

**Generic 4-parameter family of planar vector fields. Local bifurcations of an improper saddle-node singularity**

Myrna Wallace, Hernan Burgos, Jorge Billeke

A germ of vector field of the improper saddle-node type is a  $C^\infty$ -germ at  $0 \in \mathbb{R}^2$ , whose 4-jet is  $C^\infty$ -conjugate to  $y \frac{\partial}{\partial x} + (\varepsilon x^4 + bxy) \frac{\partial}{\partial y}$ ,  $\varepsilon = \pm 1$ ,  $b > 0$ . We define a submanifold  $\Sigma_{ISN}^4$  of codimension 6 in the space of  $C^\infty$ -germs, consisting of germs of improper saddle-nodes. We prove that any local 4-parameter family of planar vector fields, intersecting  $\Sigma_{ISN}^4$  transversally at  $(0, 0) \in \mathbb{R}^2 \times \mathbb{R}^4$  has a normal form:

$$\begin{cases} \dot{x} = y \\ \dot{y} = \varepsilon_1 x^4 + \mu_2 x^2 + \mu_1 x + \mu_0 + y[v + bx + c(\lambda)x^2 + d(\lambda)x^3 + O(x^4)] + y^2 Q(x, y, \lambda) \end{cases}$$

with  $\varepsilon_1 = \pm 1$ ,  $b > 0$ ,  $\lambda = (\mu_0, \mu_1, \mu_2, v) \in \mathbb{R}^4$ ,  $Q = (\|(x, y)\| + \|\lambda\|)^N$ ,  $N \in \mathbb{N}$  arbitrarily large.

We study the local bifurcations of a standard family having the origin as a fixed singularity:

$$X_\lambda^\pm = y \frac{\partial}{\partial x} + (\pm x^4 + \mu_2 x^2 + \mu_1 x + y(bx + v)) \frac{\partial}{\partial y},$$

with  $\lambda = (\mu_1, \mu_2, v) \in \mathbb{R}^3$ .

This paper continues the same line of research of R. Bogdanov [2], F. Takens [3], F. Dumortier, R. Roussarie and J. Sotomayor [4] and [5].

**Linear spaces and classical euclidean geometry**

Sandro Rajola, Maria Scafati Tallini

In a finite linear space  $(S, \mathcal{L})$  we define a metric by associating a prime number with every line. With respect to this metric we can define segments, angles, triangles and so on. We formulate and prove several statements similar to those of the classical euclidean geometry, some of which are unexpected.

**Calculative matrices revisited: algebraic properties of a generalization of cyclic matrices**

Dario Fasino

We consider the set of solutions  $A$  of the matrix equation  $A = zPAP^*$  where  $|z| = 1$  and  $P^* = P^{-1} = P^{k-1}$  for some integer  $k$ . In particular, we exhibit a suitable canonical form and some algebraic properties owned by all matrices in this set.

## **$H_W$ -Groups without "incidence property"**

Dario Pasquali-Coluzzi, Franco Eugeni

The weak join space notion originates from the classic example, related to the notion of convexity, in which, in the affine real plane, the set of points in a segment is associated to two points. In this work, we will examine the possibility to extend that notion to the finite plane of Desargues. The answer will be negative for some reasons, but, unless we could not extend notions of *weak join spaces*, the correspondent structures are really interesting and structured.

## **On the problem of asymptotic equivalence of ordinary differential equations**

Georgios I. Eleutheriadis, Moses A. Boudourides

In the first part, we discuss a variety of definitions of asymptotic equivalence of nonlinear differential systems and prove some sufficient conditions for it. In the second part, we consider the linear case and extend some known results on relative asymptotic equivalence.

## **On the strictly hyperbolic equations which are Hölder continuous with respect to time**

Massimo Cicognani

The Cauchy problem for a strictly hyperbolic equation may not be well posed in  $C^\infty$  if the coefficients are  $C^\chi$  functions of the time variable with  $\chi < 1$ . In order to obtain existence and uniqueness of the solution, one has to assume a higher regularity in space variables to compensate for the lower regularity with respect to time.

## **Approximation properties of the Picard singular integral in exponential weight spaces**

A. Leśniewicz, L. Rempulska, J. Wasiak

In this note we give some direct and inverse approximation theorems for the Picard singular integral in the exponential weight spaces and some generalized Hölder spaces.

## **Hypergroups and join spaces determined by relations**

I.G. Rosenberg

To every binary relation  $\rho$  on a set  $H$  and with full domain we associate the hypergroupoid  $\mathbb{H}_\rho$  assigning to every pair  $(x, y)$  the set of all  $z$  such that either  $(x, z) \in \rho$  or  $(y, z) \in \rho$ . We characterize all  $\rho$  such that  $\mathbb{H}_\rho$  is a semihypergroup, hypergroup and join space. We also investigate which  $\mathbb{H}_\rho$  enjoy certain standard hypergroup properties.

