Stability of fixed point sets of generalized multivalued α-ψ contraction of Ciric-Berinde type

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Abstract. The purpose of our paper is to study the existence of fixed point theorems for generalized multivalued α-ψ contraction of Ciric-Berinde type by using Hausdorff distance in metric spaces and obtain stability of fixed point sets for such multivalued contraction. Examples are providing to indicate the usefulness of our main result. Moreover, an application to single value mapping is also given.

Keywords: fixed point, multivalued α-admissible, generalized multivalued α-ψ contraction of Ciric-Berinde type, h-upper semicontinuous, Hausdorff metric, stability.

1. Introduction and preliminaries

Stability of fixed points of set valued contractions was defined by Nadler [14] and Markin [10]. The stability results for multivalued contractions have been useful in the area of generalized differential equation, discrete and continuous dynamical system. Stability of fixed points sets for multivalued mapping has been considered in ([6], [11], [13], [15], [18]).

In 2012, Samet et al. [4] introduced the notion of α-ψ contractive mappings and α-admissible mappings in metric space and gave sufficient condition for the existence of fixed points for the class of mappings. Many authors discussed the fixed point results of α-admissible mappings and gave their generalization, extensions in several works like ([1, 2, 3], [5], [12], [16], [17]). Recently, Chaudury and Bandyopadhyay [7] defined multivalued α-admissible mappings, multivalued α-ψ contractions mappings and obtained some stability results for fixed point sets associated with a sequence of multivalued mappings using Hausdorff distance in metric space.

The purpose of this paper is to introduce the concept of generalized multivalued α-ψ contraction of Ciric-Berinde type and to establish fixed point theorems for such mappings which generalizes the results of ([7], [8]). We also show that the fixed point sets of uniformly convergent sequences for the newly defined generalized multivalued α-ψ contraction of Ciric-Berinde type which are also α-admissible and h-upper semicontinuous are stable under certain condition.

Lastly, we obtain fixed point results of single valued mappings by giving applications of our main results of multivalued mappings.

Let X and Y be non-empty sets. T is said to be a multivalued mapping from X to Y if T is a function from X to the power set of Y. We denote a multivalued mapping by $T : X \to 2^y$. A point $x \in X$ is said to be a fixed point of multivalued mapping T if $x \in Tx$. We denote the set of fixed points of T by $Fix(T)$.

The following are the concepts from set valued analysis which we shall use in this paper. Let (X, d) be a metric space. Then

$$\begin{aligned}
&N(X) = \{A : A \text{ is a non-empty subset of } X\},\\
&CL(X) = \{A : A \text{ is a non-empty closed subset of } X\},\\
&C(X) = \{A : A \text{ is a non-empty compact subset of } X\} \text{ and}\\
&CB(X) = \{A : A \text{ is a non-empty closed and bounded subset of } X\}.
\end{aligned}$$

For $A, B \in CB(X)$, define the function $H : CB(X) \times CB(X) \to R^+$ by

$$H(A, B) = \max\{\delta(A, B), \delta(B, A)\},$$

where

$$\delta(A, B) = \sup\{d(a, B), a \in A\}, \quad \delta(B, A) = \sup\{d(b, A), b \in B\}$$

and

$$d(a, C) = \inf\{d(a, x), x \in C\}.$$

Note that H is called Hausdorff metric induced by the metric d.

Let $\alpha : X \times X \to [0, \infty)$ and $\psi : [0, \infty) \to [0, \infty)$ be two functions such that ψ is a continuous and non-decreasing function with $\Phi(t) = \sum_{n=1}^{\infty} \psi^n(t) < \infty$ and $\Phi(t) \to 0$ as $t \to 0$, where ψ^n denotes nth iterate of the function ψ. It is well known that $\psi(t) < t$ for all $t > 0$ and $\psi(0) = 0$ for $t = 0$.

Lemma 1.1 ([14]). *Let (X, d) be a metric space and $A, B \in C(X)$. Let $q \geq 1$. Then for each $x \in A$, there exists $y \in B$ such that $d(x, y) \leq qH(A, B)$.*

Lemma 1.2 ([8]). *Let A and B be two non-empty compact subsets of a metric space (X, d) and $T : A \to C(B)$ be a multivalued mapping. Let $q \geq 1$. Then for $a, b \in A$ and $x \in Ta$, there exists $y \in Tb$ such that $d(x, y) \leq qH(Ta, Tb)$.*

Definition 1.1 ([7],[12]). Let X be a non-empty set. A multivalued mapping $T : X \to N(X)$ is said to be multivalued α-admissible with respect to a function $\alpha : X \times X \to [0, \infty)$, if for $x, y \in X$,

$$\alpha(x, y) \geq 1 \Rightarrow \alpha(a, b) \geq 1, \quad \text{for all } a \in Tx \text{ and } b \in Ty. \tag{1.1}$$

If $T : X \to X$, a single-valued mapping then condition (1.1) of α-admissible reduces to $\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1$ for $x, y \in X$.

Example 1.1 ([7])**.** Let $X = R$, $\alpha : R\times R \to [0,\infty)$. We define $\alpha(x,y) = x^2+y^2$, where $x, y \in R$. Define $T : R \to N(X)$ by $Tx = \{\sqrt{|x|}, -\sqrt{|x|}\}$. Then T is multivalued α-admissible.

Definition 1.2 ([9])**.** Let (X, d) be a metric space and $T : X \to 2^X$ be a closed valued multifunction. We say that T is an α_*-ψ contractive multifunction whenever

$$\alpha_*(Tx,Ty)H(Tx,Ty) \le \psi(d(x,y)) \quad \text{for } x, y \in X, \tag{1.2}$$

where $\alpha_*(Tx,Ty) = \inf\{\alpha(a,b) : a \in Tx, b \in Ty\}$.

