On Nc-continuous functions

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Abstract. In this research we introduce new types of c-continuous functions by using N-open sets, also we submit some other kinds of functions by using the same sets such as N^*-open and N^{**}-open functions, we discuss the relation between them. And we show the relation between some types of normal spaces. We support our work by some facts and examples.
Keywords: Nc-continuous function, N^* c-continuous function, N^{**} c-continuous function, N^*-open function, N^{**}-open function.

1. Introduction

Fashionable, the variant forms of functions have a considerable influence in the topological spaces and many researchers have written about plentiful sorts of it, such as the researchers Gentry et al. [1] , the introduce the concept of c-continuous functions. we will use N-open sets [2] to define new types of c-continuous function called Nc-continuous, N^*c-continuous and N^{**} c-continuous function. Also, we will introduce these types of Nc-continuous functions by using N-closed sets, as well we will submit the relation between those kinds of functions. By the same way we will introduce some kinds of N-open function and the relation between them. And we will highlight on new types of N-normal spaces. We will support our work with some examples, propositions and theorems.

2. On Nc-continuous functions

Now we introduce the function of c-continuous and some types of Nc-continuous. Both of ∂ and ρ are top. spaces.

Definition 2.1 ([1],[3])**.** The function $m : (H, \partial) \longrightarrow (K, \rho)$ is c-continuous in an element h of the space (H, ∂) if to any open set J in the space (K, ρ) where $m(h) \in J$ and its complement is compact, we delimit an open set C in (H, ∂) with $h \in C, m(C) \subseteq J$. If m is c-continuous at every element in (H, ∂), hence m is c-continuous on (H, ∂).

Example 2.2. $m : (H, \partial_D) \longrightarrow (K, \rho)$ is c-continuous function.

Definition 2.3. $m : (H, \partial) \longrightarrow (K, \rho)$ is Nc-continuous function if to any element $h \in (H, \partial)$ and to any open set J in $(K, \rho), m(h) \in J$ and J^C is compact, we could delimit N-open set $C \in (H, \partial)$ such that $h \in C$ and $m(C) \subseteq J$.

Example 2.4. $m : (H, \partial_{ind}) \longrightarrow (K, \rho)$, where H is finite set, is Nc-continuous function. We found the following proposition in [4], [5] without proof, we will submit its proof here.

Proposition 2.5. If J is open set, it is an N-open set, because if J is open set in the space (H, ∂), then to any $h \in J$, we could determine an open set J in (H, ∂) where $h \in J$ and $J - J$ is finite set, so J is N-open set.

Example 2.6. Every set $\emptyset \neq J \neq H$ in the indiscrete space (H, ∂_{ind}) with H is finite set, is N-open set but not open.

Definition 2.7. Give m as a function where $m : (H, \partial) \longrightarrow (K, \rho)$, f is N^*c-continuous function if to any element$h \in (H, \partial)$ and every N-open set J in $(K, \rho), m(h)$ is a subset J and J^C is compact, we select an open set C in (H, ∂) with $m(C) \subseteq J$.

Example 2.8. $m : (H, \partial_D) \longrightarrow (K, \rho_{cof})$ where ρ is finite set, m is N^*-continuous function.

Definition 2.9. Suppose $m : (H, \partial) \longrightarrow (K, \rho)$, hence m is N^{**} c-continuous function if to any point h in (H, ∂) and every N-open set J in $(K, \rho), m(h) \subseteq J and J^C$ is compact, we could select an N-open set $C \in (H, \partial)$ with $m(C) \subseteq J$.

Example 2.10. $I_H : (H, \partial_{EX}) \longrightarrow (H, \partial_{EX})$ is N^{**} c-continuous function, X is finite set, $\Im_{EX} = \{C \subseteq H, h_0 \in C \text{ for some } h_0 \in H\} \cup \{\emptyset\}$.

Definition 2.11 ([6])**.** Give H as a space, $J \subseteq H, h$ is any element in J, then h is an N-interior point for Jif we select an N-open set C in H and $h \subseteq C \subseteq J$.

Remark 2.12. If $\{J_\alpha | \alpha \in \Lambda\}$ is a collection of N-open sets, then $U_{\alpha\in\Lambda} J_\alpha$ is also N-open.

