Edge irregularity strength of categorical product of two paths

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Abstract. For a simple graph $G = (V, E)$, a vertex labeling $\phi : V \to \{1, 2, \dots k\}$ is called k-labeling. The weight of an edge xy in G, denoted by $w_\phi(xy)$, is the sum of the labels of end vertices x and y, i.e $w_\phi(xy) = \phi(x) + \phi(y)$. A vertex k-labeling is defined to be an edge irregular k-labeling of the graph G if for every two different edges e and f, there is $w_\phi(e) \neq w_\phi(f)$. The minimum k for which the graph G has an edge irregular k-labeling is called the edge irregularity strength of G, denoted by $es(G)$.

In this paper, we determine the exact value of edge irregularity strength for categorical product of two paths.

Keywords: irregular assignment, irregularity strength, edge irregularity strength, categorical product of two graph, path.

1. Introduction

Labeled graphs are turning into an inexorably valuable group of Mathematical Models for an extensive variety of utilizations. While the subjective labelings of graph components have roused explore in assorted fields of human enquiry, for example, compromise in social brain science, electrical circuit hypothesis and vitality emergency, these labelings have prompted very multifaceted fields of use, for example, coding hypothesis issues, including the plan of good radar area codes, sync set codes; rocket direction codes and convolution codes with

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ideal autocorrelation properties. Labeled graphs have likewise been connected in deciding ambiguities in X-Ray crystallographic examination, to plan correspondence arrange tending to frameworks, in deciding ideal circuit designs, radio-Astronomy., and so on [1].

Give $G = (V, E)$ a chance to be the associated, basic and undirected diagram with vertex set V and edge set E. By a *labeling* we mean any mapping that conveys a set of diagram components to a set of numbers (normally positive whole numbers), called *labels*. On the off chance that the area is the vertex set or the edge set, the labelings are called individually *vertex labelings* or *edge labelings*. In the event that the space is $V \cup E$ then we call the labeling *total labeling*. Therefore, for an edge k-labeling $\phi : E(G) \to \{1, 2, \ldots, k\}$, the related weight of a vertex $x \in V(G)$ is

$$w_\phi(x) = \sum_{xy \in E(G)} \phi(xy),$$

where the addition is over all vertices y contiguous x, and for a total k-labeling $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$, the related weight of edge $xy \in E(G)$ is

$$wt_\phi(xy) = \phi(x) + \phi(xy) + \phi(y).$$

Chartrand *et al.* in [2] presented edge k-labeling ϕ of a graph G to such an extent that $w_\phi(x) \neq w_\phi(y)$ for all vertices $x, y \in V(G)$ with $x \neq y$. Such labelings were called *irregular assignments* and the *irregularity strength* $s(G)$ of a graph G is known as the smallest k for which G has an irregular assignment utilizing labels at generally k. This parameter has pulled in much consideration [3, 4, 5, 6].

Persuaded by the papers on the irregularity strength Bača *et al.* in [7] began to examine the $tes(G)$ and $tvs(G)$ a graph, an invariant similar to the es for total labelings. They characterize as:

Definition 1.1 ([7]). *For a graph $G = (V, E)$, a labeling $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ to be an edge irregular total k-labeling of the graph G if for each two distinct edges xy and $x'y'$ of G the edge weights $wt_\phi(xy) = \phi(x) + \phi(xy) + \phi(y)$ and $wt_\phi(x'y') = \phi(x') + \phi(x'y') + \phi(y')$ are not equal. The total edge irregularity strength, $tes(G)$, is characterized as the smallest k for which G has an edge irregular total k-labeling.*

Definition 1.2 ([7]). *For a graph $G = (V, E)$, a labeling $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ to be a vertex irregular total k-labeling of the graph G if for each two distinctive vertices x and y of G the vertex weights $wt_\phi(x) = \phi(x) + \sum_{xy \in E(G)} \phi(xy)$ and $wt_\phi(y) = \phi(y) + \sum_{xy \in E(G)} \phi(xy)$ are distinct. The total vertex irregularity strength, $tvs(G)$, is characterized as the smallest k for which G has a vertex irregular total k-labeling.*

The Ivančo and Jendrol̆'s figured the tes for complete graph and complete bipartite diagrams in [8], for the Cartesian product of two paths in [9], for the

Cartesian product of two cycles in [10], for corona product of a path with specific diagrams in [11], for huge thick graphs with $\frac{|E(G)|+2}{3} \leq \frac{\Delta(G)+1}{2}$ in [12], for the categorical product of two paths in [13] and for generalized Petersen diagram $P(n,k)$ in [14].