Definition 1.3 ([7])**.** Let (X, d) be a metric space. A multivalued mapping $T : X \to C(X)$ is called multivalued α-ψ contraction if

$$\alpha(x,y)H(Tx,Ty) \le \psi(d(x,y)) \quad \text{for } x, y \in X, \tag{1.3}$$

Remark 1.1 ([7])**.** In (1.3) of Definition 1.3, we have $\alpha(x,y)$ instead of $\alpha_*(Tx,Ty)$ which has been considered in (1.2) of Definition 1.2. $\alpha_*(Tx,Ty)$ is defined as

$$\alpha_*(Tx,Ty) = \inf\{\alpha(a,b) : a \in Tx, b \in Ty\} \quad \text{for } x, y \in X.$$

From the definition it is clear that $\alpha_*(Tx,Ty)$ is not necessarily equal to $\alpha(x,y)$ and also we cannot compare $\alpha(x,y)$ with $\alpha_*(Tx,Ty)$. Therefore Definition 1.3 is independent of Definition 1.2.

Definition 1.4. Let (X, d) be a metric space. A self mapping $T : X \to X$ is said to be h-upper semicontinuous if and only if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have $\lim_{n\to\infty} d(Tx_n, Tx) = 0$.

Definition 1.5 ([12], [17])**.** Let (X, d) be a metric space. A multivalued mapping $T : X \to C(X)$ is said to be h-upper semicontinuous if and only if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have $\lim_{n\to\infty} \delta(Tx_n, Tx) = 0$.

2. Main results

We introduce *generalized multivalued α-ψ contraction of Ciric-Berinde type* which differs from Definition 1.2 and generalization of Definition 1.3.

Definition 2.1. Let (X, d) be a metric space. A multivalued mapping $T : X \to C(X)$ is called a generalized multivalued α-ψ contraction of Ciric-Berinde type if

$$\begin{aligned} &\alpha(x,y)H(Tx,Ty) \le \psi(M(x,y)) \\ &\quad + L\min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\} \end{aligned} \tag{2.1}$$

for all $x, y \in X$ with $\alpha(x,y) \ge 1$, where $L \ge 0$ and

$$M(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}.$$

Theorem 2.1. *Let (X, d) be a complete metric space and $T : X \to C(X)$ be a generalized multivalued α-ψ contraction of Ciric-Berinde type. Also suppose that the following conditions are satisfied:*

(*i*) *T is multivalued α-admissible,*

(*ii*) *there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$,*

(*iii*) *T is h-upper semi continuous.*

Then T has a fixed point.

Proof. By the condition (ii) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$. Clearly if $x_0 = x_1$ or $x_1 \in Tx_1$, we find that x_1 is a fixed point of T and so, we can conclude the proof. Now, we assume that $x_0 \neq x_1$ and $x_1 \notin Tx_1$ and hence $d(x_1, Tx_1) > 0$. By Lemma 1.2 for $x_1 \in Tx_0$ there exists $x_2 \in Tx_1$ such that

$$d(x_1, x_2) \leq \alpha(x_0, x_1)H(Tx_0, Tx_1).$$

Applying (2.1) and using the monotone property of ψ, we have

$$\begin{aligned}
d(x_1, x_2) &\leq \alpha(x_0, x_1)H(Tx_0, Tx_1)\\
&\leq \psi\left(\max\left\{d(x_0, x_1), \frac{d(x_0, Tx_0)d(x_1, Tx_1)}{d(x_0, x_1)}, \frac{d(x_0, Tx_1) + d(x_1, Tx_0)}{2}\right\}\right)\\
&\quad + L\min\{d(x_0, Tx_0), d(x_1, Tx_1), d(x_0, Tx_1), d(x_1, Tx_0)\}\\
&\leq \psi\left(\max\left\{d(x_0, x_1), \frac{d(x_0, x_1)d(x_1, x_2)}{d(x_0, x_1)}, \frac{d(x_0, x_2) + d(x_1, x_1)}{2}\right\}\right)\\
&\quad + L\min\{d(x_0, x_1), d(x_1, x_2), d(x_0, x_2), d(x_1, x_1)\}\\
&\leq \psi\left(\max\left\{d(x_0, x_1), d(x_1, x_2), \frac{d(x_0, x_2)}{2}\right\}\right).
\end{aligned}$$

Since

$$\frac{d(x_0, x_2)}{2} \leq \frac{d(x_0, x_1) + d(x_1, x_2)}{2} \leq \max\{d(x_0, x_1), d(x_1, x_2)\},$$

it follow that

$$d(x_1, x_2) \leq \psi(\max\{d(x_0, x_1), d(x_1, x_2)\}),$$

if $\max\{d(x_0, x_1), d(x_1, x_2)\} = d(x_1, x_2)$, then we have

$$0 < d(x_1, x_2) \leq \psi(d(x_1, x_2)) < d(x_1, x_2),$$

which is a contradiction. Thus $\max\{d(x_0, x_1), d(x_1, x_2)\} = d(x_0, x_1)$ and since ψ is strictly increasing, we have

$$d(x_1, x_2) \leq \psi(d(x_0, x_1)). \tag{2.2}$$

Since T is α-admissible, from condition (ii) and $x_2 \in Tx_1$, we have $\alpha(x_1, x_2) \geq 1$. If $x_2 \in Tx_2$ then x_2 is a fixed point. Assume that $x_2 \notin Tx_2$, that is $d(x_2, Tx_2) > 0$. By Lemma 1.2 for $x_2 \in Tx_1$ there exists $x_3 \in Tx_2$ such that

