Some algebraic identities on prime near rings with generalized derivations

Phool Miyan*
Department of Mathematics
Haramaya University
P.O.Box 138, Dire Dawa
Ethiopia
phoolmiyan83@gmail.com

Seleshi Demie
Department of Mathematics
Haramaya University
P.O.Box 138, Dire Dawa
Ethiopia
seleshidemie@gmail.com

Adnew Markos
Department of Mathematics
Jimma University
P.O.Box 378, Jimma
Ethiopia
markos5397adnew@gmail.com

Leta Hailu
Department of Mathematics
Arsi University
P.O.Box 193, Assela
Ethiopia
lhbsena10@gmail.com

Abstract. The purpose of the present paper is to investigate the commutativity of a prime near ring N with a generalized derivation F associated with a nonzero derivation d satisfying one of the conditions: For some nonnegative integers p and q:

(i) $[F(x), y] = \pm y^p(x \circ y)y^q$;

(ii) $[x, F(y)] = \pm x^p(x \circ y)x^q$;

(iii) $F(x) \circ y = \pm y^p[x, y]y^q$;

(iv) $x \circ F(y) = \pm x^p[x, y]x^q$;

(v) $F(x) \circ y = \pm y^p(x \circ y)y^q$;

(vi) $[x, F(y)] = \pm x^p[x, y]x^q$;

(vii) $[F(x), y] = \pm y^p[x, y]y^q$;

*. Corresponding author

(viii) $x \circ F(y) = \pm x^p(x \circ y)x^q$,

for all $x, y \in N$. Moreover, we give an example which shows the necessity of primness hypothesis in the theorems.

Keywords: prime near ring, derivation, generalized derivation, commutativity.

MSC 2020: 16N60, 16W25, 16Y30

1. Introduction

A right near ring N is a triplet $(N, +, \cdot)$, where $+$ and $\cdot$ are two binary operations such that (i) $(N, +)$ is a group (not necessarily abelian), (ii) $(N, \cdot)$ is a semigroup, and (iii) $(x+y)\cdot z = x\cdot z + y\cdot z$, for all $x, y, z \in N$. Analogously, if instead of (iii), N satisfies the left distributive law, then N is said to be a left near ring. A near ring N is said to be zero-symmetric if $x0 = 0$, for all $x \in N$ (right distributivity yields that $0x = 0$). Throughout the paper, N represents a zero-symmetric right near ring with multiplicative center $Z(N)$. For any $x, y \in N$, the symbols $[x, y]$ and $(x \circ y)$ denote the Lie product $xy - yx$ and Jordan product $xy + yx$ respectively. A near ring N is said to be prime if $xNy = \{0\}$, for all $x, y \in N$ implies that $x = 0$ or $y = 0$. A near ring N is said to be 2-torsion free if $(N, +)$ has no element of order 2.

The notion of derivation in near rings was introduced by Bell and Mason [8]. An additive mapping $d : N \to N$ is said to be a derivation on N if $d(xy) = xd(y) + d(x)y$, for all $x, y \in N$ or equivalently in [20], $d(xy) = d(x)y + xd(y)$, for all $x, y \in N$. Motivated by the definition of derivation in near rings, Gölbaşi [13] defined generalized derivation in near rings as follows: An additive mapping $F : N \to N$ is said to be a right (resp. left) generalized derivation associated with a derivation d on N if $F(xy) = F(x)y + xd(y)$ (resp. $F(xy) = d(x)y + xF(y)$), for all $x, y \in N$. Moreover, F is said to be a generalized derivation associated with a derivation d on N if it is both a right generalized derivation as well as a left generalized derivation on N. All derivations are generalized derivations. There has been a great deal of work by various authors with some suitable constraints on derivations and generalized derivations to prime and semiprime rings (see [5, 10, 11, 12, 15, 17]). A number of authors have obtained some comparable results on near rings, (c.f. [1, 2, 4, 6, 8, 16, 19, 20]).