Proof. Since $\{J_\alpha | \alpha \in \Lambda\}$ is a combination of N-open sets in H, let $h \in U_{\alpha_i \in \Lambda} J_{\alpha_i}$, so $h \in J_{\alpha_i}$ for some $\alpha_i \in \Lambda$, therefore we can find an open set C_{α_j} in the space H and $h \in C_{\alpha_j}$, where $C_{\alpha_j} - J_{\alpha_j}$ is finite, but $C_{\alpha_j} - U_{\alpha_i \in \Lambda} J_{\alpha_i} \subseteq C_{\alpha_j} - J_{\alpha_j}$, then $C_{\alpha_j} - U_{\alpha_i \in \Lambda} J_{\alpha_i}$ is finite, hence $U_{\alpha_i \in \Lambda} J_{\alpha_i}$ is N-open set. □

We found the following proposition in [7] without proof, we will introduce its proof.

Proposition 2.13. Suppose (H, ∂) be a space, J be a set in H, so J is N-open set iff $J = J^{0N}$.

Proof. Let J be an N-open set in H, to any element h in J, $h \in J \subseteq H$, so h is an N-interior point to J, and since h is arbitrary point in J, so any point in J is an N-interior point to J. Conversely, since J is a union of whole its points which are N-interior points, so $J = U_{i \in I} C_i, C_i$ are N-open sets,$\forall i \in I$, where every N-open set from them containing at least one point of J (since every point in J is an N-interior point), so by remark (3.12), J is an N-open set. □

Theorem 2.14. *Give $m : H \longrightarrow K$, then m is Nc-continuous iff to all open set J in K with compact complement, $m^{-1}(J)$ is N-open set in H.*

Proof. If m is Nc-continuous function, J is an open set in the space K and has a compact complement, let h inside $m^{-1}(J)$, so m(h) is in J, therefore we can select an N-open set C in H with $h \in C$ and $m(C) \subseteq J$, then $G \subseteq m^{-1}(m(C)) \subseteq m^{-1}(J)$, hence h is N-interior point to $m^{-1}(J)$ which implies $m^{-1}(J)$ is N-open set (by proposition (3-13)). Conversely, if P is an open set in K with m(h) in P where h is inside H, and has a compact complement, hence $h \in m^{-1}(P)$, but $m^{-1}(P)$ is N-open subset of H (hypothesis), and $m(m^{-1}(P)) \subseteq P$, Now let V equal to $m^{-1}(P)$, then m is Nc-continuous. □

Example 2.15. $m : (H, \partial_{ind}) \longrightarrow (K, \rho)$ with H is finite, is Nc-continuous function. In the same way of proofing the previous theorem, we can proof the next proposition.

Proposition 2.16. If $m : H \longrightarrow K$, so m is $N^*(N^{**})$ c-continuous function iff to every N-open set J of K and J^C is compact, $m^{-1}(J)$ is open (N-open) set in H.

Example 2.17. A function $I_H : (H, \partial_D) \longrightarrow (H, \partial_{ind})$ where H is finite set, is $N^*(N^{**})$ c-continuous function.

Definition 2.18 ([8])**.** An element h in a space H is a condensation point to a subset P of H, when any open set C of H having the point h, $C \cap P$ is infinite. When the set P includes all its condensation points, then it is N-closed set. And P^C is N-open set.

Example 2.19. Every set in the indiscrete space (H, ∂_{ind}) where H is finite, is N-closed set. We found the next Remark in [9] without proof, we introduce its proof for completeness.

Remark 2.20. Suppose (H, ∂) is a space, hence any closed set in H is an N-closed, because when P is a closed set inside H, and $h \in H$ and h does not belong to P, so $h \in P^C$, since P is closed set then P^C is open. But $P \cap P^C = \emptyset$ which is finite, so h is not a condensation point for P, hence every condensation point for P is inside it, therefore P is N-closed.

