New types of soft ordered mappings via soft $\alpha$-open sets

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Abstract. The concept of soft topological ordered spaces is an extension of the soft topological spaces notion. The motivation of this paper is twofold: One is to generalize and extend the existing ordered maps and other is to contribute on making a general framework for studying soft topological ordered spaces. In this study, we utilize soft $\alpha$-open sets to introduce new ordered maps, which generalize existing comparable notions, namely soft $xa$-continuous, soft $xa$-open, soft $xa$-closed and soft $xa$-homeomorphism maps, for $x \in \{I, D, B\}$, via soft topological ordered spaces. We show the relationships among these concepts and discuss the equivalent conditions for each one of them. Also, we derive that an extended soft topologies notion guarantees the equivalent between the soft maps initiated herein and their counterparts of maps on topological ordered spaces. For illustration and comparison, various examples are provided.

Keywords: soft $I(D, B)\alpha$-continuous map, Soft $I(D, B)\alpha$-open map and soft $I(D, B)\alpha$-homeomorphism map.

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1. Introduction and preliminaries

By combining a topological structure $\tau$ with a partial order relation $\preceq$ on a non-empty set $X$, Nachbin [42] named the triple $(X, \tau, \preceq)$ a topological ordered space and established some topological ordered notions about it which depend on topological structure and partial order relation. In his work, he introduced the concepts of normally ordered, regularly ordered and completely ordered spaces. McCartan [37] renamed some ordered separation axioms and characterised $T_i$-ordered spaces ($i = 0, 1, 2, 3, 4$). Also, he presented interesting examples to show that $T_i$-spaces are proper generalization of $T_i$-ordered spaces, for $i = 0, 1, 2$, and derived some results which connected $T_i$-ordered axioms and some topological notions like one-point compactification and local compactness. Some scholars investigated another types of ordered spaces by replacing a partial order relation by preorder relation or any binary relation (see, for example, [29, 38, 39, 45]). As a generalization of topological ordered spaces, Das [20] introduced and studied ordered spaces. For more investigation on ordered spaces, we refer the interested readers to [3, 7, 10, 13, 17, 22, 23, 24, 28, 29, 32, 33, 34, 49].

Molotdsov [41], in 1999, came up with an idea, namely soft sets, in order to approach uncertainties and vague of data on some reality situations and phenomena. He discussed the strengths of soft set theory in compared with fuzzy theory and probability theory and investigated its applications on different fields. As a result of advantages of soft sets in overcoming incomplete data, many researchers began introducing some soft operations between soft sets and applying it in several situations arising in information science, decision making, mathematics and other related disciplines (see, for example, [1, 2, 19, 27, 44]).

To insert soft sets on topology studies, Shabir and Naz [48] formulated the soft topological spaces notion by analogy with the definition of topology. They fixed a parameters set to avoid anomaly arising in appearing various non-null soft sets. They gave elementary ideas on soft topological spaces and studied soft separation axioms. Min [40] observed some mistakes on Shabir and Naz’s work and corrected them. In this regard, we think the existence of these mistakes was due to the lack of knowledge of shape of soft open sets in soft regular spaces. Later on, the desire of obtaining a deeper understanding of soft topology prompted interested researchers to carry out many studies on soft topological notions and their features (see, for example, [18, 30, 31, 43]). [8, 9, 12, 25] investigated and corrected some errors which appeared in some previous studies. In [4], the authors gave the definition of soft $\alpha$-open sets and examined some related properties. They [5] also defined soft $\alpha$-separation axioms and characterised them. In the regard of studying generalized soft open sets, Al-shami [11] originated a concept of somewhere dense sets and investigated many of its properties.

Recently, the authors of [14] introduced a concept of soft topological ordered spaces and established the notions of increasing (decreasing) soft sets and increasing (decreasing) soft maps. Also, they defined $p$-soft $T_i$-ordered spaces.
(i = 0, 1, 2, 3, 4) depending on totally non belong relations, which defined in [26], and monotone soft neighborhoods. Based on soft \( \beta \)-open sets, some of soft ordered maps were introduced and studied in [16]. In the end of this literature, we indicate to that the authors of [1, 2, 46] defined new types of soft subsets, soft equality, soft union and intersection by relaxing the conditions on the parameters sets.

The purpose of this study is to introduce and study the concepts of soft \( x \)-continuous, soft \( x \)-open, soft \( x \)-closed and soft \( x \)-homeomorphism maps, for \( x \in \{ I, D, B \} \), via soft topological ordered spaces and to obtain a deeper understanding of them. Also, we give various examples to show the relationships among these maps and illustrate that soft \( x \)-continuous, soft \( x \)-open, soft \( x \)-closed and soft \( x \)-homeomorphism maps are strictly stronger than soft \( \alpha \)-continuous, soft \( \alpha \)-open, soft \( \alpha \)-closed and soft \( \alpha \)-homeomorphism maps, respectively, for \( x \in \{ I, D, B \} \). Furthermore, we characterize each one of the initiated soft maps and clarify a significant role of extended soft topologies on studying the interrelations between these soft maps and their counterparts of maps in topological ordered spaces.

We allocate the rest of this section to present some fundamental definitions and findings that will be needed in the sequels.

**Definition 1.1** ([41]). An ordered pair \((G, E)\) is said to be a soft set over \(X\) if \(G\) is a map of a set of parameters \(E\) into \(2^X\).

**Remark 1.2.**

(i) For short, we use the notation \(G_E\) instead of \((G, E)\).

(ii) A soft set \(G_E\) can be written as a set of ordered pairs \(G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}\).

**Definition 1.3** ([36]). A soft set \(G_E\) over \(X\) is called a null soft set, denoting by \(\emptyset\), if \(G(e) = \emptyset\), for each \(e \in E\) and is called an absolute soft set, denoting by \(X\), if \(G(e) = X\), for each \(e \in E\).

**Definition 1.4** ([6]). The relative complement of a soft set \(G_E\) is denoted by \(G_E^c\), where \(G_E^c : E \to 2^X\) is a mapping defined by \(G_E^c(e) = X \setminus G(e)\), for each \(e \in E\).

In this connection, it is worth noting that \(x \notin G_E\) does not imply that \(x \in G_E^c\).

