Stochastic optimal control model practices in development, finance and the industrial production

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Abstract. The systems mainly study in this work are dynamic to know, It have unfold all the time, besides are ask by the EDS of ITO and sometimes called models of diffusions. The basic source is the white noise, and since the systems are dynamic, the proposed decision (controls) are based on available information from controllers, it must also change over time.

Keywords: deterministic c.o.s, strong and weak formulations, dynamic programming, singular controls.

1. Introduction

The controllers choose an optimal decision from among all that are possible to achieve the best result.

Any optimization problem is called the optimal stochastic control problem, and the domain of these problems covers a variety of systems (physics, biology, economics, and management ...), just to mention a few.

In our work we will establish rigorous mathematical frameworks for the problems of the S.O.C.

Our work is organized as follows:

- firstly the formulation of deterministic S.O.C problems, and the reason for doing so is not only that the deterministic situation itself contains important results but also so that readers can see the essential deference between the deterministic case and the case of stochastic problem.

- second we will give several practical examples arise in development, finance, and industrial production, it can formulate as a model of optimal stochastic control given.

- third we present the strong and weak formulation of problems of (S.O.C) it mainly to treat during this project.

Despite not being studied in this work, the two approaches, namely the Pontryagin maximum principle, and the dynamic programming of Bellman are also main for these problems.

2. Formulation of deterministic S.O.C problem

starting with the production of the problem plan; a machine produce only one type of product, the raw materials produced by the machine, and the final products are stored.

Suppose that in time t the rate of production is $u(t)$ and the inventory level is $x(t)$, if the rate request for this product is a known function $Z(t)$ and the inventory is x_0 at the time $t = 0$, therefore the relationship between the quantities is given by:

$$\begin{cases} \dot{x}(t) = u(t) - Z(t) \\ x(0) = x_0 \end{cases} . \tag{2.1}$$

Here $x(t)$ can take positive or negative values, the product is surplus when $x(t) > 0$ and descends in the opposite direction suppose the cost of finding inventory x and the production rate u by unit time is $h(x,u)$.

A typical example of h is:

$$h(x,u) = c^+x^+ + c^-x^- + pu \tag{2.2}$$

Or $x^+ = \max \{x,0\}, x^- = \max\{-x,0\}$, $c^+, c^- \geq 0$ are:

- the marginal cost / penalty for the excess / goes down respectively and p is the unit cost of production

- the care of the chosen production $u(.)$ to minimize the total discount cost on the horizon plan $[0,T]$.

The following functional will minimize it:

$$J(u(.)) = \int_0^T e^{-\lambda t} h(x(t),u(t))dt . \tag{2.3}$$

Or λ is the discount rate, note here that $u(.)$ is a function on $[0,T]$,which is called production plan if the machine has a maximum production rate k (called production capacity), therefore for any production plan we have:

$$0 \leq u(t) \leq k , \forall t \in [0,T] . \tag{2.4}$$

If the storage size $b > 0$,then the inventory level $x(t)$ it must satisfy the constraint:

$$x(t) \leq b \tag{2.5}$$

Any production plan that verified (2.4),(2.5),(2.1) is called an eligible plan.

The problem is to minimize the cost (2.3) on all the admissible plans.

The above problem is perhaps one of the simplest examples for a (S.O.C) problem, it is deterministic because there is no doubt in the dynamic system (2.1), and (2.4), (2.5).

Now we will present the general formulation of the problem of deterministic optimal control.

Let: T , $0 < T \leq +\infty$, $x_0 \in \mathbb{R}^n$,and a metric space Γ.

The dynamics of the system under consideration are given by the following **O.D.E**:

$$\begin{cases} \dot{x}(t) = b(t, x(t), u(t)) \\ x(0) = x_0 \end{cases} \quad t \in [0, T] \tag{2.6}$$

with $b : [0, T] \times \mathbb{R}^n \times \Gamma \longrightarrow \mathbb{R}^n$ a given application.

When $T = +\infty$, $[0, T]$ is replaced by $[0.\infty)$.

A measurable application $u(.) : [0, T] \longrightarrow \Gamma$ is called control and x_0 it is called the initial state and x. The solution of (2.6) is called the state trajectory corresponds to $u(.)$.

In the example of production plan the dynamic is: (2.1), the control is the rate of production, and the state is the levels of inventory.

We assume that for all $x_0 \in \mathbb{R}^n$ and all control $u(.)$ There is a unique solution:$x(.) \equiv x(.; u(.))$ of (2.6).

