On restrained hub number in graphs

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Abstract. In this paper, we study the restrained hub number $h_r(G)$ of a graph G. We characterize the class of all graphs for which $h_r(G) = 1$. Also the relationship between cut vertices and restrained hub number are presented. The restrained hub number of the corona of two graphs is determined.

Keywords: hub number, restrained hub number, corona of two graphs.

1. Introduction

Let $G = (V, E)$ be a finite and undirected graph without loops and multiple edges. And $G = (p, q)$ graph if its with p vertices and q edges. The degree of a vertex v in a graph G denoted by $deg(v)$, and $\delta(G)(\Delta(G))$ denotes the minimum (maximum) degree among the vertices of G, respectively [2]. An end vertex is a vertex of degree one, a clique of a graph is a maximal complete subgraph, a block of a graph is a maximal nonseparable subgraph. A star is a complete bipartite graph $K_{1,p-1}$, and denoted by S_p. Given any vertex $v \in V(G)$, the graph obtained from G by removing the vertex v and all of its

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incident edges is denoted by $G-v$. For $v \in V(G)$, the open neighbourhood of v is $N(v) = \{u \in V(G) : uv \in E(G)\}$, for $S \subseteq V(G)$, $N(S) = \bigcup_{v \in S} N(v)$, the closed neighbourhood $N[v] = N(v) \cup \{v\}$, and $N[S] = N(S) \cup S$. The contraction of a vertex x in G, denoted by G/x, is being the graph obtained by deleting x and putting a clique on the (open) neighbourhood of x, (note that, this operation does not create multiple edges, if two neighbours of x are already adjacent, then they remain simply adjacent). Graphs G_1 and G_2 have disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively, their union, $G(V, E) = G_1 \cup G_2$ has as expected, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. The corona $G \circ F$ of two graphs G and F is the graph obtained by taking one copy of G of order p and p copies of F, and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of F. For every $v \in V(G)$, denote by F_v the copy of F whose vertices are attached one by one to the vertex v [1]. The distance $d(u, v)$ between two vertices u and v in a graph G is the length of a shortest path connecting them. For a vertex v of G, the eccentricity of v is $e(v) = \max\{d(v, u), u \in V(G)\}$. See [2] for terminology and notations not defined here.

M. Walsh [13] introduced the theory of hub in 2006. A hub set in a graph G is a set H of vertices in G such that any two vertices in $V(G) \setminus H$ are connected by a path whose all internal vertices lie in H. The hub number of G, denoted by $h(G)$, is the minimum size of a hub set in G. If for every pair of vertices in $V(G) \setminus H$ are also connected by a path whose all internal vertices lie in $V(G) \setminus H$, then H is called a restrained hub set and denoted by H_r. The restrained hub number $h_r(G)$ is the minimum cardinality of a restrained hub set in G [6]. For more details on the hub studies we refer to [3, 4, 5, 7, 8, 9, 10, 11, 12].

Imagine there is a graph representing an industrial city map, where each point on the map represents a building place, with an edge between two points if there's an easy walk from one to the other. Some buildings will be implemented as factories and others as transit stations. Where raw material can be transferred between any two factories by transit stations. Also, trader can move between any two non-adjacent factories to buy goods without passing any transit point, the goal is to make costs as cheaply as possible by converting as few buildings as possible into transit stations. Motivated by this along with the concept of restrained hub number and the great attention from researchers in the concept of hub number in graph, we try to develop the theory of hub by establish a new results on this theory. The following results will be useful in the proof of our results.

Theorem 1.1 ([6])**.** *Let G be any graph. Then the set H_r is restrained hub set if and only if G/H_r is complete, and $G[V(G) \setminus H_r]$ is connected.*

Theorem 1.2 ([6])**.** *Let G be a graph with at least one end vertex. Then $h_r(G) = p-2$ if and only if there exists minimum restrained hub set not containing an end vertex.*

Lemma 1.1. *If uv is an edge of a connected graph G, then $|e(u) - e(v)| \leq 1$.*

2. Main results

Proposition 2.1. *Let G be a graph. Then $h_r(G) = h(G) = 0$ if and only if G is a complete graph.*

Proof. Let $h_r(G) = 0$, and let $\{x, y\} \subseteq V(G)$. Then $H_r = \phi$ is a minimum restrained hub set of G, so there exists $xy-$ path with all internal vertices in $H_r = \phi$. Thus x is adjacent to y. Therefore, G is complete. The converse is trivial. □

Note that: If H_r is a restrained hub set of G, then $H_r \cup \{v\}$ for some $v \in V(G)$, may not be a restrained hub set of G. For example if $G \cong C_5$, then $H_r = \{x, y\}$, where $xy \in E(G)$ is a restrained hub set of G, but $H_r \cup \{z\}$, where z not adjacent to both x and y, is not a restrained hub set of G.