$$\begin{aligned}
d(x_2, x_3) &\leq \alpha(x_1, x_2) H(Tx_1, Tx_2) \\
&\leq \psi\left(\max\left\{d(x_1, x_2), \frac{d(x_1, Tx_1)d(x_2, Tx_2)}{d(x_1, x_2)}, \frac{d(x_1, Tx_2) + d(x_2, Tx_1)}{2}\right\}\right) \\
&\quad + L\min\{d(x_1, Tx_1), d(x_2, Tx_2), d(x_1, Tx_2), d(x_2, Tx_1)\} \\
&\leq \psi\left(\max\left\{d(x_1, x_2), \frac{d(x_1, x_2)d(x_2, x_3)}{d(x_1, x_2)}, \frac{d(x_1, x_3) + d(x_2, x_2)}{2}\right\}\right) \\
&\quad + L\min\{d(x_1, x_2), d(x_2, x_3), d(x_1, x_3), d(x_2, x_2)\} \\
&\leq \psi\left(\max\left\{d(x_1, x_2), d(x_2, x_3), \frac{d(x_1, x_3)}{2}\right\}\right).
\end{aligned}$$

Since

$$\frac{d(x_1, x_3)}{2} \leq \frac{d(x_1, x_2) + d(x_2, x_3)}{2} \leq \max\{d(x_1, x_2), d(x_2, x_3)\},$$

it follow that

$$d(x_2, x_3) \leq \psi(\max\{d(x_1, x_2), d(x_2, x_3)\}),$$

if $\max\{d(x_1, x_2), d(x_2, x_3)\} = d(x_2, x_3)$, then we have

$$0 < d(x_2, x_3) \leq \psi(d(x_2, x_3)) < d(x_2, x_3),$$

which is a contradiction.

Thus $\max\{d(x_1, x_2), d(x_2, x_3)\} = d(x_1, x_2)$ and since ψ is strictly increasing, we have

$$d(x_2, x_3) \leq \psi(d(x_1, x_2)) < \psi^2(d(x_0, x_1)) \tag{2.3}$$

Since $x_2 \in Tx_1$, $x_3 \in Tx_2$ and $\alpha(x_1, x_2) \geq 1$, the α-admissibility of T implies that $\alpha(x_2, x_3) \geq 1$. Continuing this process, we construct a sequence $\{x_n\}$ such that for all $n \geq 0$,

$$\begin{aligned}
&x_n \notin Tx_n, x_{n+1} \in Tx_n, \\
&\alpha(x_n, x_{n+1}) \geq 1
\end{aligned}$$

and

$$d(x_{n+1}, x_{n+2}) \leq \psi(d(x_n, x_{n+1})). \tag{2.4}$$

By repeated application (2.4) and monotonic property of ψ, we have

$$d(x_{n+1}, x_{n+2}) \leq \psi(d(x_n, x_{n+1})) \leq \psi^2(d(x_{n-1}, x_n)) \leq \cdots \leq \psi^{n+1}(d(x_0, x_1)).$$

Then by property of ψ, we have

$$\sum_n d(x_n, x_{n+1}) \leq \sum_n \psi^n(d(x_0, x_1) = \Phi(d(x_0, x_1)) < \infty \,.$$

This shows that $\{x_n\}$ is a Cauchy sequence in X. Hence, there exists $z \in X$, such that $x_n \to z$ as $n \to \infty$, that is $\lim_{n\to\infty} d(x_n, z) = 0$.

Consider

$$\begin{aligned} d(z, Tz) &\leq d(z, x_{n+1}) + d(x_{n+1}, Tz) \\ &\leq d(z, x_{n+1}) + \delta(Tx_n, Tz). \end{aligned} \tag{2.5}$$

Since $\lim_{n\to\infty} d(z, x_{n+1}) = 0$ and by using h-upper semicontinuity of T we have $\lim_{n\to\infty} \delta(Tx_n, Tz) = 0$. By letting $n \to \infty$ in the inequality (2.5), we obtain $d(z, Tz) = 0$. Since Tz is compact and hence Tz is closed, that is, $Tz = \overline{Tz}$, where $\overline{Tz}$ denotes the closure of Tz. Now $d(z, Tz) = 0$ implies that $z \in \overline{Tz} = Tz$, that is, z is a fixed point of T. □

Notice that one can relax the h-upper semicontinuity hypothesis on T, by introducing another regularity condition as shown in next theorem.

Theorem 2.2. *Let (X, d) be a complete metric space and $T : X \to C(X)$ be a generalized multivalued α-ψ contraction of Ciric-Berinde type. Also suppose that the following conditions are satisfied:*

(i) T is multivalued α-admissible,

(ii) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$,

(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n, where $x_{n+1} \in Tx_n$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.

Then T has a fixed point.

Proof. By the condition (ii) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$. Following the proof of Theorem 2.1, we obtain a sequence $\{x_n\} \subset X$ with $\lim_{n\to\infty} d(x_n, z) = 0$ for some $z \in X$ such that $x_n \notin Tx_n, x_{n+1} \in Tx_n$ and $\alpha(x_n, x_{n+1}) \geq 1$ for all n. By condition (iii), there exists a sequence $\{x_n\}$ such that $\alpha(x_n, z) \geq 1$ for all n. Now we prove that $z \in Tz$. We have

$$\begin{aligned} M(x_n, z) &= \max\left\{d(x_n, z), \frac{d(x_n, Tx_n)d(z, Tz)}{d(x_n, z)}, \frac{d(x_n, Tz) + d(z, Tx_n)}{2}\right\} \\ &\leq \max\left\{d(x_n, z), \frac{d(x_n, x_{n+1})d(z, Tz)}{d(x_n, z)}, \frac{d(x_n, Tz) + d(z, x_{n+1})}{2}\right\} \\ &\leq \max\left\{d(x_n, z), \frac{d(x_n, x_{n+1})d(z, Tz)}{d(x_n, z)}, \frac{[d(x_n, z) + d(z, Tz)] + d(z, x_{n+1})}{2}\right\}. \end{aligned}$$

