Compactness of topological spaces with grills

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Abstract. This work aimed at creation of expansion of topological structures using the concept of grill for the first time. In this respect, we obtained new findings such as αg-compact, αg-compact sets and countably αg-compact spaces. On the other hand, we studied the properties of these concepts and their relationships to each others and their previous counterparts.
Keywords: g-compact, αg-compact, αg-compact sets and countably αg-compact spaces.

1. Introduction and preliminaries

Topological constructions are currently used in applied topical areas such as artificial intelligence, information systems , economics, and data analysis, cf

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[14], [15], [16]. The concept of a grill has been introduced by Choquet [4], with the aim of generalizing the concept of a topological space. It generates finer topology which helps in investigating properties that were difficult to be deduced using the original topology. There are convenient relations between the concept of a grill and some well known concepts such as ideals, nets and filters. In [13], Roy and Mukherjee introduced and investigated the notion of a topology τ_g associated to a grill g on a topological space (X, τ). During, the past ten years, the study of continuity and compactness, nano closed sets and irresolute functions has been generalized. Using the concept of a grill and many interesting constructions, properties and characterization have been deduced, cf [1, 5, 6, 7, 8, 9]. Nassef and Azzam [10, 11] reported a new topological operators via grill. Azzam [2, 3] demonstrated generalized irresolute and quasi-irresolute functions and nano generalized closed sets via grill. Recently, Roy and Mukherjee [13] introduced g-compactness and investigated its relation with compactness. In this paper our purpose is to present some new types of compactness with grill such as αg-compactness, sets αg-compactness and countably αg-compactness. Also, some of its properties and characterizations are obtained.

Definition 1.1 ([4]). A nonempty subcollection g of a space X which carries topology τ is named a grill on this space if the following conditions are true:

(1) $\emptyset \notin g$,

(2) $A \in g$ and $A \subseteq B \subseteq X \Rightarrow B \in g$,

(3) If $A \cup B \in g$ for $A, B \subseteq X$, then $A \in g$ or $B \in g$.

Definition 1.2 ([12]). Let (X, τ) be a topological space and g be a grill on X. We define a mapping $\Phi : P(X) \to P(X)$, denoted by $\Phi_g(A, \tau)$ (for $A \in P(X)$ which stands the power set of X) or $\Phi_g(A)$ or simply by $\Phi(A)$ (when it is known which topology and grill on X we are talking about), called the operator associated with the grill g and the topology τ, and is defined by $\Phi_g(A) = \{x \in X : A \cap U \in g, \forall\ U \in \tau(x)\}$, where $\tau(x)$ stands for the collection of all open neighborhoods of x. We also define a mapping $\psi : P(X) \to P(X)$ by $\psi(A) = A \cup \Phi(A)$, for all $A \in P(X)$, which is a Kuratowski closure operator and hence induces a topology τ_g on X.

Definition 1.3 ([12]). Corresponding to a grill g on a topological space (X, τ), there exists a unique topology τ_g on X given by $\tau_g = \{U \subseteq X : \psi(X \setminus U) = X \setminus U\}$, where for any $A \subseteq X$, $\psi(A) = A \cup \Phi(A)$.

Theorem 1.4 ([12]). *(a) If g_1 and g_2 are two grills on a space (X, τ) with $g_1 \subseteq g_2$, then $\tau_{g_2} \subseteq \tau_{g_1}$.*

(b) If g is a grill on a space (X, τ) and $B \notin g$, then B is closed in (X, τ_g).

(c) For any subset A of a space (X, τ) and any grill g on X, $\Phi(A)$ is τ_g-closed.

Remark 1.5 ([12]). [12] Let (X, τ) be a topological space and g be a grill on X. Then $B(g, \tau) = \{V \setminus A : V \in \tau$ and $A \notin g\}$ is obviously an open base for τ_g.

