ON SOME APPLICATIONS OF ALGEBRAIC HYPERSTRUCTURES FOR THE MANAGEMENT OF TEACHING AND RELATIONSHIPS IN SCHOOLS

Sarka Hoskova-Mayerova[*]
University of Defence
Department of Mathematics and Physics
Kounicova 65, 66210 Brno
Czech Republic
sarka.mayerova@unob.cz

Antonio Maturo
University of Pescara-Chieti
Department of Architecture
viale Pindaro 42, 65127 Pescara
Italy
amaturo@unich.it

Abstract. In terms of school context, the effectiveness of a teaching process frequently depends to a large extent on the relationship system, which had been formed within a classroom. This paper is going to present how algebraic hyperstructures can contribute significantly to understanding the system of relationships within a classroom. Furthermore, it becomes possible to assess the impact of interventions targeted at improving the system of relationships and thus to establish undisturbed and fundamental participation of students in the learning process.

Keywords: algebraic hyperstructures, interpersonal relations.

1. Introduction

In recent decades, research on learning efficiency has attracted and still has been attracting the interest of world experts. Various experience has shown that at schools, in particular at elementary and primary level, both teacher training and pedagogical skills cannot reach required and acceptable results unless there is collaborative, friendly and positive relationship, e.g. [2, 3, 11, 25, 22].

Studying existing relationships among students is an inevitable prerequisite for planning interventions focused on reaching adequate required teaching/learning efficiency. [24, 35, 37]. Let S is a set of students of a particular scholastic classroom K. A scientific study covering the relationship among students within a classroom has lead to the determining a final set of relationships R.

∗. Corresponding author

In the past, social relationships within school environment were described by many authors (e.g. Moreno, [30, 29] Sciarra, [36]) using a set of binary sharp relationships. The most efficient teaching methods in terms of the system of mutual relationships within a classroom have also been studied in several recent papers by Delli Rocili and Maturo [7, 9, 10] and also Hoskova-Mayerova [12, 13, 14, 15].

Some of our findings concluded (Hoskova-Mayerova & Maturo, [17, 18, 19, 20] that deeper and more profound knowledge on school environment relationship can be obtained through fuzzy relationships since they consider semantic uncertainty and degree of relationship intensity. See also [8, 25, 26]. This paper continues, deepens and expands the ideas presented in our previous several works, which demonstrate different perspectives. We also present that there is another tool for representing and evaluating uncertainty; these are the algebraic hyperstructures, more flexible than the common operations, because they are multi-results, which represent the possible outcomes of an agreement between individuals. The section dealing with results covers some algebraic hyperstructures associated with existing relationships within a school context.

2. Algebraic hyperstructures associated to the Moreno approach

The theory of hyperstructures originates from the work of Marty [23] both the ideas and definitions published here have been developing particularly in the last 40 years. The hypergroup has been the most studied hyperstructure; it is a concept that generalizes the concept of a group. In the book "Prolegomena hypergroup theory" (Corsini [4]), basic results in terms of hypergroup theory are presented up to 1992. The book supplement comprises the entire bibliography. The results overview until 2003 was published in 2006 by Corsini & Leoreanu, see [5]). A very detailed bibliography with respect to hyperstructure study is available on the website: `www.aha.eled.duth.gr/Thesaurus1.1.htm.` Further review can be found in the work done by Hoskova & Chvalina and published at proceedings of the conference Algebraic Hyperstructures and Applications–AHA 2005, see [16].

Perhaps, the most important impetus for the study of algebraic hyperstructures came from the basic material "Join Geometries" by Prenowitz and Jantosciak published in 1979; [34] in addition to providing an original and general approach to the study of geometry, this work introduces the interdisciplinary view of geometry and algebra: there is shown how to trace back the study of the Euclidean geometry of a specific commutative hypergroup that satisfies a particular axiom called *incidence property*. Various other geometries, such as "Projective geometry", can also be transferred to commutative hypergroups satisfying the incidence property.

The idea of studying hypergroup applications to solve problems of uncertainty and decision-making problems in architecture and social sciences was born after a series of conferences held at the Faculty of Architecture in Italian

Pescara organized by Giuseppe Tallini in 1993. The concept was being expanded, researched and developed at various AHA (Algebraic Hyperstructures and Applications) conferences as well as in domestic seminars and conferences organized by Prof. Piergiulio Corsini in the period 1994-2014; e.g., in December 1994 and October 1995, two conferences on "Hyperstructures and their applications in the field of cryptography, geometry and uncertainty" were organized by Corsini, Eugeni and Maturo in Chieti and Pescara.

