Certain types of functions by using supra $\hat{\omega}$-open sets

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Abstract. The target of this research is to present and discuss some kinds of functions in the supra spaces and some of their strongest and weakest forms, and we will introduce the relation between these forms for each type. The results that we reached will be supported by proofs and examples.

Keywords: supra* $\hat{\omega}$-continuous function, supra* $\hat{\omega}$-irresolute function, supra* $\hat{\omega}^*$-compact function, supra* $\hat{\omega}^{**}$-compact function, strongly supra* $\hat{\omega}$-closed function.

1. Introduction

The functions are important connectors between different spaces, and they are one of the common concepts in the world of topology, we will deal with some types of functions connect between supra spaces. The first researcher who introduced the supra spaces was Mashhour, A. S [1] (1983). A good number of researchers followed him dealt with many issues in this spaces such as the researchers Vidyarani, L. and Vigneshwaran, M. where they presented supra N-compact also supra N-connected in the supra spaces [2]. Also, we will introduce some functions in this spaces like supra* $\hat{\omega}$-continuous, strongly supra* $\hat{\omega}$-continuous, supra* $\hat{\omega}$-irresolute, supra* $\hat{\omega}$-compact, supra* $\hat{\omega}^*$-compact, supra* $\hat{\omega}^{**}$-compact and supra* $\hat{\omega}$-closed, totally supra* $\hat{\omega}$-closed, strongly supra* $\hat{\omega}$-closed functions, we will give some theorems and propositions in this paper.

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2. On supra topology

At the beginning we will present the definition of supra space and some of its properties, also its relationship with the topological space. As well as the concept of supra $\hat{\omega}$-open, supra $\hat{\eta}$-open sets and the relationship between them and with the supra open sets. We will use the abbreviation "su." to express the "supra".

Definition 2.1 ([2], [3], [4])**.** Let X be a non-empty set, the su. topology μ is a sub collection of $P(X)$ in which ϕ, X and $\bigcup_{\alpha\in\wedge} W_\alpha$ are belong to μ, where $W_\alpha \in \mu$. The su. space is denoted by (X, μ), any set W in μ is su. open set, W^c is su. closed set.

Definition 2.2 ([5], [6])**.** The su. closure for a set W in a su. space X, (symbolizes it $cl^\mu(W)$) defined as: $cl^\mu(W)$=$\cap\{M|M^c \in \mu$ and $W \subseteq M\}$. While the su. interior for W, (symbolizes it $int^\mu(W)$) is defined by: $int^\mu(W)$=$\cup\{B|B \in \mu$ and $B \subseteq W\}$.

Remark 2.3 ([7])**.** Any topology is su. topology, since every topology includes ϕ,X and it is closed under the infinite union. This remark is irreversible.

Example 2.4. In the su. space (X, μ), where X= $\{1, 2, 3\}$, μ= $\{\phi, X, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$, μ is su. topology on X but not topology since $\{1, 3\} \cap \{2, 3\} = \{3\} \notin \mu$.

Definition 2.5. Any set W in (X, μ) is called su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set, if for any element $a \in W$ there is a set $B \in \mu$ containing a, with $B - W$ is countable (resp. finite). W^c is called su. $\hat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set.

Example 2.6. The set $R - \{0\}$ in the co-finite su. space (R, μ_{cof}) is su. $\hat{\omega}$-open and su. $\hat{\eta}$-open set.

Remark 2.7. Any su. open set is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open), since if (X, μ) is a su. space, $W \in \mu$ and $a \in W$, take $B \in \mu$, put $B = W$, thus $a \in B$ and $B - W = W - W = \phi$ is countable (resp. finite), so W is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set.

Remark 2.8. Any su. $\hat{\eta}$-open set is su. $\hat{\omega}$-open, because, if W is a su. $\hat{\eta}$-open subset of (X, μ), so for any $a \in W$ there is $B \in \mu$, with $a \in B$ such that $B - W$ is a finite set, accordingly it is a countable set (since every finite set is countable), therefore W is a su. $\hat{\omega}$-open set. This remark is irreversible.

Example 2.9. The set $\{x\}$ in the indiscrete su. space (Z, μ_{ind}), where Z is the set of all integers, is su. $\hat{\omega}$-open set but neither su. open set nor su. $\hat{\eta}$-open.

Definition 2.10 ([9], [10])**.** If $f^{-1}(W)$ is su. open (resp. su. closed) set in the su. space (X, μ_X) for any su. open (resp. su. closed) set W in the su. space $(Y,$

μ_Y). Then, the function f from the su. space (X, μ_X) into the space (Y, μ_Y) is called su*. continuous function.