Some results on $tes(G)$ and $tvs(G)$ can be found in [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

Joining both past changes of their irregularity strength, Marzuki, Salman and Miller [28] presented a new irregular total k-labeling of a graph G called totally irregular total k-labeling, which is required to be in the mean time vertex irregular total and further more edge irregular total. They have given an upper bond and a lower bound for totally irregular total k-labeling, indicated by $ts(G)$. The most entire late review of graph labelings is [29, 30].

Motivated by all these irregular labeling concepts Ahmad *et al.* in [31] introduce a new labeling as follows:

Definition 1.3 ([31]). *A vertex k-labeling $\phi : V \to \{1, 2, \ldots, k\}$ is characterized to be an edge irregular k-labeling of the graph G if for each two distinct edges e and f there is $w_\phi(e) \neq w_\phi(f)$, where the weight of an edge $e = xy \in E(G)$ is $w_\phi(xy) = \phi(x) + \phi(y)$. The smallest k for which the diagram G has an edge irregular k-labeling is known as the edge irregularity strength of G, meant by $es(G)$.*

The following theorem proved in [31], establishes lower bound for the edge irregularity strength of a graph G.

Theorem 1.1 ([31]). *Let $G = (V, E)$ be a simple graph with maximum degree $\Delta = \Delta(G)$. Then*

$$es(G) \geq \max\left\{\left\lceil \frac{|E(G)|+1}{2} \right\rceil, \Delta(G)\right\}.$$

In this paper, we determine the exact values of the edge irregularity strength of categorical product of two paths.

2. The categorical product of two paths

For numbers a and b let [a, b] be an interval of whole numbers x, $a \leq x \leq b$. In this area we manage a categorical product of two diagrams. The categorical product $G \times H$ of two graphs G and H is the graph with vertex set $V(G) \times V(H)$, where two vertices (u, u') and (v, v') are neighbouring if and just if u, v are contiguous in G and u', v' are adjoining in H (see e.g. [32] or [33]). On the off chance that we consider graph G as the path P_n with:

$$V(P_n) = \{x_i : i \in [1, n]\},$$
$$E(P_n) = \{x_i x_{i+1} : i \in [1, n-1]\}$$

and graph H as the path P_m with

$$V(P_m) = \{y_j : j \in [1, m]\},$$
$$E(P_m) = \{y_j y_{j+1} : j \in [1, m-1]\}$$

then

$$V(P_n \times P_m) = \{(x_i, y_j) : i \in [1, n], j \in [1, m]\}$$

is the vertex set of $P_n \times P_m$ and

$$E(P_n \times P_m) = \{(x_i, y_j)(x_k, y_l) : i, k \in [1, n], j, l \in [1, m], |i - k| = 1, |j - l| = 1\}$$

is the edge set of $P_n \times P_m$. The categorical product of two paths $P_7 \times P_5$ is delineated in figure 1. It is easy to see that , $P_n \times P_m$ is the graph of order nm and size $2(n-1)(m-1)$. As the most extreme degree $\triangle(P_n \times P_m) = 4$ at that point from Theorem 1.1 it takes after that $es(P_n \times P_m) \geq (m-1)(n-1)+1$. The primary objective of this paper is to prove equality.

