

**Convexity in normed hypervector spaces**

P. Raja, S.M. Vaezpour

In this paper we obtain some results on convexity in a normed hypervector space. We also investigate the concept of absorbing and balanced set and generalize the corresponding results of vector space. (pp. 7-16)

**Domination in the intersection graphs of rings and modules**

Sayyed Heidar Jafari, Nader Jafari Rad

In this paper we obtain domination number in the intersection graphs of ideals of rings, and intersection graphs of submodules of modules. (pp. 17-20)

**Notes for Hartley transforms of generalized functions**

S.K.Q. Al-Omari

The classical Hartley transform, originally introduced by Hartley as a real transform with a number of properties being similar to the properties of Fourier transform. In this work, we extend the Hartley transform to certain space of distributions of compact support. Further, we establish that the Hartley transform and its inverse are one to one and onto mappings in the space of Boehmians. Moreover, continuity with respect to  $\delta$  and  $\Delta$  convergence is discussed in some detail. Certain theorems are also proved. (pp. 21-30)

**Nonparametric estimation of a multivariate probability density for mixing sequences by the method of wavelets**

Narges Hosseinioun, Hassan Doosti, Hossein Ali Niroumand

The mathematical theory of wavelet and their applications in statistics have become a well-known technique for non-parametric curve estimation: see e.g. Meyer (1990), Daubachies (1992), Chui (1992), Donoho and Johnstone (1995) and Vidakovic (1999). We Consider the problem of estimation of the partial derivatives of a multivariate probability density  $f$  of mixing sequences, using wavelet-based method. Many stochastic processes and time series are known to be mixing. Under certain weak assumptions autoregressive and more generally bilinear time series models are strongly mixing with exponential mixing coefficients. The problem of density estimation from dependent samples is often considered. For instance quadratic losses were considered by Ango Nze and Doukhan (1993). Bosq (1995) and Doukhan and Loen (1990). We investigate the variance and the rate of the almost convergence of wavelet-based estimators. Rate of convergence of estimators when  $f$  belongs to the Besov space is also established. (pp. 31-40)

## Lower and upper bounds of the Čebyšev functional for the Riemann-Stieltjes integral

S.S. Dragomir, A. Sofo

Lower and upper bounds of the Čebyšev functional for the Riemann-Stieltjes integral, in the monotonicity case of one function, are given. Applications in relation with the Steffensen generalisation of the Čebyšev inequality are provided. (pp. 41-50)

## Enumeration of hypercompositional structures defined by binary relations

Ch.G. Massouros, Ch. Tsitouras

This paper deals with hyperoperations that derive from binary relations and it studies the hypercompositional structures that are created by them. It is proved that if  $\rho$  is a binary relation on a non-void set  $H$ , then the hypercomposition  $xy = \{z \in H : (x, z) \in \rho \text{ and } (z, y) \in \rho\}$  satisfies the associativity or the reproductivity only when it is total. There also appear routines that calculate (with the use of small computing power) the number of non isomorphic hypergroupoids, when the cardinality of  $H$  is finite. (pp. 51-62)

## Degeneracy of some cluster sets

C.K. Basu, B.M. Uzzal Afsan, S.S. Mandal

Using  $\theta$ -closure [39],  $\delta$ -closure [39] of a subset and  $\beta$ -open [1] sets, we initiate two new kinds of cluster sets for functions and multifunctions. An explicit expression of each kind of cluster sets are given in terms of filters and grills [38] and also several of their properties are investigated. In the process, the degeneracy of such cluster sets are used as tools to obtain new characterizations of various separation axioms. As application, such investigations ultimately provide new techniques for studying the covering property  $\beta$ -closedness [7]. (pp. 63-78)

## Inégalités de type faible pour l'opérateur maximal fractionnaire dans les espaces de Morrey par rapport à la capacité de Hausdorff

Modeste Essoh, Ibrahim Fofana, Konin Koua

We prove a boundedness property for the fractional maximal operator in the Morrey type spaces with respect to the Hausdorff content. As an application of this result, we obtain a Fefferman-Stein inequality. (pp. 79-90)

### **Inequalities for marks in multidigraphs**

S. Pirzada, U. Samee, T.A. Naikoo, Merajuddin

An  $r$ -digraph (multidigraph)  $D$  is an orientation of a multigraph that is without loops and contains at most two edges between any pair of distinct vertices. So 1-digraph is an oriented graph, and complete 1-digraph is a tournament. Define  $p_v = r(n - 1) + d_v^+ - d_v^-$ , the mark ( $r$ -score) of a vertex  $v$  in an  $r$ -digraph  $D$ , where  $d_v^+$  and  $d_v^-$ , respectively denote the outdegree and indegree of  $v$  and  $n$  is the number of vertices in  $D$ . In this paper, we obtain some stronger inequalities for marks in  $r$ -digraphs. (pp. 91-100)