**Definition 1.5** ([48]). For \(x \in X\) and a soft set \(G_E\) over \(X\), we say that \(x \in G_E\) if \(x \in G(e)\), for each \(e \in E\) and \(x \notin G_E\) if \(x \notin G(e)\), for some \(e \in E\).

**Definition 1.6** ([48]). A soft topology on a non-empty set \(X\) is a collection \(\tau\) of soft sets over \(X\) under a parameters set \(E\) satisfying the following axioms:

(i) \(\tilde{X}\) and \(\tilde{\emptyset}\) belong to \(\tau\).

(ii) The soft intersection of finite members in \(\tau\) belongs to \(\tau\).
The soft union of any members in $\tau$ belongs to $\tau$.

The triple $(X, \tau, E)$ is called a soft topological space. Every member of $\tau$ is called soft open and its relative complement is called soft closed.

**Proposition 1.7** ([48]). Let $(X, \tau, E)$ be a soft topological space. Then $\tau_e = \{G(e) : G \in \tau\}$ defines a topology on $X$, for each $e \in E$.

**Definition 1.8** ([4]). A soft subset $H \subseteq (X, \tau, E)$ is called a soft open if $H$ is soft-open, its relative complement is called soft closed.

**Proposition 1.9** ([4], [48]). For a soft subset $H \subseteq (X, \tau, E)$, we define the following four operators:

(i) $\text{int}(H)$ is the largest soft open set contained in $H$.

(ii) $\text{cl}(H)$ is the smallest soft closed set containing $H$.

(iii) $\text{int}_\alpha(H)$ is the largest soft $\alpha$-open set contained in $H$.

(iv) $\text{cl}_\alpha(H)$ is the smallest soft $\alpha$-closed set containing $H$.

**Definition 1.10** ([43]). Consider $(X, \tau, E)$ is a soft topological space and $\tau_e$ is a topology on $X$ as in the above proposition. Then $\tau^* = \{G_E : G(e) \in \tau_e, \text{for each } e \in E\}$ is a soft topology on $X$ finer than $\tau$.

In this work, we term $\tau^*$ an extended soft topology.

**Definition 1.11** ([50]). Consider $f : X \rightarrow Y$ and $\phi : A \rightarrow B$ are two maps and let $f_\phi : S(X_A) \rightarrow S(Y_B)$ be a soft map. Let $G_K$ and $H_L$ be soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then

(i) $f_\phi(G_K) = (f_\phi(G))_B$ is a soft subset of $S(Y_B)$ such that

$$f_\phi(G)(b) = \begin{cases} \bigcup_{a \in \phi^{-1}(b)} f(G(a)), & \phi^{-1}(b) \cap K \neq \emptyset \\ \emptyset, & \phi^{-1}(b) \cap K = \emptyset \end{cases}$$

for each $b \in B$.

(ii) $f_\phi^{-1}(H_L) = (f_\phi^{-1}(H))_A$ is a soft subset of $S(X_A)$ such that

$$f_\phi^{-1}(H)(a) = \begin{cases} f^{-1}(H(\phi(a))), & \phi(a) \in L \\ \emptyset, & \phi(a) \notin L \end{cases}$$

for each $a \in A$.

**Remark 1.12.** Henceforth, a soft map $f_\phi : S(X_A) \rightarrow S(Y_B)$ implies that a map $f$ of the universe set $X$ into the universe set $Y$ and a map $\phi$ of a set of parameters $A$ into a set of parameters $B$. 
Definition 1.13 ([50]). A soft map $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if $f$ and $\phi$ are injective (resp. surjective, bijective).

Proposition 1.14 ([43]). Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft map and let $G_A$ and $H_B$ be two soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then we have the following results:

(i) $G_A \subseteq f_\phi^{-1}f_\phi(G_A)$ and the equality relation holds if $f_\phi$ is injective.

(ii) $f_\phi f_\phi^{-1}(H_B) \subseteq H_B$ and the equality relation holds if $f_\phi$ is surjective.

Definition 1.15 ([4]). A soft map $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ is said to be:

(i) Soft $\alpha$-continuous if the inverse image of each soft open subset of $(Y, \theta, B)$ is a soft $\alpha$-open subset of $(X, \tau, A)$.

(ii) Soft $\alpha$-open (resp. soft $\alpha$-closed) if the image of each soft open (resp. soft closed) subset of $(X, \tau, A)$ is a soft $\alpha$-open (resp. soft $\alpha$-closed) subset of $(Y, \theta, B)$.

(iii) Soft $\alpha$-homeomorphism if it is bijective, soft $\alpha$-continuous and soft $\alpha$-open.

Definition 1.16 ([21], [43]). A soft subset $P_E$ over $X$ is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(a) = \emptyset$, for each $a \in E \setminus \{e\}$. A soft point will be shortly denoted by $P_e^x$ and we say that $P_e^x \in G_e$, if $x \in G(e)$.

Definition 1.17 ([14]). Let $\preceq$ be a partial order relation on a non-empty set $X$ and let $E$ be a set of parameters. A triple $(X, E, \preceq)$ is said to be a partially ordered soft set.

Definition 1.18 ([14]). We define an increasing soft operator $i : (SS(X)_E, \preceq) \to (SS(X)_E, \preceq)$ and a decreasing soft operator $d : (SS(X)_E, \preceq) \to (SS(X)_E, \preceq)$ as follows, for each soft subset $G_E$ of $SS(X_E)$

(i) $i(G_E) = (iG)_E$, where $iG$ is a mapping of $E$ into $X$ given by $iG(e) = i(G(e)) = \{x \in X : y \preceq x, \text{ for some } y \in G(e)\}$.

(ii) $d(G_E) = (dG)_E$, where $dG$ is a mapping of $E$ into $X$ given by $dG(e) = d(G(e)) = \{x \in X : x \preceq y, \text{ for some } y \in G(e)\}$.

Definition 1.19 ([14]). A soft subset $G_E$ of a partially ordered soft set $(X, E, \preceq)$ is said to be increasing (resp. decreasing) if $G_E = i(G_E)$ (resp. $G_E = d(G_E)$).

