On a class of Lorentzian para-Kenmotsu manifolds admitting the Weyl-projective curvature tensor of type $(1, 3)$

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Abstract. In this present paper, we consider a class of Lorentzian almost paracontact metric manifolds namely Lorentzian para-Kenmotsu (briefly LP-Kenmotsu) manifolds admitting the Weyl-projective curvature tensor of type $(1, 3)$. We study and have shown that Lorentzian para-Kenmotsu manifolds admitting a flat curvature tensor, an irrotational curvature tensor and a conservative curvature tensor are an Einstein manifolds of constant scalar curvature. Further we study Lorentzian para-Kenmotsu manifolds satisfying the curvature condition $R(X, Y)\cdot W_2 = 0$. At the end, we construct an example of a 3-dimensional Lorentzian para-Kenmotsu manifold admitting Weyl-projective curvature tensor which verifies the results discussed in the present work.

Keywords: Lorentzian paracontact manifolds, W_2-curvature tensor, Einstein manifold, scalar curvature.

1. Introduction

In 1989, K. Matsumoto [3] introduced the notion of Lorentzian paracontact and in particular, Lorentzian para-Sasakian (LP-Sasakian) manifolds. Later, these manifolds have been widely studied by many geometers such as Matsumoto and Mihai [4], Mihai and Rosca [6], Mihai, Shaikh and De [7], Venkatesha and

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Bagewadi [13], Venkatesha, Pradeep Kumar and Bagewadi [14, 15] and obtained several results of these manifolds.

In 1970 [9], Pokhariyal and Mishra introduced new tensor fields, called W_2 and E, on a Riemannian manifold. The Weyl-projective curvature tensor W_2 of type $(1,3)$ on a Riemannian manifold M with respect to Riemannian connection is given by [9]:

$$W_2(X,Y,Z,U)=R(X,Y,Z,U)+\frac{1}{n-1}[g(X,Z)S(Y,U)-g(Y,Z)S(X,U)], \tag{1}$$

where $R(X,Y)$ is the Riemannian curvature tensor and $S(X,Y)$ is the Ricci tensor of M.

The above expression can be written as:

$$W_2(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)QY - g(Y,Z)QX], \tag{2}$$

where $QX = (n-1)X$.

Further, Pokhariyal [8] studied the properties of these tensor fields on a Sasakian manifold. In 1986, Matsumoto, Ianus and Mihai have extended these concepts to almost paracontact structures and studied para-Sasakian manifolds admitting these tensor fields [5] and these results were further generalised by De and Sarkar, in 2009 [2]. In 1995, Sinha and Sai Prasad [12] defined a class of almost paracontact metric manifolds namely para-Kenmotsu (briefly P-Kenmotsu) and special para-Kenmotsu (briefly SP-Kenmotsu) manifolds and in 2015 [11], Sai Prasad and Satyanarayana have studied W_2-tensor field in an SP-Kenmotsu manifold.

In the present work, we investigate a class of Lorentzian para-Kenmotsu manifolds [1, 10] admitting the Weyl-projective curvature tensor W_2 of type $(1,3)$. The present work is organised as follows: Section 2 is equipped with some prerequisites about Lorentzian para-Kenmotsu manifolds. Further on, we consider Lorentzian para-Kenmotsu manifolds admittting the Weyl-projective curvature tensor W_2 of type $(1,3)$ and studied the geometrical properties of these manifolds, in particular, when the Weyl-projective curvature tensor W_2 is either flat (or) irrotational (or) conservative. In section 6, we studied the properties of W_2-semisymmetric Lorentzian para-Kenmotsu manifolds.

Finally, in section 7, we construct an example of a 3-dimensional Lorentzian para-Kenmotsu manifold admitting Weyl-projective curvature tensor which verifies the results discussed in the sections 3, 4 and 5.

