New separation axioms in binary soft topological spaces

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Abstract. The concept of binary soft topological spaces defined on two universal sets and a parameter set was initiated by Benchalli et al. [2]. In the present paper, new separation axioms in binary soft topological spaces are introduced and their properties, characterizations are studied. Further, the interrelationship between these new binary soft topological spaces are investigated.

Keywords: binary soft n-T_0 spaces, binary soft n-T_1 spaces, binary soft n-T_2 spaces.

1. Introduction

In the real world situations, we often come across more than one universal sets. Therefore, the structures dealing with two universal sets, simultaneously, constitutes an important role in mathematical tooling. Nithyanantha Jothi and Thangavelu [5] constructed a single structure called binary structure, which gives the information about two universal sets and initiated the concepts of binary topological spaces. Also, they have introduced the notions of binary separation axioms namely binary T_0, binary T_1 and binary T_2-spaces [6]. To solve the problems with vague and uncertainties, Molodtsov [4] proposed the concept of soft set theory. Acikgoz et al. [1] initiated the concept of binary soft set theory and defined the notions of binary soft subset, binary soft equality, binary null soft set, binary absolute soft set, union and intersection of binary soft sets. With these background, Benchalli et al. [2] initiated the notions of binary soft topological spaces and studied the basic properties such as binary soft open and binary soft closed sets, binary soft closure, binary soft interior and binary soft boundary of binary soft set. Further, they have extended their

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work on binary soft topological spaces by introducing the notions of binary soft subspaces and binary soft separation axioms [3].

In the present paper, we defined points in binary soft sets, based on which new separation axioms are defined. Also, we studied the properties, characterizations and interrelationship between these new binary soft topological spaces. Further, we obtained the relationship between binary soft topology and general topology.

2. Basic notions and notations

Definition 2.1 ([5]). Let U and V be any two non-empty sets with the power sets $P(U)$ and $P(V)$ respectively. A binary topology on U, V is a binary structure $M \subseteq P(U) \times P(V)$ that satisfies the following axioms:

1. $(\phi,\ \phi)$ and $(U,\ V) \in M$.
2. $(G_1 \cap G_2,\ H_1 \cap H_2) \in M$ whenever $(G_1,\ H_1)$, $(G_2,\ H_2) \in M$.
3. If $\{(G_\alpha,\ H_\alpha) : \alpha \in \Delta\}$ is a family of members of M, then

$$\left(\bigcup_{\alpha \in \Delta} G_\alpha,\ \bigcup_{\alpha \in \Delta} H_\alpha \right) \in M.$$

Then, the triplet $(U,\ V,\ M)$ is called a binary topological space and the elements of M are called the binary open subsets and if $G \subseteq U$, $H \subseteq V$ and $(G,\ H)' \in M$ then $(G,\ H)$ is binary closed in $(U,\ V,\ M)$.

Definition 2.2 ([1]). Let U_1, U_2 be two initial universe sets with power sets $P(U_1)$, $P(U_2)$ respectively and E be a set of parameters. A pair (F, E) is said to be a binary soft (briefly BS) set over U_1, U_2, where F is defined as $F : E \to P(U_1) \times P(U_2)$, $F(e) = (X,\ Y)$ for each $e \in E$ such that $X \subseteq U_1$, $Y \subseteq U_2$.

Definition 2.3 ([2]). Let U_1, U_2 be two initial universe sets with power sets $P(U_1)$, $P(U_2)$ respectively and E be a set of parameters. A collection τ_b of BS sets over U_1, U_2 is said to be a BS topology on U_1, U_2 if

1. $\overset{\approx}{\phi}$, $\overset{\approx}{E} \in \tau_b$.
2. arbitrary union of members of τ_b is again a member of τ_b.
3. finite intersection of members of τ_b is again a member of τ_b.

Then, (U_1, U_2, τ_b, E) is a binary soft topological space (briefly BSTS) over U_1, U_2. (G, E) is BS open in U_1, U_2 if $(G, E) \in \tau_b$ and (F, E) is BS closed in U_1, U_2 if $(F, E)' \in \tau_b$.

Throughout this paper, let (U_1, U_2, τ_b, E) denotes a BSTS unless any separation axioms are imposed on it.

