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A few thoughts of the editor-in-chief regarding the existence of the journal

Now, in the twentieth year of the journal, it seems appropriate to strike the balance. In 1987, with the substantial help of the Consortium (and the special contribution of its Executive, Doctor Liesch), we founded *Rivista di Matematica Pura e Applicata*.

Back then, not everybody was quite convinced that the initiative would be successful. We did, even if the project presented us with inevitable difficulties.

After no more than a few years, the journal exchanged articles with prominent similar publications in Europe and other continents. Its success grew when the journal changed its name to *Italian Journal*; as the number of printed pages increased, we could publish a larger number of papers in each issue and, consequently, reduce the waiting time for the authors. This process has become more obvious in later years, as the printing shop has been changed. Therefore, the number of printing pages is around or over 250, which has led to a further substantial and constant decrease in the waiting time, from around three years (and no less than two), which was the case a few years ago, to less than a year at present.

For reasons related to research, I have often the opportunity to travel abroad and I can therefore ascertain that, wherever it is read – Europe, Asia, America – our journal is well known and widely appreciated. All told, 589 papers have been published so far (171 in *Rivista*, 418 in *Italian Journal*), our contributors being from five continents, as follows:

EUROPE: Italy, Romania, Greece, The Czech Republic, Slovakia, France, Germany, Russia, Serbia, Montenegro, Great Britain.

ASIA: Iran, Thailand, China, India, Turkey, Georgia, Saudi Arabia, Korea, Jordan, Pakistan, Nepal, Vietnam, Israel, Tadjikistan, Belarus, Japan, Kuwait, Oman, Bahrain

AMERICA: U.S., Canada, Tchile, Argentina, Brasil

OCEANIA: Australia

Moreover, the journal exchanges papers with 246 other similar journals that reach institutes on all continents.

Undoubtedly, we can infer from the above presented data that the success of our journal is well established. This is a source of satisfaction for all who have worked and contributed in various degrees to its existence.

We all hope it will continue and will do whatever it takes to further, to expand this success.

PIERGIULIO CORSINI

Corsini's method and construction of join spaces

Morteza Yavari

In this paper one continues the study of hypergroups and join spaces, [12]. Different methods of constructing join spaces from hypergroups and hypervector spaces, [9], are presented in this paper. By using Corsini's method, [1], we obtain a lot of examples of join spaces. (pp. 25-36)

On upper and lower α -irresolute fuzzy multifunctions

V. Seenivasan, G. Balasubramanian

New classes of multifunctions called fuzzy upper and fuzzy lower α -irresolute (α -continuous) multifunctions in fuzzy topological spaces are introduced in this paper. We also obtain some characterizations of this class and some basic interesting properties of such fuzzy multifunctions. We discuss mutual relationship and also relationship with other existing such multifunctions. (pp. 11-24)

On random coincidence for a pair of measurable mappings

Ljubomir B. Ćirić, Marina M. Milovanovic-Arandjelovic, Nebojša T. Nikolić

Let $(X;d)$ be a Polish space and (Ω, Σ) be a measurable space. In this paper a pair of measurable mappings $f : \Omega \times X \rightarrow X$ and $T : \Omega \times X \rightarrow X$, satisfying the inequality (2) below, are introduced and investigated. It is proved that if $T(\omega, \cdot), f(\omega, \cdot)$ are continuous for all $\omega \in \Omega$, $T(\cdot, x); f(\cdot, x)$ are measurable for all $x \in X$; X is complete and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi : \Omega \rightarrow X$ such that $f(\omega, \xi(\omega)) = T(\omega, \xi(\omega))$ for all $\omega \in \Omega$. This result generalizes and extends many classical fixed point theorems. (pp. 37-44)

Nevanlinna theory in positive characteristic and applications

Abdelbaki Boutabaa, Alain Escassut

We show that Nevanlinna's main Theorem holds, with some correction, in any algebraically closed field K of positive characteristic complete with respect to an ultrametric absolute value. So we must introduce the ramification index of an analytic function. As it was made in characteristic zero, the Nevanlinna Theory in positive characteristic also holds for meromorphic functions inside an "open" disk. As applications, first we generalize some results on uniqueness for meromorphic functions in the whole field K or in an "open" disk, satisfying $f^{-1}(\{a_i\}) = g^{-1}(\{a_i\})$ for some points a_i . Next, we characterize the meromorphic solutions of the Yosida equation with constant coefficients and more precisely, we characterize the equations admitting solutions. Finally we consider the *abc*-problem for ultrametric entire functions. (pp. 45-66)

On single series representation of mock theta functions of fifth and seventh order