2. Preliminaries

An n-dimensional differentiable manifold M admitting a (1, 1) tensor field ϕ, contravariant vector field ξ, a 1-form η and the Lorentzian metric $g(X,Y)$ satisfying

$$\phi^2 X = X + \eta(X)\xi, \quad g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{3}$$

and

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(X,\xi) = \eta(X), \ rank\phi = n-1 \tag{4}$$

is called Lorentzian almost paracontact manifold [3].

In a Lorentzian almostpara contact manifold, we have

$$\Phi(X,Y) = \Phi(Y,X), \tag{5}$$

where $\Phi(X,Y) = g(X,\phi Y)$.

A Lorentzian almost paracontact manifold M is called Lorentzian para-Kenmotsu manifold if [1]

$$(\nabla_X \phi)Y = -g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{6}$$

for any vector fields X and Y on M and ∇ is the operator of covariant differentiation with respect to the Lorentzian metric g.

In the Lorentzian para-Kenmotsu manifold, the following relations hold good:

$$\nabla_X \xi = -\phi^2 X = -X - \eta(X)\xi, \tag{7}$$

and

$$(\nabla_X \eta)Y = -g(X,Y) - \eta(X)\eta(Y). \tag{8}$$

Further, on a Lorentzian para-Kenmotsu manifold M, the following relations hold [1]:

$$\begin{aligned}
&(a)\ g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y),\\
&(b)\ R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X,\\
&(c)\ R(X,Y)\xi = \eta(Y)X - \eta(X)Y;\ \text{when } X \text{ is orthogonal to } \xi,\\
&(d)\ S(X,\xi) = (n-1)\eta(X),\\
&(e)\ S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y).
\end{aligned} \tag{9}$$

A Lorentzian para-Kenmotsu manifold M is said to be an η-Einstein manifold if its Ricci tensor $S(X,Y)$ is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{10}$$

where a and b are scalar functions on M.

A Lorentzian para-Kenmotsu manifold M is said to be a generalized η-Einstein manifold if its Ricci tensor $S(X,Y)$ is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) + c\Phi(X,Y), \tag{11}$$

where a,b,c are scalar functions on M and $\Phi(X,Y) = g(\phi X,Y)$. If $c = 0$, the manifold reduces to an η-Einstein manifold.

3. Flat W_2-curvature tensor in Lorentzian para-Kenmotsu manifolds

Let us consider a Lorentzian para-Kenmotsu manifold and suppose that it admits W_2-curvature tensor which is flat. Then, we have

$$g(W_2(X,Y)Z,\phi W)=0. \tag{12}$$

In view of (1), (12) can be written as

$$g(R(X,Y)Z,\phi W)+\frac{1}{n-1}\big\{g(X,Z)S(Y,\phi W)-g(Y,Z)S(X,\phi W)\big\}=0. \tag{13}$$

Now, by putting $Y=Z=\xi$ in (13), we get

$$g(R(X,\xi)\xi,\phi W)+\frac{1}{n-1}\big\{\eta(X)S(\xi,\phi W)+S(X,\phi W)\big\}=0; \tag{14}$$

which, by using (9)(c) and (9)(d), further reduces to:

$$-g(X,\phi W)+\frac{1}{n-1}S(X,\phi W)=0, \tag{15}$$

and this implies that

$$S(X,\phi W)=(n-1)g(X,\phi W). \tag{16}$$

By replacing W by ϕW in (16) and on using (3), we get

$$S(X,W)=(n-1)g(X,W), \tag{17}$$

which on contraction gives

$$r=n(n-1). \tag{18}$$

Hence, we can state the following:

Theorem 3.1. *The Lorentzian para-Kenmotsu manifold admitting a flat W_2-curvature tensor is an Einstein manifold and is a space of constant scalar curvature.*

4. Irrotational W_2-curvature tensor in Lorentzian para-Kenmotsu manifolds

Definition 4.1. The rotation (*curl*) of a Weyl-projective curvature tensor W_2 in a Lorentzian para-Kenmotsu manifold M is defined as:

$$\begin{aligned}RotW_2&=(\nabla_U W_2)(X,Y)Z+(\nabla_X W_2)(U,Y)Z\\&\quad+(\nabla_Y W_2)(X,U)Z-(\nabla_Z W_2)(X,Y)U.\end{aligned} \tag{19}$$

