Binary soft simply* alpha open sets and continuous function

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Abstract. A topological rough approximation space is defined over two different universes using binary soft relations. A new class of binary soft set and its corresponding soft topology is obtained. Continuity functions for the newly defined set are introduced. The characteristics of continuity functions between two binary soft topological spaces and that between binary soft topological space and topological rough approximation space are examined. The proposed definitions and properties are demonstrated with suitable examples.

Keywords: soft set, binary soft set, binary soft nowhere dense, continuity mappings, approximation space.
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1. Introduction

Data involving uncertainties are present in various disciplines such as economics, engineering, social science, and medical science. Uncertainty in events complicates decision making in many aspects. To handle problems with uncertainty, the concept of fuzzy sets was first defined by Zadeh [34]. Though fuzzy set theory helped in solving problems with uncertainty, assigning membership values to a large number of data was challenging. To overcome such difficulties, the concepts of rough set and soft set were developed. Pawlak [26] first defined rough sets in 1982. These sets were related to upper and lower approximations and generally are crisp sets. Pawlak's rough sets are based on equivalence relations, but finding an equivalence relation among the elements of a set was difficult. Though different relations were used to define rough set theory, they had compli-

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cations in modelling problems with uncertainty. Hence, Moldtsov [22] initiated the theory of soft sets. It was further investigated by Maji et al. and other researchers [7, 8, 19, 20, 23]. Soft set theory has application in various fields like decision making, game theory, operations research, etc. Continuity functions of soft near open sets in soft topological spaces [2, 3], continuity functions of rough sets [28], fuzzy continuous functions [13, 17], and many other hybrid topological spaces are studied in literature [5, 24]. In addition, the relationship between soft sets and fuzzy sets was studied by Alcantud [4], and the relationship between soft, rough, and fuzzy sets was investigated by Feng et al. [11]-[12]. A review on soft set based parameter reduction and decision making was done by Sani Danjuma et al. [9].

Extension of theories of uncertainty over n number of different non-empty finite sets helps the decision maker to make many decisions at a time. This will help in the future advancements in different areas of research. When it comes to two universes, the rough set model was first studied in 1996 [33]. Following them, numerous studies utilising uncertainty theories were conducted over two universes [18], [27], [29], [30], [31], [32], [35], [36], [37].

In recent years, attempts have been made to generalise the soft sets over a single universe to two or more universes. The binary soft set was first defined and studied by Ackogz et al. [1]. Following them, Hussain [14] studied the topological properties of binary soft set. Further, binary soft mappings, separation axioms, connectedness, and other hybrid concepts like binary bipolar soft sets, fuzzy binary soft sets, etc. are studied by researchers in [6, 13, 15, 16]. Simply* alpha open sets are useful in the field of decision making as it contributes to attribute reduction. It was studied over a rough set by El Safty et al. [10]. Although it is examined in a rough set, it is appropriate to study the simply* alpha open set over the soft set since the soft set contains a parametrization tool. Simply* alpha open set is extended to soft set theory over two different universes using soft binary relations in the author's previous work [25]. In that work, BR-soft topological rough approximation space was obtained, the definition of BR-soft simply* alpha open sets, other related BR-soft sets are defined, and their basic properties are studied.

Decision-making becomes relatively simple if we can identify continuous mapping from one set of parameters to another set and continuous mapping among the universal set. In this paper, the notions of BR-soft simply* alpha continuous mapping, contra continuous, and irresolute are defined between soft topological rough approximation space and newly obtained soft topology. This can be used to deal with uncertainty and vagueness in many areas, like data analysis, machine learning, etc.

This paper is divided into three main sections. In Section 2, the basic definitions used in the paper is discussed. Section 3 deals with the BR-soft simply* alpha open sets and related topological notions. In Section 4, BR-soft simply* alpha mapping, continuous functions, and contra continuous functions are introduced along with theorems and examples followed by concluding remarks.

2. Preliminary

Definition 2.1. *Let S be the universe set, E be the parameter set, and k be the subset of the parameter set E. Then, a soft set is a mapping from a subset of a parameter set to the power set of the universe set.*

Definition 2.2. *Let (m_1, k) and (m_2, j) be two soft sets over a common universe S. Then, (m_1, k) is said to be a soft subset of (m_2, j), if k is a subset of j for all e belongs to k, $m_i(e)$ are identical approximations.*

Definition 2.3. *A soft set (m, k) over S is said to be an absolute soft set if for every e belongs to k, $m(e) = S$.*

Definition 2.4. *A relation between the sets S and T is a subset of the cartesian product $S \times T$, where $S \times T = \{(s, t) : s \in S, t \in T\}$.*

Definition 2.5. *Let S and T be two different nonempty finite sets. k be the subset of a parameter set E. A pair (m, k) or m_k is called a soft binary relation over S and T if (m, k) is a soft set (binary soft set or BR-soft set) over $S \times T$.*

(Throughout this paper, BR stands for binary.)