Daif and Bell [10] proved that if R is a prime ring, I a nonzero ideal of R and d is a derivation on R such that $d([x, y]) = \pm[x, y]$, for all $x, y \in I$, then R is commutative. Further, Dhara [12] proved that if R is a semiprime ring with a generalized derivation F associated with a derivation d satisfying $F([x, y]) = \pm[x, y]$ or $F(x \circ y) = \pm(x \circ y)$, for all $x, y \in I$, a nonzero ideal of R, then R must contain a nonzero central ideal, provided $d(I) \neq \{0\}$. Moreover, he proved that in case R is a prime ring, R must be commutative, provided $d \neq 0$. Motivated by the above results, Boua and Oukhtite [9] proved that a prime near ring N with a derivation d is a commutative ring if one of the conditions holds: (i) $d([x, y]) = \pm[x, y]$, (ii) $d(x \circ y) = \pm(x \circ y)$, for all $x, y \in N$.

Recently, Shang [19] considered the more general situations (i) $F([x,y]) = \pm x^k[x,y]x^l$, (ii) $F(x \circ y) = \pm x^k(x \circ y)x^l$, for all $x, y \in N$, $k \geq 0$, $l \geq 0$ non negative integers and proved that the prime near ring N is a commutative ring if it satisfies one of the above conditions.

In this line of investigation, we prove that a prime near ring N equipped with a generalized derivation F associated with a nonzero derivation d is a commutative ring if it satisfies one of the following conditions: For some nonnegative integers p and q: (i) $[F(x), y] = \pm y^p(x \circ y)y^q$, (ii) $[x, F(y)] = \pm x^p(x \circ y)x^q$, (iii) $F(x) \circ y = \pm y^p[x,y]y^q$, (iv) $x \circ F(y) = \pm x^p[x,y]x^q$, (v) $F(x) \circ y = \pm y^p(x \circ y)y^q$, (vi) $[x, F(y)] = \pm x^p[x,y]x^q$, (vii) $[F(x), y] = \pm y^p[x,y]y^q$ and (viii) $x \circ F(y) = \pm x^p(x \circ y)x^q$, for all $x, y \in N$.

2. Preliminary results

For developing the proof of our theorems, we shall need the following lemmas. These results appear in the case of left near rings and so it is easy to observe that they also hold for right near ring as well.

Lemma 2.1 ([14], Lemma 2.2)**.** *Let N be a near ring admitting a generalized derivation F associated with a derivation d. Then:*

(*i*) $F(x)y + xd(y) = xd(y) + F(x)y$, *for all* $x, y \in N$,

(*ii*) $F(xy) = xF(y) + d(x)y$, *for all* $x, y \in N$.

Lemma 2.2. *Let N be a near ring admitting a generalized derivation F associated with a derivation d. Then*

(*i*) $x(F(y)z + yd(z)) = xF(y)z + xyd(z)$, *for all* $x, y, z \in N$,

(*ii*) $x(yd(z) + F(y)z) = xyd(z) + xF(y)z$, *for all* $x, y, z \in N$.

Proof. (i) For all $x, y, z \in N$, we have

$$F(x(yz)) = d(x)yz + xF(yz) = d(x)yz + x(F(y)z + yd(z)). \tag{1}$$

Also

$$F((xy)z) = F(xy)z + xyd(z) = d(x)yz + xF(y)z + xyd(z). \tag{2}$$

Comparing (1) and (2), we get

$$x(F(y)z + yd(z)) = xF(y)z + xyd(z), \quad \text{for all} \quad x, y, z \in N.$$

(ii) For all $x, y, z \in N$,

$$F(x(yz)) = xF(yz) + d(x)yz = x(yd(z) + F(y)z) + d(x)yz. \tag{3}$$

On the other hand,

$$F((xy)z) = xyd(z) + F(xy)z = xyd(z) + xF(y)z + d(x)yz. \tag{4}$$

Comparing (3) and (4), we get the result.