Theorem 1.20 ([14]). If a soft map $f_\phi : (S(X_A), \preceq_1) \to (S(Y_B), \preceq_2)$ is increasing, then the inverse image of each increasing (resp. decreasing) soft subset of $\tilde{Y}$ is an increasing (resp. a decreasing) soft subset of $\tilde{X}$.
Definition 1.21 ([14]). A quadrable system \((X, \tau, E, \leq)\) is said to be a soft topological ordered space, where \((X, \tau, E)\) is a soft topological space and \((X, E, \leq)\) is a partially ordered soft set. Henceforth, the two notations \((X, \tau, E, \leq_1)\) and \((Y, \theta, F, \leq_2)\) stand for soft topological ordered spaces.

Definition 1.22 ([47]). A map \((X, \tau, \leq_1) \to (Y, \theta, \leq_2)\) is said to be:

(i) \(I\) (resp. \(D, B\)) \(\alpha\)-continuous if the inverse image of each open set is \(I\) (resp. \(D, B\)) \(\alpha\)-open.

(ii) \(I\) (resp. \(D, B\)) \(\alpha\)-open if the image of each open set is \(I\) (resp. \(D, B\)) \(\alpha\)-open.

(iii) \(I\) (resp. \(D, B\)) \(\alpha\)-closed if the image of each open set is \(I\) (resp. \(D, B\)) \(\alpha\)-closed.

(iv) \(I\) (resp. \(D, B\)) \(\alpha\)-homeomorphism if it is bijective, \(I\) (resp. \(D, B\)) \(\alpha\)-continuous and \(I\) (resp. \(D, B\)) \(\alpha\)-open.

Definition 1.23 ([15]). The composition of two soft maps \(f_\phi : (X, \tau, E, \leq_1) \to (Y, \theta, F, \leq_2)\) and \(g_\lambda : (Y, \theta, F, \leq_2) \to (Z, \nu, K, \leq_3)\) is a soft map \(f_\phi \circ g_\lambda : (X, \tau, E, \leq_1) \to (Z, \nu, K, \leq_3)\) and is given by \((f_\phi \circ g_\lambda)(P_\epsilon^x) = f_\phi(g_\lambda(P_\epsilon^x))\).

2. Soft \(I(D, B)\alpha\)-continuous maps

The purpose of this section is to define soft \(I(D, B)\alpha\)-continuity at soft point, ordinary point and on the universe set and to elucidate the relationships among them with the help of examples. The characterisations of of the given soft maps are studied and the interrelations between these soft maps and their counterparts of maps on topological ordered spaces are discussed.

Definition 2.1. A soft subset \(H_E\) of \((X, \tau, E, \leq_1)\) is said to be:

(i) Soft \(I\) (resp. Soft \(D,\) Soft \(B\)) \(\alpha\)-open if it is soft \(\alpha\)-open and increasing (resp. decreasing, balancing).

(ii) Soft \(I\) (resp. Soft \(D,\) Soft \(B\)) \(\alpha\)-closed if it is soft \(\alpha\)-closed and increasing (resp. decreasing, balancing).

Definition 2.2. A soft map \(f_\phi : (X, \tau, E, \leq_1) \to (Y, \theta, F, \leq_2)\) is called:

(i) Soft \(I\) (resp. Soft \(D,\) Soft \(B\)) \(\alpha\)-continuous at \(P_\epsilon^x \in X\) if for each soft open set \(H_F\) containing \(f_\phi(P_\epsilon^x)\), there exists a soft \(I\) (resp. soft \(D,\) soft \(B\)) \(\alpha\)-open set \(G_E\) containing \(P_\epsilon^x\) such that \(f_\phi(G_E) \subseteq H_F\).

(ii) Soft \(I\) (resp. Soft \(D,\) Soft \(B\)) \(\alpha\)-continuous at \(x \in X\) if it is soft \(I\) (resp. soft \(D,\) soft \(B\)) \(\alpha\)-continuous at each \(P_\epsilon^x\).
(iii) Soft I (resp. Soft D, Soft B) $\alpha$-continuous if it is soft I (resp. soft D, soft B) $\alpha$-continuous at each $x \in X$.

**Theorem 2.3.** A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) $\alpha$-continuous if and only if the inverse image of each soft open subset of $Y$ is a soft I (resp. soft D, soft B) $\alpha$-open subset of $X$.

**Proof.** We only present theorem’s proof in case of $f_\phi$ is soft I-continuous and the cases between parenthesis can be made similarly.

To prove the necessary part, let $G_F$ be a soft open subset of $\tilde{Y}$. Then we have the following two cases:

(i) Either $f_\phi^{-1}(G_F) = \tilde{\emptyset}$.

(ii) Or $f_\phi^{-1}(G_F) \neq \tilde{\emptyset}$. By choosing $P_\varepsilon^{x} \in X$ such that $P_\varepsilon^{x} \in f_\phi^{-1}(G_F)$, we obtain that $f_\phi(P_\varepsilon^{x}) \in G_F$. So there exists a soft I-open set $H_E$ containing $P_\varepsilon^{x}$ such that $f_\phi(H_E) \subseteq G_F$. Since $P_\varepsilon^{x}$ is chosen arbitrary, then $f_\phi^{-1}(G_F) = \bigcup_{P_\varepsilon^{x} \in f_\phi^{-1}(G_F)} H_E$.

From the two cases above, we conclude that $f_\phi^{-1}(G_F)$ is a soft I-open subset of $\tilde{X}$. To prove the sufficient part, let $G_F$ be a soft open subset of $\tilde{Y}$ containing $f_\phi(P_\varepsilon^{x})$. Then $P_\varepsilon^{x} \in f_\phi^{-1}(G_F)$. By hypothesis, $f_\phi^{-1}(G_F)$ is a soft I-open set. Since $f_\phi(f_\phi^{-1}(G_F)) \subseteq G_F$, then $f_\phi$ is a soft I-continuous map at $P_\varepsilon^{x} \in X$ and since $P_\varepsilon^{x}$ is chosen arbitrary, then $f_\phi$ is a soft I-continuous map. \qed

**Remark 2.4.** From Definition (2.2), we can note the following:

(i) Every soft I (D, B) $\alpha$-continuous map is always soft $\alpha$-continuous.

(ii) Every soft Bo-continuous map is soft Io-continuous or soft Do-continuous.

To elucidate that the converse of the two results of the remark above need not be true, we construct the following two examples.