Example 2.1. Let S denote the set of three patients $\{N, Z, C\}$, and T denote the set of three diseases $\{Typhoid(Ty), Dengue(D), Pneumonia(P)\}$. Let E be the set of parameter that define the symptoms of diseases, where $E = \{e_1(fever), e_2(breathingproblem), e_3(jointpain), e_4(headache)\}, K = \{e_1, e_2\} \subseteq E$. Let $S \times T = \{(N, Ty), (N, D), (N, P), (Z, Ty), (Z, D), (Z, P), (C, Ty), (C, D), (C, P)\}$ be the universal set. Then, soft set $(m, K) = \{(e_1, \{(N, Ty), (N, P), (Z, Ty), (Z, P), (C, Ty), (C, P)\}), (e_2, \{(N, D), (N, P), (Z, D), (Z, P), (C, D), (C, P)\})\}$ denotes patients and their symptoms along with the possibility of diseases.

Definition 2.6. *A binary relation $R_{(m(s,t))}$ on S and T induced by m_k is defined by $(s, t)R_{m(s,t)}(s_1, t_1) \iff \{(s, t)m_k(s_1, t_1)\}$ for each $(s, t), (s_1, t_1) \in S \times T$.*

Definition 2.7. *The successor neighbourhood of each (s, t) in $S \times T$ is given by $R_{m(s,t)}(s, t) = \{(s_1, t_1) \in S \times T; (s, t)R_{m(s,t)}(s_1, t_1)\}$.*

Definition 2.8. *Let m_k be a soft binary relation over $S \times T$. $G \times J \subseteq S \times T$ and $(S, T, R_{m(s,t)})$ be a rough approximation space with respect to the parameter set. The approximation operators are defined as follows:*

$$\begin{aligned}\underline{S_{apr}}(G \times J) =& \{(s, t) \in S \times T; R_{m(s,t)}(s, t) \subseteq (G \times J)\},\\ \overline{S_{apr}}(G \times J) =& \{(s, t) \in S \times T; R_{m(s,t)}(s, t) \cap (G \times J) \neq \emptyset\},\end{aligned}$$

where $\underline{S_{apr}}(G \times J)$ is the lower rough soft approximation and $\overline{S_{apr}}(G \times J)$ is the upper rough soft approximation over two different universal sets. If $\underline{S_{apr}}(G \times J) = \overline{S_{apr}}(G \times J)$, then $G \times J$ is a definable soft set. If $\underline{S_{apr}}(G \times J) \neq \overline{S_{apr}}(G \times J)$, then $(G \times J)$ is a rough soft set.

Definition 2.9. *Let $(S, T, R_{m(s,t)})$ be a rough approximation space, and τ_{BR} be a soft topology obtained from soft binary relation over S, T. Thus, $(S, T, R_{m(s,t)}, \tau_{BR})$ is said to be BR-topological rough approximation space, where the elements of τ_{BR} are BR-soft open, and its complements are closed.*

Definition 2.10. *Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space. For each $m_{ki} \subseteq m_k$, the BR-topological approximation operators are defined as follows:*

$$\underline{\tau}_{BR}(m_{ki}) = \cup\{m_{kj} \in \tau_{BR}; m_{kj} \subseteq m_{ki}\},$$
$$\overline{\tau}_{BR}(m_{ki}) = \cap\{m_{kj} \in \tau^c_{BR}; m_{ki} \subseteq m_{kj}\}.$$

In other words, $\underline{\tau}_{BR}, \overline{\tau}_{BR}$ is considered the interior and closure of the BR-topological approximation space, respectively.