From $x_n \to z$, we deduce that

$$\lim_{n\to\infty} M(x_n, z) = d(z, Tz).$$

Since T is generalized multivalued α-ψ contraction of Ciric-Berinde type, for all n we have

$$\begin{aligned} d(z,Tz) &\le d(z,x_{n+1}) + d(x_{n+1},Tz) \le d(z,x_{n+1}) + H(Tx_n,Tz) \\ &\le d(z,x_{n+1}) + \psi(M(x_n,z)) \\ &\quad + L\min\{d(x_n,Tx_n), d(z,Tz), d(x_n,Tz), d(z,Tx_n)\} \\ &\le d(z,x_{n+1}) + \psi(M(x_n,z)) \\ &\quad + L\min\{d(x_n,x_{n+1}), d(z,Tz), d(x_n,Tz), d(z,x_{n+1})\}. \end{aligned}$$

Letting $n \to \infty$ in the above inequality and $\psi(t) < t$, we have

$$d(z,Tz) \le \psi(d(z,Tz)) < d(z,Tz)$$

which implies $d(z,Tz) = 0$. Since Tz is compact and hence Tz is closed, that is, $Tz = \overline{Tz}$, where $\overline{Tz}$ denotes the closure of Tz. Now $d(z,Tz) = 0$ implies that $z \in \overline{Tz} = Tz$, that is, z is a fixed point of T. □

Remark 2.1. By taking $L = 0$ in Theorem 2.2, replacing $M(x,y)$ by $d(x,y)$, Theorem 2.2 reduces to [7, Theorem 2.1] which is as follows:

Corollary 2.1 ([7])**.** *Let (X,d) be a complete metric space and $T : X \to CL(X)$ be a multivalued α-ψ contraction. Also suppose that the following conditions are satisfied:*

(i) T is multivalued α-admissible,

(ii) there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1) \ge 1$,

(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1}) \ge 1$ for all n, where $x_{n+1} \in Tx_n$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n,x) \ge 1$ for all n.

Then T has a fixed point.

Example 2.1. Let $X = [0,\infty)$ and $d(x,y) = |x-y|$. Let define the multivalued mapping $T : X \to C(X)$ as

$$Tx = \begin{cases} \left\{1, \frac{1}{2x}\right\}, & \text{if } x > 1, \\ \left\{0, \frac{x}{8}\right\}, & \text{if } 0 \le x \le 1. \end{cases}$$

Now we define the functions $\alpha : X \times X \to [0,\infty)$ and $\psi : [0,\infty) \to [0,\infty)$ as follows:

$$\alpha(x,y) = \begin{cases} 2, & \text{if } x,y \in (0,1], \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad \psi(t) = \frac{1}{2}t.$$

Obviously, conditions (ii) of Theorem is satisfied with $x_0 = \frac{1}{2}$ and T is multivalued α-admissible. Now we show that T is generalized multivalued α-ψ contraction of Ciric-Berinde type.

Taking for $x, y \in [0,1]$, we have

$$\begin{aligned}
d(0,Ty) &= \inf\left\{0, \frac{y}{8}\right\} = 0,\\
d\left(\frac{x}{8}, Ty\right) &= \inf\left\{\left|0-\frac{x}{8}\right|, \left|\frac{x}{8}-\frac{y}{8}\right|\right\},\\
d(0,Tx) &= \inf\left\{0, \frac{x}{8}\right\} = 0,\\
d\left(\frac{y}{8}, Tx\right) &= \inf\left\{\left|0-\frac{x}{8}\right|, \left|\frac{x}{8}-\frac{y}{8}\right|\right\}.
\end{aligned}$$

Then we have

$$\begin{aligned}
H(Tx,Ty) &= \max\left\{\sup_{x\in Tx} d(x,Ty), \sup_{y\in Ty} d(y,Tx)\right\}\\
&= \max\left\{\inf\left\{\left|\frac{x}{8}\right|, \left|\frac{x}{8}-\frac{y}{8}\right|\right\}, \inf\left\{\left|\frac{y}{8}\right|, \left|\frac{y}{8}-\frac{x}{8}\right|\right\}\right\}\\
&= \left|\frac{x}{8}-\frac{y}{8}\right|.
\end{aligned}$$

Now,

$$\begin{aligned}
\alpha(x,y)H(Tx,Ty) &= 2\times\left|\frac{x}{8}-\frac{y}{8}\right|\\
&= \frac{1}{4}|x-y|\\
&\leq \frac{1}{2}|x-y|\\
&= \psi(d(x,y)\\
&\leq \psi(M(x,y)) + L\min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}.
\end{aligned}$$

Thus condition (2.1) is satisfied.

Hence all the conditions of Theorems 2.1 and 2.2 are satisfied and T has a fixed point at $x = 0$.

3. Stability of fixed point sets

Stability is a concept associated with the limiting behavior of a system. The study of the relationship between the convergence of a sequence of mappings and their fixed points, known as the stability of fixed points. A sequence of fixed point sets is said to be stable when it converges to the corresponding fixed point sets of the limiting function in the Hausdorff metric. Multivalued maps often have more fixed points than single valued maps. Therefore, the set of fixed points of multivalued mappings becomes larger and hence more interesting for study of stability.

In this section, we consider the stability of fixed point sets of the multivalued contractions mentioned in section 2.