Theorem 2.1. *For any graph G with connected subgraph $G[H_{cr}]$. H_{cr} is a connected restrained hub set of G if and only if G has the following structure:*

1. *$V(G) = M \cup N \cup H_{cr}$, where M, N and H_{cr} are disjoint.*
2. *Every vertex in N is not adjacent to any vertex in H_{cr}.*
3. *Every vertex in M is adjacent to some vertices in H_{cr}.*
4. *Every vertex in M is adjacent to every vertex in N.*
5. *$G[N]$ is complete graph.*
6. *$N \neq \phi$ or $G[M]$ is connected.*

Proof. Assume that H_{cr} is a connected restrained hub set of G, take $M = N(H_{cr})$ and $N = V(G) \backslash N[H_{cr}]$. Its clear that the sets H_{cr}, M and N are disjoint sets. And by definition of N and M the conditions 1 to 3 are satisfied . Since H_{cr} is a restrained hub set, then by Theorem 1.1, G/H_{cr} is a complete graph. Since the contraction of H_{cr} is unaffected by the adjacency of vertices in N, then clearly any vertex in N must adjacent to every vertex in $V(G) \setminus H_{cr}$, and that proves conditions 4 and 5. Now we will prove 6^{th} condition by contradiction. Let $N = \phi$ and $G[M]$ is disconnected. Then $G[V(G) \setminus H_{cr}] = G[M]$ is disconnected, and that contradicts Theorem 1.1, this completes the proof. The converse is trivial. □

Remark 2.1. Note that, H_c is a connected hub set if and only if H_c satisfied the conditions $1 - 5$ in the previous Theorem.

Corollary 2.1. *Let $G \ncong K_p$. Then $h_r(G) = 1$ if and only if there exists non cut vertex v, such that $G[V(G) \setminus N[v]]$ is complete and every vertex of $V(G) \setminus N[v]$ adjacent to every vertex of $N(v)$.*

Proposition 2.2. *Let G be a graph, and H be a minimum hub set of G. If there exists $v \in V(G)$ such that $N[v] \cap H = \phi$, then $h_r(G) = h(G)$.*

Proof. Let H be a minimum hub set of G, such that there exists $v \in V(G)$ with $N[v] \cap H = \phi$. Let $u \in V(G) \setminus H$, then there is $vu-$ path with all internal vertices in H, but v is not adjacent to any vertex in H, so v is adjacent to u. Therefore $V(G) \setminus H$ is connected, and by Theorem 1.1, H is a restrained hub set. Therefore, $h_r(G) \leq h(G)$. □

Lemma 2.1. *Let G be a disconnected graph with components $G_1, G_2, ..., G_n$. Then $h_r(G) = min\{h_k\}$, where $h_k = \sum_{i=1, i\neq k}^{n} |G_i| + h_r(G_k), k = 1, 2, ..., n$.*

Proof. Let $G_1, G_2, ..., G_n$ be the components of G, and let H_r be a minimum restrained hub set of G. If there is $\{x, y\} \subseteq (V(G) \setminus H_r)$ belongs to two different components of G, then $G[V(G)\setminus H_r]$ is disconnected, which contracts proposition 2.1. So $(V(G)\setminus H_r) \subseteq G_j$, for some $j = 1, 2, ..., n$. Hence any minimal restrained hub set H_r must contains all vertices from all components except one, and the vertices of any minimum restrained hub set of the remaining component. Thus $h_r(G) = min\{h_k\}$, where $h_k = \sum_{i=1, i\neq k}^{n} |G_i| + h_r(G_k), k = 1, 2, ..., n$. □

Corollary 2.2. *Let G be a disconnected graph. Then $h_r(G) = 1$ if and only if $\overline{G} \cong S_p$.*

Proof. Let $h_r(G) = 1$. Then by Lemma 2.1, G has two components only, one of them has just one vertex, and the second one is complete. Therefore $\overline{G} \cong S_p$. The converse is trivial. □

Proposition 2.3. *Let $G \cong K_{n_1, n_2, ..., n_k}$ with $k \geq 3$. Then*

$$h_r(G) = \begin{cases} 0, & \text{if } n_i = 1 \text{ for all } 1 \leq i \leq k\ ; \\ 1, & \text{if } n_i \leq 2 \text{ for some } 1 \leq i \leq k\ ; \\ 2, & \text{otherwise.} \end{cases}$$