3. BR-soft simply* alpha open set

Definition 3.1. *In a BR-topological rough approximation space, a BR-soft subset is called BR-soft nowhere dense, if $\underline{\tau}_{BR}(\overline{\tau}_{BR}(m_{ki})) = \emptyset$.*

Definition 3.2. *In a BR-topological rough approximation space, a BR-soft subset is said to be BR-soft alpha open if $m_{ki} \subseteq \underline{\tau}_{BR}(\overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki})))$ and is BR-soft alpha closed if $\underline{\tau}_{BR}(\overline{\tau}_{BR}(\underline{\tau}_{BR}(m_{ki}))) \subseteq m_{ki}$.*

Definition 3.3. *In a BR-topological rough approximation space, a BR-soft subset is called a BR-soft simply* alpha open set if $(m_{ki}) \in \{\emptyset, m_k, (m_{kj}) \cup (m_{kl}) : (m_{kj})$ is $BR - soft$ alpha open$, (m_{kl}) is BR - soft$ nowhere dense$\}$. The collection of BR-soft simply* alpha open set is denoted by $BR_S S^*\alpha O(m_{ki})$, the complement is BR-soft simply* alpha closed.*

Proposition 3.1. *Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space.*

i) The arbitrary union of the BR-Soft simply alpha open set is BR-Soft simply* alpha open.*

ii) Finite intersection of BR-Soft simply alpha open set is BR-Soft simply* alpha open.*

Definition 3.4. *Let $(S, T, R_{m(s,t)}, \tau_{BR})$ be a BR-topological rough approximation space. For each $m_{ki} \subseteq m_k$, where m_{ki}, m_k are BR-soft simply* alpha open sets. Then the BR-topological approximation operators are defined as follows:*

$$\overline{BR}_S(m_{ki}) = \cap\{m_{kj} \in \tau^c_{BR}; m_{ki} \subseteq m_{kj}\},$$
$$\underline{BR}_S(m_{ki}) = \cup\{m_{kj} \in \tau_{BR}; m_{kj} \subseteq m_{ki}\},$$

where $\overline{BR}_S(m_{ki}), \underline{BR}_S(m_{ki})$ are the closure and interior of BR soft simply alpha open sets in BR-topological rough approximation space respectively.*

Theorem 3.1. *A collection of BR-Soft simply* alpha open sets forms a BR-soft topology* τ^*_{BR}.

Definition 3.5. *Let* τ^*_{BR} *be a BR-soft topology obtained from the collection of BR-soft simply* alpha open sets. For each BR-soft simply* alpha open sets* $m_{ki} \subseteq m_k$, *the BR-topological approximation operators are defined as follows:*

$$\overline{BR}_S(m_{ki}) = \cap\{m_{kj} \in (\tau^*_{BR})^c; m_{ki} \subseteq m_{kj}\},$$
$$\underline{BR}_S(m_{ki}) = \cup\{m_{kj} \in \tau^*_{BR}; m_{kj} \subseteq m_{ki}\},$$

where $\overline{BR}_S(m_{ki}), \underline{BR}_S(m_{ki})$ *are the closure and interior of BR soft simply* alpha open sets in* τ^*_{BR} *respectively.*

4. Continuous mapping of BR-soft simply* alpha open set

Definition 4.1. *Let* (S, T, τ^*_{BR}, E) *be a BR-soft topological space obtained from the collection of BR-soft simply* alpha open sets and* $(U, V, R_{m(s,t)}, \tau_{BR})$ *be a soft topological approximation space. Then,* $f : (S, T, \tau^*_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ *is said to be* $BR_S S^*\alpha$*-continuous, if* $f^{-1}(m_k)$ *is BR-soft simply* alpha open set for every* $m_k \in (U, V, R_{m(s,t)}, \tau_{BR})$.

Example 4.1. Let $U = \{b, c, d\}, V = \{e, f\}$. Then $U \times V = \{(b, e), (b, f), (c, e), (c, f), (d, e), (d, f)\}$. Let the parameter sets be $E = \{e_1, e_2\}, K = \{k_1, k_2\}$ respectively, and $m_e = \{(e_1, \{(c, e), (d, e)\}), (e_2, \{(b, e), (c, f)\})\}$ be the BR-soft set. Then, the topological rough approximation space obtained is