On such case one obtains an input-output relation with input $u(.)$ and output $x(.)$.

Also call (2.6) a control system.

A special case of (2.6) is the linear case where the system to control is in the form:

$$\begin{cases} \dot{x}(t) = A(t) x(t) + B(t) u(t) \\ x(0) = x_0 \end{cases} \quad t \in [0, T] \tag{2.7}$$

with $A : [0, T] \longrightarrow \mathbb{R}^{n \times n}$ et $B : [0, T] \longrightarrow \mathbb{R}^{n \times k}$ are measurable application, sometimes we will note (2.7) by linear system with varied time.

Since $A(.)$ and $B(.)$ Depend, moreover the following system is called linear system of time invariant:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ x(0) = x_0 \end{cases} \quad t \in [0, T] \tag{2.8}$$

with: $A \in \mathbb{R}^{n \times n}$ et $B \in \mathbb{R}^{n \times k}$.

As we saw in the example of production plan, the constrained state is given by:

$$x(t) \in S(t), \forall t \in [0, T] . \tag{2.9}$$

And the control constraint is given by:

$$u(t) \in U(t), t \in [0,T], \tag{2.10}$$

where $S(t) : [0,T] \longrightarrow 2^{\mathbb{R}^n}$ and $U(t) : [0,T] \longrightarrow 2^{\Gamma}$ are multifunctional (i.e: for all $t \in [0,T]$, $S(t) \subseteq \mathbb{R}^n$ and $U(t) \subseteq \Gamma$).

We will only deal with cases where the control $u(t)$ is time invariant ; $u(t) \equiv u$.

We recall that U itself can look like a metric space, so from now we replace Γ by U.

Let : $V[0,T] = \{u : [0,T] \longrightarrow U \ : u(.) \text{ is measurable }\}$.

Any $u(.) \in V[0,T]$ is called a feasible control, by following either the functional cost to measure the execution of the control:

$$J(u(.)) = \int_0^T f(t, x(t), u(t))\, dt + h(x(T)). \tag{2.11}$$

For an application: $f : [0,T] \times \mathbb{R}^n \times U \longrightarrow \mathbb{R}^{n\times n} \to \mathbb{R}$ and $h : \mathbb{R}^n \longrightarrow \mathbb{R}$.

The first and second right-hand term (2.11) are called the initial cost and the terminal cost, respectively.

Definition 2.1. *A control $u(.)$ is called admissible and $(x(.), u(.))$ an admissible pair if:*

1) $u(.) \in V[0,T]$
2) $x(.)$ *is the only solution to equation* (2.6) *under* $u(.)$.
3) *the state (2.9) is satisfied.*
4) $t \to f(t, x(t), u(t)) \in L^1[0,T]$.

The set of all admissible controls is denoted by $U_{ad}[0,T]$.

Our problem of the c-o-determinist is as follows:

2.0.1 Problème (D.O.C)

Minimize (2.11) on $U_{ad}[0,T]$.

Our problem is finite if (2.11) admits a lower bound and it is unique if there exists $\bar{u}(.) \in U_{ad}[0,T]$ tell that:

$$J(\bar{u}(.)) = \inf_{u(.)\in U_{ad}[0,T]} J(u(.)). \tag{2.12}$$

Any $\bar{u}(.) \in U_{ad}[0,T]$ verified (2.12) is called optimal control, which corresponds to the state trajectory $\bar{x}(u) \equiv x(.;\bar{u}(.))$ et $(\bar{x}(.), \bar{u}(.))$ re called the optimal state path and pair, respectively. An important and special case is such that the control system given by (2.7) .

(2.9) is absent "there is no constrained state ($S(t) \equiv \mathbb{R}^n$) "

The constraint of the control u is $\mathbb{R}^k$,and the functional cost is of the form:

$$J(u(.)) = \frac{1}{2}\int [\langle Q(t)x(t), x(t)\rangle + \langle R(t)u(t), u(t)\rangle]dt + \frac{1}{2}\langle G\, x(T), x(T)\rangle. \tag{2.13}$$

For some suitable symmetric matrix G and matrix values $Q(.)$ and $R(.)$ The problem (D.O.C) is called problem of the quadratic linear optimal control (LQ problem).

And finally, we will notice in the example of the production plan the terminal cost h equals 0.

In general, a problem of optimal control with:

1) $h = 0$ is called the Lagrange problem.
2) $f = 0$is a Mayer's problem.
3) si: $f \neq 0$, $h \neq 0$ is a Bolza problem.