Definition 2.4 ([3]). $(\widetilde{\widetilde{Y}}, \tau_{b_Y}, E)$ is called a binary soft subspace of (U_1, U_2, τ_b, E), if $\tau_{b_Y} = \{{}^Y(G,\ E) = (G,\ E) \cap (Y,\ E) : (G,\ E) \in \tau_b\}$. Then, τ_{b_Y} is a binary soft relative topology on $\widetilde{\widetilde{Y}}$.

Definition 2.5 ([3]). A BSTS (U_1, U_2, τ_b, E) is said to be a BS normal space if for every pair of disjoint BS closed sets (F, E) and (H, E), there exist disjoint BS open sets (A, E) and (B, E) such that $(F, E) \subseteq (A, E)$ and $(H, E) \subseteq (B, E)$.

3. Main results

In this section, we have introduced new separation axioms namely binary soft n-T$_i$ $(i = 0,\ 1,\ 2,\ 3,\ 4)$ spaces and BS n-T$_i{}^*$ $(i = 0,\ 1,\ 3,\ 4)$ spaces.

Definition 3.1. In a BSTS (U_1, U_2, τ_b, E), for a point $(x, y) \in U_1 \times U_2$ and a BS subset (A, E) over $U_1,\ U_2$, where $A(e) = (X, Y)$ for each $e \in E$, we say that

i. (x, y) belongs to (A, E), denoted by $(x, y) \in (A, E)$, whenever $x \in X,\ y \in Y$ for each $e \in E$.

ii. (x, y) does not belong to (A, E), denoted by $(x, y) \notin (A, E)$ if $x \notin X$ for some $e \in E$ or $y \notin Y$ for some $e \in E$ or both.

iii. (x, y) strictly does not belong to (A, E), denoted by $(x, y) \notin^s (A, E)$ if $x \notin X$ and $y \notin Y$ for each $e \in E$.

Note that (iii) implies (ii).

Definition 3.2. A BSTS (U_1, U_2, τ_b, E) is said to be binary soft n-T$_0$ if for any pair $(x_1, y_1),\ (x_2, y_2) \in U_1 \times U_2$ of distinct points (that is, $x_1 \neq x_2$ and $y_1 \neq y_2$), there exists at least one BS open set (F, E) or (G, E) such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \notin (F, E)$ or $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \notin (G, E)$.

Definition 3.3. A BSTS (U_1, U_2, τ_b, E) is said to be binary soft n-T$_0{}^*$ if for any pair of distinct points $(x_1, y_1),\ (x_2, y_2) \in U_1 \times U_2$, there exists at least one BS open set (F, E) or (G, E) such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \in (F, E)'$ or $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \in (G, E)'$.

Theorem 3.4. *Every BS n-T$_0{}^*$ space is BS n-T$_0$.*

Proof. Let $(x_1, y_1),\ (x_2, y_2)$ be any two distinct points of a BS n-T$_0{}^*$ space (U_1, U_2, τ_b, E). Then, there exists at least one BS open set (F, E) or (G, E) such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \in (F, E)'$ or $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \in (G, E)'$. This implies $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \notin (F, E)$ or $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \notin (G, E)$.

Example 3.5. Let $U_1 = \{a_1,\ a_2\}$, $U_2 = \{b_1,\ b_2\}$ and $E = \{p_1,\ p_2\}$ with $\tau_b = \{\widetilde{\widetilde{\phi}},\widetilde{\widetilde{E}},\{(p_1,(\{a_1\},\{b_1\})),(p_2,(\{a_1\},\{b_1\}))\},\{(p_1,(\{a_2\},\{b_1\})),(p_2,(\{a_2\},\{b_1\}))\},$ $\{(p_1,(\{a_1,a_2\},\{b_1\})),(p_2,(\{a_1,a_2\},\{b_1\}))\},\ \{(p_1,(\phi,\{b_1\})),(p_2,(\phi,\{b_1\}))\}\}$.

Then, (U_1,U_2,τ_b,E) is BS n-$T_0{}^*$ as well as BS n-T_0.

Remark 3.6. Converse of the Theorem 3.4 need not be true in general.