In consequence of Bianchi's second identity for Riemannian connection ∇, (19) becomes

$$RotW_2 = -(\nabla_Z W_2)(X, Y)U. \tag{20}$$

Let us suppose that the W_2 curvature tensor is irrotational. Then Rot $W_2 = 0$ and hence from (20), we have

$$(\nabla_Z W_2)(X, Y)U = 0; \tag{21}$$

which can be written as:

$$\nabla_Z(W_2(X, Y)U) = W_2(\nabla_Z X, Y)U + W_2(X, \nabla_Z Y)U + W_2(X, Y)\nabla_Z U. \tag{22}$$

By replacing $U = \xi$ in (22), we get

$$\nabla_Z(W_2(X, Y)\xi) = W_2(\nabla_Z X, Y)\xi + W_2(X, \nabla_Z Y)\xi + W_2(X, Y)\nabla_Z \xi. \tag{23}$$

On the other hand, by substituting $Z = \xi$ in (1) and on using (9)(c) and (9)(d), we get

$$W_2(X, Y)\xi = k\big[\eta(Y)X - \eta(X)Y\big]; \tag{24}$$

where

$$k = \Big[1 - \frac{1}{n-1}\Big\{\frac{r}{n-1} - 1\Big\}\Big]. \tag{25}$$

In view of (24) and (25), (23) becomes

$$W_2(X, Y)Z = k\big[g(Y, Z)X - g(X, Z)Y\big]. \tag{26}$$

Further, equations (1) and (26) gives

$$S(Y, Z) = (n - 1)g(Y, Z), \tag{27}$$

which on contraction gives

$$r = n(n - 1). \tag{28}$$

In consequence of the above equations from (25) to (28) and (1), we get

$$R(X, Y)Z = g(Y, Z)X - g(X, Z)Y. \tag{29}$$

Thus we have the following assertion:

Theorem 4.1. *The Lorentzian para-Kenmotsu manifold admitting an irrotational W_2-curvature tensor is a space of constant curvature.*

5. Conservative W_2-curvature tensor in Lorentzian para-Kenmotsu manifolds

On differentiating (1) with respect to U, we get

$$(\nabla_U W_2)(X,Y)Z = (\nabla_U R)(X,Y)Z + \frac{1}{n-1}\big[g(X,Z)(\nabla_U Q)Y - g(Y,Z)(\nabla_U Q)(X)\big], \tag{30}$$

which on contracting becomes

$$(divW_2)(X,Y)Z = \big[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\big] + \frac{1}{2(n-1)}\big[g(X,Z)dr(X) - g(Y,Z)dr(Y)\big]. \tag{31}$$

Let us suppose that W_2-curvature tensor is conservative, i. e., $divW_2 = 0$. Then (31) can be written as:

$$\big[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)\big] = \frac{1}{2(n-1)}\big[g(Y,Z)dr(X) - g(X,Z)dr(Y)\big]. \tag{32}$$

By putting $X = \xi$ in (32), we have

$$\big[(\nabla_\xi S)(Y,Z) - (\nabla_Y S)(\xi,Z)\big] = \frac{1}{2(n-1)}\big[g(Y,Z)dr(\xi) - g(\xi,Z)dr(Y)\big]. \tag{33}$$

On the other hand, since ξ is a killing vector and the scalar curvature r remains invariant, we have $L_\xi r = 0$, where L denotes the Lie derivative.