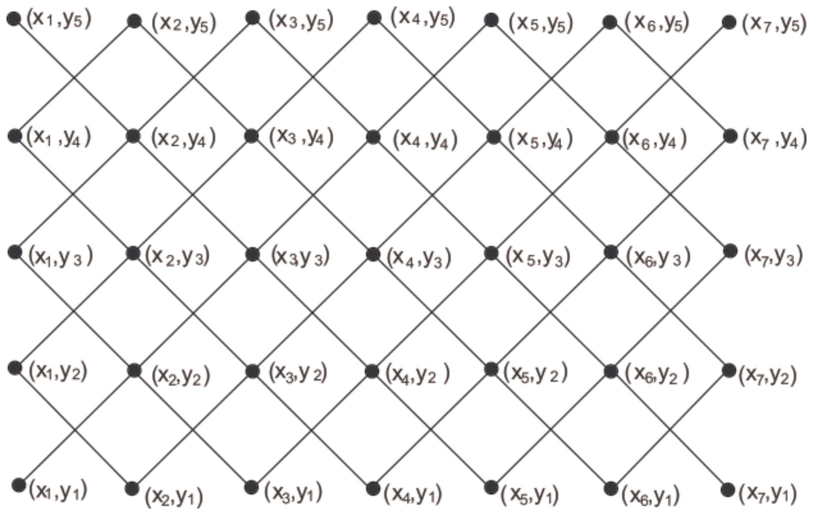


Figure 1: The Categorical Product of Two Paths $P_7 \times P_5$.

Theorem 2.1. *Let $m \geq 3$, odd and $n \geq 2$. Then*

$$es(P_n \times P_m) = (m-1)(n-1)+1.$$

Proof. Let $k = (m-1)(n-1)+1$. According to Theorem 1.1 it is enough to prove that $es(P_n \times P_m) \leq k$. For this we define a vertex labeling $\phi : V(P_n \times P_m) \to \{1, 2, \ldots, k\}$ as follows.

$$\phi((x_i, y_j)) = \begin{cases} k - n - \frac{m-1}{2} + i + \frac{j-1}{2}, & \text{for } i \text{ even and } j \text{ odd} \\ n - 2 + \frac{m-1}{2} + i + (2n-3)\frac{j-2}{2}, & \text{for } i \text{ odd and } j \text{ even} \\ i + \frac{j-1}{2}, & \text{for } i \text{ odd and } j \text{ odd} \\ i - 1 + \frac{j-2}{2}(2n-3), & \text{for } i \text{ even and } j \text{ even} \end{cases}$$

Now, we show that all the vertex labels are from 1 up to k for $1 \leq i \leq n, 1 \leq j \leq m$. i.e

$$1 \leq \phi((x_i, y_j)) \leq k$$

- For $1 \leq i \leq n$, even and $1 \leq j \leq m$, odd

$$\begin{aligned}\phi((x_i, y_j)) &= k - n - \frac{m-1}{2} + i + \frac{j-1}{2} \\ &\geq k - n - \frac{m-1}{2} + 2 = \frac{2k - 2n - m + 5}{2} \geq 1\end{aligned}$$

and

$$\begin{aligned}\phi((x_i, y_j)) &= k - n - \frac{m-1}{2} + i + \frac{j-1}{2} \\ &\leq k - n - \frac{m-1}{2} + n + \frac{m-1}{2} = k\end{aligned}$$

For $1 \leq i \leq n$, odd and $1 \leq j \leq m$, even

$$\begin{aligned}\phi((x_i, y_j)) &= n - 2 + \frac{m-1}{2} + i + (2n-3)\frac{j-2}{2} \\ &\geq n - 2 + \frac{m-1}{2} + 1 \geq 1\end{aligned}$$

and

$$\begin{aligned}\phi((x_i, y_j)) &= n - 2 + \frac{m-1}{2} + i + (2n-3)\frac{j-2}{2} \\ &\leq n - 2 + \frac{m-1}{2} + n + (2n-3)\frac{m-3}{2} \\ &= (m-1)(n-1) + 1 = k\end{aligned}$$

- For $1 \leq i \leq n$, odd and $1 \leq j \leq m$, odd

$$\begin{aligned}\phi((x_i, y_j)) &= i + \frac{j-1}{2} \geq 1, \\ \phi((x_i, y_j)) &= i + \frac{j-1}{2} \leq \frac{2n + m - 1}{2}.\end{aligned}$$