**Example 2.5.** Let the two universe sets $X = \{2, 3, 5\}$, $Y = \{7, 11, 13\}$ and the two parameters sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$. Consider a map $\phi : A \to B$ is defined as, $\phi(a_1) = \phi(a_2) = b_1$ and a map $f : X \to Y$ is defined as, $f(2) = 7$ and $f(3) = f(5) = 11$. We define a partial order relation on $X$ as $\preceq = \triangle \bigcup \{(3, 2), (2, 5), (3, 5)\}$ and we define two soft topologies $\tau$ and $\theta$ on $X$ and $Y$, respectively, as $\tau = \{\emptyset, \tilde{X}, F_A\}$ and $\theta = \{\emptyset, \tilde{Y}, H_B\}$, where $F_A = \{(a_1, \{a_2\}), (a_2, \emptyset)\}$ and $H_B = \{(b_1, \{7\}), (b_2, \emptyset)\}$. Since $f_\phi^{-1}(H_B) = \{(a_1, \{2\}), (a_2, \{2\})\}$ is a soft $\alpha$-open set, then $f_\phi : S(X_A) \to S(Y_B)$ is a soft $\alpha$-continuous map. On the other hand, $f_\phi^{-1}(H_B)$ is neither a soft Do-open nor a soft Io-open set. Hence $f_\phi$ is not soft I (soft D, soft B) $\alpha$-continuous.

**Example 2.6.** In Example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(2, 5)\}$ (resp. $\preceq = \triangle \bigcup \{(3, 2)\}$), then the soft map $f_\phi$ is soft D-continuous (resp. soft I-continuous), but is not soft B-continuous.
Definition 2.7. For a soft subset $H_E$ of $(X, \tau, E, \preceq)$, we define the following six operators:

(i) $H_E^{\text{iao}}$ (resp. $H_E^{\text{doo}}, H_E^{\text{hoo}}$) is the largest soft I (resp. soft D, soft B) $\alpha$-open set contained in $H_E$.

(ii) $H_E^{\text{iacl}}$ (resp. $H_E^{\text{docl}}, H_E^{\text{bocl}}$) is the smallest soft I (resp. soft D, soft B) $\alpha$-closed set containing $H_E$.

Lemma 2.8. We have the following three properties for a soft subset $H_E$ of $(X, \tau, E, \preceq)$:

(i) $(H_E^{\text{docl}})^c = (H_E^{\text{iao}})^c$.

(ii) $(H_E^{\text{iacl}})^c = (H_E^{\text{doo}})^c$.

(iii) $(H_E^{\text{bocl}})^c = (H_E^{\text{hoo}})^c$.

Proof. (i) $(H_E^{\text{docl}})^c = \bigcap\{F_E : F_E$ is a soft $D\alpha$-closed set containing $H_E\}^c$

$= \bigcup\{F_E : F_E$ is a soft I$\alpha$-open set contained in $H_E^c\} = (H_E^{\text{iao}})^c$.

By analogy with (i), one can prove (ii) and (iii).

Theorem 2.9. The following five properties of a soft map $f_\phi : (X, \tau, E, \preceq_1) \rightarrow (Y, \theta, F, \preceq_2)$ are equivalent:

(i) $f_\phi$ is soft $I\alpha$-continuous;

(ii) $f_\phi^{-1}(L_F)$ is a soft $D\alpha$-closed subset of $\tilde{X}$, for each soft closed subset $L_F$ of $\tilde{Y}$;

(iii) $(f_\phi^{-1}(M_F))^{\text{docl}} \subseteq f_\phi^{-1}(\text{cl}(M_F))$, for every $M_F \subseteq Y$;

(iv) $f_\phi(N_E^{\text{docl}}) \subseteq \text{cl}(f_\phi(N_E))$, for every $N_E \subseteq \tilde{X}$;

(v) $f_\phi^{-1}(\text{int}(M_F)) \subseteq (f_\phi^{-1}(M_F))^{\text{iao}}$, for every $M_F \subseteq Y$.

Proof. (i) $\Rightarrow$ (ii) : Consider $L_F$ is a soft closed subset of $\tilde{Y}$. By hypothesis, $f_\phi^{-1}(L_F)$ is a soft $I\alpha$-open subset of $\tilde{X}$ and by the fact that $f_\phi^{-1}(L_F) = (f_\phi^{-1}(L_F))^c$, we obtain that $f_\phi^{-1}(L_F)$ is soft $D\alpha$-closed as required.

(ii) $\Rightarrow$ (iii) : It follows from statement (ii) that $f_\phi^{-1}(\text{cl}(M_F))$ is a soft $D\alpha$-closed subset of $\tilde{X}$, for every $M_F \subseteq Y$. So $(f_\phi^{-1}(M_F))^{\text{docl}} \subseteq (f_\phi^{-1}(\text{cl}(M_F)))^{\text{docl}} = f_\phi^{-1}(\text{cl}(M_F))$.

(iii) $\Rightarrow$ (iv) : From the fact that $N_E^{\text{docl}} \subseteq (f_\phi^{-1}(f_\phi(N_E)))^{\text{docl}}$ and from (iii), we have $(f_\phi^{-1}(f_\phi(N_E)))^{\text{docl}} \subseteq f_\phi^{-1}(\text{cl}(f_\phi(N_E)))$.

This implies that $f_\phi(N_E^{\text{docl}}) \subseteq \text{cl}(f_\phi(N_E))$.

(iv) $\Rightarrow$ (v) : For any soft subset $M_F$ of $\tilde{Y}$, we obtain from Lemma 2.8 that $f_\phi(\tilde{X} - (f_\phi^{-1}(N_E))^{\text{iao}}) = f_\phi(((f_\phi^{-1}(N_E))^c)^{\text{docl}})$. It follows from statement
The following five properties of a soft map $f_\phi : (X, \tau, E, \leq_1) \to (Y, \theta, F, \leq_2)$ are equivalent:

(i) $f_\phi$ is soft $D_\alpha$-continuous (resp. soft $B_\alpha$-continuous);

(ii) $f_\phi^{-1}(L_F)$ is a soft $I_\alpha$-closed (resp. soft $B_\alpha$-closed) subset of $\tilde{X}$, for each soft closed subset $L_F$ of $\tilde{Y}$;

(iii) $(f_\phi^{-1}(M_F))^{I_\alpha} \subseteq \tilde{X}$ (resp. $(f_\phi^{-1}(M_F))^{B_\alpha} \subseteq \tilde{X}$), for every $M_F \subseteq Y$;

(iv) $f_\phi(N_E^{I_\alpha}) \subseteq \tilde{X}$ (resp. $f_\phi(N_E^{B_\alpha}) \subseteq \tilde{X}$), for every $N_E \subseteq \tilde{X}$;

(v) $f_\phi^{-1}(int(M_F)) \subseteq (f_\phi^{-1}(M_F))^{da\alpha}$ (resp. $f_\phi^{-1}(int(M_F)) \subseteq (f_\phi^{-1}(M_F))^{ba\alpha}$), for every $M_F \subseteq Y$.