Theorem 3.1. *Let (X,d) be a complete metric space, and $F(T_1)$, $F(T_2)$ are the fixed point sets of T_1 and T_2 respectively, where $T_i : X \to C(X)$, $i = 1,2$. Each T_i is generalized multivalued α-ψ contraction of Ciric-Berinde type as defined in Definition 2.1 with the same α, ψ and L. Also each T_i satisfies the following conditions:*

(i) *each T_i is multivalued α-admissible,*

(ii) *each T_i is h-upper semi continuous,*

(iii) *for any $x \in F(T_1)$, we have $\alpha(x,y) \geq 1$ whenever $y \in T_2x$ and for any $x \in F(T_2)$, we have $\alpha(x,y) \geq 1$ whenever $y \in T_1x$.*

Then $H(F(T_1), F(T_2)) \leq \Phi(k)$, where $k = \sup_{x\in X} H(T_1x, T_2x)$.

Proof. From Theorem 2.1, the set of fixed points of T_i $(i = 1,2)$ are non-empty, that is $F(T_i) \neq \phi$, for $i = 1,2$. Let $x_0 \in F(T_1)$, that is $x_0 \in T_1x_0$. Then by Lemma 1.1, there exists $x_1 \in T_2x_0$ such that

$$d(x_0, x_1) \leq H(T_1x_0, T_2x_0).$$

Since $x_0 \in F(T_1)$ and $x_1 \in T_2x_0$, by condition(iii) of the theorem, we have $\alpha(x_0, x_1) \geq 1$. By Lemma 1.2, for $x_1 \in T_2x_0$ there exists $x_2 \in T_2x_1$ such that

$$d(x_1, x_2) \leq \alpha(x_0, x_1) H(T_2x_0, T_2x_1).$$

Then, arguing similarly as in the proof of Theorem 2.1, we construct a sequence $\{x_n\}$ such that for all $n \geq 0$,

$$\begin{aligned} &x_{n+1} \in T_2x_n, \\ &\alpha(x_n, x_{n+1}) \geq 1, \\ &d(x_{n+1}, x_{n+2}) \leq \psi(d(x_n, x_{n+1})) \end{aligned}$$

and

$$d(x_{n+1}, x_{n+2}) \leq \psi(d(x_n, x_{n+1})) \leq \psi^2(d(x_{n-1}, x_n)) \leq \ldots \leq \psi^{n+1}(d(x_0, x_1)).$$

Then by property of ψ, we have

$$\sum_n d(x_n, x_{n+1}) \leq \sum_n \psi^n(d(x_0, x_1)) = \Phi(d(x_0, x_1)) < \infty\,.$$

This shows that $\{x_n\}$ is a Cauchy sequence in X. Since (X,d) is complete, there exists $z \in X$ such that $x_n \to z$ as $n \to \infty$.

Now, we prove that $z \in T_2 z$. For all $n \geq 0$, $x_{n+1} \in T_2 x_n$. Suppose that T_2 is h-upper semicontinuous. We have

$$\begin{aligned} d(z, T_2 z) &\leq d(z, x_{n+1}) + d(x_{n+1}, T_2 z) \\ &\leq d(z, x_{n+1}) + \delta(T_2 x_n, T_2 z). \end{aligned}$$

Since $\lim_{n\to\infty} d(z, x_{n+1}) = \lim_{n\to\infty} \delta(T_2 x_n, T_2 z) = 0$, by letting $n \to \infty$ in the above inequality, we obtain $d(z, T_2 z) = 0$. Thus $z \in T_2 z$, that is, z is a fixed point of T_2.

Using $d(x_0, x_1) \leq H(T_1 x_0, T_2 x_0)$ and the definition of k, we have

$$d(x_0, x_1) \leq H(T_1 x_0, T_2 x_0) \leq k = \sup_{x \in X} H(T_1 x, T_2 x).$$

Now using triangular inequality,

$$\begin{aligned} d(x_0, z) &\leq \sum_{i=0}^{n} (d(x_i, x_{i+1})) + d(x_{n+1}, z) \\ &\leq \sum_{i=0}^{\infty} (d(x_i, x_{i+1})) \\ &\leq \sum_{i=0}^{\infty} \psi^i (d(x_0, x_1)) \\ &\leq \sum_{i=0}^{\infty} \psi^i (k) = \Phi(k). \end{aligned}$$

Thus, given arbitrary $x_0 \in F(T_1)$, we can find $z \in F(T_2)$ for which $d(x_0, z) \leq \Phi(k)$. Reversing the roles of T_1 and T_2 we also conclude that for each $y_0 \in F(T_2)$ there exists $w \in F(T_1)$ such that $d(y_0, w) \leq \Phi(k)$. Hence $H(F(T_1), F(T_2)) \leq \Phi(k)$. □

Theorem 3.2. *Let (X, d) be a complete metric space, and $F(T_1)$, $F(T_2)$ are the fixed point sets of T_1 and T_2 respectively, where $T_i : X \to C(X)$, $i = 1, 2$. Each T_i is generalized multivalued α-ψ contraction of Ciric-Berinde type as defined in Definition 2.1 with the same α, ψ and L. Also each T_i satisfies the following conditions:*

- *(i) for any $x \in F(T_1)$, we have $\alpha(x, y) \geq 1$ whenever $y \in T_2 x$ and for any $x \in F(T_2)$, we have $\alpha(x, y) \geq 1$ whenever $y \in T_1 x$.*
- *(ii) each T_i is multivalued α-admissible,*
- *(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n, where $x_{n+1} \in T_i x_n$, $i = 1, 2$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.*

Then $H(F(T_1), F(T_2)) \leq \Phi(m)$, where $m = \sup_{x \in X} H(T_1 x, T_2 x)$.

Proof. From Theorem 2.2, the set of fixed points of T_i $(i = 1, 2)$ are non empty, that is $F(T_i) \neq \phi$, for $i = 1, 2$. Let $x_0 \in F(T_1)$, that is $x_0 \in T_1 x_0$. Arguing similarly as in the proof of Theorem 3.1, we prove that $\{x_n\}$ is a Cauchy sequence in X. Since (X, d) is complete, there exist $z \in X$ such that $x_n \to z$ as $n \to \infty$.