Remy Y. Denis, S.N. Singh, S.P. Singh

In this paper an attempt has been made to represent the fifth and seventh order mock-theta functions as the sum of two single series. We shall also establish relation between two mock-theta functions of fifth order belonging to the two different groups and also between two mock-theta functions of seventh order. In the sequel, we shall establish various transformations involving mock-theta functions and basic hypergeometric function signifying the influence of basic hypergeometric functions on mock-theta functions. The paper conclude by pointing out that the representation discussed herein belong to a general class of transformation of basic hypergeometric functions. (pp. 67-74)

On pairwise fuzzy pre-basically and pre-extremally disconnected spaces

M.K. Uma, E. Roja, G. Balasubramanian

In this paper we introduce pairwise fuzzy pre-basically disconnected spaces and pairwise fuzzy pre-extremally disconnected spaces. Besides giving examples, some interesting properties of these spaces are also given. (pp. 75-84)

On a fourth order pseudoparabolic equation

Ahmad Maher

The aim of the present paper is devoted to investigation of the Goursat problem of a fourth order pseudoparabolic equation, with the help of Riemann function. (pp. 85-92)

Filters and isomorphism theorems in residuated lattices

Lingxia Lu, Wei Yao

The aim of this paper is to study the properties of filters and fuzzy filters and establish isomorphism theorems in residuated lattices. In order to do these, the definitions of congruence relations, subresiduated lattices and homomorphisms are given and their corresponding properties are studied. Three basic isomorphism theorems are proved. (pp. 93-106)

Connes amenability of the second duals of Arens regular Banach algebras

M. Eshaghi Gordji

In this paper we study the Connes amenability of the second dual of Arens regular Banach algebras. Of course we provide a partial answer to the question posed by Volker Runde. Also we find the necessary and sufficient conditions for the second dual of an Arens regular module extension Banach algebra to be Connes amenable when the module is reflexive. (pp. 107-112)

Hyperring of matrices over a regular hyperring

A. Asokkumar, M. Velrajan

The aim of this paper is to show that a hyperring R with a hyperaddition and a hypermultiplication is regular (in the sense of Von Neumann) if and only if the hyperring of matrices over R is regular. (pp. 113-120)

Patent document classification based on mutual information feature selection

Ioana Costantea, Radu Ioan Boț, Gert Wanka

We describe a supervised text classification approach based on a greedy feature selection method, which uses a support vector machine (SVM) classifier. As feature selection method we use the mutual information. This measures the quantity of information about the categories contained by the words. To train and test the algorithm we used patent documents from the US Patent Classification System. Average break-even point (BEP) for some US Classes is reported as conclusion. (pp. 121-136)

Semi-compatibility and common fixed point theorems in Menger space

Bijendra Singh, Shishir Jain, Shobha Jain

In this paper the concept of semi-compatibility has been introduced in Menger space and has been applied to prove results on existence of unique common fixed point of four self maps. These results highlight the importance and necessity of semi-compatibility in study of fixed point theory in Menger space besides being generalization and alternative results of many existing results in this field. All the results presented in this paper are new. (pp. 137-150)

Comments on a 'Fixed point theorem in Menger space through weak compatibility'

Kubiacyzk, Bhavana Deshpande

The aim of this note is to point out that some conditions of a fixed point theorem in Menger space due to Singh and Jain can be relaxed, replaced and omitted. Of course, we also improve extend and generalize many known results in Menger spaces and metric spaces. (pp. 151-160)

On the strong defining spectrum of k -regular graphs

Doost Ali Mojdeh

In a given graph $G = (V; E)$ a set of vertices S with an assignment of colors to them is said to be a strong defining set of the vertex coloring of G , if there exist an ordering $\{v_1, v_2, \dots, v_n\}$ of the vertices of $G-S$ such that in the induced list of colours in each of the subgraphs $G-S$, $G-(S \cup \{v_1\})$, $G-(S \cup \{v_1, v_2\})$ and $G-(S \cup \{v_1, v_2, \dots, v_{n-s}\})$, there exist at least one vertex whose list of colours is of cardinality 1. A strong defining set with minimum cardinality is called a minimum strong defining set and its cardinality is the strong defining number, denoted by $sd(G, c)$. Let F be a family of k -regular graphs on n vertices and be colored with $k-1$ colors. A strong defining spectrum of F , with $k-1$ colors is $sd_{k-1}(F) = \{d \in \mathbb{C} \mid \exists G \in F, sd(G) = d\}$. We study $sd_{k-1}(F)$ on n vertices for $k=5$ and $k=6$. (pp. 161-172)