### **Improved exponential estimator for population variance using two auxiliary variables**

Rajesh Singh, Pankaj Chauhan, Nirmala Sawan, Florentin Smarandache

In this paper, exponential ratio and exponential product type estimators using two auxiliary variables are proposed for estimating unknown population variance  $S_y^2$ . Problem is extended to the case of two-phase sampling. Theoretical results are supported by an empirical study. (pp. 101-109)

### **On some classes of submanifolds satisfying Chen's equality in an Euclidean space**

Cihan Özgür, Uday Chand De

We study submanifolds satisfying Chen's equality in an Euclidean space. Firstly, we consider projectively semi-symmetric submanifolds satisfying Chen's equality in an Euclidean space. We also study submanifolds satisfying the condition  $P \cdot P = 0$ . (pp. 109-116)

### **An approach to the approximation of the inverse of a square matrix by He's homotopy perturbation method**

B. Keramati

In this paper, we present an efficient numerical algorithm for approximating the inverse of a square matrix based on homotopy perturbation method. Some numerical illustrations are given to show the efficiency of the algorithm. (pp. 117-124)

### **A new characterization of $L_2(q)$ where $q = p^n < 125$**

Li-Guan He, Gui-Yun Chen

It is a well-known topic to characterize a finite simple group by using two quantities, the order of  $G$  and  $\pi_e(G)$  in the past 30 years, where  $\pi_e(G)$  denotes the set of orders of elements in  $G$ . Recently this topic has been finished by V.D. Mazurov, et al. Here the authors will try to characterize some finite simple groups by using less quantities and have successfully characterized  $L_2(q)$ , where  $q = p^n < 125$ , by using the order of  $L_2(q)$  and the three largest element orders of  $L_2(q)$ . (pp. 125-134)

### **Improvement in estimating population mean using two auxiliary variables in two-phase sampling**

Rajesh Singh, Pankaj Chauhan, Nirmala Sawan, Florentin Smarandache

This study proposes improved chain-ratio type estimator for estimating population mean using some known values of population parameter(s) of the second auxiliary character. The proposed estimators have been compared with two-phase ratio estimator and some other chain type estimators. The performances of the proposed estimators have been supposed with a numerical illustration. (pp. 135-142)

### **A note on quasi $k$ -ideals and bi $k$ -ideals in ternary semirings**

M.K. Dubey

Dutta and Kar [3] have introduced the concept of ternary semiring. In this paper, the notion of quasi  $k$ -ideals and bi  $k$ -ideals of a ternary semiring is introduced and characterizations  $k$ -regular ternary semirings has been given. (pp. 143-150)

### **$p$ -fuzzy hypergroupoids associated to the product of $p$ -fuzzy hypergraphs**

Yuming Feng

We construct fuzzy hyperoperations from the product of  $p$ -fuzzy hypergraphs and then we prove that the fuzzy hyperstructures determined by these hyperoperations are commutative  $p^2$ -fuzzy quasi-hypergroups. Some properties of these fuzzy hyperoperations are also listed. (pp. 151-160)

### **A note on testing of hypothesis**

Rajesh Singh, Jayant Singh, Florentin Smarandache

In this paper problem of testing of hypothesis is discussed when the samples have been drawn from normal distribution. The study of hypothesis testing is also extended to Bayes set up. (pp. 161-164)

### **Some properties of $n$ -isoclinism in Lie algebras**

Foroud Parvaneh, Mohammad Reza R. Moghaddam, A. Khaksar

In 1940, P. Hall introduced the concept of isoclinism of groups and it was generalized to  $n$ -isoclinism and isologism with respect to a given variety of groups. In the present article this notion is studied in Lie algebras and give some results similar to N.S. Hekster in 1986. In particular, it is shown that every family of  $n$ -isoclinism of Lie algebras contains an  $n$ -stem Lie algebra of minimal dimension. (pp. 165-176)

## The Dirichlet BVP for the second order nonlinear ordinary differential equation at resonance

Sulkhan Mukhigulashvili

Efficient sufficient conditions are established for the solvability of the Dirichlet problem

$$\begin{aligned}u''(t) &= p(t)u(t) + f(t, u(t)) + h(t) \quad \text{for } a \leq t \leq b, \\u(a) &= 0, \quad u(b) = 0,\end{aligned}$$

where  $h, p \in L([a, b]; \mathbb{R})$  and  $f \in K([a, b] \times \mathbb{R}; \mathbb{R})$ , in the case where the linear problem

$$u''(t) = p(t)u(t), \quad u(a) = 0, \quad u(b) = 0$$

has nontrivial solutions. (pp. 177-204)

## Grüss' type inequalities for functions of selfadjoint operators in Hilbert spaces

S.S. Dragomir

Some inequalities of Grüss' type for functions of selfadjoint operators in Hilbert spaces, under suitable assumptions for the involved operators, are given.