Example 3.7. Let $U_1 = \{a_1,\ a_2\}$, $U_2 = \{b_1,\ b_2\}$ and $E = \{p_1,\ p_2\}$ with $\tau_b = \{\widetilde{\widetilde{\phi}},\widetilde{\widetilde{E}},\{(p_1,(\{a_1\},U_2)),(p_2,(U_1,\{b_1\}))\},\{(p_1,(\{a_2\},\{b_1\})),(p_2,(U_1,U_2))\},$ $\{(p_1,(\phi,\{b_1\})),(p_2,(U_1,\{b_1\}))\}\}$. Then, (U_1,U_2,τ_b,E) is BS n-T_0 but not BS n-$T_0{}^*$.

Theorem 3.8. *Let (U_1,U_2,τ_b,E) be a BS n-T_0 space and $\widetilde{\widetilde{Y}}$ be a non-empty subset of $\widetilde{\widetilde{E}}$ with $Y = (A,B)$ such that $A \subseteq U_1$ and $B \subseteq U_2$ for each $e \in E$. Then $(\widetilde{\widetilde{Y}},\tau_{b_Y},E)$ is BS n-T_0. That is, the property of BS n-T_0 is hereditary.*

Proof. Let (x_1,y_1), $(x_2,y_2) \in A \times B$ with $x_1 \neq x_2$ and $y_1 \neq y_2$. Then, there exists at least one BS open set (F,E) or (G,E) in (U_1,U_2,τ_b,E) such that $(x_1,y_1) \in (F,E)$, $(x_2,y_2) \notin (F,E)$ or $(x_2,y_2) \in (G,E)$, $(x_1,y_1) \notin (G,E)$ as (U_1,U_2,τ_b,E) is BS n-T_0. Therefore, there exists at least one BS open set ${}^Y(F,E)$ or ${}^Y(G,E)$ in $(\widetilde{\widetilde{Y}},\tau_{b_Y},E)$ such that $(x_1,y_1) \in {}^Y(F,E)$, $(x_2,y_2) \notin {}^Y(F,E)$ or $(x_2,y_2) \in {}^Y(G,E)$, $(x_1,y_1) \notin {}^Y(G,E)$.

Theorem 3.9. *The property of BS n-$T_0{}^*$ is hereditary.*

Definition 3.10. *A BSTS (U_1,U_2,τ_b,E) is said to be binary soft n-T_1 if for any pair of distinct points (x_1,y_1), $(x_2,y_2) \in U_1 \times U_2$, there exist (F,E), $(G,E) \in \tau_b$ such that $(x_1,y_1) \in (F,E)$, $(x_2,y_2) \notin (F,E)$ and $(x_2,y_2) \in (G,E)$, $(x_1,y_1) \notin (G,E)$.*

Definition 3.11. *A BSTS (U_1,U_2,τ_b,E) is said to be binary soft n-$T_1{}^*$ if for any pair of distinct points (x_1,y_1), $(x_2,y_2) \in U_1 \times U_2$, there exist (F,E), $(G,E) \in \tau_b$ such that $(x_1,y_1) \in (F,E)$, $(x_2,y_2) \in (F,E)'$ and $(x_2,y_2) \in (G,E)$, $(x_1,y_1) \in (G,E)'$.*

Theorem 3.12. *Every BS n-$T_1{}^*$ space is BS n-T_1.*

Proof. Let (x_1,y_1), (x_2,y_2) be any two distinct points of a BS n-$T_1{}^*$ space (U_1,U_2,τ_b,E). Then, there exist (F,E), $(G,E) \in \tau_b$ such that $(x_1,y_1) \in (F,E)$, $(x_2,y_2) \in (F,E)'$ and $(x_2,y_2) \in (G,E)$, $(x_1,y_1) \in (G,E)'$. This implies $(x_1,y_1) \in (F,E)$, $(x_2,y_2) \notin (F,E)$ and $(x_2,y_2) \in (G,E)$, $(x_1,y_1) \notin (G,E)$.

Example 3.13. Every discrete BSTS is BS n-$T_1{}^*$ as well as BS n-T_1.

Remark 3.14. The converse of the Theorem 3.12 need not be true in general.