Now, we show that $\frac{2n+m-1}{2} \leq mn - n - m + 2$,

For $t \geq 1, n \geq 2$, we have

$$\begin{gathered}(2t-1)(n-1) - t \geq 0, \\ 2tn - 3t - n + 1 \geq 0, \\ (2t+1)n - n - (2t+1) + 2 \geq n + t, \\ (2t+1)n - n - (2t+1) + 2 \geq \frac{2n + 2t + 1 - 1}{2}.\end{gathered} \tag{1}$$

Sine m is odd, let $m = 2t + 1$ for all $t \geq 1$, then (1) becomes

$$mn - n - m + 2 \geq \frac{2n + m - 1}{2}.$$

- For $1 \leq i \leq n$, even and $1 \leq j \leq m$, even

$$\begin{aligned}\phi((x_i, y_j)) &= i + \frac{(2n-3)(j-2)}{2} \geq 1,\\ \phi((x_i, y_j)) &= i + \frac{(2n-3)(j-2)}{2} \leq \frac{2n+m-4}{2}.\end{aligned}$$

Now, we show that $\frac{2n+m-4}{2} \leq mn - n - m + 2 = k$,

For $t \geq 1, n \geq 2$, we have

$$\begin{aligned}(4t-2)(n-1) - 2t + 3 &\geq 0,\\ 4tn - 6t - 2n + 5 &\geq 0,\\ 2tn - 2t + 1 &\geq \frac{2n + 2t + 1 - 4}{2},\\ (2t+1)n - n - (2t+1) + 2 &\geq \frac{2n + 2t + 1 - 4}{2}.\end{aligned} \tag{2}$$

Sine m is odd, let $m = 2t + 1$ for all $t \geq 1$, then (2) becomes

$$mn - n - m + 2 \geq \frac{2n + m - 4}{2}.$$

Thus all the vertex labels satisfy $1 \leq \phi((x_i, y_j)) \leq k$.

Now, we define the weights on the edges for $1 \leq i \leq n, 1 \leq j \leq m$,

$$wt_\phi((x_i, y_{j+1})(x_{i+1}, y_j)) = \begin{cases} k - 1 + 2i + (n-1)(j-1), & \text{for } i \text{ odd and } j \text{ odd}\\ k + 2i + (n-1)(j-2), & \text{for } i \text{ even and } j \text{ even}\\ 2i + 1 + (j-2)(n-1), & \text{for } i \text{ odd and } j \text{ even}\\ 2i + (j-1)(n-1), & \text{for } i \text{ even and } j \text{ odd}\end{cases}$$

$$wt_\phi((x_i, y_j)(x_{i+1}, y_{j+1})) = \begin{cases} k - 1 + 2i + (n-1)(j-1), & \text{for } i \text{ even and } j \text{ odd}\\ k + 2i + (n-1)(j-2), & \text{for } i \text{ odd and } j \text{ even}\\ 2i + (j-1)(n-1), & \text{for } i \text{ odd and } j \text{ odd}\\ 2i + 1 + (j-2)(n-1), & \text{for } i \text{ even and } j \text{ even}\end{cases}$$

One can see that the set of edge weights form a sequence of consecutive integers $\{2, 3, 4, \ldots, 2k - 1\}$. So the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling ϕ is an optimal edge irregular k-labeling. An illustration of Theorem 2 is depicted in Figure 2. In which number in black colour are vertex labels and number in blue colour shows the edge weights of the corresponding edge. This completes the proof. □

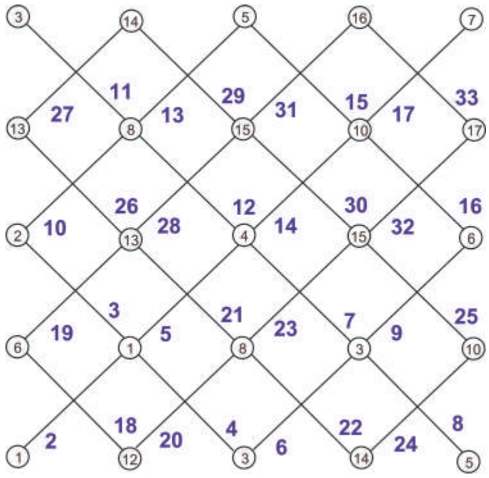


Figure 2: The Edge Irregular labeling of $P_5 \times P_5$.