Proof. Let $G = K_{n_1, n_2, ..., n_k}$ with $k \geq 3$. Then we consider the following cases:
Case 1. $n_i = 1$ for all $1 \leq i \leq k$. Then its clear that G is a complete graph, hence by Proposition 2.1, $h_r(G) = 0$.
Case 2. $n_i \leq 2$. For some $1 \leq i \leq k$, let v be any vertex in this part. Then its clear that G/v is complete and $G - v$ is connected. Thus by Theorem 1.1, $\{v\}$ is a restrained hub set, and by Proposition 2.1, $\{v\}$ it's minimum since G is not a complete graph, so $h_r(G) = 1$.
Case 3. $n_i > 2$ for all $1 \leq i \leq k$. For any vertex $x \in V(G)$, G/x is not complete because there is at least two vertices in the part containing x are not adjacent in G/x, hence $h_r(G) \geq 2$. Let u, v be two vertices in different parts. Then $G/\{u, v\}$ is complete and $G - \{u, v\}$ is connected, hence by Theorem 1.1, $\{u, v\}$ is a restrained hub set. □

Lemma 2.2. *Let G be a graph with degree sequence $\Delta, d_2, d_3, ..., d_n$, and let $v \in V(G)$ such that $deg(v) = \Delta = n - 1$. Then $h_r(G) \leq n - d_2$.*

Proof. Let $u \in V(G)$ such that $deq(u) = d_2$, and let $H_r = (V(G) \setminus N[u]) \cup \{v\}$. Then $V(G) \setminus H_r = N[u] \setminus \{v\}$, so every two non adjacent vertices $x, y \in (V(G) \setminus H_r)$ has two paths x, v, y and x, u, y with all internal vertices in H_r and in $V(G) \setminus H_r$. So H_r is a restrained hub set. Thus

$$h_r(G) \leq |(V(G) \setminus N[u]) \cup \{v\}| = n - (d_2 + 1) + 1 = n - d_2.$$

□

Theorem 2.2. *If A and B are two components of graph $G - x$ and H_r is a restrained hub set of G, then $A \subseteq H_r$ or $B \subseteq H_r$.*

Proof. Let G be a graph with a cut vertex x, A and B are two components of $G - x$. Then the following cases are considered.
Case 1. $x \in H_r$. Let $u \in A$ and $v \in B$, such that $\{u, v\} \nsubseteq H_r$. Then $G[V(G) \setminus H_r]$ is disconnected, and it has at least two components, a contradiction. So $A \subseteq H_r$ or $B \subseteq H_r$.
Case 2. $x \notin H_r$. Let $u \in A$ and $v \in B$, such that $\{u, v\} \nsubseteq H_r$. Since x is a cut vertex, it follows that x lies in every path between u and v. Hence there is no path between u and v with all internal vertices are in H_r, which is a contradiction. Thus $A \subseteq H_r$ or $B \subseteq H_r$. □

Corollary 2.3. *Let x be a cut vertex of graph G. If H_r is a restrained hub set, then H_r contains all components of $G - x$ except one.*

Proof. Suppose that x is a cut vertex in a graph G, H_r is a restrained hub set, and $G_1, G_2, ..., G_k$ be the components of $G - x$. Take $A = G_1$, $B = G_2$, then by Theorem 2.2, either $G_1 \subseteq H_r$ or $G_2 \subseteq H_r$. Now let $A \nsubseteq H_r$, and B be the next component that does not compered yet, continuo in the progress to reach the last component. Therefore, there is just one component $G_j \nsubseteq H_r$, for some $1 \leq j \leq k$. □

Corollary 2.4. *For any graph G, let C be a cut vertex set, and H_r is a restrained hub set. If $C \subseteq H_r$, or $C \cap H_r = \phi$, then H_r contains all components of $G[V(G) \setminus C]$ except one.*

Corollary 2.5. *Let G be a connected graph, and $A_i = \{v \in V(G) : e(v) = i, r(G) < i < d(G)\}$. If H_r is a restrained hub set such that $A_j \subseteq H_r$ or $A_j \cap H_r = \phi$, for some $r(G) < j < d(G)$, then $\bigcup_{k=i+1}^{d-1} A_k \subseteq H_r$ or $\bigcup_{k=r+1}^{i-1} A_k \subseteq H_r$.*