It is very clear that these three problems are mathematically equivalent.

2.1 Optimal stochastic control model given

This section is an overview of several practical problems.

2.2 Production planning

Consider the problem of optimal production planning, and suppose a filtered probability space: $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ check the usual conditions on which a standard Brownian Motion $W(t)$ is defined.

The requested process $z(t)$ is not deterministic, rather it is given by the following:

$$z(t) = z_0 + \int_0^t \xi(s)\,ds + \int_0^t \sigma(s)\,dW(s) \quad, t \in [0,T]. \tag{3.1}$$

$\xi(t)$: represents the rate requested at time t, and the term $\int_0^t \sigma(s)\,dW(s)$ represents the fluctuation.

In addition, it covers these processes to avoid damage to inventories, which they have entirely variable, we assume that $\xi(t)$ and $\sigma(t)$ are $\{\mathcal{F}_t\}_{t\geq 0}$-adapted. The integrals in (3.1) are well defined. To meet demand the factory is to adjust the rate of production all the time to accommodate the change in the situation.

Suppose the machine is trustworthy (seriously: it never arrived).

And as in the deterministic case; at time t the production rate is $u(t)$ and the inventory level is $x(t)$.

If at time $t = 0$ the inventory is x_0 then the system is modeled as follows:

$$\begin{cases} dx(t) = (u(t) - z(t))\,dt \quad, \ x(0) = x_0 \\ dz(t) = \xi(t)\,dt + \sigma(t)\,dW(t) \quad, z(0) = z_0 \end{cases} . \tag{3.2}$$

Another time, control or the rate of production $u(t)$ is an object of production capacity

$$0 \leq u(t) \leq k \ , \ t \in [0,T] \quad \textbf{A.s}. \tag{3.3}$$

There is another implicit constraint on the rate of production $u(t)$ due to the stochastic environment of the problem, namely for all time, production management to make a decision based only on past information, rather any information in the future, moreover the control or the decision; it is wrong that must be non anticipatory.

The precise mathematical translation for this constraint will study it.

Here to avoid infinite accumulation, the maximum production capacity is completely large enough to meet the demand.

Then the following minimum condition must be imposed for the problem to have a meaning:

$$E\int_0^T z(t)\,dt \;\equiv z_0T + E\int_0^T\int_0^t \xi(s)\,dsdt \;< kT. \tag{3.4}$$

On the other side the inventory status is not necessarily exceeded the size

$$b \;\; X(t)\,.\le b\;,\;\; \forall t\,\epsilon\,[0,T]\;\;,\;\; \mathbf{A.s}. \tag{3.5}$$

The total expected cost is as follows:

$$J(u(.)) = E\left\{\int e^{-\gamma t} f(x(t), u(t))\,dt + e^{-\gamma T} h(x(T))\right\}. \tag{3.6}$$

Where the first term represents the total cost of inventory and production, and the second term is the inventory penalty at the end of production (example: disposition of the cost), and finally γ is the discount rate.

The objective of production management is to choose a suitable production plan $u(.)$ tell that: (3.2), (3,3) et (3.5) are satisfied and $J(u(.))$ is minimize.

2.3 Investment and consumption

Suppose that there is a market in which $(n+1)$ possession are in trade, one of the possession is called engagement which in the price process $P_0(t)$, and we have the following **O.D.E** (déterminitse):

$$\begin{cases} dP_0(t) = r(t)P_0(t)\,dt \;\;,\; t\,\epsilon\,[0,T] \\ P_0(0) = P_0 > 0 \end{cases}. \tag{3.7}$$

With $r(t) > 0$ is called the interest rate (of the commitment), it is clear that $P_0(t)$ increase steadily over time and the commitment is more of that called a possession with risk.

The other n holdings are called stocks, with a price process: $P_1(t), \dots, P_n(t)$, satisfies the **SDE**:

$$\begin{cases} dP_i(t) = P_i(t)\{b_i(t)\,dt + \langle\sigma_i(t), dW(t)\rangle\} \\ P_I(0) = P_I > 0 \end{cases} \;\;,\; t\,\epsilon\,[0,T]. \tag{3.8}$$

With $b_i : [0,T]\times\Omega \to \mathbb{R}$ and $b_i(t) > 0$ (**p.s**) is called the rate of appreciation, and $\sigma_i : [0,T]\times\Omega \to \mathbb{R}^m$ is the volatile (inconstant) or else the stock dispersion, all these processes are assumed so that they are $\{\mathcal{F}_t\}_{t\ge 0}$-adapted.