Lemma 2.1. *Let $n \geq 2$. Then $es(P_n \times P_2) = n$.*

Proof. From Theorem 1.1 it follows that $es(P_n \times P_2) \geq n$. The existence of the optimal labeling proves the converse inequality.
Case 1. When n is odd

$$\phi((x_i, y_1)) = \begin{cases} \frac{i+1}{2}, & \text{for } \ 1 \leq i \leq n \text{ odd} \\ \frac{n-1+i}{2}, & \text{for } \ 1 \leq i \leq n \text{ even} \end{cases}$$

$$\phi((x_i, y_2)) = \begin{cases} \frac{n+i}{2}, & \text{for } \ 1 \leq i \leq n \text{ odd} \\ \frac{i}{2}, & \text{for } \ 1 \leq i \leq n \text{ even} \end{cases}$$

Case 2. When n is even

$$\phi((x_i, y_1)) = \begin{cases} \frac{i+1}{2}, & \text{for } \ 1 \leq i \leq n \text{ odd} \\ \frac{n+i}{2}, & \text{for } \ 1 \leq i \leq n \text{ even} \end{cases}$$

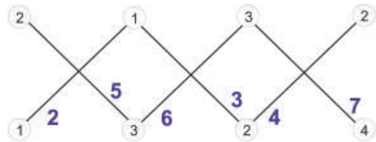
$$\phi((x_i, y_2)) = \begin{cases} \frac{n+i-1}{2}, & \text{for } \ 1 \leq i \leq n \text{ odd} \\ \frac{i}{2}, & \text{for } \ 1 \leq i \leq n \text{ even} \end{cases}$$

Now for all $n \geq 2$, the weight on the edges are define as follows.
$wt_\phi((x_i, y_1)(x_{i+1}, y_2)) = i + 1$, for $1 \leq i \leq n$, odd
$wt_\phi((x_{i+1}, y_1)(x_i, y_2)) = i + 1$, for $1 \leq i \leq n$,even
$wt_\phi((x_i, y_2)(x_{i+1}, y_1)) = n + i$, for $1 \leq i \leq n$,odd
$wt_\phi((x_i, y_1)(x_{i+1}, y_2)) = n + i$, for $1 \leq i \leq n$,even. □

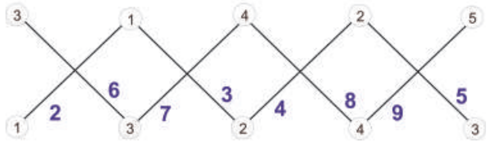
An illustration of Lemma 1 is depicted in Figure 3.

Theorem 2.2. *Let $m \geq 4$, even and $n \geq 3$. Then*

$$es(P_n \times P_m) = (m - 1)(n - 1) + 1.$$



(b) Edge irregular labeling of $P_4 \times P_2$



(a) Edge irregular labeling of $P_5 \times P_2$

Figure 3: The Edge Irregular labeling of $P_5 \times P_2$ and $P_4 \times P_2$.

Proof. Let $k = (m-1)(n-1)+1$. According to Theorem 1.1 it is enough to prove that $es(P_n \times P_m) \leq k$. For this we define a vertex labeling $\phi : V(P_n \times P_m) \to \{1, 2, \ldots, k\}$ as follows

$$\phi((x_i, y_j)) = \begin{cases} \frac{1}{2}[2i + j - 3], & \text{for } \ i \text{ even and } j \text{ odd} \\ \frac{1}{2}[2i + (j-2)(2n-3)], & \text{for } \ i \text{ odd and } 1 \leq j \leq m-2 \text{ even} \\ \frac{1}{2}[2mn - 3m - 4n + 10], & \text{for } \ i \equiv 3(mod4) \text{ and } j = m \\ \frac{1}{2}[2mn - 3m - 4n + 8], & \text{for } \ i \equiv 1(mod4) \text{ and } j = m \end{cases}$$