Lemma 2.3 ([7], Theorem 2.1). *If a prime near ring N admits a nonzero derivation with $d(N) \subseteq Z(N)$, then N is a commutative ring.*

3. Main results

Theorem 3.1. *Let N be a prime near ring. If there exist non negative integers $p \geq 0$, $q \geq 0$ and F is a generalized derivation on N associated with a nonzero derivation d satisfying one of the following:*

(i) $[F(x), y] = \pm y^p(x \circ y)y^q$, *for all* $x, y \in N$,

(ii) $[x, F(y)] = \pm x^p(x \circ y)x^q$, *for all* $x, y \in N$,

then N is a commutative ring.

Proof of Theorem 3.1. (i) Suppose that

$$[F(x), y] = y^p(x \circ y)y^q, \quad \text{for all } x, y \in N. \tag{5}$$

Replacing x by xy in (5) and using $(xy \circ y) = (x \circ y)y$, we have

$$[F(xy), y] = y^p(xy \circ y)y^q = y^p(x \circ y)y^{q+1} = [F(x), y]y,$$
$$F(xy)y - yF(xy) = F(x)y^2 - yF(x)y, \quad \text{for all } x, y \in N. \tag{6}$$

Applying the definition of F and Lemma 2.1, we get

$$(F(x)y + xd(y))y - y(xd(y) + F(x)y) = F(x)y^2 - yF(x)y, \quad \text{for all } x, y \in N.$$

Invoking Lemma 2.2, we obtain

$$F(x)y^2 + xd(y)y - yxd(y) - yF(x)y = F(x)y^2 - yF(x)y, \quad \text{for all } x, y \in N,$$

which reduces to

$$xd(y)y = yxd(y), \quad \text{for all } x, y \in N. \tag{7}$$

Substituting zx in place of x for $z \in N$ in (7) and using (7), we find that

$$zxd(y)y = yzxd(y) = zyxd(y), \quad \text{for all } x, y, z \in N.$$

This implies that $[y, z]xd(y) = 0$, for all $x, y, z \in N$, i.e., $[y, z]Nd(y) = \{0\}$, for all $y, z \in N$. Since N is prime, we get

$$[y, z] = 0, \quad \text{for all } y, z \in N \text{ or } d(y) = 0, \quad \text{for all } y \in N. \tag{8}$$

But $d \neq 0$, we have

$$[y, z] = 0, \quad \text{for all } y, z \in N, \tag{9}$$

replacing y by $yd(x)$, for any $x, y \in N$ in (9), we get

$$[yd(x), z] = 0, \text{ for all } x, y, z \in N,$$

which reduces to

$$[y, z]d(x) + y[d(x), z] = 0, \text{ for all } x, y, z \in N. \tag{10}$$

Using (8) in (10) and N is zero-symmetric, we get

$$y[d(x), z] = 0, \text{ for all } x, y, z \in N. \tag{11}$$

Again replacing y by $d(r)y$, for any $r \in N$ in (11), we have

$$d(r)y[d(x), z] = 0, \text{ for all } r, x, y, z \in N.$$

This implies that

$$d(r)N[d(x), z] = \{0\}, \text{ for all } r, x, z \in N.$$

Since N is prime, we get

$$d(r) = 0, \text{ for all } r \in N \text{ or } [d(x), z] = 0 \text{ for all } x, z \in N.$$

But $d \neq 0$, we have

$$[d(x), z] = 0, \text{ for all } x, z \in N.$$

This implies that

$$d(x) \in Z(N), \text{ for all } x \in N.$$

$$d(N) \subseteq Z(N).$$

Therefore, by Lemma 2.3, N is a commutative ring.

Now, we taking

$$[F(x), y] = -y^p(x \circ y)y^q, \text{ for all } x, y \in N. \tag{12}$$

Replacing x by xy in (12) and using $(xy \circ y) = (x \circ y)y$, we have

$$[F(xy), y] = -y^p(xy \circ y)y^q = -y^p(x \circ y)y^{q+1} = [F(x), y]y,$$

$$F(xy)y - yF(xy) = F(x)y^2 - yF(x)y, \text{ for all } x, y \in N. \tag{13}$$

Equation (13) is the same as equation (6). Now arguing in the similar manner, we can obtain the result.