Example 2.11. $f : (X, \mu_D) \longrightarrow (Y, \mu_Y)$ is su*. continuous function.

Definition 2.12. Let V be any su. open subset of the su. space (Y, μ_Y), and a be any point in the su. space (X, μ_X) with $f(a) \in V$, if there is a su. $\hat{\omega}$-open subset W of X containing a, and $f(W) \subseteq V$, then $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called a su*. $\hat{\omega}$-continuous function at the point a. When f is su*. $\hat{\omega}$-continuous at any point in its domain, then it is called a su*. $\hat{\omega}$-continuous function.

Example 2.13. The function f from the excluded point su. space (X, μ_{EX}) where X is a countable set into any su. space (Y, μ_Y) is su*. $\hat{\omega}$-continuous function, where $\mu_{EX} = \{W \subseteq X, a_\circ \notin W$ for some $a_\circ \in X\} \cup \{X\}$.

Definition 2.14. Let V be any su. open subset of the su. space (Y, μ_Y), and a be any point in the su. space (X, μ_X) with $f(a) \in V$, if there is a su. $\hat{\eta}$-open subset W of X containing a, and $f(W) \subseteq V$, then $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called a su*. $\hat{\eta}$-continuous function at the point a. When f is su*. $\hat{\eta}$-continuous at any point in its domain, then it is called a su*. $\hat{\eta}$-continuous function.

Example 2.15. The function $f : (X, \mu_{EX}) \longrightarrow (Y, \mu_Y)$ is su*. $\hat{\omega}$-continuous function.

Proposition 2.16 *The infinite union of su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets is also su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open).*

Proof. Let $\{U_\alpha | \alpha \in \wedge\}$ be a collection of su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets in a su. space X, and let $x \in \bigcup_{\alpha\in\wedge} U_\alpha$, so there is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set U_{α_j} for some $\alpha_j \in \wedge$ with $x \in U_{\alpha_j}$, from definition of su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set, there is a su. open set G_j in X containing x and $G_j - U_{\alpha_j}$ is countable (resp. finite), since $G_j - \bigcup_{\alpha\in\wedge} U_\alpha \subseteq G_j - U_{\alpha_j}$, then $G_j - \bigcup_{\alpha\in\wedge} U_\alpha$ is countable (resp. finite), therefore $\bigcup_{\alpha\in\wedge} U_\alpha$ is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set.

Lemma 2.17. *The finite union of su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set.*

Proof. Let W_1 and W_2 be su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets in a su. space (X, μ_X), and $a \in W_1 \cup W_2 \Longrightarrow a \in W_1$ or $a \in W_2$, if $a \in W_1 \Longrightarrow$ there is $B \in \mu$ with $a \in B$ and $B - W_1$ is countable (resp. finite), but $B - (W_1 \cup W_2) \subseteq B - W_1 \Longrightarrow B - (W_1 \cup W_2)$ is countable (resp. finite)$\Longrightarrow W_1 \cup W_2$ is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open). By the same way if $a \in W_2$.

Definition 2.18. A point a is a su. $\hat{\omega}$-interior (resp. su. $\hat{\eta}$-interior) point for a subset W of the su. space (X, μ), if there is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set B in X, in which $a \in B \subseteq W$.

Example 2.19. Any point in any subset of the co-finite su. space (Z, μ_{cof}) is su. $\hat{\omega}$-interior point.

Proposition 2.20. *A subset W of (X, μ) is su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set iff all the points in W are su. $\hat{\omega}$-interior (resp. su. $\hat{\omega}$-interior) points to it.*

Proof. Suggest $a \in W$, where W is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) subset of X, since each set is a subset of itself, so a is su. $\hat{\omega}$-interior (resp. su. $\hat{\eta}$-interior) point to W. conversely, since W is a union of its points, and every point in W is su. $\hat{\omega}$-interior (resp. su. $\hat{\eta}$-interior) point to it, that follows for each $a \in W$ there is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set B_a in X, in which $a \in B_a \subseteq W$, in order that W is a union of the su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) sets B_a for any a in W. Hence W is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) set (by proposition (2.16)).

Example 2.21. Any set in the co-finite su. space (Z, μ_{cof}) is su. $\hat{\omega}$-open set, so every point in this set is su. $\hat{\omega}$-interior point.