Corollary 1.6 ([12])**.** *For any grill g on a topological (X,τ), $\tau \subseteq B(g,\tau) \subseteq \tau_g$.*

Definition 1.7 ([13])**.** Let g be a grill on a topological space (X,τ). A cover $\{U_\gamma : \gamma \in \Lambda\}$ of X is said to be a g-cover if there exists a finite subset Λ_0 of Λ such that $X \setminus \bigcup_{\gamma\in\Lambda_0} U_\gamma \notin g$. A cover which is not a $g-$cover of X is named a $\overline{g}$-cover.

Definition 1.8. A subset A of a space (X,τ) is called α-open if

$$A \subseteq Int(cl(Int(A))).$$

Definition 1.9 ([6])**.** Let (X,τ) be a topological space. A cover $U = \{U_\gamma : \gamma \in \Lambda\}$ of X is said to be α-open cover if each member of U is an α-open set of X.

2. αg-compact spaces

We give a brief expansion of compactness using the grill concept.

Definition 2.1. Let g be a grill on a topological space (X,τ). Then (X,τ) is said to be αg-compact space if every α-open cover of X is a g-cover.

Example 2.2. Let $X = [0,\infty[$, $\tau = \{\emptyset, X\}\cup\{]r,\infty[: r \geq 0\}$. (X,τ) is g-compact space. Then it is αg-compact space because if $\{[r_\lambda,\infty[: r_\lambda \geq 0, \lambda \in \Lambda\}$ is an α-open cover of X, then there exist $\lambda_0 \in \Lambda$ such that $r_{\lambda_0} = 0$ and $X \setminus [r_{\lambda_0},\infty[= \emptyset \notin g$.

Remark 2.3. Any open set in (X,τ) is an α-open. But the converse need not be true.

Example 2.4. Let $X = \{a,b,c\}, \tau = \{\emptyset, \{a\}, X\}$. Clearly $\{a,b\}$ is an α-open set which is not open set.

Remark 2.5. Every α-compact space (X,τ) is clearly αg-compact for any grill g on X.

Proof. Indeed, if $\{U_\beta : \beta \in \Lambda\}$ is any α-open cover of X of an α-compact space (X,τ), then there exists a finite subcover $\{U_\beta : \beta \in \Lambda_0\}$ of X. Since $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta = \emptyset \notin g$, then (X,τ) is αg-compact. □

Proposition 2.6. *Let $g = P(X) \setminus \emptyset$, then αg-compactness of a space (X,τ) reduces to the compactness and α-compactness of (X,τ).*

Proof. First, we want to show that every α-compact (X,τ) is compact space. Let $\{U_\beta : \beta \in \Lambda\}$ be any open cover of X. Since every open set is α-open, then $\{U_\beta : \beta \in \Lambda\}$ is α-open cover of X. Since (X,τ) is α-compact space, then there exists a finite subcover $\{U_\beta : \beta \in \Lambda_0\}$ of X and (X,τ) is compact space. Now let (X,τ) be αg-compact, where g=$P(X) \setminus \emptyset$. Let $\{U_\beta : \beta \in \Lambda\}$ be any α-open cover of X. Since (X,τ) is αg-compact space, then there exists a finite subset Λ_0 of Λ such that $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \notin g$. Since g=$P(X) \setminus \emptyset$, then $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta = \emptyset$. Then $\{U_\beta : \beta \in \Lambda_0\}$ is a finite subcover of X. So (X,τ) is α-compact space and compact space. □

Proposition 2.7. *Let $g = P(X) \setminus \emptyset$ be a grill on a space (X, τ) and the space (X, τ_g) is αg-compact. Then (X, τ) is α-compact and compact (as $\tau \subseteq \tau_g$) and hence is αg-compact.*

Proof. Let (X, τ_g) be αg-compact space, where g=$P(X) \setminus \emptyset$. Now we want to show that (X, τ) is α-compact space. Let $\{U_\beta : \beta \in \Lambda\}$ be any $\tau\alpha$-open cover of X. Since $\tau \subseteq \tau_g$, then $\{U_\beta : \beta \in \Lambda\}$ be $\tau_g\alpha$-open cover of X. Since (X, τ_g) be αg-compact space, then there exists a finite subset Λ_0 of Λ such that $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \notin g$. Since g=$P(X) \setminus \emptyset$, then $X \setminus U_{\beta\in\Lambda_0} U_\beta = \emptyset$. Then $\{U_\beta : \beta \in \Lambda_0\}$ is a finite subcover of X. So (X, τ) is α-compact space and hence compact space. □

Remark 2.8. Every αg-compact space (X, τ) is clearly g-compact for any grill g on X.