(ii) By hypothesis,

$$[x, F(y)] = x^p(x \circ y)x^q, \text{ for all } x, y \in N. \tag{14}$$

Substituting yx in place of y in (14), we get

$$[x, F(yx)] = x^p(x \circ yx)x^q = x^p(x \circ y)x^{q+1} = [x, F(y)]x,$$
$$xF(yx) - F(yx)x = xF(y)x - F(y)x^2, \text{ for all } x, y \in N,$$
$$x(F(y)x + yd(x)) - (yd(x) + F(y)x)x = xF(y)x - F(y)x^2, \text{ for all } x, y \in N.$$

Using Lemma 2.2, we obtain

$$xF(y)x + xyd(x) - yd(x)x - F(y)x^2 = xF(y)x - F(y)x^2, \text{ for all } x, y \in N.$$

This reduces to,

$$xyd(x) = yd(x)x, \text{ for all } x, y \in N. \tag{15}$$

Replacing y by zy, where $z \in N$ in (15) and using it again, we arrive at

$$xzyd(x) = zyd(x)x = zxyd(x), \text{ for all } x, y, z \in N,$$

which implies that $[x, z]yd(x) = 0$, for all $x, y, z \in N$, i.e., $[x, z]Nd(x) = \{0\}$. The primness of N gives that $[x, z] = 0$ or $d(x) = 0$, for all $x \in N$. Since d is a nonzero derivation on N, then we have,

$$[x, z] = 0, \text{ for all } x, z \in N. \tag{16}$$

Similar proof follows from equation (9).

Now, we taking

$$[x, F(y)] = -x^p(x \circ y)x^q, \text{ for all } x, y \in N. \tag{17}$$

Substituting yx in place of y in (17), we get

$$[x, F(yx)] = -x^p(x \circ yx)x^q = -x^p(x \circ y)x^{q+1} = [x, F(y)]x.$$

Now arguing in the similar manner as above, we can obtain the result.

Theorem 3.2. *Let N be a prime near ring. If there exist non negative integers $p \geq 0$, $q \geq 0$ and F is a generalized derivation on N associated with a nonzero derivation d satisfying one of the following:*

(i) $F(x) \circ y = \pm y^p[x, y]y^q$, *for all* $x, y \in N$,

(ii) $x \circ F(y) = \pm x^p[x, y]x^q$, *for all* $x, y \in N$,

then N is a commutative ring.

Proof of Theorem 3.2. (i) Assume that

$$F(x) \circ y = y^p[x, y]y^q, \text{ for all } x, y \in N. \tag{18}$$

Replacing x by xy in (18) and using $[xy, y] = [x, y]y$, we get

$$F(xy) \circ y = y^p[xy, y]y^q = y^p[x, y]y^{q+1} = (F(x) \circ y)y,$$

which implies that

$$(F(x)y + xd(y))y + y(xd(y) + F(x)y) = (F(x)y + yF(x))y, \text{ for all } x, y \in N.$$

Applying Lemma 2.2, we obtain

$$F(x)y^2 + xd(y)y + yxd(y) + yF(x)y = F(x)y^2 + yF(x)y, \text{ for all } x, y \in N,$$

which reduces to,

$$yxd(y) = -xd(y)y, \text{ for all } x, y \in N. \tag{19}$$

Substituting zx for x in (19), where $z \in N$, we have

$$yzxd(y) = -zxd(y)y = (-z)(xd(y)y) = (-z)(-yxd(y)) = (-z)((-y)xd(y)).$$

Replacing y by $-y$ in the above expression, we find that

$$-yzxd(-y) = (-y)zxd(-y) = (-z)yxd(-y) = -zyxd(-y), \text{ for all } x, y, z \in N.$$

The last expression yields that $[y, z]xd(-y) = 0$, for all $x, y, z \in N$. This implies that

$$[y, z]Nd(-y) = \{0\}, \text{ for all } y, z \in N.$$

By primness of N, we get $[y, z] = 0$, for all $y, z \in N$ or $d(-y) = 0$, for all $y \in N$. Taking $d(-y) = 0$, for all $y \in N$, this imply that $d(y) = 0$, for all $y \in N$. But $d \neq 0$, so we have,$[y, z] = 0$, for all $y, z \in N$. Hence, by the same argument as in the proof of Theorem 3.1, we conclude that N is a commutative ring.