$$\begin{aligned}\tau_{BR} = &\{\emptyset, \{(e_1, \{(d, e)\})\}, \{(e_1, \{(c, e)\})\}, \{(e_2, \{(c, f)\})\}, \{(e_2, \{(b, e)\})\},\\ &\{(e_1, \{(c, e), (d, e)\})\}, \{(e_1, \{(d, e)\}), (e_2, \{(c, f)\})\}, \{(e_1, \{(d, e)\}),\\ &(e_2, \{(b, e)\})\}, \{(e_1, \{(c, e)\}), (e_2, \{(c, f)\})\}, \{(e_1, \{(c, e)\}), (e_2, \{(b, e)\})\},\\ &\{(e_2, \{(b, e), (c, f)\})\}, \{(e_1, \{(c, e), (d, e)\}), (e_2, \{(c, f)\})\}, \{(e_1, \{(c, e),\\ &(d, e)\}), (e_2, \{(b, e)\})\}, \{(e_1, \{(d, e)\}), (e_2, \{(b, e), (c, f)\})\}, \{(e_1, \{(c, e)\}),\\ &(e_2, \{(b, e), (c, f)\})\}, \{(e_1, \{(c, e), (d, e)\}), (e_2, \{(b, e), (c, f)\})\}\}.\end{aligned}$$

Let $S = \{2, 3, 5\}, T = \{4, 6\}$, and $S \times T = \{(2, 4), (2, 6), (3, 4), (3, 6), (5, 4), (5, 6)\}$. Let $m_k = \{(k_1, \{(3, 4), (5, 4)\}), (k_2, \{(2, 4), (3, 6)\})\}$ be the BR-soft set. Then, the topological rough approximation space obtained is

$$\begin{aligned}\tau_{BR} = &\{\emptyset, \{(e_1, \{(5, 4)\})\}, \{(e_1, \{(3, 4)\})\}, \{(e_2, \{(3, 6)\})\}, \{(e_2, \{(2, 4)\})\},\\ &\{(e_1, \{(3, 4), (5, 4)\})\}, \{(e_1, \{(5, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1, \{(5, 4)\}),\\ &(e_2, \{(2, 4)\})\}, \{(e_1, \{(3, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1, \{(3, 4)\}), (e_2, \{(2, 4)\})\},\\ &\{(e_2, \{(2, 4), (3, 6)\})\}, \{(e_1, \{(3, 4), (5, 4)\}), (e_2, \{(3, 6)\})\}, \{(e_1, \{(3, 4),\\ &(5, 4)\}), (e_2, \{(2, 4)\})\}, \{(e_1, \{(5, 4)\}), (e_2, \{(2, 4), (3, 6)\})\}, \{(e_1, \{(3, 4)\}),\\ &(e_2, \{(2, 4), (3, 6)\})\}, \{(e_1, \{(3, 4), (5, 4)\}), (e_2, \{(2, 4), (3, 6)\})\}\}.\end{aligned}$$

Then, the collection of BR-soft simply*alpha open sets forming soft topology $\tau_{BR} = \tau^*_{BR}$, where $\emptyset$ is the BR-soft nowhere dense set. Here, $w : S \times T \to U \times V$ and $p : E \to K$ are defined as $w(2,4) = (c,f); w(2,6) = (b,e); w(3,4) = (d,f); w(3,6) = (d,e); w(5,4) = (b,f); w(5,6) = (c,e); p(e_1) = k_1; p(e_2) = k_2$.

Let $m_k = \{(e_2, \{(b,e),(c,f)\})\}$ be a BR-soft open set in $U \times V$ and $f : (S, T, \tau^*_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ is a BR-soft mapping. Then, $f^{-1}(m_k) = \{(e_2, \{(2,4),(3,6)\})\}$ is BR-soft simply*alpha open set in $S \times T$. Therefore, f is a BR-soft simply*alpha- continuous function.

Example 4.2. Consider Example 4.1, where τ'_{BR}, τ''_{BR} be two BR soft topological spaces obtained when $\{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}, \{(e_1, \{(5,6)\})\}$ are taken as BR-soft nowhere dense sets, respectively. Let $m_k = \{(e_1, \{(d,e)\})\}$ be a BR-soft open set in τ''_{BR}. Then, $f^{-1}(m_k) = \{(e_1, \{(5,6)\})\}$ is not a BR-soft simply* alpha open set in τ'_{BR}. Therefore, f is not a BR-soft simply*alpha- continuous function.

Theorem 4.1. *For the class of BR-soft simply*alpha continuous functions, the following are equivalent:*

i) f *is BR-soft simply*alpha continuous function.*

ii) $f^{-1}(m_k)$ *is BR-soft simply*alpha closed for every BR-soft closed set* m_k.