Theorem 2.22. *The function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is su***. $\hat{\omega}$-continuous function iff $f^{-1}(V)$ is su. $\hat{\omega}$-open set in X, for any $V \in \mu_Y$.*

Proof. Suppose V is a su. open set in Y with $f(a) \in V$, where $a \in X \Longrightarrow f^{-1}(f(a)) \in f^{-1}(V) \Longrightarrow a \in f^{-1}(V) \Longrightarrow f(a) \in f(f^{-1}(V)) \subseteq V$, therefore just by taking W (required set in definition (2.12)) equal to $f^{-1}(V)$ we get the required. Conversely, if f is su*. $\hat{\omega}$-continuous function and $V \in \mu_Y$ with $f(a) \in V$, in which a is any point in X, since f is su*. $\hat{\omega}$-continuous, thus there is a su. $\hat{\omega}$-open set W in X where $a \in W$ and $f(W) \subseteq V \Longrightarrow f(a) \in f(W) \subseteq V \Longrightarrow a \in W \subseteq f^{-1}(f(W)) \subseteq f^{-1}(V)$, then a is a su. $\hat{\omega}$-interior point to $f^{-1}(V)$, then $f^{-1}(V)$ is a su. $\hat{\omega}$-open set in X (by proposition (2.20)).

Example 2.23. The function $f : (X, \mu_{EX}) \longrightarrow (X, \mu_{EX})$ is su*. $\hat{\omega}$-continuous function.

Corollary 2.24. *Any function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is su***. $\hat{\omega}$-continuous if and only if $f^{-1}(V)$ is su. $\hat{\omega}$-closed set in X for each $V^c \in \mu_Y$.*

Proof. Let f be a su*. $\hat{\omega}$-continuous function and $V^c \in \mu_Y$, thus $f^{-1}(V^c)$ is su. $\hat{\omega}$-open set in X, since $(f^{-1}(V))^c = f^{-1}(V^c)$, hence $(f^{-1}(V))^c$ is su. $\hat{\omega}$-open set, thus $((f^{-1}(V))^c)^c = f^{-1}(V)$ is su. $\hat{\omega}$-closed set in X. Conversely, take V as a su. open set in Y so V^c is su. closed set and then $f^{-1}(V)^c$ is su. $\hat{\omega}$-closed (Given) which is equal to $(f^{-1}(V))^c$, and $((f^{-1}(V))^c)^c = f^{-1}(V)$ is su. open set of X, therefore f is su*. $\hat{\omega}$-continuous function.

Example 2.25. The function $f : (X, \mu_{ind}) \longrightarrow (Y, \mu_Y)$ is su*. $\hat{\omega}$-continuous function, where X is a finite set.

Definition 2.26. Whenever the inverse image of any $V \in \mu_Y$, is su. $\hat{\eta}$-open set in a su. space (X, μ_X), so f from (X, μ_x) into (Y, μ_Y) is called a su*. $\hat{\eta}$-continuous function.

Example 2.27. The function f from the included point su. space (X, μ_I) into the discrete su. space (X, μ_D) where X is a finite set, is su*. $\hat{\eta}$-continuous function, where $\mu_I = \{W \subseteq X | a_\circ \in W$, for some $a_\circ \in X\} \cup \{\phi\}$.

Proposition 2.28. *If the inverse image of all su. closed set in a su. space (Y, μ_Y) is su. $\hat{\eta}$-closed in a su. space (X, μ_X), then f from the space X into the space Y is su*. $\hat{\eta}$-continuous function.*

Example 2.29. The function f from the discrete su. space (X, μ_D) into any su. space (Y, μ_Y) is su*. $\hat{\eta}$-continuous function.

Remark 2.30. Any su*. $\hat{\eta}$-continuous function f is su*. $\hat{\omega}$-continuous, because if we take $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ as a su*. $\hat{\eta}$-continuous function and $W \in \mu_Y$, then $f^{-1}(W)$ is a su. $\hat{\eta}$-open set in (X, μ_X), hence it is su. $\hat{\omega}$-open (by Remark 2.8), therefore f is su*. $\hat{\omega}$-continuous.

Example 2.31. The function f from the indiscrete su. space (Z, μ_{ind}) into any su. space (Y, μ_Y) is su*. $\hat{\omega}$-continuous function but not su*. $\hat{\eta}$-continuous.

Definition 2.32. Whenever $f^{-1}(M) \in \mu_X$ for any su. $\hat{\omega}$-open set M in (Y, μ_Y), hence $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is strongly su*. $\hat{\omega}$-continuous function.

Example 2.33. The function $f : (X, \mu_X) \longrightarrow (X, \mu_{ind})$ where $f(a) = d$ for any $a \in X$, satisfying the preceding definition.