Arguing in the similar manner as above, we can obtain the results for $F(x) \circ y = -y^p[x, y]y^q$, for all $x, y \in N$.

(ii) By hypothesis, we have

$$x \circ F(y) = x^p[x, y]x^q, \text{ for all } x, y \in N. \tag{20}$$

Substituting yx for y in (20), we have

$$x \circ F(yx) = x^p[x, yx]x^q = x^p[x, y]x^{q+1} = (x \circ F(y))x,$$
$$x(F(y)x + yd(x)) + (F(y)x + yd(x))x = xF(y)x + F(y)x^2.$$

Applying Lemma 2.1 and Lemma 2.2, the last expression yields that

$$xF(y)x + xyd(x) + (yd(x) + F(y)x)x = xF(y)x + F(y)x^2.$$

This implies that

$$xyd(x) = -yd(x)x, \text{ for all } x, y \in N. \tag{21}$$

Replacing y by zy for $z \in N$ in (21), we obtain

$$xzyd(x) = -zyd(x)x = (-z)(yd(x)x) = (-z)(-xyd(x)) = (-z)((-x)yd(x)).$$

Substituting $-x$ in place of x, we arrive at $[x, z]yd(-x) = 0$, for all $x, y, z \in N$. This implies that $[x, z]Nd(-x) = \{0\}$, for all $x, z \in N$. By primness of N, we get $[x, z] = 0$, for all $x, z \in N$ or $d(-x) = 0$, for all $x \in N$. Taking $d(-x) = 0$, for all $x \in N$, this imply that $d(x) = 0$, for all $x \in N$. But $d \neq 0$, we have $[x, z] = 0$, for all $x, z \in N$. Hence, by the same argument as in the proof of Theorem 3.1, we conclude that N is a commutative ring.

Arguing in the similar manner as above, we can obtain the result for $x \circ F(y) = -x^p[x, y]x^q$, for all $x, y \in N$.

Theorem 3.3. *Let N be a prime near ring. If there exist non negative integers $p \geq 0$, $q \geq 0$ and F is a generalized derivation on N associated with a nonzero derivation d satisfying one of the following:*

(i) $F(x) \circ y = \pm y^p(x \circ y)y^q$, *for all* $x, y \in N$,

(ii) $[x, F(y)] = \pm x^p[x, y]x^q$, *for all* $x, y \in N$,

then N is a commutative ring.

Proof of Theorem 3.3. (i) Assume that

$$F(x) \circ y = y^p(x \circ y)y^q, \text{ for all } x, y \in N. \tag{22}$$

Replacing x by xy in (22), we get

$$F(xy) \circ y = y^p(xy \circ y)y^q = y^p(x \circ y)y^{q+1} = (F(x) \circ y)y,$$
$$F(xy)y + yF(xy) = F(x)y^2 + yF(x)y,$$
$$(F(x)y + xd(y))y + y(xd(y) + F(x)y) = F(x)y^2 + yF(x)y.$$

Applying Lemma 2.2, the above expression reduces to

$$yxd(y) = -xd(y)y, \text{ for all } x, y \in N. \tag{23}$$

Equation (23) is same as Equation (19), arguing in the similar manner as in Theorem 3.2, we can get the result.

Arguing in the similar manner as above, we can obtain the result for $F(x) \circ y = -y^p(x \circ y)y^q$, for all $x, y \in N$.

