Pricing European call options with default risk under a jump-diffusion model via FFT transform

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Abstract. This paper considers the pricing of European call options with default risk within the framework of reduced-form model. We model the stock price and the default intensity by two dependent jump-diffusion models with common jumps. By using a Girsanov theorem, we give the explicit expression for the Fourier transform of the price of call options with default risk.
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1. Introduction

Vulnerable option is a kind of option with credit risk. Credit risk is the risk that the counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract. There are two primary approaches for pricing credit derivatives, the structural approach and the reduced-form approach. Structural models, initially proposed by Black and Scholes (1973) and Merton (1974), could give an intuitive understanding for the credit risk by specifying a firm value process. Reduced-form models, introduced by Jarrow and Turnbull (1995), Duffie and Singleton (1999), and others, focus directly on the modeling of the default probability. This methodology does not intend to explain the default of a firm by means of an economic construction. Instead, the time of default is defined as the first jump time of a point process. Comparing with structural models, reduced-form models are more flexible and tractable in the real market. For more information on reduced-form models, we refer the interested reader to Bielecki and Rutkowski (2004) and Dong et al. (2014).

Extending the corporate bond default model of Merton (1974), Johnson and Stulz (1987) firstly proposed the conception of vulnerable option and investigated the option pricing with credit risk based on a structural model. Hull and White (1995) derived the price of vulnerable option by adopting a reduced-form

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approach under the assumption that the underlying asset and the counterparty asset were independent of each other. Extending Hull and White (1995), Klein (1996) relaxed the assumption of independence condition and deduced option pricing via a martingale method. By generalizing the results of Klein (1996), Klein and Inglis (2001) considered the stochastic default boundary which depends on options and counterparty debts for the discussion of option pricing. Wu and Dong (2019) investigated the pricing of European vulnerable option under a correlated diffusion process.

Most of the literature on vulnerable options assume that the dynamics of the assets follow the log-normal diffusion process. However, this assumption ignores sudden shocks in price due to the arrival of important new information. The purpose of this study is to provide a new pricing model for vulnerable options, where the dynamics of the underlying asset and the default intensity follow jump-diffusion processes with common jumps. The paper is organized as follows. In Section 2, we present the pricing model. In Section 3, we derive the price of the vulnerable options. Section 4 presents some numerical analysis by using FFT. Section 5 concludes.

2. The model

Consider a continuous-time model with a finite time horizon $\mathcal{T} = [0, T]$ with $T < \infty$. Let $\{\Omega, \Im, \{\Im_t\}_{0\le t\le T}, Q\}$ be a filtered complete probability space, where Q is the risk neutral measure such that the discounted asset price processes are martingales, and $\{\Im_t\}_{0\le t\le T}$ is a filtration satisfying the usual conditions. Throughout the paper, it is assumed that all random variables are well defined on this probability space and $\Im_T-$measurable.

Assume that the dynamics of the process B_t for the bank account are described by

$$dB_t = r_t B_t dt, \quad B_0 = 1,$$

where the interest rate r_t is given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma_1 dW_1(t). \tag{2.1}$$

Here, $\kappa > 0, \theta > 0, \sigma_1 > 0$ are constants; $\{W_1(t), t \ge 0\}$ is a standard Brownian motion. From (2.1), we have

$$\int_t^T r_s ds = \theta(T-t) + (r_t - \theta)D(t,T) + \int_t^T \sigma_1 D(s,T)dW_1(s), \tag{2.2}$$

where $D(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa}$.