Now, we prove that $z \in T_2 z$. For all $n \geq 0$, $x_{n+1} \in T_2 x_n$. Therefore $d(x_{n+1}, T_2 z) \leq H(T_2 x_n, T_2 z)$. By (iii), $\alpha(x_n, z) \geq 1$ for all n. Hence we have for all n,

$$d(x_{n+1}, T_2 z) \leq \alpha(x_n, z) H(T_2 x_n, T_2 z).$$

Letting $n \to \infty$ in the above inequality and $\psi(t) < t$, we have

$$d(z, T_2 z) \leq \psi(d(z, T_2 z)) \leq d(z, T_2 z),$$

which implies that $d(z, T_2 z) = 0$. Thus $z \in T_2 z$, that is, z is a fixed point of T_2.

Using $d(x_0, x_1) \leq H(T_1 x_0, T_2 x_0)$ and the definition of k, we have

$$d(x_0, x_1) \leq H(T_1 x_0, T_2 x_0) \leq m = \sup_{x \in X} H(T_1 x, T_2 x).$$

Now using triangular inequality,

$$\begin{aligned}
d(x_0, z) &\leq \sum_{i=0}^{n} (d(x_i, x_{i+1})) + d(x_{n+1}, z) \\
&\leq \sum_{i=0}^{\infty} (d(x_i, x_{i+1})) \\
&\leq \sum_{i=0}^{\infty} \psi^i(d(x_0, x_1)) \\
&\leq \sum_{i=0}^{\infty} \psi^i(k) = \Phi(m).
\end{aligned}$$

Thus, given arbitrary $x_0 \in F(T_1)$, we can find $z \in F(T_2)$ for which $d(x_0, z) \leq \Phi(m)$. Reversing the role of T_1 and T_2 we also conclude that for each $y_0 \in F(T_2)$ there exists $w \in F(T_1)$ such that $d(y_0, w) \leq \Phi(m)$. Hence $H(F(T_1), F(T_2)) \leq \Phi(m)$. □

Lemma 3.1 ([7]). *Let (X, d) be a complete metric space. If $\{T_n : X \to C(X) : n \in \mathbb{N}\}$ be a sequence of multivalued α-admissible with the same α and which is uniformly convergent to a multivalued mapping $T : X \to C(X)$, then T is multivalued α-admissible if the following condition is satisfied:*

$$\alpha(x_n, y_n) \geq 1, \quad \textit{for every } n \in \mathbb{N} \Rightarrow \alpha(a, b) \geq 1, \tag{3.1}$$

where $x_n \to a$ and $y_n \to b$ as $n \to \infty$.

Proof. Let $\alpha(x, y) \geq 1$, for some $x, y \in X$. Suppose $a \in Tx$ and $b \in Ty$ be arbitrary. Since $T_n \to T$ uniformly, there exist two sequences $\{x_n\}$ in $\{T_n x\}$ and $\{y_n\}$ in $\{T_n y\}$, such that $x_n \to a$ and $y_n \to b$ as $n \to \infty$. Since $\alpha(x, y) \geq 1$ and T_n is multivalued α-admissible for each $n \in \mathbb{N}$, it follows that $\alpha(x_n, y_n) \geq 1$ for each $n \in \mathbb{N}$. Thus by (3.1), it follows that $\alpha(a, b) \geq 1$. Thus for $x, y \in X$, we have

$$\alpha(x, y) \geq 1 \Rightarrow \alpha(a, b) \geq 1, \quad \text{where } a \in Tx \text{ and } b \in Ty.$$

Hence T is multivalued α-admissible. □

Lemma 3.2. *Let (X, d) be a complete metric space. If $\{T_n : X \to C(X) : n \in \mathbb{N}\}$ be a sequence of generalized multivalued α-ψ contraction of Ciric-Berinde type which is uniformly convergent to a multivalued mapping $T : X \to C(X)$, then T is generalized multivalued α-ψ contraction of Ciric-Berinde type.*

Proof. Since each T_n is generalized multivalued α-ψ contraction of Ciric-Berinde type for every $n \in \mathbb{N}$, therefore

$$\begin{aligned}
&\alpha(x, y) H(T_n x, T_n y) \\
&\leq \psi\left(\max\left\{d(x, y), \frac{d(x, T_n x) d(y, T_n y)}{d(x, y)}, \frac{d(x, T_n y) + d(y, T_n x)}{2}\right\}\right) \\
&\quad + L \min\{d(x, T_n x), d(y, T_n y), d(x, T_n y), d(y, T_n x)\}.
\end{aligned}$$

Since the sequence $\{T_n\}$ is uniformly convergent to T and ψ is continuous, taking limit $n \to \infty$ in the above inequality, we get

$$\begin{aligned}
\alpha(x, y) H(Tx, Ty) &\leq \psi\left(\max\left\{d(x, y), \frac{d(x, Tx) d(y, Ty)}{d(x, y)}, \frac{d(x, Ty) + d(y, Tx)}{2}\right\}\right) \\
&\quad + L \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\},
\end{aligned}$$

for all $x, y \in X$. Hence T is generalized multivalued α-ψ contraction of Ciric-Berinde type. □

Lemma 3.3. *Let (X, d) be a complete metric space. If $\{T_n : X \to C(X) : n \in \mathbb{N}\}$ be a sequence of h-upper semicontinuous which is uniformly convergent to a multivalued mapping $T : X \to C(X)$, then T is h-upper semicontinuous mapping.*