Proof. Trivial. □

Theorem 2.9. *Let g be a grill on a topological space (X, τ). Then (X, τ_g) is αg-compact if (X, τ) is αg-compact (where τ_g is discrete topology on X).*

Proof. Let $\{U_\beta : \beta \in \Lambda\}$ be a basic $\tau_g\alpha$-open cover of X. Then by definition of a base for τ_g, we get for each $\beta \in \Lambda$, $U_\beta = V_\beta \setminus H_\beta$, where $V_\beta \in \tau$ and $H_\beta \notin g$. Then $\{V_\beta : \beta \in \Lambda\}$ is a τ-open cover of X. since every open set is α-open, then $\{V_\beta : \beta \in \Lambda\}$ is a $\tau\alpha$-open cover of X. Since (X, τ) is αg-compact, then there exists a finite subset Λ_0 of Λ such that $X \setminus \bigcup_{\beta\in\Lambda_0} V_\beta \notin g$. Then $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta = X \setminus \bigcup_{\beta\in\Lambda_0}(V_\beta \setminus H_\beta) \subseteq X \setminus (\bigcup_{\beta\in\Lambda_0} V_\beta \setminus \bigcup_{\beta\in\Lambda_0} H_\beta) \subseteq (X \setminus \bigcup_{\beta\in\Lambda_0} V_\beta) \cup (\bigcup_{\beta\in\Lambda_0} H_\beta) \notin g$. So (X, τ_g) is αg-compact. □

Remark 2.10. Let g be a grill on a topological space (X, τ). Then (X, τ_g) is g-compact if (X, τ) is αg-compact.

Proof. Trivial. □

3. Grills and compactness of Hausdorff spaces

We will establish the relation between grill and α-quasi H-closed.

Definition 3.1. A topological space (X, τ) is said to be α-quasi H-closed (α-QHC, in short) if for every α-open cover U of X, there is a finite sub-collection U_0 of U such that X=$\cup\{Cl(u) : u \in U_0\}$. A Hausdorff α-quasi H-closed space is called an αH-closed space.

Proposition 3.2. *Let g be a grill on a topological space (X, τ), such that $\tau \setminus \emptyset \subseteq g$. If (X, τ) is αg-compact then (X, τ) is α-QHC*

Proof. Let $\{U_\beta : \beta \in \Lambda\}$ be any α-open cover of X. Since (X, τ) is αg-compact, then there exists a finite subset Λ_0 of Λ such that $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \notin g$. Then $Int(X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta)$=$\emptyset$. For otherwise, $Int(X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta) \in \tau \setminus \emptyset \subseteq g$ and hence

$X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \in g$, a contradiction. It follows that, $X \setminus Cl(X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta)^c = \emptyset$. Hence, $X = \bigcup_{\beta\in\Lambda_0} Cl(U_\beta)$ and (X, τ) is α-QHC space. □

Proposition 3.3. *Let (X, τ) be an α-QHC space, where τ is a discrete topology on X. Then (X, τ) is αg_δ-compact, where g_δ is the grill given by $g_\delta = \{A \subseteq X : IntCl(A) \neq \emptyset\}$.*

Proof. Let (X, τ) be an α-QHC space. Let $U = \{U_\beta : \beta \in \Lambda\}$ be any α-open cover of X. Then there is a finite sub-collection $U_0 = \{U_\beta : \beta \in \Lambda_0\}$ of U such that $X = \cup\{Cl(U_\beta) : U_\beta \in U_0\}$. Then $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \notin g_\delta$. Otherwise, $X \setminus \bigcup_{\beta\in\Lambda_0} U_\beta \in g_\delta$ and hence $X \setminus ClInt(\bigcup_{\beta\in\Lambda_0} U_\beta) \neq \emptyset$. Thus $X \setminus \bigcup_{\beta\in\Lambda_0} Cl(U_\beta) \neq \emptyset$, a contradiction. Therefore (X, τ) is αg_δ-compact. □

Definition 3.4. Let (X, τ) be a topological space and g be a grill on X. Then the space X is said to be αg-regular if for any α-closed set F in X with $x \notin F$, there exists disjoint α-open sets U and V such that $x \in U$ and $F \setminus V \notin g$.