Let us recall some fundamental definitions on algebraic hyperstructures. For more information, see the Vougiouklis book [40] and the papers (Jafarpour and Cristea [21], Chvalina and Hoskova [6], Massouros and Massouros [27, 28], Nikolaidou P. & Vougiouklis T. [31, 41] Novák [32, 33]; Vougiouklis [38]; Vougiouklis et al. [39, 42, 43]) and many others.

Definition 2.1. *Let S be a nonempty set. A function $\alpha : S \times S \to P(S)$, where $P(S)$ is the family of subsets of S, is said to be:*

- *a hyperoperation on S, if $\forall x, y \in S, \alpha(x, y) \neq \emptyset$;*
- *a partial hyperoperation on S, if $\exists x, y \in S, \alpha(x, y) = \emptyset$;*
- *commutative on S, if $\forall x, y \in S, \alpha(x, y) = \alpha(y, x)$;*
- *closed, if $\forall x, y \in S, \alpha(x, y) \supseteq \{x, y\}$;*
- *open, if $\forall x, y \in S, (x \neq y) \Rightarrow \alpha(x, y) \cap \{x, y\} = \emptyset$;*
- *idempotent, if $\forall x \in S, \alpha(x, x) = \{x\}$;*
- *reproductive, if $\forall x, y \in S, \exists u, v \in S : y \in \alpha(x, u) \cap \alpha(v, x)$.*

The pair (S, α), with S hyperoperation (resp. partial hyperoperation) on S is said to be a *hypergroupoid* (resp. *partial hypergroupoid*). Usually the set α(x, y) is written x α y and is called the hyperproduct of x by y (with respect to the hyperoperation α). If H and K are subsets of S then the set hyperproduct H α K is the union of the hyperproducts x α y with x∈H, y∈K.
A hypergroupoid (S, α) is said to be:

- a *quasi-hypergroup*, if the reproductive property is valid, i.e., $\forall x \in S$, $x\alpha S = S = S\alpha x$;
- a *semi-hypergroup*, if the following *associative property* is valid: $\forall x, y, z \in S$, $(x\alpha y)\alpha z = x\alpha(y\alpha z)$
- a *hypergroup*, if it is a quasi-hypergroup and a semi-hypergroup.

3. New results

Let ρ be a Moreno binary relation on the set S of students in a school class. Let us introduce the following definition:

Definition 3.1. *Let $\otimes$ be a binary operation, i. e. an operation in $\{0,1\}$, and ρ a reflexive relation on S. We define:*

- *active hyperoperation (eventually partial hyperoperation) associated with $(\otimes,\rho)$ the function $\otimes^a_\rho\colon (x,y)\in S\times S\to x\otimes^a_\rho y=\{z\in S\colon (x\,\rho\,z)\otimes(y\,\rho\,z)=1\}$;*
- *passive hyperoperation (eventually partial hyperoperation) associated with $(\otimes,\rho)$ the function $\otimes^p_\rho\colon (x,y)\in S\times S\to x\otimes^p_\rho y=\{z\in S : (z\,\rho\,x)\otimes(z\,\rho\,y)=1\}$;*
- *circular hyperoperation (eventually partial hyperoperation) associated with $(\otimes,\rho)$ the function $\otimes^c_\rho\colon (x,y)\in S\times S\to x\otimes^c_\rho y=\{z\in S : (x\,\rho\,z)\otimes(z\,\rho\,y)=1\}$;*
- *inverse circular hyperoperation (eventually partial hyperoperation) associated with $(\otimes,\rho)$ the function $\otimes^i_\rho\colon (x,y)\in S\times S\to x\otimes^i_\rho y=\{z\in S : (z\,\rho\,x)\otimes(y\,\rho\,z)=1\}$.*

Let us consider, to set the ideas, the active hyperoperation. As ρ is reflexive, the possible cases that can arise are those shown in the following Table 1.

ρ	x	y	u	v	w	t
x	1	0,1	0	1	0	1
y	0,1	1	0	0	1	1

Table 1:

3.1 Particular cases of active hyperoperations

Let $\otimes$ be the union $\cup$. Then $(x\,\rho\,z)\otimes(y\,\rho\,z)=\max\{(x\,\rho\,z),\,(y\,\rho\,z)\}$ and from Table 1 we can see that $(x\,\rho\,z)\otimes(y\,\rho\,z)=1$ for $z\in\{x,y,v,w,t\}$. In particular, $\forall x,y\in S, x\cup^a_\rho y\supseteq\{x,y\}$. Then $(S,\cup^a_\rho)$ is a closed quasi-hypergroup. As $\cup$ is commutative, the associate active hyperoperation is commutative.