Let S_t be the value of the asset at time t. Let τ denote the default time of the writer of the option with default intensity process λ_t. Suppose that the dynamics of the stock price S_t and default intensity λ_t follow

$$\frac{dS_t}{S_{t^-}} = (r_t - (\rho_1+\rho_2)\xi)dt + \sigma_2 dW_2(t) + d\Bigg(\sum_{i=1}^{N_1(t)+N_2(t)} (e^{Y_i} - 1)\Bigg), \tag{2.3}$$

and

$$d\lambda_t = a(b - \lambda_t)dt + \sigma_3 dW_3(t) + d \sum_{i=1}^{N_1(t)+N_3(t)} Z_i, \tag{2.4}$$

where a, b, σ_2, σ_3 are all positive constants; $\{W_2(t), t \geq 0\}$ and $\{W_3(t), t \geq 0\}$ are two standard Brownian motions; $\{N_1(t), t \geq 0\}$, $\{N_2(t), t \geq 0\}$ and $\{N_3(t), t \geq 0\}$ are three mutually independent Poisson processes with arrival rates ρ_1, ρ_2 and ρ_3, respectively; $\{Y_i, i \geq 1\}$ is a sequence of independent identically distributed random variables with common density function given by f_y and $\xi = E(e^{Y_1})$; $\{Z_i, i \geq 1\}$ is also a sequence of independent identically distributed random variables with common density function given by f_z. Moreover, we suppose that $\{N_1(t), t \geq 0\}$,$\{N_2(t), t \geq 0\}$,$\{N_3(t), t \geq 0\}$,$\{Z_i, i \geq 1\}$ and $\{Y_i, i \geq 1\}$ are mutually independent. Finally, we assume that the covariance matrix of the Brownian motion $(W_1(t), W_2(t), W_3(t))$ is

$$\begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix} t$$

This model assumes that the firm value and the default intensity have common jumps, which describe the sudden changes in stock prices and default intensity due to the revealing of important new information which has a large effect on them. Note that, the default intensity λ_t can take negative values with positive probability. However, in practical applications, due to the low volatility, the probability λ_t takes negative values can be considered negligible.

We now specify the information structure of our model. Let $\Im_t = \mathcal{G}_t \vee \mathcal{H}_t$, where $\mathcal{G}_t = \Im_t^s \vee \Im_t^r \vee \Im_t^\lambda$ with $\Im_t^s = \sigma(S_s, s \leq t), \Im_t^r = \sigma(r_s, s \leq t), \Im_t^\lambda = \sigma(\lambda_s, s \leq t)$ and $\mathcal{H}_t = \sigma(1_{\{\tau \leq s\}}, s \leq t)$.

Let $J(t) = \sum_{i=1}^{N_1(t)+N_3(t)} Z_i$. Then from (2.3) and (2.4), we can obtain

$$S_T = S_t e^{\int_t^T (r_s - (\rho_1+\rho_2)\xi - \frac{1}{2}\sigma_2^2)ds + \int_t^T \sigma_2 dW_2(s) + \sum_{i=N_1(t)+N_2(t)}^{N_1(T)+N_2(T)} Y_i}, \tag{2.5}$$

and

$$\begin{aligned} \int_t^T \lambda_s ds &= b(T-t) + (\lambda_t - b)D_1(t,T) + \int_t^T \sigma_3 D_1(s,T)dW_3(s) \\ &\quad + \int_t^T D_1(s,T)dJ(s), \end{aligned} \tag{2.6}$$

where $D_1(t,T) = \frac{1-e^{-a(T-t)}}{a}$.

3. Pricing options with credit risk

In this section we consider the pricing of the European style option with credit risk. Assume that the recovery rate is a constant ω. When the writer of the

European option defaults, the payoff is given by ω times the payoff of the default-free option at maturity. By risk-neutral pricing theorem, the valuation of the vulnerable European call option at time t,with strike price K and maturity T is given by

$$C(t,T,K) = E\left[e^{-\int_t^T r_s ds}\left(\omega(S_T-K)^+ 1_{\{\tau\le T\}} + (S_T-K)^+ 1_{\{\tau>T\}}\right)\Big|\Im_t\right]$$