Then, the relation

$$\begin{aligned}(\nabla_\xi S)(Y,Z) &= \xi S(Y,Z) - S(\nabla_\xi Y, Z) - S(Y, \nabla_\xi Z)\\ &= (L_\xi S)(Y,Z) - S(\nabla_Y \xi, Z) - S(Y, \nabla_Z \xi)\end{aligned}$$

yields

$$(\nabla_\xi S)(Y,Z) = 0. \tag{34}$$

Then from (34) and (33), we have

$$\begin{aligned}&-\big[\nabla_Y S(\xi,Z) - S(\nabla_Y \xi, Z) - S(\xi, \nabla_Y Z)\big]\\ &= \frac{1}{2(n-1)}\big[g(Y,Z)dr(\xi) - \eta(Z)dr(Y)\big].\end{aligned} \tag{35}$$

By using (7), (9)(d) and $dr(\xi) = 0$ the above expression becomes

$$\begin{aligned}&[-\nabla_Y\{(n-1)\eta(Z)\} + S(-\phi^2 Y, Z) + (n-1)\eta(\nabla_Y Z)]\\ &= \frac{-1}{2(n-1)}[\eta(Z)dr(Y)].\end{aligned} \tag{36}$$

On simplifying (36), we get

$$[(n-1)g(Y,Z) - S(Y,Z)] = \frac{-1}{2(n-1)}[\eta(Z)dr(Y)]. \tag{37}$$

By putting $Z = \phi Z$ in (37), we get

$$(n-1)g(Y,\phi Z) = S(Y,\phi Z) \tag{38}$$

and this implies that

$$S(Y,Z) = (n-1)g(Y,Z), \tag{39}$$

which on contracting gives

$$r = n(n-1), \tag{40}$$

where

$$r = \sum_{i=1}^{3} \epsilon_i \; S(e_i, e_i) \text{ and } \epsilon_i = g(e_i, e_i), \text{ which is constant.} \tag{41}$$

So, one can state that:

Theorem 5.1. *A Lorentzian para-Kenmotsu manifold admitting a conservative W_2-curvature tensor is an Einstein manifold and it is of constant scalar curvature.*

6. W_2-semisymmetric para-Kenmotsu manifolds

We define a W_2-semisymmetric Lorentzian para-Kenmotsu manifold as:

Definition 6.1. An n-dimensional Lorentzian para-Kenmotsu manifold is called W_2-semisymmetric if its W_2-curvature tensor satisfies the condition

$$R(X,Y) \cdot W_2 = 0, \tag{42}$$

where $R(X,Y)$ is considered to be the derivation of tensor algebra at each point of the manifold for tangent vectors X and Y.

It can be easily shown that, on Lorentzian para-Kenmotsu manifold the W_2-curvature tensor satisfies the condition

$$W_2(X,Y,Z,\xi) = 0. \tag{43}$$

In consequence of $QX = hX$, (1) becomes

$$W_2(X,Y)Z = R(X,Y)Z + \frac{h}{(n-1)}\{g(X,Z)Y - g(Y,Z)X\}. \tag{44}$$

In view of (9)(a) and (44), we obtain

$$\eta(W_2(X,Y)Z) = \Big(1 - \frac{h}{n-1}\Big)\big\{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\big\}. \tag{45}$$

Replacing Z by ξ in (45), we have

$$\eta(W_2(X,Y)\xi) = 0. \tag{46}$$

Now

$$\begin{aligned}(R(X,Y)\cdot W_2)(Z,U)V &= R(X,Y)W_2(Z,U)V - W_2(R(X,Y)Z,U)V \\ &\quad - W_2(Z,R(X,Y)U)V - W_2(Z,U)R(X,Y)V.\end{aligned} \tag{47}$$

Using $R(X,Y)\cdot W_2 = 0$ in the above equation, we obtain $R(X,Y)W_2(Z,U)V - W_2(R(X,Y)Z,U)V - W_2(Z,R(X,Y)U)V - W_2(Z,U)R(X,Y)V = 0$. By taking the inner product of the above relation with ξ, we get

$$\begin{aligned}&g(R(X,Y)W_2(Z,U)V,\xi) - g(W_2(R(X,Y)Z,U)V,\xi) \\ &- g(W_2(Z,R(X,Y)U)V,\xi) - g(W_2(Z,U)R(X,Y)V,\xi) = 0.\end{aligned} \tag{48}$$