Proof of Theorem 4.1. i) $\implies$ ii). Let m_k be a BR-soft closed set over $U \times V$. Then, $(m_k)^c \in SO(U \times V)$. Hence, $f^{-1}((m_k)^c) \in BR_S S^* \alpha O(S \times T)$. That is, $(f^{-1}(m_k))^c \in BR_S S^* \alpha O(S \times T)$ which implies $f^{-1}(m_k) \in BR_S S^* \alpha C(S \times T)$.

ii) $\implies$ i). Let $m_k \in SO(U \times V)$. Then, $(m_k)^c \in SC(U \times V)$. So, $f^{-1}((m_k)^c) \in BR_S S^* \alpha C(S \times T)$. That is, $(f^{-1}(m_k))^c \in BR_S S^* \alpha C(S \times T)$ implies that $f^{-1}(m_k) \in BR_S S^* \alpha O(S \times T)$. Thus, f^{-1} is $BR_S S^* \alpha$-continuous.

Theorem 4.2. *For a BR-soft simply*alpha continuous function* $f : (S, T, \tau^*_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$, *where* m_k *is any BR-soft subset.*

i) $\underline{BR}(f^{-1}(\underline{\tau}_{BR}(m_k))) \subseteq f^{-1}(m_k)$

ii) $\underline{BR}(f^{-1}(\underline{\tau}_{BR}(m_k))) \subseteq f^{-1}(\underline{\tau}_{BR}(m_k))$

iii) $f^{-1}(\underline{\tau}_{BR}(m_k)) \subseteq \overline{BR}(f^{-1}(\underline{\tau}_{BR}(m_k)))$

Proof of Theorem 4.2. *i)* Since f is BR-soft simply* alpha continuous, and $\underline{\tau}_{BR}(m_k)$ is BR-soft open in $U \times V$, $f^{-1}(\underline{\tau}_{BR}(m_k))$ is BR-soft simply* alpha open in $S \times T$. We know that $\underline{\tau}_{BR}(m_k) \subseteq m_k$, which implies that $f^{-1}(\underline{\tau}_{BR}(m_k)) \subseteq f^{-1}(m_k)$. Since BR-soft simply* alpha interior of m_k is the largest open subset of m_k. Thus, $\underline{BR}(f^{-1}(\underline{\tau}_{BR}(m_k))) \subseteq f^{-1}(m_k)$. Hence the proof.

ii) The proof is obvious from (i).

iii) Let m_k be any BR-soft subset, and $\underline{\tau}_{BR}(m_k)$ is the largest BR-soft open subset of m_k. Since f is BR-soft simply* alpha continuous, $f^{-1}(\underline{\tau}_{BR}(m_k))$ is BR-soft simply* alpha open. Thus, we have

$$f^{-1}(\underline{\tau}_{BR}(m_k)) \subseteq \overline{BR}(f^{-1}(\underline{\tau}_{BR}(m_k))).$$

Definition 4.2. *Let (S, T, τ^*_{BR}, E) be a BR-soft topological space obtained from the collection of BR-soft simply* alpha open sets and $(U, V, R_{m(s,t)}, \tau_{BR})$ be a soft topological rough approximation space. Then, $f : (S, T, \tau^*_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ is said to be BR soft semi-continuous, if $f^{-1}(m_k)$ is BR-soft semi open set for every $m_k \in (U, V, R_m(s,t), \tau_{BR})$.*

Definition 4.3. *Let (S, T, τ^*_{BR}, E) be a BR-soft topological space obtained from the collection of BR-soft simply* alpha open sets and $(U, V, R_{m(s,t)}, \tau_{BR})$ be a soft topological rough approximation space. Then, $f : (S, T, \tau^*_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ is said to be BR soft beta-continuous, if $f^{-1}(m_k)$ is BR-soft beta open set for every $m_k \in (U, V, R_{m(s,t)}, \tau_{BR})$.*

Theorem 4.3. *Every BR-soft semi continuous is BR-soft simply*alpha-continuous.*

Proof of Theorem 4.3. Under a BR-soft semi continuous function, the inverse image of every BR-soft open set is BR-soft semi open. Since every BR-soft semi open is BR-soft simply*alpha open, inverse image of BR-soft open set is BR-soft simply*alpha open.

Theorem 4.4. *Every BR-soft simply*alpha-continuous function is BR-soft beta continuous.*

Proof of Theorem 4.4. Under a BR-soft beta continuous function, the inverse image of every BR-soft open set is BR-soft beta open. Since every BR-soft beta open is BR-soft simply*alpha open, inverse image of BR-soft open set is BR-soft simply*alpha open.