Then from Corollary 5.1.1 of Bielecki and Rutkowski (2004), we obtain the following expression:

$$\begin{aligned} C(t,T,K) &= \omega E\left[e^{-\int_t^T r_s ds}(S_T-K)^+\Big|\mathcal{G}_t\right] \\ &\quad +(1-\omega)1_{\{\tau>t\}}E\left[e^{-\int_t^T (r_s+\lambda_s)ds}(S_T-K)^+\Big|\mathcal{G}_t\right] \\ &\doteq \omega C_1(t,T,K) + 1_{\{\tau>t\}}(1-\omega)C_2(t,T,K) \end{aligned}$$

where

$$\begin{aligned} C_1(t,T,K) &= E\left[e^{-\int_t^T r_s ds}(S_T-K)^+|\mathcal{G}_t\right], \\ C_2(t,T,K) &= E\left[e^{-\int_t^T (r_s+\lambda_s)ds}(S_T-K)^+|\mathcal{G}_t\right]. \end{aligned}$$

Since it is difficult to compute $C_1(t,T,K)$ and $C_2(t,T,K)$, we will investigate the Fourier transform of the option price. We adopt the Fourier methods, introduced in Carr and Madan (1999), to investigate the option price. Following the notation in Carr and Madan (1999), we write $k = ln(K)$. For $a > 0$, define

$$\begin{aligned} c(t,T,k) &= e^{ak}C(t,T,K) \\ &\doteq \omega c_1(t,T,k) + 1_{\{\tau>t\}}(1-\omega)c_2(t,T,k), \end{aligned}$$

where

$$c_1(t,T,k) = e^{ak}C_1(t,T,K),\ c_2(t,T,k) = e^{ak}C_2(t,T,K).$$

Define

$$\begin{aligned} \zeta(u,t,T) &= \int_{-\infty}^{+\infty} e^{iuk}c(t,T,k)dk \\ &\doteq \omega\zeta_1(u,t,T) + 1_{\{\tau>t\}}(1-\omega)\zeta_2(u,t,T), \end{aligned}$$

where

$$\zeta_1(u,t,T) = \int_{-\infty}^{+\infty} e^{iuk}c_1(t,T,k)dk,\ \zeta_2(u,t,T) = \int_{-\infty}^{+\infty} e^{iuk}c_2(t,T,k)dk.$$

Proposition 3.1. *For $a > 0$, we have*

$$\zeta_1(u,t,T) = \frac{P(t,T)\eta^T(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u},$$

where

$$P(t,T) = E\left[e^{-\int_t^T r_s ds}\middle|\Im_t\right] = exp(-r_t D(t,T) + A(t,T)),$$

with $A(t,T) = (\theta - \frac{\sigma_1^2}{2\kappa^2})(D(t,T) - (T-t)) - \frac{\sigma_1^2}{4\kappa}D^2(t,T)$ *, and*

$$\begin{aligned}\eta^T(v,t,T) &= e^{iv(s_t+\wedge(t,T)-\int_t^T \rho_{12}\sigma_1\sigma_2 D(u,T)du - \int_t^T \sigma_1^2 D^2(u,T)du)} \\ &\quad \times e^{-\int_t^T(\frac{\sigma_2^2 v^2}{2}+\frac{\sigma_1^2 D^2(u,T)}{2}+iv\rho_{12}\sigma_1\sigma_2 D(u,T))du} e^{(\rho_1+\rho_2)(T-t)(E[e^{ivY_1}]-1)},\end{aligned}$$

with $s_t = \ln S_t$ *and* $\wedge(t,T) = \theta(T-t) + (r_t-\theta)D(t,T) - (\frac{\sigma_2^2}{2} + (\rho_1+\rho_2)\xi)(T-t)$.