**Proof.** The proof is similar to that of Theorem (2.9). \qed

**Theorem 2.11.** Let $\tau^*$ be an extended soft topology on $X$. Then a soft map $g_\phi : (X, \tau^*, E, \leq_1) \to (Y, \theta, F, \leq_2)$ is soft $I$ (resp. soft $D$, soft $B$) $\alpha$-continuous if and only if a map $g : (X, \tau^*, \leq_1) \to (Y, \theta_{\phi(e)}, \leq_2)$ is $I$ (resp. $D$, $B$) $\alpha$-continuous.

**Proof.** Necessity: Let $U$ be an open subset of $(Y, \theta_{\phi(e)}, \leq_2)$. Then there exists a soft open subset $G_F$ of $(Y, \theta, F, \leq_2)$ such that $G(\phi(e)) = U$. Since $g_\phi$ is a soft $I$ (resp. soft $D$, soft $B$) $\alpha$-continuous map, then $g_\phi^{-1}(G_F)$ is a soft $I$ (resp. soft $D$, soft $B$) $\alpha$-open set. From Definition (1.11), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau, E, \leq_1)$ is given by $g_\phi^{-1}(G(e)) = g^{-1}(G(\phi(e)))$, for each $e \in E$. By hypothesis, $\tau^*$ is an extended soft topology on $X$, we obtain that a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of $(X, \tau, \leq_1)$ is $I$ (resp. $D$, $B$) $\alpha$-open. Hence a map $g$ is $I$ (resp. $D$, $B$) $\alpha$-continuous.

Sufficiency: Let $G_F$ be a soft open subset of $(Y, \theta, F, \leq_2)$. Then from Definition (1.11), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau^*, E, \leq_1)$ is given by $g_\phi^{-1}(G(e)) = g^{-1}(G(\phi(e)))$, for each $e \in E$. Since a map $g$ is $I$ (resp. $D$, $B$) $\alpha$-continuous, then a subset $g^{-1}(G(\phi(e)))$ of $(X, \tau^*, \leq_1)$ is $I$ (resp. $D$, $B$) $\alpha$-open. By hypothesis, $\tau^*$ is an extended soft topology on $X$, we obtain that $g_\phi^{-1}(G_F)$ is a soft $I$ (resp. soft $D$, soft $B$) $\alpha$-open subset of $(X, \tau^*, E, \leq_1)$. Hence a soft map $g_\phi$ is soft $I$ (resp. soft $D$, soft $B$) $\alpha$-continuous. \qed
Let \( (X, \tau, E, \leq_1) \rightarrow (Y, \theta, F, \leq_2) \) be a surjective soft \( \alpha \)-continuous. If \( \leq_1 \) is linearly ordered, then \( \theta \) is the soft indiscrete topology.

3. Soft \( I(D, B)\alpha \)-open and soft \( I(D, B)\alpha \)-closed maps

In this part, we establish the notions of soft \( I(D, B)\alpha \)-open and soft \( I(D, B)\alpha \)-closed maps and show the relationships among them. Also, we give the equivalent conditions for each one of these soft maps and investigate the interrelations between these soft maps and their counterparts of maps on topological ordered spaces.

**Definition 3.1.** A soft map \( f_\phi : (X, \tau, E, \leq_1) \rightarrow (Y, \tau, F, \leq_2) \) is called:

(i) Soft \( I \) (resp. Soft \( D \), Soft \( B \)) \( \alpha \)-open if the image of every soft open subset of \( X \) is a soft \( I \) (resp. soft \( D \), soft \( B \)) \( \alpha \)-open subset of \( Y \).

(ii) Soft \( I \) (resp. Soft \( D \), Soft \( B \)) \( \alpha \)-closed if the image of every soft closed subset of \( X \) is a soft \( I \) (resp. soft \( D \), soft \( B \)) \( \alpha \)-closed subset of \( Y \).

**Remark 3.2.** From Definition (3.1), we can note the following:

(i) Every soft \( I(D, B) \) \( \alpha \)-open map is soft \( \alpha \)-open.

(ii) Every soft \( I(D, B) \) \( \alpha \)-closed map is soft \( \alpha \)-closed.

(iii) Every soft \( I \alpha \)-open (resp. soft \( \alpha \alpha \)-closed) map is soft \( I \alpha \)-open or soft \( \alpha \beta \)-open (resp. soft \( \alpha \alpha \)-closed or soft \( \alpha \beta \)-closed).

In the following two examples, we show that the converse of the three statements of remark above fails.

**Example 3.3.** Let the two universe sets \( X, Y \), the two parameters sets \( A, B \) and a map \( f : X \rightarrow Y \) be the same as in Example (2.5). Consider a map \( \phi : A \rightarrow B \) is defined as, \( \phi(a_m) = b_m \), for \( m \in \{1, 2\} \). We define a partial order relation on \( Y \) as \( \leq = \Delta \cup \{(7, 11), (11, 13), (7, 13)\} \) and we define two soft topologies \( \tau \) and \( \theta \) on \( X \) and \( Y \), respectively, as \( \tau = \{\emptyset, X, F_A\} \) and \( \theta = \{\emptyset, Y, H_B\} \), where \( F_A = \{(a_1, \{2, 3\}), (a_2, \{5\})\} \) and \( H_B = \{(b_1, \{11\}), (b_2, \{11\})\} \). Since \( f_\phi(F_A) = \{(b_1, \{7, 11\}), (b_2, \{11\})\} \) is a soft \( \alpha \)-open set, then \( f_\phi : \! S(X_A) \rightarrow S(Y_B) \) is a soft \( \alpha \)-open map and since \( f_\phi(F_A) = \{(b_1, \{11\}), (b_2, \{7, 11\})\} \) is a soft \( \alpha \)-closed set, then \( f_\phi : \! S(X_A) \rightarrow S(Y_B) \) is a soft \( \alpha \)-closed map. On the other hand, \( f_\phi(F_A) \) is neither a soft \( \alpha \beta \)-open nor a soft \( \alpha \alpha \)-open set and \( f_\phi(F_A) \) is neither a soft \( \alpha \beta \)-closed nor a soft \( \alpha \alpha \)-closed set. Hence \( f_\phi \) is not a soft \( I(D, B) \) \( \alpha \)-open map and not a soft \( I(D, B) \) \( \alpha \)-closed map.