Here $W\,(t)$ is a standard Brownian motion defined on some "fixed" complete filtered probability spaces ; $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}\,, P)$.

The diffusion term in (3.8) return the stock price fluctuation, the latter is called risk possession and we have:

$$Eb_i\,(t) > r(t) > 0\ , \forall t \in [0, T]\ \ ,\ 1 \leq i \leq\ n. \tag{3.9}$$

It is a very natural assumption, whereas in the opposite case none should invest in risky stocks, however, the following discussion is not dependent on these assumption.

Now we consider an investor with total opulence at time $t \geq 0$ is denoted by $x(t)$.

Suppose that it decides to contain $N_i(t)$ portions of the i -th possession $(i = 0, \ldots, n)$ at time t , then:

$$x(t) = \sum_{i=0}^{n} N_i(t) P_i(t)\ \ ,\ t \geq 0. \tag{3.10}$$

Let $c(t)$ the consumption rate, therefore:

$$\begin{aligned} x(t+\Delta t) - x(t) &= \sum_{i=0}^{n} N_i(t)\left[P_i(t+\Delta t) - P_i(t)\right] \\ &+ \sum_{i=0}^{n} \mu_i(t) N_i(t) P_i((t)\Delta t - c(t)\ \Delta t. \end{aligned} \tag{3.11}$$

When $\Delta t \to 0$ we obtain::

$$\begin{cases} dx\,(t) = \sum\limits_{i=0}^{n} N_i(t) dP_i(t) + \sum\limits_{i=0}^{n} \mu_i(t) N_i(t) P_i((t) dt - c(t) dt \\ = \left\{ r(t) N_0(t) P_0((t) dt + \sum\limits_{i=0}^{n} \left[b_i(t) + \mu_i(t)\right] N_i(t) P_i((t) - c(t) \right\} dt \\ + \left\langle \sum\limits_{i=0}^{n} \sigma_i(t) u_i(t)\ ,\ dW(t) \right\rangle \end{cases} \tag{3.12}$$

or

$$u_i(t) \triangleq N_i(t) P_i(t)\ \ ,\ i = 0, 1, \ldots, n \tag{3.13}$$

Note that the value of the investor opulence market is in the i-ème commitment / stock.

When $u_i(t) < 0, (i = 0, 1, \ldots, n)$ the investor is completely selling the i-th stock.

When $u_0(t) < 0$, the investor is borrowing the quantity $|u_0(t)|$ with rate $r(t)$, it is clear that by changing $u_i(t)$, the investor changes the allocation of his wealth in the $(n+1)$ possession.

Note $u(t) = (u_1(t)\ , \ldots,\ u_n(t))$ the investor's portfolio.

We cannot include the allocation to the commitment in the portfolio, as it will only be determined by the allocation to stocks, which gives total opulence.

As in the case of stochastic production planning any eligible portfolio is required so that must be non-anticipatory.

Now for $x(0) = x_0 > 0$, the investor will choose the investment portfolio $u(.)$ and the consumption plan $c(.)$ so that:

$$x(t) > 0 \ , \ \forall t \in [0,T] \quad \mathbf{A.s}$$

and that the utility of delivery:

$$J(u(.)\ ,\ c(.)) = E\left\{\int_0^T e^{-\gamma t}\varphi(c(t))\,dt + e^{-\gamma T}h(x(T))\right\}. \tag{3.15}$$

Is to maximize, with $\gamma > 0$ is the discount rate, $\varphi(c)$ the instantaneous utility for consumption c and $e^{-\gamma T}h(x(T))$ is the discount utility which is derived from what we're looking for.

We may also impose some constraints for the problem above.

For example a constraint can be bounded with a cost vene and then it has the constraints:

$$u_i(t) \ \geq -L_i \ , \ \forall \ t \in [0,T] \quad \mathbf{A.s} \quad , \ i = 0, 1, \ldots, n.$$

For some $L_i > 0$ and if $L_i = 0$ sa implies that the short sale is very low.

2.4 The strong and weak formulation of (S.O.C) problems

The examples presented in the previous section have certain common characteristics: there is a diffusion system, which is described by the stochastic differential equation of Itô, there are many other decisions that can influence the dynamics of the system. The objective is to optimize (maximize or minimize) the criteria for selecting a non-anticipatory decision among those meeting all the constraints. These problems are called stochastic optimal control problems. We will now present two mathematical formulations (strong and weak formulations) of the optimal stochastic control problems in the two subsections, respectively.