Proof. In the presence of a stochastic interest rate, we will define the forward-neutral measure Q^T equivalent to the risk-neutral measure Q by

$$\frac{dQ^T}{dQ} = \frac{P(T,T)}{P(0,T)B_T} = \frac{e^{-\int_0^T r_s ds}}{P(0,T)}.$$

where $P(t,T)$ denotes the value at time t of a T-maturity zero coupon bond whose face value is 1. It is well known that

$$P(t,T) = exp(-r_t D(t,T) + A(t,T))$$

and $P(t,T)$ satisfies

$$dP(t,T) = r_t P(t,T)dt - \sigma_1 D(t,T)P(t,T)dW_1(t).$$

So, the Radon-Nikodym derivative is given by

$$\frac{dQ^T}{dQ} = e^{-\int_0^T \sigma_1 D(t,T)dW_1(t) - \frac{1}{2}\int_0^T \sigma_1^2 D^2(t,T)dt}.$$

Girsanov's theorem implies that

$$W_1^T(t) = W_1(t) + \int_0^t \sigma_1 D(u,T)du; \quad W_2^T(t) = W_2(t) + \int_0^t \rho_{12}\sigma_1 D(u,T)du$$

are two standard Brownian motions under Q^T with the correlation coefficient ρ_{12}.

Therefore,

$$\begin{aligned}S_T &= S_t \exp(\wedge(t,T) + \int_t^T \sigma_2 dW_2^T(u) + \int_t^T \sigma_1 D(u,T)dW_1^T(u) \\ &\quad - \int_t^T \rho_{12}\sigma_1\sigma_2 D(u,T)du - \int_t^T \sigma_1^2 D^2(u,T)du + \sum_{i=N_1(t)+N_2(t)}^{N_1(T)+N_2(T)} Y_i).\end{aligned}$$

Moving to the forward measure yields

$$C_1(t,T,K) \quad = \quad P(t,T)E^T\Big[(S_T-K)^+|\mathcal{G}_t\Big].$$

In order to derive the formula for $\zeta_1(u,t,T)$, we first derive the expression for the characteristic function of s_T conditional on $\mathcal{G}_t$. Let $f^T(s)$ be the density function conditional on s_t under Q^T, then we have

$$\begin{aligned}
\eta^T(v,t,T) &= E^T\Big[e^{ivs_T}\Big|\mathcal{G}_t\Big] \\
&= e^{iv(s_t+\wedge(t,T)-\int_t^T \rho_{12}\sigma_1\sigma_2 D(u,T)du-\int_t^T \sigma_1^2 D^2(u,T)du)} \\
&\quad \times E^T\Big[e^{iv(\int_t^T \sigma_2 dW_2^T(u)+\int_t^T \sigma_1 D(u,T)dW_1^T(u)+\sum_{j=N_1(t)+N_2(t)}^{N_1(T)+N_2(T)} Y_j)}\Big|\mathcal{G}_t\Big] \\
&= e^{iv(s_t+\wedge(t,T)-\int_t^T \rho_{12}\sigma_1\sigma_2 D(u,T)du-\int_t^T \sigma_1^2 D^2(u,T)du)} \\
&\quad \times e^{-\int_t^T(\frac{\sigma_2^2v^2}{2}+\frac{\sigma_1^2D^2(u,T)}{2}+iv\rho_{12}\sigma_1\sigma_2D(u,T))du} \times e^{(\rho_1+\rho_2)(T-t)(E[e^{ivY_1}]-1)}
\end{aligned}$$

Hence,

$$\begin{aligned}
\zeta_1(u,t,T) &= \int_{-\infty}^{+\infty} e^{iuk}c_1(t,T,K)dk \\
&= \int_{-\infty}^{+\infty}\int_k^{+\infty} e^{(iu+a)k}P(t,T)(e^s-e^k)f^T(s)dsdk \\
&= \frac{P(t,T)\eta^T(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u}.
\end{aligned}$$