Putting $X = \xi$ in (48) and then using (9)(b), we obtain

$$\begin{aligned}&- W_2(Z,U,V,Y) - \eta(Y)\eta(W_2(Z,U)V) + \eta(Z)\eta(W_2(Y,U)V) \\ &+ \eta(U)\eta(W_2(Z,Y)V) + \eta(V)\eta(W_2(Z,U)Y) - g(Y,Z)\eta(W_2(\xi,U)V) \\ &- g(Y,U)\eta(W_2(Z,\xi)V - g(Y,V)\eta(W_2(Z,U)\xi = 0.\end{aligned} \tag{49}$$

In consequence of (45), the above equation gives

$$\begin{aligned}-W_2(Z,U,V,Y) + \eta(V)\Big(1 - \frac{h}{n-1}\Big)&\big[\eta(Z)g(U,Y) - \eta(U)g(Y,Z)\big] \\ + g(Y,Z)\Big(1 - \frac{h}{n-1}\Big)&\big[g(U,V) + \eta(U)\eta(V)\big] \\ - g(Y,U)\Big(1 - \frac{h}{n-1}\Big)&\big[\eta(Z)\eta(V) + g(Z,V)\big] \\ - g(Y,V)\Big(1 - \frac{h}{n-1}\Big)&\big[\eta(Z)\eta(U) - \eta(U)\eta(Z)\big] = 0,\end{aligned} \tag{50}$$

which on simplification, we obtain

$$W_2(Z,U,V,Y) = \Big(1 - \frac{h}{n-1}\Big)\big[g(Y,Z)g(U,V) - g(Y,U)g(Z,V)\big], \tag{51}$$

which implies

$$W_2(Z,U)V = \Big(1 - \frac{h}{n-1}\Big)\big[g(U,V)Z - g(Z,V)U\big]. \tag{52}$$

Thus in view of (44) and (52), we obtain

$$R(Z,U)V = g(U,V)Z - g(Z,V)U. \tag{53}$$

Thus we have the following:

Theorem 6.1. *A Lorentzian para-Kenmotsu manifold satisfying* $R(X,Y)\cdot W_2 = 0$ *is a space of constant curvature.*

7. Example of a 3-dimensional Lorentzian para-Kenmotsu manifold

Example 7.1. We consider a 3-dimensional manifold $M_3 = \{(x, y, z) \in R^3\}$, where (x, y, z) are the standard coordinates in R^3. Let e_1, e_2 and e_3 be the vector fields on M_3 given by

$$e_1 = x\frac{\partial}{\partial x} = \xi, \quad e_2 = x\frac{\partial}{\partial y}, \quad e_3 = x\frac{\partial}{\partial z}. \tag{54}$$

Clearly, $\{e_1, e_2, e_3\}$ is a set of linearly independent vectors for each point of M_3 and hence form a basis of $\chi(M_3)$.

The Lorentzian metric $g(X, Y)$ is defined by:

$$g(e_i, e_j) = \begin{cases} -1, & \text{if } \ i = j = 1 \\ 1, & \text{if } \ i = j = 2 \ or \ 3 \\ 0, & \text{if } \ i \neq j; \ i, \ j = 1, 2, 3. \end{cases}$$

Let η be the 1-form defined by:

$$\eta(Z) = g(Z, e_1), \text{ for any } Z \ \in \ \chi(M_3).$$

Let ϕ be a (1, 1)-tensor field on M_3 defined by:

$$\phi(e_1) = 0, \phi(e_2) = -e_2, \phi(e_3) = -e_3 \text{ and}$$
$$\phi^2(e_1) = 0, \phi^2(e_2) = e_2, \phi^2(e_3) = e_3.$$

The linearity of ϕ and $g(X, Y)$ yields that

$$\eta(e_1) = -1, \ \phi^2(Z) = Z + \eta(Z)e_1 \ ; and$$
$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y)$$

for any vector fields $X, \ Y, \ Z \ \in \ \chi(M_3)$.