Proposition 3.5. *Let g be a grill on a Hausdorff space (X, τ). If (X, τ) is αg-compact then it is αg-regular.*

Proof. Let F be any α-closed subset of X and $x \notin F$. Since X is a Hausdorff space, then for each $y \in F$ there exists two disjoint open sets U_x and V_y such that $x \in U_x$ and $y \in V_y$. Thus, $\{V_y : y \in F\} \cup \{X \setminus F\}$ is an α-open cover of X. Since (X, τ) is αg-compact, then there exists finitely many points $y_1, y_2, y_3, ..., y_n \in F$ such that $X \setminus [(\bigcup_{i=1}^n V_{y_i}) \cup (X \setminus F)] \notin g$. Let $G = X \setminus \bigcup_{i=1}^n ClV_{y_i}$ and $H = \bigcup_{i=1}^n V_{y_i}$. Then G and H are disjoint non-empty α-open sets in X such that $x \in G$(Since $x \notin ClV_{y_i}$ for all $i = 1, 2, 3, ..., n$) and $F \setminus H = F \cup [X \setminus \bigcup_{i=1}^n V_{y_i}] = X \setminus [(\bigcup_{i=1}^n V_{y_i}) \cup (X \setminus F)] \notin g$. So (X, τ) is αg-regular. □

Corollary 3.6. *Let g be a grill on a Hausdorff space (X, τ) such that $\tau \setminus \emptyset \subseteq g$. If (X, τ) is αg-compact then it is αH-closed and αg-regular.*

Proof. Direct consequence of Propositions 3.2. and 3.5. □

Theorem 3.7. *Let U be an α-open subbase for a topological space (X, τ). If X has an open $\bar{g}$-cover then there is a $\bar{g}$-cover of X, which consists of elements of U.*

Proof. Let C be the collection of all open $\bar{g}$-covers of X. Clear that C is partially ordered set. Then by hypothesis C is non-empty. Let $\{P_\beta\}$ be a linearly ordered (chain) subset of C. Then $\bigcup_\beta P_\beta$ is a covering of X. We claim that it is open $\bar{g}$-cover of X. For, if not, then there exist $G_1, G_2, G_3, ..., G_n \in \bigcup_\beta P_\beta$ such that $X \setminus \bigcup_{i=1}^n G_i \notin g$. Now, there exists a $P_{\beta_0} \in C$ such that $G_1, G_2, G_3, ..., G_n \in P_{\beta_0}$. Thus $P_{\beta_0} \notin C$, a contradiction. Consequently by Zorn's lemma, C contains a maximal element P. Thus if H is an open set and $H \notin P$, then there exist finitely many $G_1, G_2, G_3, ..., G_n \in P$ such that $X \setminus (H \cup G_1 \cup G_2 \cup ... \cup G_n) \notin g$. It is clear that the family of open sets which do not belong to P form a filter.

To complete the proof it is sufficient to show that $U \cap P$ is a $\bar{g}$-cover of X. Let $x \in X$. Since P is an open cover of X, there exists an open set $G \in P$ such that $x \in G$. Since U is α-open subbase for (X, τ), there exist $H_1, H_2, ..., H_n \in U$ such that $x \in H_1 \cap H_2 \cap ... \cap H_n \subseteq G$. It follows that there exists an H_i (for some $i = 1, 2, ..., n$) such that $H_i \in P$. For otherwise, if $H_i \notin P$ for all $i = 1, 2, ..., n$, then $\bigcap_{i=1}^{n} H_i \in \tau \setminus P(i.e, \bigcap_{i=1}^{n} H_i \notin P)$. Thus $G \in \tau \setminus P$ and $G \notin P$, a contradiction. Therefore $x \in H_i \in U \cap P$ and consequently, $U \cap P$ is a $\bar{g}$-cover of X. □

4. αg-compact sets relative to a space

Definition 4.1. Let g be a grill on a topological space (X, τ), a subset A of a space (X, τ) is said to be αg-compact relative to X if for every cover $\{U_\lambda : \lambda \in \Lambda\}$ of A by α-open sets of X, there exists a finite subset Λ_0 of Λ such that $A \setminus \bigcup_{\lambda \in \Lambda_0} U_\lambda \notin g$.