Proof. Since each T_n is h-upper semicontinuous for all $n \geq 1$. Then by definition of h-upper semicontinuous mapping for each $x \in X$ and $x_n \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have

$$\lim_{n\to\infty} \delta(T_n(x_n), T_n(x)) = 0.$$

Since $T_n \to T$ uniformly, letting $n \to \infty$, we have for each $x \in X$ and $x_n \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have $\lim_{n\to\infty} \delta(Tx_n, Tx) = 0$, which implies that T is h-upper semicontinuous mapping. □

Theorem 3.3. *Let (X, d) be a complete metric space. Let $\{T_n : X \to C(X) : n \in \mathbb{N}\}$ be a sequence of generalized multivalued α-ψ contraction of Ciric-Berinde type which are also multivalued α-admissible and h-upper semicontinuous with the same α, ψ and L is uniformly convergent to a multivalued mapping $T : X \to C(X)$. Also suppose that the following hold:*

(i) if $\{x_n\}$ and $\{y_n\}$ are two sequences in X with $x_n \to a$ and $y_n \to b$ as $n \to \infty$, then $\alpha(x_n, y_n) \geq 1$, for every $n \in \mathbb{N} \Rightarrow \alpha(a, b) \geq 1$,

(ii) for every $n \in \mathbb{N}$, for any $x \in F\{T_n\}$, we have $\alpha(x, y) \geq 1$ whenever $y \in Tx$, and for any $x \in F(T)$, we have $\alpha(x, y) \geq 1$ whenever $y \in T_n x$.

Then $H(F(T_n), F(T)) \to 0$ as $n \to \infty$, that is, the fixed point of T_n are stable.

Proof. By Lammas 3.1, 3.2 and 3.3, it follows that T is generalized multivalued α-ψ contraction of Ciric-Berinde type, multivalued α-admissible and h-upper semicontinuous. Let $k_n = \sup_{x \in X} H(T_n x, Tx)$. Since the sequence $\{T_n\}$ is uniformly convergent to T on X. Therefore

$$\lim_{n \to \infty} k_n = \lim_{n \to \infty} \sup_{x \in X} H(T_n x, Tx) = 0.$$

Using Theorem 3.1, we get

$$H(F(T_n), F(T)) \leq \Phi(k_n), \quad \text{for every } n \in \mathbb{N}.$$

Since ψ is continuous and $\Phi(t) \to 0$ as $t \to 0$, we have

$$\lim_{n \to \infty} H(F(T_n), F(T)) \leq \lim_{n \to \infty} \Phi(k_n) = 0,$$

that is $\lim_{n \to \infty} H(F(T_n), F(T)) = 0$. Hence the proof is complete. □

Theorem 3.4. *Let (X, d) be a complete metric space. Let $\{T_n : X \to C(X) : n \in \mathbb{N}\}$ be a sequence of generalized multivalued α-ψ contraction of Ciric type which are also multivalued α-admissible with the same α, ψ and L is uniformly convergent to a multivalued mapping $T : X \to C(X)$. Also suppose that the following hold:*

(i) if $\{x_n\}$ and $\{y_n\}$ are two sequences in X with $x_n \to a$ and $y_n \to b$ as $n \to \infty$, then $\alpha(x_n, y_n) \geq 1$, for every $n \in \mathbb{N} \Rightarrow \alpha(a, b) \geq 1$,

(ii) for every $n \in \mathbb{N}$, for any $x \in F\{T_n\}$, we have $\alpha(x, y) \geq 1$ whenever $y \in Tx$ and for any $x \in F(T)$, we have $\alpha(x, y) \geq 1$ whenever $y \in T_n x$.

Then $H(F(T_n), F(T)) \to 0$ as $n \to \infty$, that is, the fixed point of T_n are stable.

Proof. By Lammas 3.1 and 3.2, it follows that T is generalized multivalued α-ψ contraction of Ciric-Berinde type and multivalued α-admissible.
Let $m_n = \sup_{x\in X} H(T_n x, Tx)$. Since the sequence $\{T_n\}$ is uniformly convergent to T on X. Therefore

$$\lim_{n\to\infty} m_n = \lim_{n\to\infty} \sup_{x\in X} H(T_n x, Tx) = 0.$$

Using Theorem 3.2, we get

$$H(F(T_n), F(T)) \leq \Phi(m_n), \quad \text{for every } n \in \mathbb{N}.$$

Since ψ is continuous and $\Phi(t) \to 0$ as $t \to 0$, we have

$$\lim_{n\to\infty} H(F(T_n), F(T)) \leq \lim_{n\to\infty} \Phi(m_n) = 0,$$

that is $\lim_{n\to\infty} H(F(T_n), F(T)) = 0$. Hence the proof is complete. □

Example 3.1. Let $X = [0, \infty)$ and $d(x, y) = |x - y|$. Let define the multivalued mappings $T : X \to CL(X)$ as

$$T_n x = \begin{cases} \left\{1 + \frac{1}{n}, \frac{1}{2x} + \frac{1}{n}\right\}, & \text{if } x > 1, \\ \left\{\frac{1}{n}, \frac{1}{n} + \frac{x}{8}\right\}, & \text{if } 0 < x \leq 1, \\ \{0\}, & \text{if } x = 0. \end{cases}$$

Now we define the functions $\alpha : X \times X \to [0, \infty)$ and $\psi : [0, \infty) \to [0, \infty)$ as follows:

$$\alpha(x, y) = \begin{cases} 2, & \text{if } x, y \in (0, 1], \\ 0, & \text{otherwise}, \end{cases} \quad \text{and} \quad \psi(t) = \frac{1}{2}t.$$

Each T_n is multivalued α-admissible. $T_n \to T$ as $n \to \infty$. Then T is define by

$$Tx = \begin{cases} \left\{1, \frac{1}{2x}\right\}, & \text{if } x > 1, \\ \left\{0, \frac{x}{8}\right\}, & \text{if } 0 \leq x \leq 1. \end{cases}$$