Theorem 2.21. *$m : H \longrightarrow K$ is Nc-continuous function iff for any closed and compact set P of K, its inverse image is N-closed set in H.*

Proof. Give m as Nc-continuous function and P is closed and compact set in K, so P^C is an open set of K and its complement is compact (given), since m is Nc-continuous function so $m^{-1}(P^C)$ is N-open set in the space H, but $m^{-1}(P^C) = (m^{-1}(P))^c$, then $((m^{-1}(P))^c)^c = m^{-1}(P)$ is N-closed set of H. Conversely, suppose J is an open set in K and J^C is compact, $m^{-1}(J^C) = (m^{-1}(J))^c$ is N-closed subset of the space H, and the complement of the last one is equal to $m^{-1}(J)$ is N-open set of H, therefore m is Nc-continuous function. □

Example 2.22. $I_H : (H, \partial_{ind}) \longrightarrow (H, \partial)$ where H is finite, is Nc-continuous function. We can proof the following two propositions by the same way of proof the last theorem.

Proposition 2.23. If $m : H \longrightarrow K$ is N^*c-continuous function iff for every N-closed and compact subset P of K, its inverse image is closed set of H.

Example 2.24. $I_H : (H, \partial_D) \longrightarrow (H, \partial_D)$ where H is finite set, is N^*c-continuous function.

Proposition 2.25. If $m : H \longrightarrow K$ is N^{**}c-continuous function iff to any N-closed and compact set P of K, its inverse image is N-closed set of H.

Example 2.26. The function in example (3-24) satisfies the previous proposition.

Remark 2.27. 1. If m is c-continuous function, then it is Nc-continuous.
2. If m is N^*-continuous function, so m is c-continuous.
3. When m is N^*-continuous function, hence it is N^{**}c-continuous.
4. When m is c-continuous function, it is not necessarily N^{**}c-continuous.
5. If m is N^{**}c-continuous function, it is not needs to be c-continuous.
6. If m is N^*-continuous function, hence it is Nc-continuous.
7. When m is N^{**}c-continuous function, so it is Nc-continuous.
The converses of (1, 2, 3, 6, 7) is untrue, the next example shows that.

Example 2.28. 1. If H is a finite set, then $m : (H, \partial_{ind}) \longrightarrow (H, \partial_D)$ is Nc-continuous function but not c-continuous.

2. Give H as a finite set, then $I_H : (H, \partial_{ind}) \longrightarrow (H, \partial_{ind})$ is c-continuous function but not N^*-c-continuous.

3. Suppose H is a finite set, $m : (H, \partial_{ind}) \longrightarrow (H, \partial_D)$ is N^{**}c-continuous function but not N^*-c-continuous.

4. $m : (\Re, \partial_D) \longrightarrow (\Re, \partial_{ind})$ is c-continuous function but not N^{**} c-continuous.

5. Give H as a finite set, then $m : (H, \partial_{ind}) \longrightarrow (H, \partial_D)$ is N^{**}c-continuous function but not c-continuous.

6. $m : (H, \partial_{ind}) \longrightarrow (H, \partial_{ind})$ where H is finite, it is Nc-continuous function but not N^*c-continuous.

7. $m : (H, \partial_{ind}) \longrightarrow (H, \partial_{ind})$ where H is finite, it is Nc-continuous function but not N^{**}c-continuous.

Definition 2.29 ([9])**.** (H, ∂) is NT_1-space if to all distinct points q,r in it, we could select two N-open sets J,G with $q \in J, r$ is not in J, and $r \in C, q$ is not in C.

Example 2.30. The discrete space $(\Re, \partial_D)$ is NT_1-space. We found the following proposition in [9] without proof, we introduce its proof for completeness.

Proposition 2.31. The space (H, ∂) is NT_1-space iff each singleton point in it is N-closed set.

Proof. If H is NT_1-space, r is not in H, to any $q \in H$ then $q \neq r$, but H is NT_1-space, so choose N-open sets J, C in H and q inside J, r does not belong to J and r belong to C, q does not belong to C, so $C \cap \{q\} = \emptyset$ which is finite set, so r is not condensation point for $\{q\}$, since r is arbitrary, so $\{q\}$ contains each of its condensation points and then it is N-closed set. Contrariwise, if $\{q\}$ is an N-closed set to any $q \in H$, r is any element in H with q not equal to r, so $\{q\}$, $\{r\}$ are N-closed subsets in H (by hypothesis), hence $\{q\}^c$ is N-open subset from H where r is in $\{q\}^c$,q is not in $\{q\}^c$ and $\{r\}^c$ is N-open subset from H such that q in $\{r\}^c$,r is not in $\{r\}^c$ and therefore H is NT_1-space. □