2.5 Strong formulation

Let be the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$,on which a standard Brownian motion $W(.)$ Is defined.

Consider the following controlled SDE:

$$\begin{cases} dx(t) = b(t, x(t), u(t))dt + \sigma(t, x(t), u(t))dW(t)\,, \\ x(0) = x_0 \in \mathbb{R}^n \ , \end{cases} \tag{4.1}$$

With:

$$b : [0,T] \times \mathbb{R}^n \times U \to \mathbb{R}^n \ , \ \sigma : [0,T] \times \mathbb{R}^n \times U \to \mathbb{R}^{n\times m}.$$

U : separable metric space.and $t \in [0, \infty[$,fixed.

The function $u(.)$ is called control representing the action, decision, or policy of the decision makers (the controllers).

At any time, the controller is well informed about certain information (as specified by the filtration of information $\{\mathcal{F}_t\}_{t\geq 0}$ of what happened at that time, but not able to predict what will happen next due to the uncertainty of the system (consequently, for all t the controller cannot exercise his decision $u(t)$ before time$t$ actually).

This nonanticipative restriction in mathematical terms can be represented as "$u(.)$ is $\{\mathcal{F}_t\}_{t\geq 0}$-adapted " (we recall that any process $\{\mathcal{F}_t\}_{t\geq 0}$-adapted is progressively measurable). Know, control $u(.)$ Is taken from the set:

$$\mathcal{U}[0,T] \overset{D}{=} \left\{ u : [0,T] \times \Omega \to U \;\;,\; u(.) \text{ est } \{\mathcal{F}_t\}_{t\geq 0} - \text{adapté} \right\}.$$

Any $u(.) \in \mathcal{U}[0,T]$ is called feasible control, moreover there are some constrained states. For all $u(.) \in \mathcal{U}[0,T]$ the equation (4.1) is with random coefficients.

Let $S(t) : [0,T] \to 2^{\mathbb{R}^n}$ be a multifunction.

The forced state may be granted by:

$$x(t) \in S(t) \;,\; \forall t \in [0,T] \quad \textbf{A.s}. \tag{4.2}$$

Note that other types of constrained state are also possible, the following one introduces the functional cost:

$$J(u(.)) = E \int_0^T f(t, x(t), u(t))\, dt + h(x(T)). \tag{4.3}$$

Definition 2.2. *Let* $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ *a filtered probability space,* $W(t)$ *a* $\mathcal{F}_t$*-Brownien motion*

A control $u(.)$ Is called an admissible control (in the strong sense) and $(x(.), u(.))$ an eligible peer (in the strong sense) if:

1) $u(.) \in \mathcal{U}[0,T]$.
2) $x(.)$ Is the unique solution of (4.1) .
3) some imposed constraint states (for example (4.1)) are satisfied.
4) $f\ (., x(.), u(.)) \in L^1_{\mathcal{F}}(0, T, \mathbb{R})$ et $h(x(T)) \in L^1_{\mathcal{F}}(\Omega, \mathbb{R})$.

The set of all admissible controls is noted by: $\mathcal{U}^F_{ad}[0,T]$.

Our problem of stochastic optimal control under the strong formulation can be indicated as follows:

Problem(SF): Minimize (4.3) on $\mathcal{U}^F_{ad}[0,T]$.

Our objective is to find: $\bar{u}(.) \in \mathcal{U}^F_{ad}[0,T]$, "if there is" tell that:

$$J(\bar{u}(.)) = \inf_{u(.)\in\mathcal{U}^F_{ad}[0,T]} J(u(.)). \tag{4.4}$$

Our problem is S-finite if the right side of (4.4) is finite, and it is said to (unique) S-solved if there is a (unique) $\bar{u}(.) \in \mathcal{U}_{ad}^{F}[0,T]$ such as (4.4) holds.

Any $\bar{u}(.) \in \mathcal{U}_{ad}^{F}[0,T]$ satisfactory (4.4) is called F-optimal control. The corresponding state process of $\bar{x}(.)$ And the state control pair $(\bar{x}(.), \bar{u}(.))$ are called an F-optimal state process and F-optimal pair, respectively.

2.6 Weak formulation

We note that in the strong formulation, the underlying filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ as well as the Brownian motion $W(t)$ are all fixed.

