship between a fuzzy soft topological space and rough approximate operators based on fuzzy soft relation is further established. (pp. 912–925)



### New classes of uniformly convex functions of fractional power on Banach space

Zainab E. Abdulnaby

The aim of this paper is to define new certain subclasses of analytic functions of fractional parameters in the well-known unit disk  $\mathbb{U}$ . Then introduce and study a new integral operator type fractional in the sense of Noor integral on Banach space. In addition, some of its applications are discussed by utilizing a Owa-Hadamard product. (pp. 926–933)

### Asymptotic stability analysis of nonlinear systems with impulsive effects and disturbance input

Xingkai Hu, Linru Nie

In this paper, a sufficient condition for asymptotic stability of nonlinear systems with impulse time window is derived, which avoids solving linear matrix inequalities. For the system with disturbance input and bounded gain error due to limit of equipment and technology in practical applications, another sufficient condition is also obtained. Numerical examples are carried out to validate effectiveness of the proposed results. (pp. 934–943)

### Numerical simulation of nonlinear fractional integrodifferential equations of Volterra type via power series expansion

A. Ale'damat, A. Alrawajfi, A. Talafha, A. Alhabahbeh, A. Atewi

In this article, an effective recent analytical treatment is presented to solve a certain class of nonlinear fractional integrodifferential equations of Volterra type based on the residual error functions. The solution methodology of the fractional power series (FPS) approach is to replace the n-term truncated solution by generalized fractional power series to minimize the residual error function through the derivation of those functions under the Caputo concept. Anyhow, the approximate solution is obtained directly in a rapidly convergent fractional power series without needed to linearization, perturbation, or discretization. Numerical examples are performed to show the validity and reliability of the FPS method. Numerical analysis of the results indicates that the RPS approach is simple, efficient and systematic tool in solving fractional nonlinear issues arising in applied mathematics, physics and engineering. (pp. 944–957)

### Qualitative behavior of a SIRS epidemic model with vaccination on heterogeneous networks

A. Assadouq, H. El Mahjour, A. Settati

This paper studies the dynamics of a SIRS epidemic model with varying population size and vaccination in a complex network. Using an analytical method, we mainly investigate the stability of the model according to the threshold  $\mathcal{R}_0$ . That is, if  $\mathcal{R}_0$  is less than one, then the disease will die out. Alternatively, the system admits a unique endemic equilibrium which is globally asymptotically stable if  $\mathcal{R}_0 > 1$ . Moreover, we investigate the case when  $\mathcal{R}_0 = 1$ . Finally, some numerical simulations are provided to illustrate the effectiveness of the theoretical results. (pp. 958–974)

### The non-zero divisor graph of a ring

Sameer Kadem, Ali Aubad, Abdulrahman H. Majeed

Let  $R$  be a ring, we associate a simple graph  $\Phi(R)$  to  $R$ , with vertices  $V(R) = R \setminus \{0, 1, -1\}$ , where distinct vertices  $x, y \in V(R)$  are adjacent if and only if either  $xy \neq 0$  or  $yx \neq 0$ . In this paper, we prove that if  $\Phi(R)$  is connected such that  $R \not\cong Z_2 \times Z_4$  then the diameter of  $\Phi(R)$  is almost 2. Also, we will pay specific attention to investigate the connectivity of certain rings such that, the ring of integers modulo  $n$ ,  $Z_n$  is connected, reduced ring and matrix ring. (pp. 975–983)