□

Proposition 3.2. *For $a>0$, we have*

$$\zeta_2(u,t,T)=\frac{P^\lambda(t,T)\eta^\lambda(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u},$$

where

$$\begin{aligned}
P^\lambda(t,T) &= e^{-(\theta+b)(T-t)-(r(t)-\theta)D(t,T)-(\lambda(t)-b)D_1(t,T)} \\
&\quad \times e^{(\rho_1+\rho_3)\int_t^T\int_{-\infty}^{+\infty}(e^{-D_1(u,T)}-1)f_z(z)dzdu} \\
&\quad \times e^{-\frac{1}{2}\int_t^T(\sigma_1^2D^2(u,T)+2\rho_{13}\sigma_1\sigma_3D(u,T)D_1(u,T)+\sigma_3^2D_1^2(u,T))du}
\end{aligned}$$

and

$$\begin{aligned}
\eta^\lambda(v,t,T) &= e^{iv(s_t+\wedge(t,T)-\int_t^T(\rho_{12}\sigma_2M_1(u)+\rho_{13}\sigma_2M_2(u)+\sigma_1D(u,T)M_1(u))du)} \\
&\quad \times e^{-\frac{v^2}{2}\sigma_2^2(T-t)-\int_t^T\frac{\sigma_1^2}{2}v^2D(u,T)du+\int_t^T iv\sigma_1\sigma_2D(u,T)du} \\
&\quad \times e^{\int_t^T(\rho_1^\lambda(s)+\rho_2)(E[e^{ivY_1}]-1)ds}
\end{aligned}$$

with

$$M_1(t) = \sigma_1 D(t,T) + \rho_{13}\sigma_3 D_1(t,T), \qquad M_2(t) = \sigma_3 D_1(t,T) + \rho_{23}\sigma_1 D(t,T).$$

Proof. Define

$$\frac{dQ^\lambda}{dQ} = \frac{e^{-\int_0^T(\lambda_s+r_s)ds}}{E[e^{-\int_0^T(\lambda_s+r_s)ds}]}$$

From Eqs. (2.5)-(2.6) and some calculations, we can have

$$\begin{aligned} P^\lambda(t,T) &= E\left[e^{-\int_t^T(r_s+\lambda_s)ds}\middle|\mathcal{G}_t\right] \\ &= e^{-(\theta+b)(T-t)-(r(t)-\theta)D(t,T)-(\lambda(t)-b)D_1(t,T)} \\ &\quad\times e^{(\rho_1+\rho_3)\int_t^T\int_{-\infty}^{+\infty}(e^{-D_1(u,T)}-1)f_z(z)dzdu} \\ &\quad\times e^{-\frac{1}{2}\int_t^T(\sigma_1^2D^2(u,T)+2\rho_{13}\sigma_1\sigma_3D(u,T)D_1(u,T)+\sigma_3^2D_1^2(u,T))du}. \end{aligned}$$

Then

$$\begin{aligned} \frac{dQ^\lambda}{dQ} &= e^{-\int_0^T\sigma_1D(u,T)dW_1(u)-\int_0^T\sigma_3D_1(u,T)dW_3(u)-\frac{1}{2}\int_0^T\sigma_1^2D^2(u,T)du} \\ &\quad\times e^{-\frac{1}{2}\int_0^T\sigma_3^2D_1^2(u,T)du-\rho_{13}\int_0^T\sigma_1\sigma_3D(u,T)D_1(u,T)du} \\ &\quad\times e^{-\int_0^T D_1(u,T)dJ(u)-(\rho_1+\rho_3)\int_0^T\int_{-\infty}^{+\infty}(e^{-D_1(u,T)}-1)f_Z(z)dzdu} \end{aligned}$$

and Girsanov's theorem implies that

$$W_1^\lambda(t) = W_1(t) + \int_0^t \sigma_1 D(u,T)du + \int_0^t \rho_{13}\sigma_3 D_1(u,T)du$$