Proposition 2.34. *Give f as a function from the su. space (X, μ_X) into the su. space (Y, μ_Y), when $f^{-1}(V)$ is su. closed set in the su. space (X, μ_X) for any su. $\hat{\omega}$-closed set V in the su. space (Y, μ_Y), then f is strongly su*. $\hat{\omega}$-continuous function.*

Example 2.35. The function $f : (R, \mu_{ind}) \longrightarrow (R, \mu_D)$ is strongly su*. $\hat{\omega}$-continuous function, where f is a constant function.

Definition 2.36. If $f^{-1}(M) \in \mu_X$ of each su. $\hat{\eta}$-open set M in the su. space (Y, μ_Y), then the function f from X into Y is called strongly su*. $\hat{\eta}$-continuous function.

Example 2.37. The identity function from the co-countable su. space (X, μ_{coc}) into the same su. space where X is uncountable set, is strongly su*. $\hat{\eta}$-continuous function.

Proposition 2.38. *The function f from the su. space (X, μ_X) into the su. space (Y, μ_Y), is strongly su*. $\hat{\eta}$-continuous function, whenever the inverse image of any su. $\hat{\eta}$-closed set in Y is su. closed set in X.*

Example 2.39. The function $f : (X, \mu_D) \longrightarrow (Y, \mu_Y)$ is also strongly su*. $\hat{\eta}$-continuous function.

Remark 2.40. Every strongly su*. $\hat{\omega}$-continuous function is strongly su*. $\hat{\eta}$-continuous function, because if $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is strongly su*. $\hat{\omega}$-continuous function and W is su. $\hat{\eta}$-open set in Y, then it is su. $\hat{\omega}$-open set (by remark (2.8)), but f is strongly su*. $\hat{\omega}$-continuous function, so $f^{-1}(W) \in \mu_X$, which implies f is strongly su*. $\hat{\eta}$-continuous function. The converse is incorrect.

Example 2.41. The identity function from the co-finite su. topology defined on a countable set into the indiscrete su. topology defined on the same set, is strongly su*. $\hat{\eta}$-continuous function but not strongly su*. $\hat{\omega}$-continuous function.

Definition 2.42. Whenever the inverse image for any su. $\hat{\omega}$-open set in (Y, μ_Y) is su. $\hat{\omega}$-open set in (X, μ_X), so that $f : X \longrightarrow Y$ is called su*. $\hat{\omega}$-irresolute function.

Example 2.43. A function f from the included point su. space (Z, μ_I) into the co-finite su. space (Z, μ_{cof}) is su*. $\hat{\omega}$-irresolute function.

Proposition 2.44. *If $f^{-1}(V)$ is su. $\hat{\omega}$-closed set in the su. space (X, μ_X) for any su. $\hat{\omega}$-closed set V in the su. space (Y, μ_Y), then the function f from the su. space (X, μ_X) into the su. space (Y, μ_Y) is su*. $\hat{\omega}$-irresolute function.*

Example 2.45. The identity function I_X from the excluded point su. space (X, μ_{EX}) into the same su. space is su*. $\hat{\omega}$-irresolute function.

Definition 2.46. When the inverse image of any su. $\hat{\eta}$-open set in (Y, μ_Y) is su. $\hat{\eta}$-open set in (X, μ_X), hence the function f from X into Y is called su*. $\hat{\eta}$-irresolute function.

Example 2.47. The function f from elective su. topology on an infinite set to the indiscrete su. topology on the same set in which $f(a) = d$ for any a in the domain, is su*. $\hat{\eta}$-irresolute function.

Proposition 2.48. *If $f^{-1}(V)$ is su. $\hat{\eta}$-closed set of the su. space (X, μ_X) for any su. $\hat{\eta}$-closed set V in the su. space (Y, μ_Y), hence the function f from the space X into the space Y is su*. $\hat{\eta}$-irresolute function.*

Example 2.49. The function f from a su. space (X, μ_X) into a su. space (Y, μ_Y) in which X is finite, is su*. $\hat{\eta}$-irresolute function.

Remark 2.50. There is no relation between su*. $\hat{\omega}$-irresolute and su*. $\hat{\eta}$-irresolute function.

Example 2.51. The identity function I_R from the co-finite su. space (R, μ_{cof}) into the co-countable su. space (R, μ_{coc}) is su*. $\hat{\omega}$-irresolute but not su*. $\hat{\eta}$-irresolute function.

Definition 2.52 ([8])**.** A point a in (X, μ) is a condensation point to a subset M of X if $W \cap M$ is uncountable set for all $W \in \mu$ containing a.