Example 3.15. Let $U_1 = \{a_1, a_2\}$, $U_2 = \{b_1, b_2\}$ and $E = \{p_1, p_2\}$ with $\tau_b = \{\widetilde{\widetilde{\phi}}, \widetilde{\widetilde{E}}, (\Psi_1, E), (\Psi_2, E), (\Psi_3, E), (\Psi_4, E), (\Psi_5, E), (\Psi_6, E), (\Psi_7, E)\}$ where, $(\Psi_1, E) = \{(p_1, (U_1, \{b_1\})), (p_2, (U_1, \{b_1\}))\}$, $(\Psi_2, E) = \{(p_1, (\{a_2\}, U_2)), (p_2, (\{a_2\}, U_2))\}$, $(\Psi_3, E) = \{(p_1, (U_1, \{b_2\})), (p_2, (U_1, \{b_2\}))\}$, $(\Psi_4, E) = \{(p_1, (\{a_2\}, \{b_1\})), (p_2, (\{a_2\}, \{b_1\}))\}$, $(\Psi_5, E) = \{(p_1, (\{a_2\}, \{b_2\})), (p_2, (\{a_2\}, \{b_2\}))\}$, $(\Psi_6, E) = \{(p_1, (U_1, \phi)), (p_2, (U_1, \phi))\}$, $(\Psi_7, E) = \{(p_1, (\{a_2\}, \phi)), (p_2, (\{a_2\}, \phi))\}$. Then, (U_1, U_2, τ_b, E) is BS n-T_1 but not BS n-$T_1{}^*$.

Theorem 3.16. *The property of BS n-T_1 is hereditary.*

Proof. Let (x_1, y_1), $(x_2, y_2) \in A \times B$ with $x_1 \neq x_2$ and $y_1 \neq y_2$. Then, there exist (F, E), $(G, E) \in \tau_b$ such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \notin (F, E)$ and $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \notin (G, E)$. Thus, there exist BS open sets ${}^Y(F, E)$ and ${}^Y(G, E)$ in $(\widetilde{\widetilde{Y}}, \tau_{b_Y}, E)$ such that $(x_1, y_1) \in {}^Y(F, E)$, $(x_2, y_2) \notin {}^Y(F, E)$ and $(x_2, y_2) \in {}^Y(G, E)$, $(x_1, y_1) \notin {}^Y(G, E)$.

Theorem 3.17. *The property of BS n-$T_1{}^*$ is hereditary.*

Definition 3.18. A BSTS (U_1, U_2, τ_b, E) is said to be binary soft n-T_2 if for any pair of distinct points (x_1, y_1), $(x_2, y_2) \in U_1 \times U_2$, there exist disjoint BS open sets (F, E) and (G, E) such that $(x_1, y_1) \in (F, E)$ and $(x_2, y_2) \in (G, E)$.

Example 3.19. Every discrete BSTS is BS n-T_2.

Theorem 3.20. *The property of BS n-T_2 is hereditary.*

Proof. Let (x_1, y_1), $(x_2, y_2) \in A \times B$ be a pair of distinct points. Now, (x_1, y_1), $(x_2, y_2) \in U_1 \times U_2$ are distinct. Since (U_1, U_2, τ_b, E) is BS n-T_2, there exist disjoint BS open sets (F, E) and (G, E) in (U_1, U_2, τ_b, E) such that $(x_1, y_1) \in (F, E)$ and $(x_2, y_2) \in (G, E)$. Therefore, there exist disjoint BS open sets ${}^Y(F, E)$ and ${}^Y(G, E)$ in $(\widetilde{\widetilde{Y}}, \tau_{b_Y}, E)$ such that $(x_1, y_1) \in {}^Y(F, E)$ and $(x_2, y_2) \in {}^Y(G, E)$.

Definition 3.21. A BSTS (U_1, U_2, τ_b, E) is said to be binary soft n-regular if for every point $(x, y) \in U_1 \times U_2$ and for every BS closed set (F, E) with $(x, y) \notin^s (F, E)$, there exist disjoint BS open sets (A, E) and (B, E) such that $(x, y) \in (A, E)$ and $(F, E) \subseteq (B, E)$.

Theorem 3.22. *Binary soft n-regularity is a hereditary property.*

Proof. Let $(x, y) \in A \times B$ and (F, E) be BS closed in $(\widetilde{\widetilde{Y}}, \tau_{b_Y}, E)$ such that $(x, y) \notin^s (F, E)$. Then, there exists a BS closed set (H, E) in (U_1, U_2, τ_b, E), so that $(F, E) = {}^Y(H, E)$, which implies $(x, y) \notin^s (H, E)$. Since, (U_1, U_2, τ_b, E) is BS n-regular, there exist disjoint BS open sets (U, E) and (V, E) such that

$(x, y) \in (U, E)$ and $(H, E) \tilde{\subseteq} (V, E)$. Thus, there exist disjoint BS open sets ${}^Y(U, E)$ and ${}^Y(V, E)$ in $(\overset{\approx}{Y}, \tau_{b_Y}, E)$ such that $(x, y) \in {}^Y(U, E)$ and $(F, E) \tilde{\subseteq} {}^Y(V, E)$.