and

$$W_3^\lambda(t) = W_3(t) + \int_0^t \sigma_3 D_1(u,T)du + \int_0^t \rho_{13}\sigma_1 D(u,T)du$$

are standard Brownian motions under Q^λ, and the intensity of the jump process $N_1^\lambda(t)$ is given by $\rho_1^\lambda(t) = \rho_1\int_{-\infty}^{+\infty}e^{-zD_1(t,T)}f_Z(z)dz$. Therefore, under the measure Q^λ,

$$\begin{aligned} S_T &= S_t e^{\wedge(t,T)+\int_t^T\sigma_2D(u,T)dW_2^\lambda(u)+\int_t^T\sigma_1D(u,T)dW_1^\lambda(u)-\int_t^T\rho_{12}\sigma_2M_1(u)du} \\ &\quad\times e^{-\int_t^T\rho_{23}\sigma_2M_2(u)du-\int_t^T\sigma_1D(u,T)M_1(u)du+\sum_{j=N_1^\lambda(t)+N_2(t)}^{N_1^\lambda(T)+N_2(T)}Y_j}. \end{aligned}$$

Similar to deriving $\eta^T(v,t,T)$, we have that the characteristic function of s_T under Q^λ is given by

$$\begin{aligned} \eta^\lambda(v,t,T) &= e^{iv(s_t+\wedge(t,T)-\int_t^T\rho_{12}\sigma_2M_1(u)du-\int_t^T\rho_{23}\sigma_2M_2(u)du-\int_t^T\sigma_1D(u,T)M_1(u)du)} \\ &\quad\times e^{-\frac{v^2}{2}\sigma_2^2(T-t)-\int_t^T\frac{\sigma_1^2}{2}v^2D(u,T)du+\int_t^T iv\sigma_1\sigma_2D(u,T)du} \\ &\quad\times e^{\int_t^T(\rho_1^\lambda(s)+\rho_2)(E[e^{ivY_1}]-1)ds}. \end{aligned}$$

Therefore,

$$\begin{aligned}
\zeta_2(u,t,T) &= \int_{-\infty}^{+\infty} e^{iuk} c_2(t,T,K)dk \\
&= P^{\lambda}(t,T)\int_{-\infty}^{+\infty}\int_{-\infty}^{s} e^{(iu+a)k}(e^s - e^k) f^{\lambda}(s)dkds \\
&= \frac{P^{\lambda}(t,T)\eta^{\lambda}(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u},
\end{aligned}$$

where $f^{\lambda}(s)$ is the density function conditional of s_t under Q^{λ}. □

From Propositions 3.1, 3.2, we can directly obtain the following result.

Corollary 3.1. *For $a > 0$, we have*

$$\begin{aligned}
\zeta(u,t,T) &= \omega\zeta_1(u,t,T) + 1_{\{\tau>t\}}(1-\omega)\zeta_2(u,t,T) \\
&= \omega\frac{P(t,T)\eta^{T}(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u} \\
&\quad +1_{\{\tau>t\}}(1-\omega)\frac{P^{\lambda}(t,T)\eta^{\lambda}(u-i(a+1),t,T)}{a^2+a-u^2+i(2a+1)u}.
\end{aligned}$$

4. FFT for vulnerable European option pricing

In this section, we shall carry out some numerical calculations for the pricing of options. An approach based on the fast Fourier transform (FFT) is widely used to numerically evaluate a price of a European-style call option. The main advantage of the FFT approach is that it computes the discrete Fourier transform (DFT) faster than other approaches. For the details of the fast Fourier transform, we refer to Carr and Madan (1999).