Thus, for $e_1 = \xi$, the structure (ϕ, ξ, η, g) defines an Lorentzian almostpara-contact structure on M_3.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g. Then we have

$$[e_1, e_2] = e_2, [e_1, e_3] = e_3, [e_2, e_3] = 0.$$

The Koszul's formula is defined by

$$\begin{aligned} 2g(\nabla_X Y, Z) = {} & Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) \\ & - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]). \end{aligned} \tag{55}$$

By using the above Koszul's formula and on taking $e_1 = \xi$, we get the following:

$$\begin{aligned} & \nabla_{e_1} e_1 = 0, \nabla_{e_1} e_2 = 0, \nabla_{e_1} e_3 = 0; \\ & \nabla_{e_2} e_1 = -e_2, \nabla_{e_2} e_2 = -e_1, \nabla_{e_2} e_3 = 0; \\ & \nabla_{e_3} e_1 = -e_3, \nabla_{e_3} e_2 = 0, \nabla_{e_3} e_3 = -e_1. \end{aligned} \tag{56}$$

From the above calculations, we see that the manifold under consideration satisfies all the properties of Lorentzian para-Kenmotsu manifold i.e., $\nabla_X \xi = -\Phi^2 X = -X - \eta(X)\xi$ and $(\nabla_X \phi)Y = -g(\phi X, Y)\xi - \eta(Y)\phi X$ for all $e_1 = \xi$. Thus, the manifold M_3 under consideration with the structure (ϕ, ξ, η, g) is a Lorentzian para-Kenmotsu manifold.

It is known that

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z. \tag{57}$$

Then by using (56) and (57), the non-vanishing components of the curvature tensor are obtained as:

$$\begin{aligned} &R(e_1,e_2)e_1 = e_2, R(e_1,e_2)e_2 = e_1, R(e_1,e_3)e_1 = e_3,\\ &R(e_1,e_3)e_3 = e_1, R(e_2,e_3)e_2 = -e_3, R(e_2,e_3)e_3 = e_2. \end{aligned} \tag{58}$$

With the help of above expressions of the curvature tensors, it follows that

$$R(X,Y)Z = g(Y,Z)X - g(X,Z)Y. \tag{59}$$

This proves that the manifold M_3 under consideration is a Lorentzian para-Kenmotsu manifold and it admits a Weyl-projective curvature tensor.

Let X, Y and Z be any three vector fields given by:

$$X = a_1e_1 + a_2e_2 + a_3e_3, Y = b_1e_1 + b_2e_2 + b_3e_3, Z = c_1e_1 + c_2e_2 + c_3e_3; \tag{60}$$

where a_i, b_i, c_i are all non-zero real numbers, for all $i = 1,2,3$.

By putting $Z = \xi = e_1$ in (2) and on using (60), we get that

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y = a_1b_2e_2 + a_1b_3e_3 - a_2b_1e_2 - a_3b_1e_3;$$

and hence we get that $W_2(X,Y)\xi = 0$, where $QX = (n-1)X$.

Further, in view of (2) and (60), we get

$$\begin{aligned} R(X,Y)Z = g(Y,Z)X - g(X,Z)Y &= (c_1e_2 + c_2e_1)(a_1b_2 - a_2b_1)\\ &+ (a_1b_3 - a_3b_1)(c_1e_3 + c_3e_1) + (a_2b_3 - a_3b_2)(c_3e_2 - c_2e_3); \end{aligned}$$

and hence we have $W_2(X,Y)Z = 0$.

This proves that the manifold M_3 under consideration is a Lorentzian para-Kenmotsu manifold and it admits a flat Weyl-projective curvature tensor.

From the equation (58), we obtain the Ricci tensors and the scalar curvatures as follows: $S(e_1,e_1) = -2$, $S(e_2,e_2) = 2$, $S(e_3,e_3) = 2$ and $r = 6$, where $S(X,Y) = \sum_{i=1}^{3} \epsilon_i \, g(R(e_i,X)Y,e_i)$, $r = \sum_{i=1}^{3} \epsilon_i \, S(e_i,e_i)$ and $\epsilon_i = g(e_i,e_i)$.

The above arguments verifies the results discussed in sections 3, 4 and 5.

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