(ii) Suppose that

$$[x, F(y)] = x^p[x, y]x^q, \text{ for all } x, y \in N. \tag{24}$$

Substituting yx for y in (24), we obtain

$$[x, F(yx)] = x^p[x, yx]x^q = x^p[x, y]x^{q+1} = [x, F(y)]x,$$
$$xF(yx) - F(yx)x = xF(y)x - F(y)x^2, \text{ for all } x, y \in N.$$

Applying the definition of F and Lemma 2.1(i), the above expression yields that

$$x(F(y)x + yd(x)) - (yd(x) + F(y)x)x = xF(y)x - F(y)x^2, \text{ for all } x, y \in N.$$

Using Lemma 2.2, we get

$$xF(y)x + xyd(x) - yd(x)x - F(y)x^2 = xF(y)x - F(y)x^2, \text{ for all } x, y \in N.$$

The above expression reduces to

$$xyd(x) = yd(x)x, \text{ for all } x, y \in N. \tag{25}$$

Since Equation (25) is same as Equation (15), arguing in the similar manner as in Theorem 3.1, we can get the result.

Arguing in the similar manner as above, we can obtain the result for $[x, F(y)] = -x^p[x, y]x^q$, for all $x, y \in N$.

Theorem 3.4. *Let N be a prime near ring. If there exist non negative integers $p \geq 0$, $q \geq 0$ and F is a generalized derivation on N associated with a nonzero derivation d satisfying one of the following:*

(i) $[F(x), y] = \pm y^p[x, y]y^q$*, for all* $x, y \in N$*,*

(ii) $x \circ F(y) = \pm x^p(x \circ y)x^q$*, for all* $x, y \in N$*,*

then N is a commutative ring.

Proof of Theorem 3.4. (i) Assume that

$$[F(x), y] = y^p[x, y]y^q, \text{ for all } x, y \in N. \tag{26}$$

Replacing x by xy in (26) and using $[xy, y] = [x, y]y$, we find that

$$[F(xy), y] = y^p[xy, y]y^q = y^p[x, y]y^{q+1} = [F(x), y]y.$$

This implies that

$$(F(x)y + xd(y))y - y(xd(y) + F(x)y) = F(x)y^2 - yF(x)y, \text{ for all } x, y \in N.$$

Using Lemma 2.2, we get

$$F(x)y^2 + xd(y)y - yxd(y) - yF(x)y = F(x)y^2 - yF(x)y, \text{ for all } x, y \in N,$$

which reduces to

$$xd(y)y = yxd(y), \text{ for all } x, y \in N. \tag{27}$$

Since Equation (27) is same as Equation (15), arguing in the similar manner as in Theorem 3.1, we get the result.

Arguing in the similar manner as above, we can obtain the result for $[F(x), y] = -y^p[x, y]y^q$, for all $x, y \in N$.

(ii) By hypothesis

$$x \circ F(y) = x^p(x \circ y)x^q, \text{ for all } x, y \in N. \tag{28}$$

Replacing y by yx in (28) and using $(x \circ yx) = (x \circ y)x$, we get

$$x \circ F(yx) = x^p(x \circ yx)x^q = x^p(x \circ y)x^{q+1} = (x \circ F(y))x,$$
$$x(F(y)x + yd(x)) + (yd(x) + F(y)x)x = xF(y)x + F(y)x^2.$$

Applying Lemma 2.2(i), the above expression gives

$$xF(y)x + xyd(x) + yd(x)x + F(y)x^2 = xF(y)x + F(y)x^2,$$

which reduces to,

$$xyd(x) = -yd(x)x, \text{ for all } x, y \in N. \tag{29}$$

Since Equation (29) is same as Equation (21), arguing in the similar manner as in Theorem 3.2, we can get the result.

Arguing in the similar manner as above, we can obtain the result for $x \circ F(y) = -x^p(x \circ y)x^q$, for all $x, y \in N$.

The following example demonstrates that the primness hypothesis in the Theorems 3.1, 3.2, 3.3 and 3.4 is not superfluous.