Definition 4.4. *Let (S, T, τ'_{BR}, E), and (U, V, τ''_{BR}, E) be two BR-soft topological spaces obtained from the collection of BR-soft simply*alpha open sets. Then, $f : (S, T, \tau'_{BR}, E) \to (U, V, \tau''_{BR}, E)$ is BR-soft simply* alpha-irresolute, if $f^{-1}(m_k)$ is BR-soft simply* alpha open for every BR-soft simply* alpha open set $m_k \in \tau''_{BR}$.*

Example 4.3. Consider Example 4.1, where τ'_{BR} be a BR-soft topological space obtained from the collection of BR-soft simply*alpha open sets where $\{e_1, \{(3,4),(5,4)\}, (e_2, \{(2,4),(3,6)\})\}$ is BR-soft open and $\{(e_1, \{(5,6)\}), (e_2, \{(2,6)\})\}$ is a BR-soft nowhere dense set. Let τ''_{BR} be a BR-soft topological space obtained from the collection of BR-soft simply*alpha open sets where $\{e_1, \{(c,e),(d,e),(d,f)\}, (e_2, \{(b,e),(b,f),(c,f)\})\}$ is BR-soft open and empty set is BR-soft nowhere dense set.

Let $m_k = \{e_1, \{(d,e),(d,f)\},(e_2,\{(b,e),(b,f)\})\}$ be BR-soft simply* alpha open set in τ''_{BR}. Then, $f^{-1}(m_k) = \{(e_1,\{(5,4),(5,6)\}),(e_2,\{(2,6),(3,6)\})\}$ is also BR-soft simply* alpha open set in τ'_{BR}.

Example 4.4. Consider Example 4.1, where τ'_{BR} is a BR-soft topological space obtained from the collection of BR-soft simply*alpha open sets where $\{(e_1,\{(5,6)\}),(e_2,\{(2,6)\})\}$ is a BR-soft nowhere dense set.

Similarly, τ''_{BR} is a BR-soft topological space obtained from the collection of BR-soft simply*alpha open sets where $\{(e_1,\{(d,f)\})\}$ is a BR-soft nowhere dense.

Let $m_k = \{e_1,\{(b,f),(c,e)\},(e_2,\{(c,f),(d,e)\})\}$ and $f^{-1}(m_k) = \{(e_1,\{(5,4),(5,6)\}),(e_2,\{(2,4),(3,6)\})\}$. Here, m_k is BR-soft simply*alpha open set in τ''_{BR} but $f^{-1}(m_k)$ is not a BR-soft simply*alpha open set in τ'_{BR}. Thus, f is not a BR-soft simply* alpha-irresolute.

Theorem 4.5. *Every BR-soft simply* alpha-irresolute is BR-soft simply* alpha-continuous.*

Proof of Theorem 4.5. Let $f : (S,T,\tau'_{BR},E) \to (U,V,\tau''_{BR},E)$ is BR-soft simply* alpha-irresolute. Let m_k be a BR-soft open set in (U,V,τ''_{BR},E). Then m_k is BR-soft simply*alpha open set in (U,V,τ''_{BR},E). Since f is BR-soft simply* alpha-irresolute mapping, $f^{-1}(m_k)$ is a BR-soft simply* alpha open set in (S,T,τ'_{BR},E). Hence f is BR-soft simply* alpha-continuous mapping.

Definition 4.5. *A mapping $f : (S,T,\tau'_{BR},E) \to (U,V,\tau''_{BR},E)$ is said to be a BR-soft simply*alpha open map if the image of every BR-soft open set in (S,T,τ'_{BR},E) is BR-soft simply* alpha open in (U,V,τ''_{BR},E).*

Definition 4.6. *A mapping $f : (S,T,\tau'_{BR},E) \to (U,V,\tau''_{BR},E)$ is said to be a BR-soft simply*alpha closed map if the image of every BR-soft closed set in (S,T,τ'_{BR},E) is a BR-soft simply* alpha closed set in (U,V,τ''_{BR},E).*

Definition 4.7. *A map $f : (S,T,\tau'_{BR},E) \to (U,V,R_{m(s,t)},\tau_{BR})$ is called contra BR-soft simply* alpha continuous if $f^{-1}(m_k)$ is BR-soft simply* alpha closed in (S,T,τ'_{BR},E) for every BR-soft open set m_k of $(U,V,R_{m(s,t)},\tau_{BR})$.*

Example 4.5. Considering Example 4.1, $f : (S,T,\tau'_{BR},E) \to (U,V,R_{m(s,t)},\tau_{BR})$ be a map where τ_{BR} is the topological rough approximation space over $U \times V$. Then, BR-soft topology over $S \times T$ is obtained by taking $\{(e_1,\{(5,6)\}),(e_2,\{(2,6)\})\}$ as nowhere dense set. Let $m_k = \{(e_2,\{(c,f)\})\}$ is BR-soft open in τ_{BR} and $f^{-1}(m_k) = \{(e_2,\{(2,4)\})\}$ is BR-soft simply* alpha closed. Thus, f is contra BR-soft simply* alpha continuous.