## ***F*-hyperrings**

A. Hasankhani, M.M. Zahedi

The concept of *F*-hyperrings, *F*-subhyperrings, *F*-hyperideals, fuzzy sub-*F*-hyperrings, fuzzy *F*-hyperideals and fuzzy *FR*-congruence relations are defined and some results are obtained. In particular, it is shown there is a bijection between the set of all fuzzy *F*-hyperideals of an *F*-hyperring *R* and the set of all fuzzy *FR*-congruence relations on *R*.

## **Hypergroups associated with hypergraphs**

Violeta Leoreanu, Laurențiu Leoreanu

A regular reversible hypergroup is associated with a hypergraph and some properties concerning its subhypergroups are given.

## **Some new sequence spaces, their duals and matrix transformations**

Mursaleen, A.K. Gaur, A.H. Saifi

Let  $\ell_\infty$ ,  $c$  and  $c_0$  be the Banach spaces of bounded, convergent and null sequences respectively. Let  $\Delta_r x = (k^r \Delta x_k)_{k=1}^\infty$ ,  $r < 1$ , where  $\Delta x_k = x_k - x_{k+1}$ . In this paper, we define the sequence space

$$\ell_\infty(\Delta_r p) = \{x - (x_k) : \Delta_r, x \in \ell_\infty(p)\},$$

where  $\ell_\infty(p) = \{x = (x_k) : \sup_k |x_k|^{p_k} < \infty\}$  for an arbitrary sequence of positive reals  $p = (p_k)_{k=1}^\infty$ . We find the  $\alpha$ -, and  $\beta$ -duals of our new sequence space and some matrix transformations are also considered.

## **Hypergroups induced by paths of a direct graph**

I.G. Rosenberg

To any connected oriented graph on  $V$  we assign two hypergroupoids  $\mathcal{H}_1$  and  $\mathcal{H}_2$  on  $V$ . Both assign to each pair  $(x, y)$  of distinct vertices the set  $x \circ y$  of all vertices on  $x - y$  paths. For every vertex  $x$  the hypergroupoid  $\mathcal{H}_1$  associates  $x \circ x = \{x\}$  while in  $\mathcal{H}_2$  the set  $x \circ x$  is the set of all vertices on oriented circuits through  $x$ . We give necessary and sufficient conditions for  $\mathcal{H}_i$  to be a hypergroup. We also characterize  $\mathcal{H}_i$  that are hemijoin spaces, a noncommutative version of a join space.

## **Oscillation theorems for nonlinear difference equations with damped term**

Błażej Szmanda

Some new oscillation theorems for the difference equations of the form

$$\Delta(r_n \Delta u_n) + p_n \Delta u_n + q_n f(u_{n-k}) = 0$$

are established.

## Una argomentazione euristica probabilistica sulla successione di Collatz Enzo Barone

We give a heuristic probabilistic argument to support the Collatz's conjecture.

## On the Perron-Bellman theorem for $C_0$ -semigroups and periodic evolutionary processes in Banach spaces

M. Reghiş, C. Buş

It is proved that a periodic evolutionary process  $U$  of linear operators on a Banach space  $X$  is uniformly exponentially stable if and only if

$$\sup_{t \geq 0} \left| \int_0^t U(t, x) f(s) ds \right| < \infty, \quad \forall f \in C_0(\mathbb{R}_+, X),$$

where  $C_0(\mathbb{R}_+, X)$  is the Banach space of all continuous functions  $f : \mathbb{R}_+ \rightarrow X$  such that  $\lim_{t \rightarrow \infty} f(t) = 0$ , with "sup" norm. As application, it is shown that  $U$  is uniformly exponentially stable if and only if

$$\sup_{t \geq 0} \left| \int_0^t U(t, x) f(s) ds \right| < \infty, \quad \forall f \in AP(\mathbb{R}_+, X),$$

where  $AP(\mathbb{R}_+, X)$  is the space of  $X$ -valued almost periodic functions on  $\mathbb{R}_+$ . These results contain, in particular, some results of Neerven [9]. On the other hand, it is known (see Balint [2]), that if  $A$  is a bounded linear operator on  $X$ , then the semigroup  $(T(t))_{t \geq 0}$  generated of  $A$  is uniformly exponentially stable if and only if

$$\sup_{t \geq 0} \left| \int_0^t e^{i\rho s} T(s)x ds \right| < \infty, \quad \forall \rho \in \mathbb{R}, \quad \forall x \in X.$$

In this paper are also proved that Balint's result cannot be extended for a general  $C_0$ -semigroup.

## Su alcuni $K$ -insiemi di $PG(3, q)$ di classe $[0, 1, m, n]_1$

Osvaldo Ferri, Stefania Ferri

Si prova che un  $(q+1)^2$ -insieme  $K$  di  $PG(3, q)$ , di classe  $[0, 1, m, n]_1$ , rispetto alle rette e tale che per ogni suo punto passino  $q-1$  rette unisecanti  $K$  ed, al più, due  $n$ -secanti  $K$ , è una quadrica iperbolica.

We prove that a  $(q+1)^2$ -set  $K$  of  $PG(3, q)$ , of class  $[0, 1, m, n]_1$ , with respect to the lines, such that each point of  $K$  there are  $q-1$  lines tangent to  $K$  and, at most, two  $n$ -secants  $K$ , is hyperbolic quadric.