Case 1. When n is odd

$$\phi((x_i, y_j)) = \begin{cases} k - \frac{1}{2}[(n-1)(m-j) - i + 1], & \text{for } \ i \text{ odd and } j \text{ odd} \\ k - \frac{1}{2}[(n-1)(m-j+1) - i], & \text{for } \ i \text{ even and } j \text{ even} \end{cases}$$

Case 2. When n is even

$$\phi((x_i, y_j)) = \begin{cases} k - \frac{1}{2}[(n-1)(m-j) - i], & \text{for } \ i \text{ odd and } j \text{ odd} \\ k - \frac{1}{2}[(n-1)(m-j) + n - i], & \text{for } \ i \text{ even and } j \text{ even} \end{cases}$$

Now we show that all the vertex labels are from 1 up to k for $1 \leq i \leq n, 1 \leq j \leq m$. i.e

$$1 \leq \phi((x_i, y_j)) \leq k.$$

- For $1 \leq i \leq n$, even and $1 \leq j \leq m$, odd

$$\begin{aligned} \phi((x_i, y_j)) &= \frac{1}{2}[2i + j - 3] \geq 1 \\ \phi((x_i, y_j)) &= \frac{1}{2}[2i + j - 3] \leq \frac{1}{2}[2n + m - 4] < k \end{aligned}$$

- For $1 \le i \le n$, odd and $1 \le j \le m-2$, even

$$\begin{aligned}
\phi((x_i, y_j)) &= \frac{1}{2}[2i + (j-2)(2n-3)] \ge 1 \\
\phi((x_i, y_j)) &= \frac{1}{2}[2i + (j-2)(2n-3)] \\
&\le \frac{1}{2}[2n + (m-4)(2n-3)] \\
&= mn - n - m + 2 - \frac{1}{2}[4n + m - 8] \\
&= k - \frac{1}{2}[4n + m - 8] < k
\end{aligned}$$

- For $i \equiv 3(mod4)$ and $j = m$

$$\begin{aligned}
\phi((x_i, y_j)) &= \frac{1}{2}[2mn - 3m - 4n + 10] > 1 \\
\phi((x_i, y_j)) &= \frac{1}{2}[2mn - 3m - 4n + 10] \\
&= mn - n - m + 2 - \frac{1}{2}[2n + m - 6] \\
&= k - \frac{1}{2}[2n + m - 6] < k
\end{aligned}$$

- For $i \equiv 1(mod4)$ and $j = m$

$$\begin{aligned}
\phi((x_i, y_j)) &= \frac{1}{2}[2mn - 3m - 4n + 8] > 1 \\
\phi((x_i, y_j)) &= \frac{1}{2}[2mn - 3m - 4n + 8] \\
&= mn - n - m + 2 - \frac{1}{2}[2n + m - 4] \\
&= k - \frac{1}{2}[2n + m - 4] < k
\end{aligned}$$

- For n odd, $1 \le i \le n$, odd and $1 \le j \le m$, odd

$$\begin{aligned}
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j)] + \frac{i-1}{2} \\
&\ge k - \frac{1}{2}[(n-1)(m-1)] = \frac{1}{2}[mn - n - m + 3] > 1 \\
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j)] + \frac{i-1}{2} \\
&\le k - \frac{n-1}{2} + \frac{n-1}{2} = k
\end{aligned}$$

- For n odd, $1 \leq i \leq n$, even and $1 \leq j \leq m$, even

$$\begin{aligned}
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m+1-j)] + \frac{i}{2} \\
&\geq k - \frac{1}{2}[(n-1)(m-1)] + 1 = \frac{1}{2}[mn - n - m + 5] > 1 \\
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m+1-j)] + \frac{i}{2} \\
&\leq k - \frac{n-1}{2} + \frac{n-1}{2} = k
\end{aligned}$$

- For n even, $1 \leq i \leq n$, odd and $1 \leq j \leq m$, odd

$$\begin{aligned}
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j) - 1] + \frac{i-1}{2} \\
&\geq k - \frac{1}{2}[(n-1)(m-1) - 1] = \frac{1}{2}[mn - n - m + 4] > 1 \\
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j) - 1] + \frac{i-1}{2} \\
&\leq k - \frac{n-1}{2} + \frac{n-1}{2} = k
\end{aligned}$$