Theorem 4.2. *The following are equivalent for a subset A of X:*

1-A is αg-compact relative to X, where the grill $g = P(X) \setminus \emptyset$;

2-A is α-compact relative to X.

Proof. 1. $\Longrightarrow$ 2. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a cover of A by α-open sets of X. Since A is αg-compact relative to X, then there exists a finite subset Λ_0 of Λ such that $A \setminus \bigcup_{\lambda \in \Lambda_0} U_\lambda \notin g$. Since $g = P(X) \setminus \emptyset$, then $A \setminus \bigcup_{\lambda \in \Lambda_0} U_\lambda = \emptyset$. Then $A = \bigcup_{\lambda \in \Lambda_0} U_\lambda$, thus A is α-compact relative to X.

2. $\Longrightarrow$ 1. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a cover of A by α-open sets of X. Since A is α-compact relative to X, then there exists a finite subcover $\{U_\lambda : \lambda \in \Lambda_0\}$ of A (i.e, $A \subseteq \bigcup_{\lambda \in \Lambda_0} U_\lambda$). Then $A \setminus \bigcup_{\lambda \in \Lambda_0} U_\lambda = \emptyset \notin g$, thus A is αg-compact relative to X. □

Proposition 4.3. *Let g be a grill on a topological space (X, τ), if $A_i, i = 1, 2$ are αg-compact subsets relative to a space (X, τ), then $A_1 \cup A_2$ is αg-compact relative to X.*

Proof. Let $\{U_\lambda : \lambda \in \Lambda\}$ be a cover of $A_1 \cup A_2$ by α-open sets of X. Then it is an α-open cover of A_i for $i = 1, 2$. Since A_i is αg-compact relative to X, then there exists a finite subset Λ_1 of Λ such that $A_1 \setminus \bigcup_{\lambda \in \Lambda_1} U_\lambda \notin g$ and there exists a finite subset Λ_2 of Λ such that $A_2 \setminus \bigcup_{\lambda \in \Lambda_2} U_\lambda \notin g$. Since $(A_1 \setminus \bigcup_{\lambda \in \Lambda_1} U_\lambda) \cup (A_2 \setminus \bigcup_{\lambda \in \Lambda_2} U_\lambda) \supseteq (A_1 \cup A_2) \setminus \bigcup_{\lambda \in \Lambda_1 \cup \Lambda_2} U_\lambda \notin g$. Then there exists a finite subset $\Lambda_1 \cup \Lambda_2$ of Λ such that $(A_1 \cup A_2) \setminus \bigcup_{\lambda \in \Lambda_1 \cup \Lambda_2} U_\lambda \notin g$, thus $A_1 \cup A_2$ is αg-compact relative to X. □

Theorem 4.4. *Let (X, τ) be a space with a grill g on X. A is αg-compact relative to X, then $(A, \tau/A)$ is g/A-compact.*

Proof. Let $\{(U_\lambda \cap A) : \lambda \in \Lambda\}$ be τ/A-open cover of A, where $U_\lambda \in \tau$ for each $\lambda \in \Lambda$. Now $\{U_\lambda : \lambda \in \Lambda\}$ is a cover of A by α-open subsets of X. Since A is