Lemma 2.53. *Whenever a subset M of a su. space (X, μ) contains all its condensation points, then it is su. $\hat{\omega}$-closed subset of X.*

Proof. Suppose M contains all its condensation points, to prove M is su. $\hat{\omega}$-closed set, that means to prove M^c is su. $\hat{\omega}$-open set. Let $x \in M^c$, so it is not a condensation point to M (from hypothesis), then there is $U \in \mu$ with $x \in U$ and either $U \cap M = \phi$ or $U \cap M$ is countable, if $U \cap M = \phi$, then $x \in U \subseteq M^c$, and since U is su. open set then it is su. $\hat{\omega}$-open (by remark (2.7)), therefore x is a su. $\hat{\omega}$-interior point to M^c, that means M^c is a su. $\hat{\omega}$-open set, and then M is su. $\hat{\omega}$-closed set in X. Now if $U \cap M$ is countable, let $G = U - M$, so $x \in G \subseteq M^c$, to prove G is su. $\hat{\omega}$-open set, let $y \in G$, so there is a su. open set U in X containing y and $U - G$ is countable, thus G is su. $\hat{\omega}$-open set, and then M^c is $\hat{\omega}$-open set, therefore M is su. $\hat{\omega}$-closed.

Example 2.54. The set of all irrational numbers Q^c in the co-finite su. space (R, μ_{cof}) is not su. $\hat{\omega}$-closed set, but the set of all rational numbers Q is su. $\hat{\omega}$-closed.

Lemma 2.55. *1- Any su. closed set M in (X, μ) is su. $\hat{\omega}$-closed.*

Proof. Consider M is a su. closed set in X and $a \in X$ with $a \notin M$, so $a \in M^c \in \mu$, but $M \cap M^c = \phi$ is countable set, then a is not condensation point to M, since a is arbitrary point in X, hence M includes all its condensation points, therefore M is su. $\hat{\omega}$-closed in X (by lemma (2.53)).

2- Any su. closed set M in (X, μ) is su. $\hat{\eta}$-closed.

Proof. Consider M is a su. closed set in X, so M^c is su. open set in X, and by (remark (2.7)) it is su. $\hat{\eta}$-open set, therefore $(M^c)^c$ is su. $\hat{\eta}$-closed set.

Example 2.56. In the excluded point su. space (X, μ_{EX}) where X is a finite set, every set is su. $\hat{\omega}$-closed and su. $\hat{\eta}$-closed set but not all these sets are su. closed, for instance $\{x\}$ is su. $\hat{\omega}$-closed and su. $\hat{\eta}$-closed set but not su. closed.

Remark 2.57. Every su. $\hat{\eta}$-closed set is su. $\hat{\omega}$-closed, since if M is su. $\hat{\eta}$-closed set in a su. space (X, μ_X) then M^c is su. $\hat{\eta}$-open set and the by remark (2.8) it is su. $\hat{\omega}$-open, hence $(M^c)^c = M$ is su. $\hat{\omega}$-closed.

Example 2.58. The set $Z - \{x\}$ in the indiscrete su. space (Z, μ_{ind}) is su. $\hat{\omega}$-closed but not su. $\hat{\eta}$-closed set.

Remark 2.59. Any su*. continuous function is su*. $\hat{\omega}$-continuous and su*. $\hat{\eta}$-continuous function.

Definition 2.60. A class $\{W_\alpha | \alpha \in \wedge\}$ of su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) subsets of (X, μ) is called a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) cover to a subset S of X whenever $S \subseteq \bigcup_{\alpha \in \wedge} W_\alpha$, when $S = X$, then $\{W_\alpha | \alpha \in \wedge\}$ is a su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) cover to X.

Definition 2.61. A set S in (X, μ) is a su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) if any su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) cover for S possesses a finite sub cover, when $X = S$, then X is su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) space.

Example 2.62. Let (R, μ_{ind}) be the indiscrete su. space where $\mu_{ind} = \{\phi, R\}$, it is su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) space. Since any su. $\hat{\omega}$-open (resp. su. $\hat{\eta}$-open) cover can be reduced to a finite sub cover.

Remark 2.63. Every su. $\hat{\omega}$-compact set is su. $\hat{\eta}$-compact, because if $G = \{W_\alpha | \alpha \in \wedge\}$ is an $\hat{\eta}$-open cover for a set M in a su. space (X, μ) where each $W_\alpha, \alpha \in \wedge$ is su. $\hat{\eta}$-open set in X and hence it is su. $\hat{\omega}$-open set (by remark (2.8)), but M is su. $\hat{\omega}$-compact, then G owns a finite sub cover, so M is su. $\hat{\eta}$-compact set. This remark is irreversible.