**Example 3.4.** In Example above, if we replace only the partial order relation by \( \leq = \Delta \cup \{(7, 11)\} \) (resp. \( \leq = \Delta \cup \{(11, 13)\} \), then the soft map \( f_\phi \) is soft \( I \alpha \)-open and soft \( I \alpha \)-closed (resp. soft \( \alpha \beta \)-open and soft \( \alpha \beta \)-closed), but is not soft \( I \alpha \)-open and soft \( \alpha \beta \)-closed.
Theorem 3.5. The following three properties of a soft map $f_\phi: (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:

(i) $f_\phi$ is soft $I\alpha$-open;

(ii) $\text{int}(f_\phi^{-1}(M_F)) \subseteq f_\phi^{-1}(M_F^{i\alpha})$, for every $M_F \subseteq Y$;

(iii) $f_\phi(\text{int}(N_E)) \subseteq (f_\phi(N_E))^{i\alpha}$, for every $N_E \subseteq X$.

Proof. (i) $\Rightarrow$ (ii): Given a soft subset $M_F$ of $Y$, it is obvious that $\text{int}(f_\phi^{-1}(M_F))$ is a soft open subset of $X$. Then, by hypothesis, it follows that $f_\phi(\text{int}(f_\phi^{-1}(M_F)))$ is a soft $I\alpha$-open subset of $\tilde{Y}$. Since $f_\phi(\text{int}(f_\phi^{-1}(M_F))) \subseteq f_\phi(f_\phi^{-1}(M_F)) \subseteq M_F$, then $\text{int}(f_\phi^{-1}(M_F)) \subseteq f_\phi^{-1}(M_F^{i\alpha})$.

(ii) $\Rightarrow$ (iii): Given a soft subset $N_E$ of $X$, from (ii), we obtain that $\text{int}(f_\phi^{-1}(f_\phi(N_E))) \subseteq f_\phi^{-1}((f_\phi(N_E))^{i\alpha})$.

Since $\text{int}(N_E) \subseteq f_\phi^{-1}((f_\phi(N_E))) \subseteq f_\phi^{-1}((f_\phi(N_E))^{i\alpha})$, then $f_\phi(\text{int}(N_E)) \subseteq (f_\phi(N_E))^{i\alpha}$ as required.

(iii) $\Rightarrow$ (i): Let $G_E$ be a soft open subset of $X$. Then $f_\phi(\text{int}(G_E)) = f_\phi(G_E) \subseteq (f_\phi(G_E))^{i\alpha}$. Hence $f_\phi$ is a soft $I\alpha$-open map.

The following theorem can be proved in a similar manner.

Theorem 3.6. The following three properties of a soft map $f_\phi: (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:

(i) $f_\phi$ is soft $D\alpha$-open (resp. soft $B\alpha$-open);

(ii) $\text{int}(f_\phi^{-1}(M_F)) \subseteq f_\phi^{-1}(M_F^{d\alpha})$ (resp. $\text{int}(f_\phi^{-1}(M_F)) \subseteq f_\phi^{-1}(M_F^{b\alpha})$), for every $M_F \subseteq Y$;

(iii) $f_\phi(\text{int}(N_E)) \subseteq (f_\phi(N_E))^{d\alpha}$ (resp. $f_\phi(\text{int}(N_E)) \subseteq (f_\phi(N_E))^{b\alpha}$), for every $N_E \subseteq X$.

Theorem 3.7. The following three statements hold for a soft map $f_\phi: (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$:

(i) $f_\phi$ is soft $I\alpha$-closed if and only if $(f_\phi(G_E))^{i\alpha\text{cl}} \subseteq f_\phi(\text{cl}(G_E))$, for every $G_E \subseteq X$.

(ii) $f_\phi$ is soft $D\alpha$-closed if and only if $(f_\phi(G_E))^{d\alpha\text{cl}} \subseteq f_\phi(\text{cl}(G_E))$, for every $G_E \subseteq X$.

(iii) $f_\phi$ is soft $B\alpha$-closed if and only if $(f_\phi(G_E))^{b\alpha\text{cl}} \subseteq f_\phi(\text{cl}(G_E))$, for every $G_E \subseteq X$. 

Proof. We only give a proof for the first statement and the others follow similar lines.

Necessity: Since $f_\phi$ is soft I$\alpha$-closed, then $f_\phi(\text{cl}(G_E))$ is a soft I$\alpha$-closed subset of $\widetilde{Y}$ and since $f_\phi(G_E) \subseteq f_\phi(\text{cl}(G_E))$, then $(f_\phi(G_E))^\text{I\alpha cl} \subseteq f_\phi(\text{cl}(G_E))$.

Sufficiency: Consider $H_E$ is a soft closed subset of $\widetilde{X}$.
Then $f_\phi(H_E)^\text{I\alpha cl}(f_\phi(H_E))^\text{I\alpha cl} = f_\phi(H_E)$. Therefore $f_\phi(H_E) = (f_\phi(H_E))^\text{I\alpha cl}$. This means that $f_\phi(H_E)$ is a soft I$\alpha$-closed set. Hence the proof is complete.

Theorem 3.8. The following three statements hold for a bijective soft map $f_\phi : (X, \tau, E, \preceq_1) \rightarrow (Y, \theta, F, \preceq_2)$:

(i) $f_\phi$ is soft I (resp. soft D, soft B) $\alpha$-open if and only if $f_\phi$ is soft D (resp. soft D, soft B) $\alpha$-closed.

(ii) $f_\phi$ is soft I (resp. soft D, soft B) $\alpha$-open if and only if $f_\phi^{-1}$ is soft I (resp. soft D, soft B) $\alpha$-continuous.