αg-compact relative to X, then there exists a finite subset Λ_0 of Λ such that $A \setminus \bigcup_{\lambda\in\Lambda_0} U_\lambda \notin g$. Thus $A \cap [A \setminus \bigcup_{\lambda\in\Lambda_0} U_\lambda] \notin A \cap g$. Since $A \setminus \bigcup_{\lambda\in\Lambda_0}(U_\lambda \cap A) = A \setminus \bigcup_{\lambda\in\Lambda_0} U_\lambda = A \cap [A \setminus \bigcup_{\lambda\in\Lambda_0} U_\lambda] \notin A \cap g$, then there exists a finite subset Λ_0 of Λ such that $A \setminus \bigcup_{\lambda\in\Lambda_0}(U_\lambda \cap A) \notin g/A$. Thus $(A, \tau/A)$ is g/A-compact. □

Example 4.5. The converse of Theorem 4.4 is not true. Indeed, let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}\}$. The set $U = \{a, b\}$ is an α-open set in τ. Let $A = \{b, c\}$ and $\tau/A = \{A, \emptyset\}$. Obviously, $U \cap A = \{b\}$ not open in τ/A.

5. Countably αg-compact spaces

Definition 5.1. Let g be a grill on a topological space (X, τ), a space (X, τ) is said to be countably αg-compact if for every countable α-open cover $\{U_n : n \in N\}$ of X there exists a finite subset N_0 of N such that $X \setminus \bigcup_{U\in N_0} U_n \notin g$, where N denotes the set of positive integers.

Proposition 5.2. *Let g be a grill on a topological space (X, τ), if the space (X, τ) is countably αg-compact, then for any countable family $\{f_n : n \in N\}$ of α-closed sets of X such that*
$\bigcap\{f_n : n \in N\} = \emptyset$, there exists a finite subset N_0 of N such that
$\bigcap\{f_n : n \in N_0\} \notin g$.

Proof. Let $\{f_n : n \in N\}$ be a countable family of α-closed sets of X such that $\bigcap\{f_n : n \in N\} = \emptyset$. Then $\{X \setminus f_n : n \in N\}$ is a countable α-open cover of X. Then, there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n\in N_0}(X \setminus f_n) \notin g$. This leads to $\bigcap_{n\in N_0}[X \setminus (X \setminus f_n)] \notin g$ (i.e, $\bigcap_{n\in N_0} f_n \notin g$). □

Proposition 5.3. *If (X, τ) is countably αg-compact, g and $\acute{g}$ are two grills on X such that $g \supseteq \acute{g}$ then (X, τ) is countably $\alpha\acute{g}$-compact.*

Proof. Let $\{U_n : n \in N\}$ be a countable α-open cover of X. Since (X, τ) is a countably αg-compact, then there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n\in N_0} U_n \notin g$. Since $g \supseteq \acute{g}$, thus $X \setminus \bigcup_{n\in N_0} U_n \notin \acute{g}$. Thus (X, τ) is countably $\alpha\acute{g}$-compact. □

Theorem 5.4. *If $g = P(X) \setminus \emptyset$ is the grill on the space (X, τ), then the following are equivalent*

1- The space (X, τ) is countably α-compact;

2- The space (X, τ) is countably αg-compact.

Proof. 1. $\Longrightarrow$ 2.

Let $\{U_n : n \in N\}$ be a countable α-open cover of X. By 1., there exists a finite subcover $\{U_n : n \in N_o\}$ of X. Then $X \setminus \bigcup_{n\in N_0} U_n = \emptyset \notin g$. Thus (X, τ) is countably αg-compact.

2. $\Longrightarrow$ 1.

Let $\{U_n : n \in N\}$ be a countable α-open cover of X. By 2., there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n \in N_0} U_n \notin g$. Since $g = P(X) \setminus \emptyset$, then $X \setminus \bigcup_{n \in N_0} U_n = \emptyset$. This means that $\{U_n : n \in N_0\}$ is a finite subcover of X. Thus (X, τ) is countably α-compact. □

Proposition 5.5. *If (X, τ) is countably α-compact, then (X, τ) is countably αg-compact, where g is a grill on X.*

Proof. It is clear. □

Definition 5.6. A space (X, τ) is called α-Lindelöf if and only if every α-open cover of X has a countable subcover.