Let $u_j = \eta(j-1)$. Following Carr and Madan (1999), an approximation for $C(0,T,k)$ is

$$\begin{aligned}
C(0,T,k) &\approx \frac{e^{-ak}}{\pi}\Bigg(\omega\sum_{j=1}^{N} e^{-iu_jk}\zeta_1(u_j)\eta P(0,T) \\
&\quad +(1-\omega)\sum_{j=1}^{N} e^{-iu_jk}\zeta_2(u_j)\eta P^{\lambda}(0,T)\Bigg).
\end{aligned}$$

The FFT returns N values of modified logarithmic strike k given as follows: $k_v = -b + h(v-1), v = 1,\cdots,N$, where $b = \frac{1}{2}Nh$.

In order to apply FFT, we let $\eta h = \frac{2\pi}{N}$. To obtain an accurate integration with larger values of η, we incorporate Simpson's rule weightings into our summation. From Simpson's rule weightings , we obtain European call option prices as

$$C(0,T,k_v) \approx \frac{e^{-ak_v}}{\pi}\left(\omega\sum_{j=1}^{N} e^{-\frac{2\pi i(j-1)(i-1)}{N}} e^{ibu_j}\frac{\eta}{3}[3+(-1)^j - w_{j-1}]P(0,T)\right.$$
$$\left. + (1-\omega)\sum_{j=1}^{N} e^{-\frac{2\pi i(j-1)(i-1)}{N}} e^{ibu_j}\frac{\eta}{3}[3+(-1)^j - w_{j-1}]P^{\lambda}(0,T)\right),$$

where w_n is the Kronecker delta function that is unity for $n = 0$ and zero otherwise. The above summation is an exact application of the FFT.

In what follows, we give a numerical example. Assume the parameters are as follows: $\omega = 0.4, \kappa = 0.3, \theta = 0.05, r_0 = 0.02, a = 0.2, b = 0.02, \lambda_0 = 0.5, \sigma_1 = 0.2, \sigma_2 = 0.15, \sigma_3 = 0.25, \rho_1 = \rho_2 = \rho_3 = 0.25, \rho_{12} = 0.7, \rho_{13} = 0.5, \rho_{23} = 0.6, S_0 = 100, T = 1$, the densities f_y and f_z are given by $f_y(y) = 10e^{-20y}1_{\{y>0\}} + 10e^{-20y}1_{\{y<0\}}$, $f_z(z) = 5e^{-5z}, z > 0$. The numerical results for the option prices are presented in Tables 1, 2. From them we can see that the convergence rate of the FFT is comparatively fast.

Table 1 presents the relationship between the option price and K. From it we can see that the option price decreases with the strike K. This is because a high value of K leads to a decreasing probability that S_T is larger than K.

Table 1: Prices calculated by FFT

K	$N = 512$	$N = 1024$	$N = 2048$	$N = 4096$
90	8.2413	8.2415	8.2415	8.2415
95	6.5219	6.5221	6.5222	6.5223
100	5.2344	5.2346	5.2346	5.2346
105	4.4651	4.4652	4.4652	4.4652
110	3.5442	3.5343	3.5343	3.5343

Table 2 represents the impact of the jump intensity of the common jumps on the option price. From it we can observe that the price increases with λ_0. This is because a high value of λ_0 leads to an increasing volatility of S_t and λ_t, and S_t is more sensitive to λ_0.

Table 2: Impact of jump intensity of option prices for $K = 100$

λ_0	$N = 512$	$N = 1024$	$N = 2048$	$N = 4096$
0.4	4.4154	4.4155	4.4156	4.4156
0.5	4.8732	4.8733	4.8733	4.8733
0.6	5.2344	5.2346	5.2346	5.2346
0.7	5.7612	5.7613	5.7613	5.7613
0.8	6.1507	6.1508	6.1509	6.1509

5. Conclusions

In this paper, we consider a jump-diffusion model to analyze a vulnerable European call option within the reduced-form framework. We assume the default intensity and the stock price are modelled by two jump-diffusion processes with common jumps. The jump components describe the impact of macro-economy on the asset price and the default intensity. We adopt the measure of change and the fast Fourier transform (FFT) method to value options. Numerical examples illustrate the practicality of the method.

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