Proof. Let G be a connected graph, $A_i = \{v \in V(G) : e(v) = i, r(G) < i < d(G)\}$, and H_r be a restrained hub set such that $A_j \subseteq H_r$ or $A_j \cap H_r = \phi$, for some $r(G) < j < d(G)$. Now, take $x \in A_u$, and $y \in A_l$, where $l < i < u$. Then by Lemma 1.1, every $xy-$ path has at least one vertex from A_i, hence A_i is a cut set, moreover $\bigcup_{k=i+1}^{d-1} A_k$ and $\bigcup_{k=r+1}^{i-1} A_k$, are in two different components of $G[V(G) \setminus A_i]$. So by Corollary 2.4, $\bigcup_{k=i+1}^{d-1} A_k \subseteq H_r$ or $\bigcup_{k=r+1}^{i-1} A_k \subseteq H_r$. □

Corollary 2.6. *Let H_r be a restrained hub set of a graph G. Then $V(G) \setminus H_r$ lies in one block from blocks of G.*

Proof. Let G be any graph, H_r be a restrained hub set of G. By contradiction, suppose that A, B be two different blocks of G suppose that there exist two vertices x and y belongs to A, B, respectively. Let $u \in A$ be a cut vertex such that $d(y, u) \leq d(y, z)$ for all $z \in A$. Then x and y belongs to two different components of $G - u$, thus by Theorem 2.2, $x \in H_r$ or $y \in H_r$, and that is a contradiction. $\square$

Theorem 2.3. *Let $G(n_1, q_1)$ and $F(n_2, q_2)$ be two graphs. Then*

$$h_r(G \circ F) = \begin{cases} n_1n_2 + h_r(G), & \text{if } n_1 > h_r(G) + s + t; \\ (n_1 - 1)(n_2 + 1) + (1 - t), & \text{if } n_1 \leq h_r(G) + s + t. \end{cases}$$

Where $t = \lfloor \frac{\delta(C)+1}{s} \rfloor$, and C is a component of F with maximum order s, and with largest number of edges if there is more than one.

Proof. Let $G(n_1, q_1)$ and $F(n_2, q_2)$ be two graphs, and let the copies of F are $F_1, F_2, ..., F_{n_1}$ are incident to vertices $v_1, v_2, ..., v_{n_1}$ of graph G respectively. Then every vertex in $V(G)$ is a cut vertex of the graph $G \circ F$. Therefore, $(F_i \cup \{v_i\}), i = 1, 2, ...n_1$, are blocks of the graph $G \circ F$. Let H_r be a minimum restrained hub set of $G \circ F$, so by Corollary 2.6, $V(G \circ F) \setminus H_r$ lies in one block of $G \circ F$. If $(V(G \circ F) \setminus H_r) \subseteq G$. Then its clear that all paths between any two vertices in $V(G)$ are consists from vertices of $V(G)$ it self, so any minimum restrained hub set for G will not changed in $G \circ F$. Therefore, $h_r(G \circ F) = n_1n_2 + h_r(G)$. But if $(V(G \circ F) \setminus H_r) \subseteq Fi \cup \{v_i\}$, for some $i = 1, 2, ..., n_1$, (say $F_1 \cup \{v_1\}$), then consider that $F_1 \cong \bigcup_{i=1}^{m} C_i$, where C_i are the components of F_1, let C be a component of F_1 with maximum order s, and with largest number of edges if there is more than one. Now we have to discuss the following cases:

Case 1. C is not complete graph. Then $H_r = (V(G \circ F) \setminus V(C))$ is a restrained hub set of $(G \circ F)$ and its minimum, since $v_1 + C$ not complete subgraph of $G \circ F$. Thus,

$$|H_r| = |(V(G \circ F) \setminus V(C))| = (n_1 - 1)(n_2 + 1) + 1 + (n_2 - s).$$

Therefore, $h_r(G \circ F) = \min\{n_1n_2 + h_r(G), (n_1 - 1)(n_2 + 1) + 1 + (n_2 - s)\}$. But

$$\begin{aligned} (n_1 - 1)(n_2 + 1) + 1 + (n_2 - s) \leq n_1n_2 + h_r(G) \iff \\ n_1n_2 + n_1 - n_2 - 1 + 1 + n_2 - s \leq n_1n_2 + h_r(G) \iff \\ n_1 \leq h_r(G) + s. \end{aligned}$$

Therefore,

$$h_r(G \circ F) = \begin{cases} n_1n_2 + h_r(G), & \text{if } n_1 > h_r(G) + s; \\ (n_1 - 1)(n_2 + 1) + 1, & \text{if } n_1 \leq h_r(G) + s. \end{cases}$$