Theorem 3.23. *For any BSTS (U_1, U_2, τ_b, E), the following statements are equivalent:*

i. (U_1, U_2, τ_b, E) *is BS n-regular.*

ii. *For any $(x, y) \in U_1 \times U_2$ and $(G, E) \tilde{\in} \tau_b$ with $(x, y) \in (G, E)$, there exists BS open set (A, E) such that $(x, y) \in (A, E) \tilde{\subseteq} \overline{\overline{(A, E)}} \subseteq (G, E)$.*

iii. *For any $(x, y) \in U_1 \times U_2$ and BS closed set (F, E) with $(x, y) \notin^s (F, E)$, there exists BS open set (B, E) such that $(x, y) \in (B, E)$ and $\overline{\overline{(B, E)}} \tilde{\cap} (F, E) = \overset{\approx}{\phi}$.*

Proof. $(i) \Rightarrow (ii)$:

Let $(x, y) \in U_1 \times U_2$ and $(G, E) \tilde{\in} \tau_b$ with $(x, y) \in (G, E)$. Then, $(G, E)'$ is BS closed with $(x, y) \notin^s (G, E)'$. Since (U_1, U_2, τ_b, E) is BS n-regular, there exist $(A, E), (B, E) \tilde{\in} \tau_b$ such that $(x, y) \in (A, E)$, $(G, E)' \tilde{\subseteq} (B, E)$ and $(A, E) \tilde{\cap} (B, E) = \overset{\approx}{\phi}$. Now, $(A, E) \tilde{\subseteq} (B, E)'$, implies $\overline{\overline{(A, E)}} \tilde{\subseteq} \overline{\overline{(B, E)'}} = (B, E)'$ as $(B, E)'$ is BS closed. Also, $(B, E)' \tilde{\subseteq} (G, E)$. Therefore, $(x, y) \in (A, E) \tilde{\subseteq} \overline{\overline{(A, E)}} \subseteq (G, E)$.

$(ii) \Rightarrow (iii)$:

Let $(x, y) \in U_1 \times U_2$ and (F, E) be a BS closed set with $(x, y) \notin^s (F, E)$. Then $(F, E)'$ is BS open with $(x, y) \in (F, E)'$. By (ii), there exists $(B, E) \tilde{\in} \tau_b$ such that $(x, y) \in (B, E) \tilde{\subseteq} \overline{\overline{(B, E)}} \tilde{\subseteq} (F, E)'$. Therefore, $(x, y) \in (B, E)$ and $\overline{\overline{(B, E)}} \tilde{\cap} (F, E) = \overset{\approx}{\phi}$.

$(iii) \Rightarrow (i)$: Let $(x, y) \in U_1 \times U_2$ and (F, E) be a BS closed set with $(x, y) \notin^s (F, E)$. By (iii), there exists $(B, E) \tilde{\in} \tau_b$ such that $(x, y) \in (B, E)$ and $\overline{\overline{(B, E)}} \tilde{\cap} (F, E) = \overset{\approx}{\phi}$, implies $(F, E) \tilde{\subseteq} \overline{\overline{(B, E)}}'$. Further, (B, E) and $\overline{\overline{(B, E)}}'$ are disjoint BS open sets. Therefore, (U_1, U_2, τ_b, E) is BS n-regular.
Thus, all the above three statements are equivalent.

Definition 3.24. A BSTS (U_1, U_2, τ_b, E) is said to be binary soft n-T_3 (respectively binary soft n-$T_3{}^*$) if it is both binary soft n-regular and binary soft n-T_1 (respectively binary soft n-regular and binary soft n-$T_1{}^*$).

Remark 3.25. Every BS n-$T_3{}^*$ space is BS n-T_3, but not conversely, by Theorem 3.12 and Remark 3.14.