Theorem 4.6. *Let arbitrary union of BR-soft simply* alpha open set is BR-soft simply* alpha open. Then, the following statements are equivalent for a map $f : (S,T,\tau'_{BR},E) \to (U,V,R_{m(s,t)},\tau_{BR})$*

i) f is BR-soft simply alpha contra continuous.*

ii) *For every BR-soft closed set m_k of $(U, V, R_{m(s,t)}, \tau_{BR})$, $f^{-1}(m_k)$ is BR-soft simply* alpha open in (S, T, τ'_{BR}, E).*

Proof of Theorem 4.6. i) $\Longrightarrow$ ii). Let m_k be BR-soft closed set of $(U, V, R_{m(s,t)}, \tau_{BR})$ over $U \times V$. Then, $U \times V - m_k$ is BR-soft open in $(U, V, R_{m(s,t)}, \tau_{BR})$. By (i), $f^{-1}(U \times V - m_k) = S \times T - f^{-1}(m_k)$ BR-soft simply* alpha closed in $S \times T$. Thus $f^{-1}(m_k)$ is BR-soft simply* alpha open in (S, T, τ'_{BR}, E).

ii) $\Longrightarrow$ i). Let m_{ki} be BR-soft open in $(U, V, R_{m(s,t)}, \tau_{BR})$. Then, $U \times V - m_{ki}$ is BR-soft closed in $(U, V, R_{m(s,t)}, \tau_{BR})$. By (ii), $f^{-1}(U \times V - m_{ki})$= $f^{-1}(U \times V - m_{ki}) = S \times T - f^{-1}(m_{ki})$ is BR-soft simply* alpha open in $S \times T$. Thus, $f^{-1}(m_{ki})$ is BR-soft simply* alpha closed in (S, T, τ'_{BR}, E).

Theorem 4.7. *Let $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ be contra BR-soft simply* alpha continuous. Then, $\underline{BR}(f^{-1}(\overline{\tau}_{BR}(m_k))) \subset f^{-1}(m_k)$ for every m_k in $(U, V, R_{m(s,t)}, \tau_{BR})$.*

Proof of Theorem 4.7. Let f be a contra BR-soft simply* alpha continuous function. Let $\overline{\tau}_{BR}(m_k)$ is BR-soft closed in $(U, V, R_{m(s,t)}, \tau_{BR})$. Then $f^{-1}(\overline{\tau}_{BR}(m_k))$ is BR-soft simply* alpha open in (S, T, τ'_{BR}, E). Also, we know that $\underline{BR}(m_k) \subset m_k$, such that $\underline{BR}(f^{-1}(\overline{\tau}_{BR}(m_k))) \subset f^{-1}(m_k)$.

Theorem 4.8. *If $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ is contra BR-soft simply* alpha continuous, then the following statements hold:*

i) *f is contra BR-soft simply* alpha continuous*

ii) *For every $(s, t) \in S \times T$ and every BR-soft closed set m_k of $(U, V, R_{m(s,t)}, \tau_{BR})$ containing $f(s, t)$, there exists a BR-soft simply* alpha open set m_{ki} such that $(s, t) \in m_{ki}$ and $f(m_{ki}) \subseteq m_k$, if arbitrary union of BR-soft simply* alpha open sets is BR-soft simply* alpha open.*

iii) *The inverse image of each BR-soft open set in $(U, V, R_{m(s,t)}, \tau_{BR})$ is BR-soft simply* alpha closed in (S, T, τ'_{BR}, E).*

Proof of Theorem 4.8. i) $\Longrightarrow$ ii). Let f be a contra BR-soft simply* alpha continuous. Let $(s, t) \in S \times T$ and m_k be a BR-soft closed set in $U \times V$ containing $f(s, t)$. So, $(s, t) \in f^{-1}(m_k)$, which is BR-soft simply* alpha open in $S \times T$. Let $f^{-1}(m_k) = m_{ki}$. Hence, $(s, t) \in m_{ki}$. Thus, $f(m_{ki}) = ff^{-1}(m_k) \subset m_k$.