Theorem 2.32. *If (K, ρ) is T_1-space and $m : H \longrightarrow K$ is Nc-continuous and one to one function, hence (H, ∂) is NT_1-space.*

Proof. If h is in H, so $\{m(h)\}$ is a subset of K but K is a NT_1-space, so $\{m(h)\}$ is closed and compact set in K, $\{m(h)\}^C$ is open in K with closed and compact complement, $m^{-1}(\{m(h)\}^C) = m^{-1}(K - \{m(h)\}) = H - m^{-1}(\{m(h)\}) = H - \{h\}$ which is N-open subset from H (because m is Nc-continuous function), hence h is N-closed subset from H, therefore H is NT_1-space (proposition (3.31)). The following proposition can be proof by the same way of the proof for previous theorem. □

Proposition 2.33. When K is NT_1-space and $m : H \longrightarrow K$ is N^*c-continuous and one to one function, then H is T_1-space.

Proposition 2.34. When K is NT_1-space and $m : H \longrightarrow K$ is N^{**}c-continuous and one to one function, then H is NT_1-space.

Example 2.35. $m : (H, \partial_D) \longrightarrow (K, \rho_D)$, satisfying (3-32), (3-33) and (3-34).

Definition 2.36 ([9], [10], [11])**.** (H, ∂) is N-normal space if to any closed sets O and , $O \cap P = \emptyset$, we could delimit two N-open sets J, C such that $J \cap C = \emptyset$, with $O \subseteq J$ and $P \subseteq C$.

Example 2.37. The space (H, ∂_{ind}) with H is finite set, is N-normal space.

Definition 2.38 ([12])**.** (H, ∂) is N^*-normal space if to all N-closed sets O and P where $O \cap P = \emptyset$, there are two open sets J, C and $J \cap C = \emptyset$, with $O \subseteq J$ and $P \subseteq C$.

Example 2.39. The discrete space (H, ∂_D) where H is finite set, is N^*-normal space.

Definition 2.40 ([12])**.** *A* space X is N^{**}-normal space if for each N-closed sets F and H with $F \cap H = \emptyset$, we can find N-open sets U and G where $U \cap G = \emptyset$, with $F \subseteq U$ and $H \subseteq G$.

Example 2.41. The discrete space $(\mathbb{X}, \mathfrak{L}_D)$ where $\mathbb{X}$ is finite set, is N^{**}-normal space.

Remark 2.42. 1. If H is normal space, so it is N-normal ([12]).
2. When H is N^*-normal space, then it is normal ([12]).
3. If H is normal space, it is not necessarily $N^{(**)}$-normal.
4. If H is $N^{(**)}$-normal space, it is not needs to be normal.
5. When H is N^*-normal space, hence it is $N^{(**)}$-normal.
6. When HN^*-normal space, so it is N-normal.
7. If H is $N^{(**)}$-normal space, then it is N-normal.
The converse of (1, 2, 5, 6, 7) is not true.

Example 2.43. 1. Let $H = \{2, 3, 4\}$ and $\partial = \{\emptyset, H, \{2\}, \{2, 3\}, \{2, 4\}\}$, hence (H, ∂) is N-normal space but not normal.

2. Let $H = \{1, 2, 3\}$ and $\partial = \{\emptyset, H, \{1\}, \{2\}, \{1, 2\}\}$, so (H, ∂) is normal space while not N^*-normal.

3. $(\mathcal{R}, \partial_u)$ is normal space while not N^{**}-normal.

4. Let $H = \{2, 3, 4\}$ and $\partial = \{\emptyset, H, \{2\}, \{2, 3\}, \{2, 4\}\}$, then (H, ∂) is N^{**}-normal space but not normal.

5. The indiscrete space (H, ∂_{ind}) where $H = \{1, 2, 3\}$ is N^{**}-normal space but not N^*-normal.

6. Let $H = \{2, 3, 4\}$ and $\partial = \{\emptyset, H, \{2\}, \{2, 3\}, \{2, 4\}\}$, then (H, ∂) is N-normal space but not N^*-normal.