Let $\otimes$ be the intersection $\cap$. Then $(x\,\rho\,z)\otimes(y\,\rho\,z)=\min\{(x\,\rho\,z),\,(y\,\rho\,z)\}$ and from Table 1 we can see that $(x\,\rho\,z)\otimes(y\,\rho\,z)=1$ for $z=t$.

The function $\cup^a_\rho$ is a commutative partial hyperoperations and is an hyperoperations if and only if the following condition holds: $\forall x,y\in S$, $\exists t\in S : x\,\rho\,t$ and $y\,\rho\,t$. The student t can be defined as "*a passive mediator*" between x and y. So each pair of students must have at least one passive mediator.

Let $\otimes$ be the implication $\to$. Then $(x\,\rho\,z)\otimes(y\,\rho\,z)=\max\{1-(x\,\rho\,z),\,(y\,\rho\,z)\}$ and from Table 1 we can see that $(x\,\rho\,z)\otimes(y\,\rho\,z)=1$ for $z\in\{y,u,w,t\}$. In particular, $\forall x,y\in S, x\to^a_\rho y\supseteq\{y\}$. Then $(S,\cup^a_\rho)$ is a hypergroupoid.

Similar considerations can be made for the passive hyperoperation (resp. partial hyperoperation) associated with $(\otimes,\ \rho)$. It is sufficient to observe that it is reduced to active hyperoperation (resp. partial hyperoperation) associated with $(\otimes,\ \rho^{-1})$.

3.2 Particular case of circular hyperoperations

The circular hyperoperation (resp. partial hyperoperation) associated with $(\otimes, \rho)$ leads to the consideration of the paths of length 2 of the digraph (S, ρ), and to consider the binary operation on the arcs of each paths. As the inverse circular hyperoperation (resp. partial hyperoperation) associated with $(\otimes,\ \rho)$ reduces to the circular hyperoperation (resp. partial hyperoperation) associated with $(\otimes, \rho^{-1})$, it is sufficient to study the properties of the function $\otimes_p^c$.

Let $\otimes$ be the union $\cup$. Then $(x\rho z) \otimes (z\,\rho\, y) = \max\{(x\,\rho\, z),\ (z\,\rho\, y)\}$. As ρ and the inverse ρ^{-1} are reflexive, $\forall x, y \in S, x \cup_p^c y \supseteq \{x, y\}$. Then $(S, \cup_p^c)$ is a closed quasi-hypergroup.

Let $\otimes$ be the intersection $\cap$. Then $(x\,\rho\, z) \otimes (z\,\rho\, y) = \ min\{(x\,\rho\, z),\ (z\,\rho\, y)\}$. As the reflexivity of ρ, $(x\,\rho\, z) \otimes (z\,\rho\, y) = 1$ if $x, \rho\, y$ or there is a path of length 2 of consecutive vertices x, z, y with $z \neq x$ and $z \neq y$.

The function $\otimes_p^c$ is a hyperoperation if and only if the following condition hold: $\forall x, y \in S$, $(x(-\rho)y) \Rightarrow z \in S\colon x\,\rho\, z$ and $z\,\rho\, y$. The student z can be defined as"an intermediate mediator" between x and y. So each pair of students must have at least one intermediate mediator.

Let $\otimes$ be the implication $\rightarrow$. Then $(x\,\rho\, z) \otimes (z\,\rho\, t) = \max\{1-(x\,\rho\, z),\ (z\,\rho\, y)\}$. As ρ and the inverse ρ^{-1} are reflexive, $\forall x, y \in S$, $x \rightarrow_\rho^c y \supseteq \{y\}$. Then $(S, \rightarrow_\rho^a)$ is a hypergroupoid.

4. Conclusions and perspective of research

From Sections 1 we can see that the significant algebraic hyperstructures that can be associated with the relations system of a school class are very numerous. In some of our papers (see e.g., Hoskova-Mayerova and Maturo, [17, 18, 19]) many other types of hyperstructures, from points of view different from those considered in this work, have also been examined.

In the context of Moreno's binary approach it is also possible to directly construct algebraic hyperstructures in S by making interviews to the ordered pairs (x, y) of students, making the first element of the pair, x, assume, the role of indicating classmates that he considers suitable for an activity (at least 2) and the second element of the pair, y, the possibility to choose which elements indicated by x are accepted (at least 1).

If n is the number of elements of the class, to implement this procedure it is necessary to propose $2n$ meetings for interviews (as for a football league with n teams). This way to obtain hyperstructures associated with the class is therefore very significant but rather time consuming.

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