ii) $\Longrightarrow$ i). Let m_k be BR-soft closed in $U \times V$. Let $(s, t) \in f^{-1}(m_k)$. Thus, $f(s, t) \in m_k$. Hence, there exists a BR-soft simply* alpha open set m_{kj} containing (s, t) such that $f(m_{kj}) \subset m_k$. That is, $(s, t) \in m_{kj} \subset f^{-1}(m_k)$. Therefore, $f^{-1}(m_k)$ is a BR-soft simply*alpha open set in (S, T, τ'_{BR}, E).

iii) $\Longrightarrow$ i. The proof is obvious.

Theorem 4.9. *Let $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ be a contra BR-soft simply* alpha continuous function and $g : (U, V, R_{m(s,t)}, \tau_{BR}) \to (P, Q, \tau''_{BR}, E)$ be a BR-soft continuous function. Then $g \circ f : (S, T, \tau'_{BR}, E) \to (P, Q, \tau''_{BR}, E)$ is contra BR-soft simply* alpha continuous.*

Proof of Theorem 4.9. Let m_k be any BR-soft open set over $P \times Q$. Since g is BR-soft continuous, $g^{-1}(m_k)$ is BR-soft open cover over $U \times V$. Since f is contra BR-soft simply* alpha continuous, $f^{-1}(g^{-1}(m_k)) = (g \circ f)^{-1}(m_k)$ is BR-soft simply* alpha closed over $S \times T$. Thus $g \circ f$ is a contra BR-soft simply* alpha continuous function.

Theorem 4.10. *Let $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ be a BR-soft simply* alpha irresolute and $g : (U, V, R_{m(s,t)}, \tau_{BR}) \to (P, Q, \tau''_{BR}, E)$ be a contra BR-soft simply* alpha continuous function. Then $(g \circ f) : (S, T, \tau'_{BR}, E) \to (P, Q, \tau''_{BR}, E)$ is contra BR-soft simply* alpha continuous function.*

Proof of Theorem 4.10. Let m_k be any BR-soft open set over $P \times Q$. Since g is contra BR-soft simply* alpha continuous, $g^{-1}(m_k)$ is BR-soft simply* alpha closed over $U \times V$. Since f is BR-soft simply* alpha irresolute, $f^{-1}(g^{-1}(m_k)) = (g \circ f)^{-1}(m_k)$ is BR-soft simply* alpha closed over $S \times T$. Thus $g \circ f$ is a contra BR-soft simply* alpha continuous function.

Definition 4.8. *A map $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ is called perfectly BR-soft simply* alpha continuous if $f^{-1}(m_k)$ is BR-soft simply* alpha clopen in (S, T, τ'_{BR}, E) for every BR-soft open in $(U, V, R_{m(s,t)}, \tau_{BR})$.*

Theorem 4.11. *Every perfectly BR-soft simply* alpha continuous is contra BR-soft simply* alpha continuous.*

Proof of Theorem 4.11. Let $f : (S, T, \tau'_{BR}, E) \to (U, V, R_{m(s,t)}, \tau_{BR})$ be perfectly BR-soft simply* alpha continuous. Let m_k be BR-soft closed in $(U, V, R_{m(s,t)}, \tau_{BR})$. Then, $f^{-1}(m_k)$ is BR-soft simply* alpha clopen, and hence $f^{-1}(m_k)$ is BR-soft simply* alpha open. Thus, f is contra BR-soft simply* alpha continuous.

The converse of the above theorem need not be true, as can be seen from the following example:

Example 4.6. From Example 4.5, it is shown that $f^{-1}(m_k)$ is BR-soft simply* alpha closed but not BR-soft simply* alpha open.

5. Conclusion

A new class of binary soft sets, namely BR-soft simply* alpha open sets, was studied over two different universes. This is followed by the study of the continuous functions of the defined new class of set. Definitions of BR-soft simply* alpha continuous function, BR-soft simply* alpha contra continuous, BR-soft simply* alpha perfectly continuous, BR-soft simply* alpha open map, BR-soft simply* alpha closed map, and BR-soft simply* alpha irresolute are introduced and studied. The properties and results of the definitions are illustrated with examples. The definitions of such a new class of sets and the study of their continuous functions can lead to simplification in the decision making process in various fields of research and may help in further developments. In addition to this study, other topological properties of the defined set are being studied.

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