(iii) $f_\phi$ is soft D (resp. soft I, soft B) $\alpha$-closed if and only if $f_\phi^{-1}$ is soft I (resp. soft D, soft B) $\alpha$-continuous.

Proof. For the sake of economy, we only give proofs of cases outside the parenthesis for the three statements above and the cases between parenthesis can be made similarly.

(i) To prove the necessary condition, let $H_E$ be a soft closed subset of $\widetilde{X}$ and consider $f_\phi$ is a soft I$\alpha$-open map. Then $H_E^c$ is soft open and $f_\phi(H_E^c)$ is soft I$\alpha$-open. It follows from the bijectiveness of $f_\phi$, that $f_\phi(H_E^c) = [f_\phi(H_E)]^c$. This automatically implies that $f_\phi(H_E)$ is soft D$\alpha$-closed. Thus $f_\phi$ is a soft D$\alpha$-closed map. In a similar manner, we can prove the sufficiency condition.

(ii) Necessity: Let $G_E$ be a soft open subset of $\widetilde{X}$ and consider $f_\phi$ is a soft I$\alpha$-open map. Then $f_\phi(G_E)$ is soft I$\alpha$-open. It follows from the bijectiveness of $f_\phi$, that $f_\phi(G_E) = (f_\phi^{-1})^{-1}(G_E)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_E)$ is soft I$\alpha$-open. Thus $f_\phi^{-1}$ is a soft I$\alpha$-continuous map. In a similar manner, we can prove the sufficiency condition.

(iii) The proof of this statement comes immediately from (i) and (ii) above.

$\square$

Theorem 3.9. Let $\theta^*$ be an extended soft topology on $Y$ and $\phi$ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \preceq_1) \rightarrow (Y, \theta^*, F, \preceq_2)$ is soft I (resp. soft D, soft B) $\alpha$-open if and only if a map $g : (X, \tau, E, \preceq_1) \rightarrow (Y, \theta^*_{\phi(e)}, \preceq_2)$ is I (resp. D, B) $\alpha$-open.
Proof. Necessity: Let $U$ be an open subset of $(X, \tau, \leq_1)$ and $\phi(e) = f$. Then there exists a soft open subset $G_E$ of $(X, \tau, E, \leq_1)$ such that $G(e) = U$. Since $g_\phi$ is a soft I (resp. soft D, soft B) \(\alpha\)-open map, then $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) \(\alpha\)-open set. From Definition (1.11), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta, F, \leq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e \in \phi^{-1}(f)} g(G(e))$, for each $f \in F$. By hypothesis, $\theta^*$ is an extended soft topology on $Y$, a subset $\bigcup_{e \in \phi^{-1}(f)} g(G(e)) = g(U)$ of $(Y, \theta_\phi(e), \leq_2)$ is I (resp. D, B) \(\alpha\)-open. Hence a soft map $g_\phi$ is I (resp. D, B) \(\alpha\)-open.

Sufficiency: Let $G_E$ be a soft open subset of $(X, \tau, E, \leq_1)$. Then from Definition (1.11), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta^*, F, \leq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e \in \phi^{-1}(f)} g(G(e))$, for each $f \in F$. Since a map $g$ is I (resp. D, B) \(\alpha\)-open, then a subset $\bigcup_{e \in \phi^{-1}(f)} g(G(e))$ of $(Y, \theta^*_\phi(e), \leq_2)$ is I (resp. D, B) \(\alpha\)-open. By hypothesis, $\theta^*$ is an extended soft topology on $Y$, $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) \(\alpha\)-open subset of $(Y, \theta^*, F, \leq_2)$. Hence a soft map $g_\phi$ is soft I (resp. soft D, soft B) \(\alpha\)-open. \(\blacksquare\)

The result above is restated in case of a soft I (resp. soft D, soft B) \(\alpha\)-closed map. One can prove them similarly and so the proof will be omitted.

Theorem 3.10. Let $\theta^*$ be an extended soft topology on $Y$ and $\phi$ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \leq_1) \to (Y, \theta^*, F, \leq_2)$ is soft I (resp. soft D, soft B) \(\alpha\)-closed if and only if a map $g : (X, \tau_\epsilon, \leq_1) \to (Y, \theta^*_\phi(e), \leq_2)$ is I (resp. D, B) \(\alpha\)-closed.

Proposition 3.11. Consider $\tau$ is not the indiscrete topology on $X$. If an injective soft map $f_\phi : (X, \tau, E, \leq_1) \to (Y, \theta, F, \leq_2)$ is soft $\alpha$-open or soft $\alpha$-closed, then $\leq_2$ is not linearly ordered.

Proposition 3.12. Let $f_\phi : (X, \tau, E, \leq_1) \to (Y, \theta, F, \leq_2)$ and $g_\lambda : (Y, \theta, F, \leq_2) \to (Z, \upsilon, K, \leq_3)$ be two soft maps. Then then following properties hold, for $x \in \{I, D, B\}$.

(i) If $f_\phi$ is a soft $x\alpha$-continuous map and $g_\lambda$ is a soft continuous map, then $g_\lambda \circ f_\phi$ is a soft $x$-continuous map.

(ii) If $f_\phi$ is a soft open (resp. soft closed) map and $g_\lambda$ is a soft $x\alpha$-open (resp. $x\alpha$-closed) map, then $g_\lambda \circ f_\phi$ is a soft $x$-open (resp. $x$-closed) map.

(iii) If $g_\lambda \circ f_\phi$ is a soft $x$-open map and $f_\phi$ is surjective soft continuous, then $g_\lambda$ is a soft $x$-open map.

(iv) If $g_\lambda \circ f_\phi$ is a soft closed map and $g_\lambda$ is an injective soft $x$-continuous map, then $f_\phi$ is a soft $y$-closed map, where $(x, y) \in \{(I, D), (D, I), (B, B)\}$. 
4. Soft \( I(D, B) \alpha \)-homeomorphism maps

The concepts of soft \( I(D, B) \alpha \)-homeomorphism maps are introduced and their main properties are discussed. Some examples are constructed to illustrate the relationships among the initiated soft maps.