Example 2.64. The set Z with the indiscrete su. space is su. $\hat{\eta}$-compact space, while it is not su. $\hat{\omega}$-compact space.

Definition 2.65. Whenever the inverse image of each su. compact subset of the su. space (Y, μ_Y) is su. compact subset of the su. space (X, μ_X), then the function f from $(X, \mu_X\ )$ into (Y, μ_Y) is called a su*. compact function.

Definition 2.66. Whenever the inverse image of each su. compact subset of the su. space (Y, μ_Y) is su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) subset of the su. space (X, μ_X), then the function f from $(X, \mu_X\ )$ into (Y, μ_Y) is called a su*. $\hat{\omega}^*$-compact (resp. su*. $\hat{\eta}^*$-compact) function.

Example 2.67. 1- The identity function I_R from the co-finite su. space (R, μ_{cof}) into the discrete su. space (R, μ_D) is su*. compact.

2- The identity function I_R from (R, μ_{ind}) into the same space is su*. $\hat{\omega}^*$-compact function.

3- The function $f : (R, \mu_{ind}) \longrightarrow (R, \mu_R)$ is su*. $\hat{\eta}^*$-compact function.

Remark 2.68. Any su*. $\hat{\omega}^*$-compact function is su*. $\hat{\eta}^*$-compact, since if $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a su*. $\hat{\omega}$-compact function, and K is a su. compact set in the su. space (Y, μ_Y), so $f^{-1}(K)$ is su. $\hat{\omega}$-compact set in the supra space (X, μ_X) (because f is su*. $\hat{\omega}^*$-compact function) and by remark (2.63), $f^{-1}(K)$ is su. $\hat{\eta}$-compact set, therefore f is su*. $\hat{\eta}^*$-compact function.

Example 2.69. The identity function $I_Z : (Z, \mu_{ind}) \longrightarrow (Z, \mu_{ind})$ is su$*$. $\hat{\eta}^*$-compact but not su*. $\hat{\omega}^*$-compact function.

Definition 2.70. Whenever $f^{-1}(K)$ is su. compact set in (X, μ_X) for any su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) subset K in (Y, μ_Y), then $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called su*. $\hat{\omega}$-compact (resp. su*. $\hat{\eta}$-compact) function.

Example 2.71. 1- The function $f : (R, \mu_{ind}) \longrightarrow (R, \mu_{cof})$ is su*. $\hat{\omega}$-compact function.

2- The function $f : (X, \mu_{cof}) \longrightarrow (Y, \mu_Y)$ is su*. $\hat{\eta}$-compact function.

Remark 2.72. Every su*. $\hat{\eta}$-compact function is su*. $\hat{\omega}$-compact. Since if $f : X \longrightarrow Y$ is su*. $\hat{\eta}$-compact function and K is su. $\hat{\omega}$-compact set in Y, consequently, K is su. $\hat{\eta}$-compact set (by remark (2.63)), but f is su*. $\hat{\eta}$-compact function, hence $f^{-1}(K)$ is su. compact set in X, so f is su*. $\hat{\omega}$-compact.

Definition 2.73. Whenever $f^{-1}(K)$ is su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) set in (X, μ_X) for any su. $\hat{\omega}$-compact (resp. su. $\hat{\eta}$-compact) set K in (Y, μ_Y), hence $f : X \longrightarrow Y$ is called su*. $\hat{\omega}^{**}$-compact (resp. su*. $\hat{\eta}^{**}$-compact) function.

Example 2.74. 1- If X is a countable set, then $I_X : (X, \mu_{ind}) \longrightarrow (X, \mu_{ind})$ is a su*. $\hat{\omega}^{**}$-compact function.

2- The function $f : (X, \mu_{ind}) \longrightarrow (X, \mu_X)$, where X is a finite set, is su*. $\hat{\eta}^{**}$-compact function.