Each T_n is generalized multivalued α-ψ contraction of Ciric-Berinde type and T is also. Let $x, y \in (0, 1]$,

$$\begin{aligned} H(T_n x, T_n y) &= \max\left\{ \sup_{x\in Tx} d(x, Ty), \sup_{y\in Ty} d(y, Tx) \right\} \\ &= \max\left\{ \inf\left\{ \left|\frac{x}{8}\right|, \left|\frac{x}{8} - \frac{y}{8}\right| \right\}, \inf\left\{ \left|\frac{y}{8}\right|, \left|\frac{y}{8} - \frac{x}{8}\right| \right\} \right\} \\ &= \left|\frac{x}{8} - \frac{y}{8}\right|. \end{aligned}$$

Therefore,

$$\alpha(x, y) H(T_n x, T_n y) \leq \psi(M(x, y)) + L \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}.$$

Thus conditions of Theorem 3.3 and Theorem 3.4 are satisfied. $F(T_1) = \{0, 1\}$ and $F(T_n) = \{0\}$ for $n \geq 2$. Hence $\mathrm{H}(F(T_n), F(T)) \to 0$ as $n \to \infty$.

4. Application to single valued mappings

In this section we obtain some fixed point results for single valued mappings by an application of the corresponding results of section 2.

Theorem 4.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a single valued mapping. Suppose that for all $x, y \in X$,*

$$\alpha(x,y)d(Tx,Ty) \leq \psi\left(\max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}\right) + L\min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\} \tag{4.1}$$

where the conditions α, ψ and L are same as in Definition 2.1. Also suppose that the following conditions are satisfied:

(i) T is α-admissible,

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$,

(iii) T is h-upper semi continuous.

Then T has a fixed point.

Proof. We know that for every $x \in X, \{x\}$ is compact in X. Now, we define multivalued mapping $F : X \to C(X)$ as $Fx = \{Tx\}$, for $x \in X$. Let $x_0, y_0 \in X$ such that $\alpha(x_0, y_0) \geq 1$. Then by α-admissible of T, we have $\alpha(Tx_0, Ty_0) \geq 1$, that is, $\alpha(x_1, y_1) \geq 1$, where $x_1 \in Fx_0 = \{Tx_0\}$ and $y_1 \in Fy_0 = \{Ty_0\}$. Therefore, for $x_0, y_0 \in X$,

$$\alpha(x_0, y_0) \geq 1 \Rightarrow \alpha(x_1, y_1) \geq 1, \quad \text{where } x_1 \in Fx_0 \text{ and } y_1 \in Fy_0,$$

that is, F is a multivalued α-admissible mapping.

Let $x, y \in X$. Then by using (4.1), we have

$$\begin{aligned}\alpha(x,y)H(Fx,Fy) &= \alpha(x,y)d(Tx,Ty) \\ &\leq \psi\left(\max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{d(x,y)}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}\right) \\ &\quad + L\min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\} \\ &\leq \psi\left(\max\left\{d(x,y), \frac{d(x,Fx)d(y,Fy)}{d(x,y)}, \frac{d(x,Fy)+d(y,Fx)}{2}\right\}\right) \\ &\quad + L\min\{d(x,Fx), d(y,Fy), d(x,Fy), d(y,Fx)\},\end{aligned}$$

that is, F satisfy condition (2.1). Therefore, F is a generalized multivalued α-ψ contraction of Ciric-Berinde type of the Theorem 2.1.

Suppose there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Let $x_1 \in Fx_0 = \{Tx_0\}$. Then $\alpha(x_0, Tx_0) \geq 1$ means $\alpha(x_0, x_1) \geq 1$. Therefore, there exists $x_0 \in X$ and $x_1 \in Fx_0$ such that $\alpha(x_0, x_1) \geq 1$.

Then by h-upper semi continuity of T for $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have $\lim_{n\to\infty} d(Tx_n, Tx) = 0 = \lim_{n\to\infty} \delta(Fx_n, Fx) = 0$, where $Fx_n = \{Tx_n\}$. Therefore for $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} d(x_n, x) = 0$, we have $\lim_{n\to\infty} \delta(Fx_n, Fx) = 0$, that is F is h-upper semi continuous. So, all the conditions of Theorem 2.1 are satisfied and hence F has a fixed point z in X. Then $z \in Fz = \{Tz\}$, that is, $z = Tz$. Hence z is a fixed point of T in X. □

Theorem 4.2. *Let (X, d) be a complete metric space and $T : X \to X$ be a single valued mapping. Suppose that* (4.1) *is satisfied, where the conditions α, ψ and L are same as in Definition 2.1. Also suppose that the following conditions are satisfied:*

(*i*) *T is α-admissible,*

(*ii*) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$,*

(*iii*) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n, where $x_{n+1} \in Tx_n$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.*

Then T has a fixed point.

Proof. Similarly as in the proof of Theorem 4.1, we define the multivalued mapping $F : X \to C(X)$ as $Fx = \{Tx\}$, for $x \in X$ and we prove that there exists $x_0 \in X$ and $x_1 \in Fx_0$ such that $\alpha(x_0, x_1) \geq 1$, also prove that F is a multivalued α-admissible mapping, which satisfies (2.1). So all the conditions of Theorem 2.2 are satisfied and hence F has a fixed point z in X. Then $z \in Fz = \{Tz\}$, that is, $z = Tz$. Hence z is a fixed point of T in X. □

5. Conclusion

In fixed point theory, most of works have been derived for α-ψ contractions and α-admissible conditions for different mappings defined on various spaces. A multivalued version of α-ψ contractions and α-admissible mapping was introduced in [4]. We introduce generalized multivalued α-ψ contraction of Ciric-Berinde type which is different from other mentioned contractions. This paper deals with fixed point theorems and stability of fixed point sets associated with a sequence of multivalued mappings.

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