In certain situations, it will be practical or necessary to vary $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ as well as $W(t)$ and to consider them as parts of the control. this is the case, for example, when we apply the dynamic programming method to solve a stochastic optimal control problem initially under the strong formulation. Therefore, we need another formulation of the problem.

Definition 2.3. *The six-tuple* $q = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P, W(.), u(.))$ *is called an admissible control (in the weak sense) and* $(x(.), u(.))$ *an admissible peer (in the weak sense) if*

A1) $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ is a filtered probability space satisfying the usual conditions ;

A2) $W(t)$ a standard Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$;

A3) $u(.)$ Is a process $\{\mathcal{F}_t\}_{t\geq 0}$-adapted on $(\Omega, \mathcal{F}, P)$ with values in $\mathcal{U}$

A4) $x(.)$ Is the only solution of equation (4.1) on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ under $u(.)$;

A5) certain prescribed state constraints (example (4.2)) are satisfactory ;

A6) $f(., x(.), u(.)) \in L^1_{\mathcal{F}}(0,T;\mathbb{R})$ and $h(x(T)) \in L^1_{\mathcal{F}_T}(\Omega;\mathbb{R})$ .Here, the spaces $L^1_{\mathcal{F}}(0,T;\mathbb{R})$ and $L^1_{\mathcal{F}_T}(\Omega;\mathbb{R})$ are defined on the given filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ associated with the six-tuple q .

The set of all admissible controls is designated by $\mathcal{U}_{ad}^{f}[0,T]$. Sometimes we can write $u(.) \in \mathcal{U}_{ad}^{f}[0,T]$ instead of $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P, W(.), u(.)) \in \mathcal{U}_{ad}^{f}[0,T]$ if it is clear from the context that the weak formulation is under review. our problem of stochastic optimal control in the framework of the weak formulation can be indicated as follows:

Problem (Wf): Minimize (4.3) on $\mathcal{U}_{ad}^{f}[0,T]$.

Our objective is to find: $\bar{q} \in \mathcal{U}_{ad}^{f}[0,T]$, "if there is" tell that:

$$J(\bar{q}) = \inf_{q \in \mathcal{U}_{ad}^{f}[0,T]} J(q) . \tag{4.5}$$

As in the strong formulation our problem is finite if the right side of (4.5) is finite.

We can point out here that the strong formulation is indeed in the practical world, but the weak formulation is sometimes auxiliary, mathematically for the objective model, and generally we have problems with the strong formulation.

We emphasize here that the strong formulation is that which follows from Practical World, while the weak formulation sometimes serves as an auxiliary but also an effective mathematical model aiming at solving problems in the end with the strong formulation.One of the main reasons for which this work is perhaps that the objective of a stochastic control problem is to minimize the mathematical expectation of a random variable which depends only on the distribution of the processes. So, if the solutions of (4.1) in the different probability spaces have the same probability distribution, we have more freedom in choosing a suitable probability space to work with. the weak formulation fails if none of the coefficients given are also random (ie that they depend on ω explicitly), because in this case, the probability space must be specified and fixed a priori.

And as in the deterministic case, the following types of equations of state are of particular interest:

$$\begin{cases} dx(t) = [A(t)x(t) + B(t)u(t)]dt + \sum[C_j(t)x(t) + D_j(t)u(t)]dW^j(.), \\ x(0) = x_0 \ , \end{cases} \tag{4.6}$$

Or $A(.), B(.), C_j(.), D_j(.)$ are the appropriate size values matrix (suitable), the functional cost is quadratic:

$$\begin{aligned} J \quad (u(.)) = E \Bigg\{ \frac{1}{2} \int_0^T & [\langle Q(t)\, x(t), x(t)\rangle + \langle R(t)\, u(t), u(t)\rangle]\, dt \\ & + \frac{1}{2} \langle G\ x(T), x(T)\rangle \Bigg\}. \end{aligned} \tag{4.7}$$

The problems **(SF)** or **(WF)** with the equation of state (4.6) and the functional cost (4.7) is called quadratic linear stochastic optimal control problem **(LQSOCP)**.

We note that in this formulation $T > 0$ is a constant value, therefore the problems **(SF)** or **(WF)** are also mentioned as a problem with fixed duration (fixed).

Remark 2.1. everywhere in this work when the type of formulation (strong or weak) under study is clear from the context, we will simply refer to a control as "admissible control" and the set of controls admissible by $\mathcal{U}_{ad}[0,T]$,omitting the prefixes **W** or **S** .

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