Proposition 2.75. *Every su. $\hat{\omega}$-closed* (*resp. su. closed, su. $\hat{\eta}$-closed*) *set in a su. $\hat{\omega}$-compact* (*resp. su. compact, su. $\hat{\eta}$-compact*) *space (X, μ_X) is a su. $\hat{\omega}$-compact* (*resp. su. compact* [11], *su. $\hat{\eta}$-compact*) *set.*

Proof. Consider (X, μ_X) is a su. $\hat{\omega}$-compact space and M is a su. $\hat{\omega}$-closed set in X, let $\{W_\alpha\}_{\alpha \in \wedge}$ be a su. $\hat{\omega}$-open cover for $M \Longrightarrow M \subseteq \bigcup_{\alpha \in \wedge} W_\alpha$, but $X = M \cup M^c \Longrightarrow X \subseteq \{\bigcup_{\alpha \in \wedge} W_\alpha\} \cup M^c$ and since M is su. $\hat{\omega}$-closed set in X, then M^c is su. $\hat{\omega}$-open, this means $\{W_\alpha | \alpha \in \wedge, M^c\}$ is a su. $\hat{\omega}$-open cover for X, but X is su. $\hat{\omega}$-compact, then any su. $\hat{\omega}$-open cover for X possesses a finite sub cover, so $X \subseteq (\bigcup_{i=1}^{n} W_{\alpha_i}) \cup M^c$, but $M \subseteq X \Longrightarrow M \subseteq \bigcup_{i=1}^{n} W_{\alpha_i} \cup M^c$, since $M \cap M^c = \phi \Longrightarrow M \subseteq \bigcup_{i=1}^{n} W_{\alpha_i}$, then $\{W_{\alpha_i}\}_{i=1}^{n}$ is a finite sub cover from the su. $\hat{\omega}$-open cover $\{W_\alpha\}_{\alpha \in \wedge}$ for M, therefore M is a su. $\hat{\omega}$-compact.

Example 2.76. Every su. $\hat{\omega}$-closed set in the su. space (X, μ_{EX}) is su. $\hat{\omega}$-compact set.

Remark 2.77. There is no relation between su*. $\hat{\eta}^{**}$-compact and su*. $\hat{\omega}^{**}$-compact function.

Example 2.78. The function $f : (Z, \mu_{ind}) \longrightarrow (Z, \mu_Z)$ is su*. $\hat{\eta}^{**}$-compact but not su*. $\hat{\omega}^{**}$-compact function.

Definition 2.79. If for each two non-equal points a, d in the su. space (X, μ) there are disjoint su. $\hat{\omega}$-open sets W, B in X with $a \in W$ and $d \in B$, then the su. space (X, μ) is called su. $\hat{\omega}T_2$-space.

Remark 2.80. Every su. T_2-space is su. $\hat{\omega}T_2$-space, since if X is su. T_2-space then for each non-equal points a,d in X there are two su. open sets W, B in X with $W \cap B = \phi$ and $a \in W, d \in B$, then by remark (2.7) X will be su. $\hat{\omega}T_2$-space. This remark is irreversible.

Example 2.81. The excluded point su. space (X, μ_{EX}), where X is finite set is su. $\hat{\omega}T_2$-space, but not su. T_2-space.

Definition 2.82. Let $a \neq d$ be two points in the su. space (X, μ), if there are disjoint su. $\hat{\eta}$-open sets W, B in X with $a \in W$ and $d \in B$, then (X, μ) is called su. $\hat{\omega}T_2$-space.

Example 2.83. The discrete su. space (X, μ_D) is su. $\hat{\eta}T_2$-space.

Remark 2.84. Every su. $\hat{\eta}T_2$-space is su. $\hat{\omega}T_2$-space, because if $a \neq d$ are points in X, so there exist two su. $\hat{\eta}$-open sets W, B in X with $a \in W, d \in B$ and $W \cap B = \phi$, but by remark (2.8) every su. $\hat{\eta}$-open set is su. $\hat{\omega}$-open, hence X is su. $\hat{\omega}T_2$-space.

Example 2.85. The indiscrete su. space (X, μ_{ind}) is su. $\hat{\omega}T_2$-space, but not su. $\hat{\omega}T_2$-space, where X is countable set.

Proposition 2.86. *If the function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is su*. $\hat{\omega}$-continuous (resp. strongly su*. $\hat{\omega}$-continuous, su*. $\hat{\omega}$-irresolute)-function, so the image of each su. $\hat{\omega}$-compact (resp. su. compact, su. $\hat{\omega}$-compact) set in (X, μ_X) is a su. compact (resp su. $\hat{\omega}$-compact) set in (Y, μ_Y).*