Definition 2.44 ([13])**.** $m : H \to K$ is open function when $m(J)$ is open set in K to all open set J in H.

Example 2.45. $m : (H, \partial_D) \to (K, \partial_D)$ is open function.

Definition 2.46 ([14])**.** $m : H \to K$ is N-open function if to all open set J in $H, m(J)$ is N-open subset from K.

Example 2.47. $I_H : (H, \partial_{ind}) \to (H, \partial_{ind})$, where H is finite, is N-open function.

Definition 2.48. $m : H \to K$ is $N^*(N^{**})$-open function when to any N-open subset J in $H, m(J)$ is open (N-open) set from K.

Example 2.49. 1. $I_H : (H, \partial_{ind}) \to (H, \partial_{ind})$, where H is finite, is N^{**}-open function.

2. $m : (\mathcal{R}, \partial_D) \to (\mathcal{R}, \partial_D)$ is N^*-open function.

Remark 2.50. 1. Every open function is N-open.
2. Every N^*-open function is open.
3. Not every open function is N^{**}-open.
4. Not every N^{**}-open function is open.
5. Every N^*-open function is N-open.
6. Every N^{**}-open function is N-open.

The converse of $(1, 2, 5, 6)$ is untrue, we give the next example to that.

Example 2.51. 1. $I_H : (H, \partial_D) \to (H, \partial_D)$, where H is finite, is N-open function but not open.

2. $I_H : (H, \partial_{ind}) \to (H, \partial_{ind})$, where H is finite, is open function but not N^*-open.

3. $I_H : (H, \partial_{ind}) \to (H, \partial_{cof})$ is open function but not N^{**}-open.

4. $I_H : (H, \partial_D) \to (H, \partial_{ind})$, where H is finite, is N^{**}-open function but not open.

5. $I_H : (H, \partial_{ind}) \to (H, \partial_{cof})$, where H is finite set, is N-open function but not N^*-open.

6. $I_H : (H, \partial_{ind}) \to (H, \partial_{cog})$ where $m(h) = h$ to any $h \in H, H$ is finite set, is N-open function but not N^{**}-open.

Definition 2.52 ([15])**.** (H, ∂_D) is T_2-space if to every not equal points q, r in H, we could find two open sets J, C with $q \in J, r \in C$ and $J \cap C = \emptyset$.

Example 2.53. (H, ∂) is T_2-space.

Definition 2.54. (H, ∂_D) is NT_2-space if to not equal element $q, r \in H$, there are two N-open sets J, C in the space H, where $q \in J, r \in C$ and $J \cap C = \emptyset$.

Theorem 2.55. *If $m : H \to K$ is Nc-continuous, open and surjective where H is N^*-normal space and K is T_1-space, hence K is T_2-space.*

Proof. Suppose $q \neq r$ are elements in K, because K is T_1-space, $\{q\}, \{r\}$ are closed sets in K, and they are compact since they are finite sets, then the inverse image of these sets are N-closed sets in H (Theorem (3.21)), H is N^*-normal space, then $\exists$ two open sets J, C in H, where $(m^{-1}(\{q\})) \subseteq J$ and $(m(\{r\})) \subseteq C$, hence $m(m(\{q\})) \subseteq m(J)$, $m(m^{-1}(\{r\})) \subseteq m(C)$, where $m(m(\{q\}))$ and $m(m^{-1}(\{r\}))$ are open subsets from K (because m is open), $m(J) \cap m(C) = m(J \cap C) = m(\emptyset) = \emptyset$, so K is T_2-space. □

Definition 2.56 ([16])**.** If $m : H \to K$ is N-continuous if to any open subset J in H, $m(J)$ is N-open set in H.

Example 2.57. When H is finite set, $m : (H, \partial_{ind}) \to (H, \partial_D)$, is N-continuous function.

Definition 2.58. If $m : H \to K$ is N^*-continuous if to any N-open J subset in K, $m^{-1}(J)$ is open set in H.

Example 2.59. $m : (H, \partial_D) \to (H, \partial_{ind})$ with H is finite set, is N^*-continuous function.

Definition 2.60. If $m : H \to K$ is N^{**}-continuous if to all N-open subset J from K, $m^{-1}(J)$ is N-open subset from H.