A theorem of finite groups having only two non-normal subgroups

Huaguo Shi, Guiyun Chen

We prove that a finite 2-group G contains exactly two non-normal subgroups if and only if $G = \langle u, v \mid u^{2^n} = v^2 = 1, v^{-1}uv = u^{1+2^{n-1}}, n \geq 3 \rangle$. (pp. 173-178)

A note on a multiplication formula for the multiple gamma function Γ_n

J. Choi, H.M. Srivastava

The theory of the multiple Gamma functions Γ_n was studied in about 1900 and was revived in the study of the determinants of the Laplacians in about the middle of the 1980s. Ever since then, it has found many interesting applications in a variety of subjects such as number theory, theoretical and mathematical physics, and geometry. Here the authors show how nicely and simply a multiplication formula for the n -ple Γ_n can be proved. (pp. 179-188)

A common fixed point theorem in uniformly convex Banach space

Sushil Sharma, Prashant Tilwankar

In this paper we improve results of Sharma and Bambaria by using the property (E.A) and replacing the completeness of subspace by closed subset. (pp. 189-196)

Linearizing transformation and exact solutions of nonlinear equations in mathematical physics

O.H. El-Kalaawy, R.S. Ibrahim

In this paper, we investigate the integrability for some nonlinear partial differential equations (NPDEs). The concepts are developed by firstly discussing the integrability of the Kortweg de-Vries-Burgers (KdV-B) equation. The method is based upon a linearization principle which can be applied on nonlinearities which have a polynomial form. We illustrate the potential of the method by finding solutions of the KdV-mKdV equation and break soliton equation in (2+1) dimensions which play an important role in mathematical physics. (pp. 197-204)

Nonlinear degenerate equations with natural growth terms and L^1 data

L. Aharouch, Y. Akdim

In this paper, we shall be concerned with the existence result of the quasi-linear degenerate elliptic equations of the form,

$$Au + H(x, u, \nabla u) = f,$$

where A is a Leray-Lions operator from $W_0^{1,p}(\Omega, w)$ into its dual. On the nonlinear lower order term $H(x, u, \nabla u)$, we assume that it's a Carathéodory function having natural growth with respect to $|\nabla u|$, but without assume the sign condition. The right hand side f belongs to $L^1(\Omega)$. (pp. 205-220)

Nonparametric estimation of the derivative of a density by the method of wavelets with negatively dependent sequences

Hassan Doosti

We propose a method of estimation of the derivatives of probability density based wavelets methods for a sequence of negatively dependent random variables with a common one-dimensional probability density function and obtain an upper bound on L_p -losses for the such estimators. (pp. 221-228)

Pure characterization of the projective special linear groups

M.R. Darafsheh

For a finite group G let $\omega(G)$ be the set of orders of elements of G . Let $h(\Omega)$ denote the number of non-isomorphic groups with $\omega(G) = \Omega$, where Ω is a subset of \mathbb{N} . For convenience we write $h(\omega(G)) = h(G)$. We call a group G recognizable or characterizable by its element orders if $h(G) = 1$. In this paper we are interested in the problem of element orders and characterization property of the projective special linear groups. We first explain how one can obtain $\omega(PSL_n(q))$ for given n and q , where q is a prime power. Then we use a well-known method to prove the simple group $PSL_5(9)$ is recognizable by the set of its element orders. (pp. 229-244)

The rule behind the occurrence of prime numbers

Matteo Arpe, Claudia La Chioma

In this paper we present a methodology to identify the occurrence of prime numbers in their apparently random sequence. We prove that each prime number contains all the needed information to identify the subsequent primes. In particular:

1. every prime number p determines a vector $\mathbf{d}^{(p)}$ whose components are symmetric but a tail of three points;
2. the couple $(p, \mathbf{d}^{(p)})$ identifies uniquely the primes $\{p_i\}_{i=1}^n$ for some n , the related vectors $\{\mathbf{d}^{(p_i)}\}_{i=1}^n$ and the set of coprimes with q prime for all $q \leq p$;
3. dimension and mass of $\mathbf{d}^{(p)}$ are uniquely determined from dimension and mass of $\mathbf{d}^{(q)}$, $q = \sup\{a \in \mathbb{N} \text{ s.t. } a \text{ is prime and } a \leq p\}$;
4. symmetry and three{points tail are preserved by all the $\mathbf{d}^{(p)}$.

We underline that the methodology presented here is an inductive one and that its by-product is the complete determination of prime numbers by the only knowledge of a couple $(p; \mathbf{d}^{(p)})$. (pp. 245-260)

Several types of n -ary subhypergroups

V. Leoreanu-Fotea

In this paper, we introduce several types of n -ary subhypergroups in a commutative n -ary hypergroup, we give examples and establish some connections among them. (pp. 261-274)