Example 3.1. Let S be a zero-symmetric right near ring. Let us consider

$$N = \left\{ \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} \mid 0, a, b \in S \right\}.$$

It is easy to verify that N is a non prime zero-symmetric right near ring with respect to matrix addition and matrix multiplication. Define mappings $F, d : N \to N$ by

$$F\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } d\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then, F is a nonzero generalized derivation associated with a nonzero derivation d on N satisfying for some nonnegative integers p and q:

(i) $[F(x), y] = \pm y^p(x \circ y)y^q$;

(ii) $[x, F(y)] = \pm x^p(x \circ y)x^q$;

(iii) $F(x) \circ y = \pm y^p[x, y]y^q$;

(iv) $x \circ F(y) = \pm x^p[x, y]x^q$;

(v) $F(x) \circ y = \pm y^p(x \circ y)y^q$;

(vi) $[x, F(y)] = \pm x^p[x, y]x^q$;

(vii) $[F(x), y] = \pm y^p[x, y]y^q$;

(viii) $x \circ F(y) = \pm x^p(x \circ y)x^q$, for all $x, y \in N$.

However, N is not commutative.

4. Concluding remarks

In this paper, the class of near rings involving generalized derivations satisfying some differential identities has been studied. We proved commutativity of prime near rings with differential identities on generalized derivations. This work can be further studied by considering multiplicative generalized derivations on prime near rings and semiprime near rings along with examples that illustrates the necessity of the assumptions used which is left for future work.

Acknowledgement

The authors are very thankful to the referees for their valuable suggestions and comments.

References

[1] A. Ali, H. E. Bell, P. Miyan, *Generalized derivations on prime near rings*, Int. J. Math. Mat. Sci., Article ID 170749, 2013 (2013), 5 pages.

[2] A. Ali, H. E. Bell, P. Miyan, *Generalized derivations on prime near rings II*, Afr. Mat., 26 (2015), 275-282.

[3] A. Ali, P. Miyan, I. Huque, A. Markos, *Some commutativity theorems for prime near rings with generalized derivations*, J. Seybold Report, 15 (2020), 578-585.

[4] N. Argaç, *On prime and semiprime near rings with derivations*, Internat. J. Math. Math. Sci., 20 (1997), 737-740.

[5] M. Ashraf, A. Boua, A. Raji, *On derivations and commutativity in prime near rings*, J. Taibah Univ. Sci., 8 (2014), 301-316.

[6] K. I. Beidar, Y. Fong, X. K. Wang, *Posner and Herstein theorems for derivations of prime near rings*, Comm. Algebra, 24 (1996), 1581-1589.

[7] H. E. Bell, *On derivations in near rings II*, Kluwer Academic Publishers Netherlands, 426 (1997), 191-197.

[8] H. E. Bell, G. Mason, *On derivations in near rings*, North.-Holl. Math. Stud., 137 (1987), 31-35.

[9] A. Boua, L. Oukhtite, *Derivations on prime near rings*, Int. J. Open Probl. Comput. Sci. Math., 4 (2011), 162-167.

[10] M. N. Daif, H. E. Bell, *Remarks on derivations on semiprime rings*, Internat. J. Math. Math. Sci., 15 (1992), 205-206.

[11] H. E. Bell, M. N. Daif, *On commutativity and strong commutativity preserving maps*, Canad. Math. Bull., 37 (1994), 443-447.

[12] B. Dhara, *Remarks on generalized derivations in prime and semiprime rings*, Int. J. Math. Math. Sci., Article ID 646587, 2010 (2010), 6 pages.

[13] O. Gölbaşi, *On generalized derivations of prime near rings*, Hacet. J. Math. Stat., 35 (2006), 173-180.

[14] O. Gölbaşi, *Notes on prime near rings with generalized derivation*, Southeast Asian Bull. Math., 30 (2006), 49–54.

[15] I. N. Herstein, *A note on derivations*, Canad. Math. Bull., 21 (1978), 369-370.

[16] M. Hongan, *On near rings with derivation*, Math. J. Okayama Univ., 32 (1990), 89-92.

[17] E. C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc., 8 (1957), 1093-1100.

[18] M. Samman, L. Oukhtie, A. Boua, *A study of near rings with generalized derivations*, J. Taibah Univ. Sci., 9 (2015), 407-413.

[19] Y. Shang, *A note on the commutativity of prime near rings*, Algebra Colloq., 22 (2015), 361-366.

[20] X. K. Wang, *Derivations in prime near rings*, Proc. Amer. Math. Soc., 121 (1994), 361-366.

Accepted: December 30, 2023