Case 2. C is complete graph. Then $v_1 + C$ is complete subgraph of $G \circ F$, so $(V(G \circ F) \setminus \{C \cup v_1\}$ is a minimum restrained hub set of $G \circ F$. Thus,

$$h_r(G \circ F) = \min\{n_1n_2 + h_r(G), (n_1 - 1)(n_2 + 1) + (n_2 - s)\}.$$

Therefore,

$$h_r(G \circ F) = \begin{cases} n_1n_2 + h_r(G), & \text{if } n_1 > h_r(G) + s + 1; \\ (n_1 - 1)(n_2 + 1), & \text{if } n_1 \leq h_r(G) + s + 1. \end{cases}$$

The two formulas can be merged in one formula as the following:

Let $t = \lfloor \frac{\delta(C)+1}{s} \rfloor$. Then

$$h_r(G \circ F) = \begin{cases} n_1n_2 + h_r(G), & \text{if } n_1 > h_r(G) + s + t; \\ (n_1 - 1)(n_2 + 1) + (1 - t), & \text{if } n_1 \leq h_r(G) + s + t. \end{cases}$$

□

Lemma 2.3. *Let G be a graph with at least two internal vertices, and let $F = G[V(G) - E_n(G)]$, where $E_n(G)$ is the set of all end vertices of G. Then $h_r(G) = h_r(F) + |E_n(G)|$.*

Proof. Let G be a graph has at least two internal vertices, $F = G[V(G) - E_n(G)]$, and let H_r be a minimum restrained hub set of F.

Claim: $S_r = H_r \cup E_n$ is a minimum restrained hub set of G. Its clear that S_r is a restrained hub set, since the vertices of $G - S_r$ are the same vertices of $F - H_r$. Now, we will show that S_r is minimum, let $S_r - v$ be a restrained hub set, either $v \in H_r$ or $v \in E_n(G)$. If $v \in H_r$, then $H_r - v$ is a restrained hub set of F and this contradicts the minimality of H_r. If $v \in E_n$ then by Theorem 1.2, $|S_r - v| = p - 2$, so $|S_r| = p - 1$ which is a contradiction. Therefore, S_r is a minimum restrained hub set of G, thus $h_r(G) = |H_r \cup E_n(G)| = h_r(F) + |E_n(G)|$. □

Theorem 2.4. *Let G be a graph, and let $A \subseteq V(G)$ such that $G[V(G)/A]$ is a tree, where $G[A]$ is a nontrivial connected subgraph of G. Then $h_r(G) = h_r(G[A]) + |V(G)| - |A|$.*

Proof. Let G be a graph, and let $A \subseteq V(G)$ such that $G[V(G)/A]$ is a tree, where $G[A]$ is a nontrivial connected subgraph of G. Then by Lemma 2.3,

$$\begin{aligned} h_r(G) &= h_r(G_1) + |E_n(G)| \\ &= h_r(G_2) + |E_n(G_1)| + |E_n(G)| \\ &= h_r(G_3) + |E_n(G_2)| + |E_n(G_1)| + |E_n(G)| \\ &\dots \\ &= h_r(G[A]) + |V(G)| - |A| \end{aligned}$$

where $G_1 = G[V(G) - E_n(G)]$, $G_{i+1} = G[V(G_i) - E_n(G_i)]$, and $E_n(G_i)$ are the end vertices of G_i that not in A. □

Corollary 2.7. *For any non trivial tree T, $h_r(T) = p - 2$.*

Proof. Let T be any tree, take $A = \{x, y\}$, where $xy \in E(T)$. Then its clear that $G[V(T)/A]$ is a tree, hence by Theorem 2.4,

$$h_r(T) = h_r(T[A]) + |V(T)| - |A| = 0 + p - 2 = p - 2.$$

□

Corollary 2.8. *Let F be a forest of order p. Then*

$$h_r(F) = \begin{cases} p-1, & if F \cong N_p \\ p-2, & if F \ncong N_p. \end{cases}$$

Proof. Let F be any forest of order p, if $F \cong N_P$, then $h_r(F) = p - 1$, while if not, then F can written as, $F \cong \bigcup_{i=1}^{n} T_i, |T_i| = p_i, i = 1, 2, ..., n$, with at least one non trivial tree T_k. Therefore, by Lemma 2.1,

$$\begin{aligned} h_r(F) &= \sum_{i=1, i\neq k}^{n} p_i + h_r(T_k), \\ &= \sum_{i=1, i\neq k}^{n} p_i + (p_k - 2) \quad \text{(by Corollary 2.7)} \\ &= \sum_{i=1}^{n} p_i - 2 \\ &= p - 2. \end{aligned}$$

Thus, we get that $h_r(F) = \begin{cases} p-1, & if F \cong N_p \\ p-2, & if F \ncong N_p. \end{cases}$ □

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