Example 2.61. $m : (H, \partial_D) \to (H, \partial_{ind})$ with H is finite set, is $N^{*}*$-continuous function.

Theorem 2.62. *Let $m : (H, \partial) \to (K, \rho), m_* : (H, \partial) \to (K, \rho^*)$, and $i : (K, \rho') \to (K, \rho^*)$, and suppose that ρ^* contains a base consist from open sets having compact complement in ρ' . Whenever i is continuous function, i^{-1} is Nc-continuous. m is Nc-continuous if m_* is Nc-continuous, where $m_*(h) = m(h)$, for any h inside H.*

Proof. Suppose m is Nc-continuous function and J is open set in (K, ρ^*), since ρ^* contains a base consist from open and compact sets in ρ' , so J^C is compact, and $\rho^* \subset \rho'$, so $J \subset \rho'$ and because m is Nc-continuous, so $m^{-1}(J)$ is N-open set in H, but $m_*(h) = m(h)$, for any h in H, so $m_*^{-1}(J)$ is N-open set in H, therefore m_* is Nc-continuous function. Now to prove that i is continuous function, suppose J is open set in (K, ρ^*), so it is an open set (K, ρ'), hence i is continuous function (since $i^{-1}(J) = J$). To manifestation that i^{-1} is Nc-continuous, suppose J is an open subset from (K, ρ') and its complement is compact, so $J \subset \rho^*$, then J is N-open subset from (K, ρ^*) (by proposition (3-5)), so i^{-1} is Nc-continuous function (since $(i^{-1})^{-1}(J) = i(J) = J$). □

References

[1] K.R. Gentry, H.B. Hoyle, *C-continuous functions*, 1970.

[2] A. AL-Omari, T. Noiri, M. Noorani, MD. Salmi, *Characterization of strongly compact spaces*, International Journal of Mathematics and Mathematical Sciences, 2009, 1-9.

[3] Ch. Konstadilaki-Savvopoulou, I.L. Reilly, *On almost-N-continuous functions*, J. Austral. Math. Soc., 1 (1995), 118-130.

[4] H.J.A. Hussein, *New characterization of topological transitivity*, Iraqi Journal of Science, 54 (2013), 817-821.

[5] S.H.H. Al-Janabi, *On some types of N-functions*, Al-Qadisiyah Journal of Pure Science, 19 (2017), 1-20.

[6] R.A. Hussain, H.I. Nasser, *On connected space by using N-open set*, Journal of Al-Qadisiyah for Computer Science and Mathematics, 4 (2012), 1-19.

[7] I.A. Al-Saeed, *Weak N-open sets*, Ibn Al-Haitham Journal for Pure and Applied Science, 29 (2016), 267-276.

[8] S.A. Ghour, H. Saleh, *Quasi N-open sets and related compactness concepts in bitopological spaces*, Proyecciones (Antofagasta), 37 (2018), 491-501.

[9] E.R. Ali, R.A. Hussain, *On dimension theory by using N-open sets*, Journal of Babylon University/Pure and Applied Sciences, 23 (2015), 595-604.

[10] E. Rosas, C. Carpintero, J. Moreno, *More on upper and lower almost nearly I-continuous multifunction*, International Journal of Pure and Applied Mathematics, 117 (2017), 521-537.

[11] C. Carpintero, J. Pacheco, N. Rajesh, E. Rosas, S. Saranyasri, *Almost nearly ω-continuious multifunctions*, Eurasian Bulletin of Mathematics, 1 (2017), 24-33.

[12] N.H. Hajee, *Characterization of covering dimension*, Journal of Thi-Qar Science, 6 (2017), 126-132.

[13] R.A. Hussain, Z.A.A. Nasser, *On the ωb-separation axioms*, International Journal of Science and Research, 6 (2017), 78-96.

[14] S.H. Hamza, F.M. Majhool, *Strongly N-proper function*, Journal of Al-Qadisiyah for Computer Science and Mathematics, 3 (2011), 1-14.

[15] R.A. Hussain, Z.A.A. Nasser, *On the ωb-compactspace*, Journal of Advance in Mathematics, 13 (2017), 7295-7301.

[16] S. Al Ghour, S. Samarah, *Co-compact open sets and continuity*, Journal of Abstract and Applied Analysis, (2012), 1-9.

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