Theorem 5.7. *If (X, τ) is a countably αg-compact and α-Lindelöf space, then (X, τ) is αg-compact, where g is a grill on (X, τ).*

Proof. Let $\{U_\lambda : \lambda \in \Lambda\}$ be an α-open cover of X. Since (X, τ) is α-Lindelöf, there exists a countable subset Λ_1 of Λ such that $X = \bigcup_{\lambda \in \Lambda_1} U_\lambda$. But (X, τ, g) is countably αg-compact and hence there exists a finite subset Λ_0 of Λ_1 such that $X \setminus \bigcup_{\lambda \in \Lambda_0} U_\lambda \notin g$. Thus (X, τ) is αg-compact. □

Theorem 5.8. *Let $f : (X, \tau, g) \to (Y, \sigma)$ be an α-irresolute surjection. If (X, τ, g) is countably αg-compact, then $(Y, \sigma, f(g))$ is countably $\alpha f(g)$-compact.*

Proof. Let $\{V_n : n \in N\}$ be a countable α-open cover of Y. Since f is α-irresolute, then $\{f^{-1}(V_n) : n \in N\}$ is a countable α-open cover of X and hence there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n \in N_0} f^{-1}(V_n) \notin g$. Sice f is surjective, we have $Y \setminus \bigcup_{n \in N_0} V_n = f[X \setminus \bigcup_{n \in N_0} f^{-1}(V_n)] \notin f(g)$. Thus $(Y, \sigma, f(g))$ is countably $\alpha f(g)$-compact. □

Theorem 5.9. *Let $f : (X, \tau, g) \to (Y, \sigma)$ be an α-continuous surjection. If (X, τ, g) is countably αg-compact, then $(Y, \sigma, f(g))$ is countable $f(g)$-compact.*

Proof. Let $\{V_n : n \in N\}$ be a countable open cover of Y. Then $\{Y \setminus V_n : n \in N\}$ is a countable closed subsets of Y. since f is α-continuous , then $\{f^{-1}(Y \setminus V_n) : n \in N\}$ is a countable α-closed subsets of X. Then $\{X \setminus f^{-1}(Y \setminus V_n) : n \in N\}$ is a countable α- open subsets of X. Since $X \setminus f^{-1}(Y \setminus V_n) = f^{-1}(V_n)$, then $\{f^{-1}(V_n) : n \in N\}$ be a countable α-open cover of X. Since (X, τ, g) is countably αg-compact, there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n \in N_0} f^{-1}(V_n) \notin g$. Thus $f(X \setminus \bigcup_{n \in N_0} f^{-1}(V_n)) \notin f(g)$, this leads to $Y \setminus \bigcup_{n \in N_0} V_n \notin f(g)$ and hence $(Y, \sigma, f(g))$ is $f(g)$-compact. □

Theorem 5.10. *If $f : (X, \tau) \to (Y, \sigma, g)$ is pre-α-open bijection and (Y, σ, g) is countably αg-compact, then (X, τ) is countably $\alpha f^{-1}(g)$-compact.*

Proof. Since f is pre-α-open, then $f(F)$ is α-open in (Y, σ, g) for every α-open set F in (X, τ). Since f is bijection, then $f^{-1} : (Y, \sigma, g) \to (X, \tau)$ exists and an α-irresolute surjection. Therefore, the proof follows from the last theorem. □

Theorem 5.11. *If (X, τ, g) is countably αg-compact and g is superset of τ, then (X, τ) is feebly compact.*

Proof. Let $\{U_n : n \in N\}$ be a countable open cover of X, so it is α-open cover of X. Since (X, τ, g) is countably αg-compact, there exists a finite subset N_0 of N such that $X \setminus \bigcup_{n\in N_0} U_n \notin g$. Since $\tau \subseteq g$, then $X \setminus \bigcup_{n\in N_0} U_n \notin \tau$. Thus $Int(X \setminus \bigcup_{n\in N_0} U_n) = \emptyset$, then $X \setminus Cl(\bigcup_{n\in N_0} U_n) = \emptyset$. So $X = Cl\{\cup U_n : n \in N_0\}$ and hence (X, τ) is feebly compact. □

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Accepted: 5.11.2018