Theorem 3.26. *Binary soft n-T_3 (respectively binary soft n-$T_3{}^*$) is a hereditary property.*

Proof. Follows from the Theorems 3.16 and 3.22 (respectively Theorems 3.17 and 3.22).

Definition 3.27. A BS normal space (U_1, U_2, τ_b, E) which is also BS n-T_1 (respectively BS n-$T_1{}^*$) is said to be a BS n-T_4 space (respectively BS n-$T_4{}^*$ space).

Remark 3.28. Every BS n-$T_4{}^*$ space is BS n-T_4, but converse need not be true in general, follows from the Theorem 3.12 and Remark 3.14.

Theorem 3.29. *Binary soft n-T_4 (respectively binary soft n-$T_4{}^*$) is a closed hereditary property.*

Proof. Follows from the Theorem 4.12 [2], and Theorem 3.16 (respectively Theorem 3.17).

Theorem 3.30. *Every BS n-T_1 space is BS n-T_0.*

Remark 3.31. The converse of the Theorem 3.30 need not be true in general, by Example 3.7.

Theorem 3.32. *Every BS n-T_2 space is BS n-$T_1{}^*$.*

Proof. Let (U_1, U_2, τ_b, E) be a BS n-T_2 space. Therefore, for every pair of distinct points (x_1, y_1), $(x_2, y_2) \in U_1 \times U_2$, there exist (F, E), $(G, E) \in \tau_b$ such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \in (G, E)$ and $(F, E) \cap (G, E) = \overset{\approx}{\phi}$, which implies $(x_2, y_2) \in (F, E)'$, $(x_1, y_1) \in (G, E)'$.

Corollary 3.33. *Every BS n-T_2 space is BS n-T_1.*

Remark 3.34. The converse of Corollary 3.33 need not be true in general, by Example 3.15.

Remark 3.35. A BS n-$T_1{}^*$ space need not be a BS n-T_2 space.

Remark 3.36. A BS n-T_1 space need not be a BS n-$T_0{}^*$ space.

Example 3.37. Let $U_1 = \{a_1, a_2\}$, $U_2 = \{b_1, b_2\}$ and $E = \{p_1, p_2\}$ with $\tau_b = \{\overset{\approx}{\phi}, \overset{\approx}{E}, \{(p_1, (U_1, \{b_1\})), (p_2, (U_1, \{b_1\}))\}, \{(p_1, (U_1, \{b_2\})), (p_2, (U_1, \{b_2\}))\}, \{(p_1, (U_1, \phi)), (p_2, (U_1, \phi))\}\}$. Then, (U_1, U_2, τ_b, E) is BS n-T_1 but not BS n-$T_0{}^*$.

Definition 3.38. A BS set (F, E) such that$F(e) = (X, Y)$ for each $e \in E$ is said to be BS singleton if both X and Y are singleton sets.

Remark 3.39. From Example 3.15, it is clear that in a BS n-T_1 space, every BS singleton set need not be BS closed.

Remark 3.40. If every BS singleton set is BS closed then the BSTS need not be BS n-T_1.

Example 3.41. Let $U1 = \{a\}$, $U_2 = \{b_1, b_2\}$, $E = \{p_1, p_2\}$ and $\tau_b = \{\overset{\approx}{\phi}, \overset{\approx}{E}$, $\{(p_1, (\phi, \{b_1\})), (p_2, (\phi, \phi))\}$, $\{(p_1, (\phi, \{b_1\})), (p_2, (\phi, \{b_1\}))\}$, $\{(p_1, (\phi, \{b_1\})), (p_2, (\phi, \{b_2\}))\}$, $\{(p_1, (\phi, \{b_1\})), (p_2, (\phi, U_2))\}$, $\{(p_1, (\phi, \{b_2\})), (p_2, (\phi, \phi))\}$, $\{(p_1, (\phi, \{b_2\})), (p_2, (\phi, \{b_1\}))\}$, $\{(p_1, (\phi, \{b_2\})), (p_2, (\phi, \{b_2\}))\}$, $\{(p_1, (\phi, \{b_2\})), (p_2, (\phi, U_2))\}$, $\{(p_1, (\phi, U_2)), (p_2, (\phi, \phi))\}$, $\{(p_1, (\phi, U_2)), (p_2, (\phi, \{b_1\}))\}$, $\{(p_1, (\phi, U_2)), (p_2, (\phi, \{b_2\}))\}$, $\{(p_1, (\phi, U_2)), (p_2, (\phi, U_2))\}$, $\{(p_1, (\phi, \phi)), (p_2, (\phi, \{b_1\}))\}$, $\{(p_1, (\phi, \phi)), (p_2, (\phi, \{b_2\}))\}$, $\{(p_1, (\phi, \phi)), (p_2, (\phi, U_2))\}\}$. Then, (U_1, U_2, τ_b, E) is not BS n-T_1 in which all the BS singleton sets are BS closed.