- For n even, $1 \leq i \leq n$, even and $1 \leq j \leq m$, even

$$\begin{aligned}
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j) + n - 2] + \frac{i-2}{2} \\
&\geq \frac{1}{2}[mn - n - m + 4] > 1 \\
\phi((x_i, y_j)) &= k - \frac{1}{2}[(n-1)(m-j) + n - 2] + \frac{i-2}{2} \\
&\leq k - \frac{n-2}{2} + \frac{n-2}{2} = k
\end{aligned}$$

Thus, all the vertex labels satisfy $1 \leq \phi((x_i, y_j)) \leq k$.
Now, we define the weights on the edges for $1 \leq i \leq n, 1 \leq j \leq m$,

$$wt_\phi((x_i, y_{j+1})(x_{i+1}, y_j)) = \begin{cases} 2i + j(n-1) - n + 1, & \text{for } \ i \text{ odd} \\ & \text{and } 1 \leq j \leq m-2 \text{ odd} \\ m(n-1) - 2n + 4 + i, & \text{for } \ i \equiv 3(mod 4) \\ & \text{and } j = m-1 \\ m(n-1) - 2n + 3 + i, & \text{for } \ i \equiv 1(mod 4) \\ & \text{and } j = m-1 \end{cases}$$

$$wt_\phi((x_{i+1}, y_j)(x_{i+2}, y_{j+1})) = \begin{cases} 2i + j(n-1) - n + 3, & \text{for } \ i \text{ odd} \\ & \text{and } 1 \leq j \leq m-2 \text{ odd} \\ m(n-1) - 2n + 3 + i, & \text{for } \ i \equiv 3(mod 4) \\ & \text{and } j = m-1 \\ m(n-1) - 2n + 4 + i, & \text{for } \ i \equiv 1(mod 4) \\ & \text{and } j = m-1 \end{cases}$$

- For $1 \leq i \leq n$, odd and $1 \leq j \leq m-2$, odd

$$wt_\phi((x_i, y_{j+1})(x_{i+1}, y_{j+2})) = 2i + j(n-1) - n + 2,$$
$$wt_\phi((x_{i+1}, y_{j+2})(x_{i+2}, y_{j+1})) = 2i + j(n-1) - n + 4,$$
$$wt_\phi((x_i, y_{j+2})(x_{i+1}, y_{j+1})) = 2k - (m-j-1)(n-1) + i.$$

- For $1 \leq i \leq n-1$, odd and $1 \leq j \leq m$, odd

$$wt_\phi((x_i, y_j)(x_{i+1}, y_{j+1})) = 2k - (m-j)(n-1) + i,$$

- For $1 \leq i \leq n-2$, odd and $1 \leq j \leq m$, odd

$$wt_\phi((x_{i+1}, y_{j+1})(x_{i+2}, y_j)) = 2k - (m-j)(n-1) + i + 1,$$

- For $1 \leq i \leq n-2$, odd and $1 \leq j \leq m-2$, odd

$$wt_\phi((x_{i+1}, y_{j+1})(x_{i+2}, y_{j+2})) = 2k - (m-j-1)(n-1) + i + 1.$$

It is easy to check that the set of edge weights form a sequence of integers

$$\{2, 3, 4, \ldots, mn - m - n + 2\} \cup \{mn - m - n + 4, mn - m - n + 5, \ldots, 2k\}.$$

So, the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling ϕ is an optimal edge irregular k-labeling. This completes the proof. □

3. Conclusion

This paper presented a new graph trademark, the edge irregularity strength, as a modification of the notable irregularity strength, add total edge irregularity strength and total vertex irregularity strength. In this paper we determined the exact an incentive for categorical product of two paths. Truth be told, it is by all accounts an extremely difficult issue to locate the correct an incentive for the edge irregularity strength of graphs. In future we are interested to compute the edge irregularity strength of categorical Product of cycle and path graph.

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Accepted: 6.07.2019