Proof. Let f be a su*. $\hat{\omega}$-continuous function and W be a su. $\hat{\omega}$-compact set in X, take $\{V_\alpha\}_{\alpha \in \wedge}$ to be a su. open cover to $f(W)$, where each $V_\alpha \in \mu_Y, \alpha \in \wedge$. then $f(W) \subseteq \bigcup_{\alpha \in \wedge} V_\alpha$, but f is su*. $\hat{\omega}$-continuous, hence $W \subseteq f^{-1}(f(W)) \subseteq f^{-1}(\bigcup_{\alpha \in \wedge} V_\alpha) = \bigcup_{\alpha \in \wedge} f^{-1}(V_\alpha)$, then $\{f^{-1}(V_\alpha)\}_{\alpha \in \wedge}$ is su. $\hat{\omega}$-open cover for W, since W is su. $\hat{\omega}$-compact, so each su. $\hat{\omega}$-open cover to it possesses a finite sub cover, hence $W \subseteq \bigcup_{i=1}^{n} f^{-1}(V_{\alpha_i})$ and by take the image of both sides we get, $f(W) \subseteq \bigcup_{i=1}^{n} f(f^{-1}(V_{\alpha_i})) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}, \alpha \in \wedge$ which means that $\{V_{\alpha_i}\}_{i=1}^{n}, \alpha \in \wedge$ is a sub cover from the su. open cover $\{V_\alpha\}_{\alpha \in \wedge}$, so $f(W)$ is a su. compact subset of Y.

The rest of the possibilities can be proved by the same way.

Example 2.87. Let $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ in which Y is finite, hence the image of any su. $\hat{\omega}$-compact set in the space X is also su. $\hat{\omega}$-compact set in the space Y.

Definition 2.88. Whenever $f(M)$ is su. closed [12] (resp. su. $\hat{\omega}$-closed, su. $\hat{\eta}$-closed) set in (Y, μ_Y) for every su. closed set M in (X, μ_X), thus the function $f : X \longrightarrow Y$ is called a su*. closed (su*. $\hat{\omega}$-closed, su*. $\hat{\eta}$-closed) function.

Example 2.89. The function $f : (R, \mu_R) \longrightarrow (R, \mu_D)$ is a su*. $\hat{\omega}$-closed and su*. closed function.

Example 2.90. The function $f : (X, \mu_X) \longrightarrow (Y, \mu_D)$ is su*. $\hat{\eta}$-closed function.

Remark 2.91. Every su*. $\hat{\eta}$-closed function is su*. $\hat{\omega}$-closed, that is if $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a su*. $\hat{\eta}$-closed function, and if M is a su. closed subset of X, so $f(M)$ is su. $\hat{\eta}$-closed subset of Y (since f is su*. $\hat{\eta}$-closed function), follows that $f(M)$ is su. $\hat{\omega}$-closed (by remark (2.57)), so f is su*. $\hat{\omega}$-closed function.

Example 2.92. The identity function $I_Z : (Z, \mu_D) \longrightarrow (Z, \mu_{ind})$ is su*. $\hat{\omega}$-closed function but not su*. $\hat{\eta}$-closed function.

Definition 2.93. 1- If the image of every su. $\hat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set M in (X, μ_X) is su. closed set in (Y, μ_Y), then $f : X \longrightarrow Y$ is called totally su*. $\hat{\omega}$-closed (resp. totally su*. $\hat{\eta}$-closed) function.

2- If the image of every su. $\hat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set M in (X, μ_X) is su. $\hat{\omega}$-closed (resp. su. $\hat{\eta}$-closed) set in (Y, μ_Y), then $f : X \longrightarrow Y$ is called strongly su*. $\hat{\omega}$-closed (resp. strongly su*. $\hat{\eta}$-closed) function.

Example 2.94. The function $f : (X, \mu_{EX}) \longrightarrow (Y, \mu_D)$ is totally su*. $\hat{\omega}$-closed function.

Example 2.95. The function $f : (X, \mu_X) \longrightarrow (X, \mu_D)$ is totally su*. $\hat{\eta}$-closed function, also it is strongly su*. $\hat{\eta}$-closed function.

Example 2.96. The identity $I_X : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is strongly su*. $\hat{\omega}$-closed.

Remark 2.97. Any totally su*. $\hat{\omega}$-closed function is totally su*. $\hat{\eta}$-closed, that is if $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a totally su*. $\hat{\omega}$-closed function, and M is a su. $\hat{\eta}$-closed set in (X, μ_X) hence it is su. $\hat{\omega}$-closed (by remark (2.57)), then $f(M)$ is su. closed set in Y (since f is totally su*. $\hat{\omega}$-closed function), therefore f is totally su*. $\hat{\eta}$-closed function.

Example 2.98. The identity function $I_R : (R, \mu_c of) \longrightarrow (R, \mu_U)$ is totally su*. $\hat{\eta}$-closed function, while it is not totally su*. $\hat{\omega}$-closed.

Remark 2.99. there is no relation between strongly su*. $\hat{\omega}$-closed and strongly su*. $\hat{\eta}$-closed function.

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