(pp. 205-222)

## Some divisible matrix groups

Y. Kemprasit, N. Triphop, A. Wasanawichit

A group  $G$  is said to be *divisible* if for any  $x \in G$  and any positive integer  $n$ , there exists an element  $y \in G$  such that  $x = y^n$ . For a positive integer  $n > 1$ , let  $G_n(\mathbb{R})$  be the group under multiplication of all invertible  $n \times n$  matrices over  $\mathbb{R}$ . For distinct positive integers  $p, q \leq n$ , let  $G(n, p, q)$  be the subgroup of  $G_n(\mathbb{R})$  consisting of all  $A \in G_n(\mathbb{R})$  with  $A_{ii} > 0$  for all  $i \in \{1, \dots, n\}$  and  $A_{ij} = 0$  for distinct  $i, j$  such that  $(i, j) \neq (p, q)$ . Also, let  $U(n)[L(n)]$  be the subgroup of  $G_n(\mathbb{R})$  consisting of all upper [lower] triangular matrices  $A \in G_n(\mathbb{R})$  with  $A_{ii} > 0$  for all  $i \in \{1, \dots, n\}$ . The purpose of this paper is to show that the matrix groups  $G(n, p, q)$ ,  $U(n)$  and  $L(n)$  are all divisible. (pp. 223-228)

## Fuzzy subnexuses

A. Saeidi Rashkolia, A. Hasankhani

The notion of fuzzy prime subnexus of a nexus is defined, and some related results are obtained. In particular, by considering the concept of homomorphism, some theorems about the coimage and preimage are proved. Finally, the notion of Quotient nexus induced by a fuzzy subnexus is introduced and some its properties are investigated. (pp. 229-242)

### On a class of Chinese hyperrings

Sanja Jančić Rašović

In this paper we have shown one construction of the Chinese hyperrings, using the class of multiendomorphisms of the starting ring  $(R, +, \cdot)$  or the class of multiendomorphisms of its additive group  $(R, +)$ . (pp. 243-254)

### Some results on analogous continued fraction of Ramanujan

Bhaskar Srivastava

We give a differential for  $\frac{1}{C(q)}$  and prove an identity which is analogous to Ramanujan's Entry 3.2.7. We also give a simpler proof for Entry 9(v). (pp. 255-260)

### On the hyperBanach spaces

P. Raja, S.M. Vaezpour

In this paper we are going to define hyperBanach spaces and prove some interesting theorems such as open mapping theorem, closed graph theorem, and uniform boundedness principal in these spaces. Also we define a quasinorm over hyper-vector spaces that converts a factor hypervector space into a normed hyper vector space. (pp. 261-272)

### On the qualitative behaviors of solutions to a kind of nonlinear third order differential equations with retarded argument

Cemil Tunç

In this paper, with use of a Lyapunov functional, we discuss stability and boundedness of solutions to a kind of nonlinear third order differential equation with retarded argument:

$$\begin{aligned} & x'''(t) + h(x(t), x'(t), x''(t), x(t-r(t)), x'(t-r(t)), x''(t-r(t)))x''(t) \\ & + g(x(t-r(t)), x'(t-r(t))) + f(x(t-r(t))) \\ & = p(t, x(t), x'(t), x(t-r(t)), x'(t-r(t)), x''(t)), \end{aligned}$$

when  $p(t, x(t), x'(t), x(t-r(t)), x'(t-r(t)), x''(t)) = 0$  and  $\neq 0$ , respectively. Our results include and improve some well-known results in the literature. An example is also given to illustrate the importance of results obtained and the topic.

(pp. 273-284)

### **Hyperaction of semigroups and monoids**

M.K. Sen, Reza Ameri, Goutam Ghowdhury

The purpose of this note is the study of hyper action as a generalization of action of a monoid on a set. In this regards, first we introduce the notion of hyperactions of type 1 and type 2 and then we study the basic properties of this notion. In particular, we investigate the relationship between hyperactions and non-deterministic automata. (pp. 285-294)

### **Une remarque sur certaines formes de Weierstrass**

Titem Harrache, Nouressadat Touafek

Dans cette note, on s'intéresse à la recherche d'une forme de Weierstrass de courbes de genre 1. On souligne sur des exemples l'intérêt de la méthode pour mettre en évidence les points de torsion ou d'ordre infini et pour obtenir des formes de Weierstrass tempérées. (pp. 295-308)

### **Non periodic solutions of fourth order nonlinear equations**

Bruno Bianchini

In this paper we obtain a result of existence of solutions for a 4th order nonlinear equation subject to non periodic boundary conditions. (pp. 309-326)

### **On eight order mock theta function**

S. Ahmad Ali

In the present paper we have introduced partial mock theta functions of orders eight. We have established certain identities relating these functions with those of different orders. (pp. 327-332)