**Definition 4.1.** A bijective soft map \( g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2) \) is called soft \( I \) (resp. soft \( D \), soft \( B \)) \( \alpha \)-homeomorphism if it is soft \( I \alpha \)-continuous and soft \( I \alpha \)-open (resp. soft \( D \alpha \)-continuous and soft \( D \alpha \)-open, soft \( B \alpha \)-continuous and soft \( B \alpha \)-open).

**Remark 4.2.** From Definition (4.1), we can note the following:

(i) Every soft \( I \) (soft \( D \), soft \( B \)) \( \alpha \)-homeomorphism map is soft \( \alpha \)-homeomorphism.

(ii) Every soft \( B \alpha \)-homeomorphism map is soft \( I \alpha \)-homeomorphism or soft \( D \alpha \)-homeomorphism.

To elucidate that the two items of the remark above are not conversely, we give two examples below.

**Example 4.3.** Let \( X = \{3, 5, 7, 11\} \) be a universe set and \( A = \{a_1, a_2\} \) be a parameters set. Consider \( \phi : A \to A \) and \( f : X \to X \) are both identity maps. We define two partial order relations on \( X \) and \( Y \), respectively, as \( \preceq_1 = \bigtriangleup \cup \{(7, 5)\} \) and \( \preceq_2 = \bigtriangleup \cup \{(3, 11)\} \) and we define two soft topologies \( \tau \) and \( \theta \) on \( X \) and \( Y \), respectively, as \( \tau = \{\emptyset, X, F_A\} \) and \( \theta = \{\emptyset, Y, F_A, H_A\} \), where \( F_A = \{(a_1, \{3\}), (a_2, \{5, 7\})\} \) and \( H_A = \{(a_1, \{3, 5\}), (a_2, \{5, 7\})\} \). Then one can readily check that a soft map \( f_\phi : S(X_A) \to S(Y_B) \) is soft \( \alpha \)-homeomorphism. On the other hand, \( f_\phi(F_A) = F_A \) is not a soft \( I \alpha \)-open set and \( f_\phi^{-1}(H_A) = H_A \) is not a soft \( D \alpha \)-open set. Hence \( f_\phi \) is not soft \( I \) (soft \( D \), soft \( B \)) \( \alpha \)-homeomorphism.

**Example 4.4.** In Example above, if we replace only the partial order relation \( \preceq_2 \) by \( \preceq = \bigtriangleup \cup \{(5, 11)\} \), then the soft map \( f_\phi \) is soft \( D \alpha \)-homeomorphism, but is not soft \( B \alpha \)-homeomorphism. Also, if we replace only the partial order relation \( \preceq_1 \) by \( \preceq = \bigtriangleup \cup \{(11, 3)\} \), then the soft map \( f_\phi \) is soft \( I \alpha \)-homeomorphism, but is not soft \( B \alpha \)-homeomorphism.

**Theorem 4.5.** If a bijective soft map \( f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2) \) is soft \( I \alpha \)-continuous (resp. soft \( D \alpha \)-continuous, soft \( B \alpha \)-continuous), Then the following three statements are equivalent:

(i) \( f_\phi \) is soft \( I \alpha \)-homeomorphism (resp. soft \( D \alpha \)-homeomorphism, soft \( B \alpha \)-homeomorphism);

(ii) \( f_\phi^{-1} \) is soft \( I \alpha \)-continuous (resp. soft \( D \alpha \)-continuous, soft \( B \alpha \)-continuous);

(iii) \( f_\phi \) is soft \( D \alpha \)-closed (resp. soft \( I \alpha \)-closed, soft \( B \alpha \)-closed).
**Proof.** (i) ⇒ (ii) Since \( f_\phi \) is a soft I\( \alpha \)-homeomorphism (resp. soft D\( \alpha \)-homeomorphism, soft B\( \alpha \)-homeomorphism) map, then \( f_\phi \) is soft I\( \alpha \)-open (resp. soft D\( \alpha \)-open, soft B\( \alpha \)-open). It follows from item (ii) of Theorem (3.8), that \( f_\phi^{-1} \) is soft I\( \alpha \)-continuous (resp. soft D\( \alpha \)-continuous, soft B\( \alpha \)-continuous).

(ii) ⇒ (iii) The proof follows from item (iii) of Theorem (3.8).

(iii) ⇒ (i) It sufficient to prove that \( f_\phi \) is a soft I\( \alpha \)-open (resp. soft D\( \alpha \)-open, soft B\( \alpha \)-open) map. This follows from item (i) of Theorem (3.8). □

**Theorem 4.6.** Let \( \tau^* \) and \( \theta^* \) be extended soft topologies on \( X \) and \( Y \), respectively. Then a soft map \( g_\phi : (X, \tau^*, E, \succeq_1) \to (Y, \theta^*, F, \succeq_2) \) is soft I (resp. soft D, soft B) \( \alpha \)-homeomorphism if and only if a map \( g : (X, \tau^*_e, \succeq_1) \to (Y, \theta^*_{\phi(e)}, \succeq_2) \) is I (resp. D, B) \( \alpha \)-homeomorphism.

**Proof.** The proof is obtained immediately from Theorem (2.11) and Theorem (3.9) □

**Proposition 4.7.** Let the two soft topologies \( \tau \) and \( \theta \) on \( X \) and \( Y \), respectively, do not belong to \{soft discrete topology, soft indiscrete topology\}. If a soft map \( f_\phi : (X, \tau, E, \succeq_1) \to (Y, \theta, F, \succeq_2) \) is soft B\( \alpha \)-homeomorphism, then \( \succeq_1 \) and \( \succeq_2 \) is not linearly ordered.

**Conclusion**

Al-shami et al. [14] combined a soft partially ordered set with a soft topological space to constitute a soft topological ordered space concept. As a contribution of this, we have established the concepts of soft \( x\alpha \)-continuous, soft \( x\alpha \)-open, soft \( x\alpha \)-closed and soft \( x\alpha \)-homeomorphism maps, for \( x \in \{I, D, B\} \). We have completely described these concepts and have deduced some results which connect the initiated soft maps with those maps via topological ordered spaces. To some extent there is a similarity between the results obtained herein and those presented in [15]. The newly ordered maps initiated herein will be an important basis for the further developments on soft topological ordered spaces. In our upcoming research, we intend to give more soft ordered topological concepts and investigate their properties.

**Conflict of interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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