Proposition 3.42. *In (U_1, U_2, τ_b, E), for any $e \in E$,*

1. $\tau_{b_e} = \{F(e) : (F, E) \in \tau_b\}$ *is a binary topology on U_1, U_2.*
2. $\tau_e = \{X \subseteq U_1 : (G, E) \in \tau_b$ *and* $G(e) = (X, Y)\}$ *is a topology on U_1.*
3. $\tau_e = \{Y \subseteq U_2 : (G, E) \in \tau_b$ *and* $G(e) = (X, Y)\}$ *is a topology on U_2.*

Example 3.43. In Example 3.7, $\tau_{b_{e_1}} = \{(\phi, \phi), (U_1, U_2), (\{a_1\}, U_2), (\{a_2\}, \{b_1\}), (\phi, \{b_1\})\}$ is a binary topology on U_1, U_2. $\tau_{e_1} = \{\phi, U_1, \{a_1\}\{a_2\}\}$ is a topology on U_1 and $\tau_{e_1} = \{\phi, U_2, \{b_1\}, \{b_2\}\}$ is a topology on U_2.

Theorem 3.44. *If (U_1, U_2, τ_b, E) is BS n-T_0^*, then for any $e \in E$, (U_1, U_2, τ_{b_e}) is binary T_0.*

Proof. Since (U_1, U_2, τ_b, E) is BS n-T_0^*, for any pair of distinct points (x_1, y_1), (x_2, y_2), there exists at least one BS open set (F, E) or (G, E) such that $(x_1, y_1) \in (F, E)$, $(x_2, y_2) \in (F, E)'$ or $(x_2, y_2) \in (G, E)$, $(x_1, y_1) \in (G, E)'$. This implies, for any $e \in E$, there exists at least one binary open set $F(e)$ or $G(e)$ such that $(x_1, y_1) \in F(e)$, $(x_2, y_2) \in F(e)'$ or $(x_2, y_2) \in G(e)$, $(x_1, y_1) \in G(e)'$.

Theorem 3.45. *If (U_1, U_2, τ_b, E) is BS n-T_1^* (respectively BS n-T_2), then for any $e \in E$, (U_1, U_2, τ_{b_e}) is binary T_1 (respectively binary T_2).*

Remark 3.46. From Example 3.7, it follows that if (U_1, U_2, τ_b, E) is BS n-T_0, then for $e \in E$, (U_1, U_2, τ_{b_e}) may not be a binary T_0-space.

Remark 3.47. If (U_1, U_2, τ_b, E) is BS n-T_1, then for $e \in E$, (U_1, U_2, τ_{b_e}) may not be binary T_1.

Example 3.48. Let $U_1 = \{a_1, a_2, a_3\}$, $U_2 = \{b_1, b_2\}$, $E = \{p_1, p_2\}$ and $\tau_b = \{\overset{\approx}{\phi}, \overset{\approx}{E}, \{(p_1, (U_1, \{b_1\})), (p_2, (U_1, \{b_1\}))\}, \{(p_1, (U_1, \{b_2\})), (p_2, (U_1, \{b_2\}))\},$

$\{(p_1,\ (U_1, \phi)), (p_2,\ (U_1, \phi))\}\}$. Then (U_1, U_2, τ_b, E) is BS n-T_1 but (U_1, U_2, τ_{b_e}) is not binary T_1 for any $e \in E$.

Remark 3.49. Let (U_1, U_2, τ_b, E) be a BSTS with initial universal sets U_1, U_2 and parameter set E. Then the number of all possible BS subsets over U_1, U_2 is given by $(|P(U_1)| \times |P(U_2)|)^{|E|}$, where, $|P(U_1)|$, $|P(U_2)|$ and $|E|$ are the cardinalities of the power set of U_1, power set of U_2 and E respectively.

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