THREE-DIMENSIONAL AIR QUALITY ASSESSMENT SIMULATIONS INSIDE SKY TRAIN PLATFORM WITH AIRFLOW OBSTACLES ON HEAVY TRAFFIC ROAD

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Abstract. Air pollutant levels in Bangkok are generally high in street tunnels. They are particularly elevated in almost closed street tunnels such as an area the Bangkok sky train platform with high traffic volume where dispersion is limited. This area has no air quality measurement stations even though there is a high percentage of people living around this vicinity. We are interested to conduct a research the Bangkok sky train platform due to the traffic density and enormous polluted areas. Therefore, we proposed a numerical modeling of air pollution concentration in sky train platform with airflow obstacles on heavy traffic road as an approximated solution of the three-dimensional advection-diffusion equation by using the finite difference methods. Our research presentation is based on how air pollution model depends on the flow of air pollution and wind directions including the governing equation of the corresponding three-dimensional advection-diffusion equation is presented. This also includes the initial condition and boundary conditions of traffic and polluted areas. In order to illustrate the performance of the model, the numerical experiments are presented. The comparison between the two methods and the simulations of air pollution control are proposed. The three-dimensional advection-diffusion equation is solved by using the Forward Time, Centered Space (FTCS) and Forward Time, Backward Space (FTBS) schemes. The results obtained indicate that the FTCS method provides a better result than FTBS method. Furthermore, the proposed experimental variations of the boundary condition in the entrance gate do affect the air pollutant concentration of each floor.

Keywords: air pollutant concentration, finite difference techniques, air quality, heavy traffic, sky train platform, tunnel.

1. Introduction

Currently, Thailand is facing a rapid growth in both agriculture and industry resulting in Bangkok being the center of prosperity in all aspects with rapid population increase in a blink of an eye, followed by a high demand for travel and transportation. This creates an intensifying traffic congestion and air pollution that derives from the rapid increase of cars and vehicles. Air pollution is one of the main and biggest problems and Bangkok has reached a critical level with hazardous substances in some areas because pollution is created by human beings and natural phenomena damaging the environment and human well-being. Not only is air pollution hazardous locally but it is one of the world's biggest killers and issue because people faced health problems such as asthma, bronchitis, cancer, etc. If people are constantly exposed to high levels of dust, they may suffer from illnesses such as silicosis or asbestosis. So it can be said that air pollution from traffic is tremendously serious especially a Bangkok sky train platform than any other areas. The volume of carbon monoxide and nitrogen oxides in this area is higher than the standard volume. This issue should be realized for the study interest and further research to find solutions to reduce pollution.

In [1], the Kriging method for regression analysis can be used to analytically relate the mass emission rate of carbon monoxide and nitrogen dioxide at the Bangkok Mass Transit System (BTS). The results indicate that the concentration of carbon monoxide exceed the Bangkok standard volume and the concentration of nitrogen dioxide does not exceed the Bangkok standard volume. Undeniably, the air pollutant concentration was related to the traffic flow pattern, traffic characteristic, street geometries, and human activities. The traffic flow is the main pollution source in many urban areas. It causes more ambient air pollutants such as carbon monoxide (CO), sulfur dioxide (SO2), nitrogen dioxide (NO2), nitrogen oxides (NOx), volatile organic compounds (VOCs), ozone (O3), particulate matter (PM10), benzene, heavy metals, and respirable particulate matter (PM2.5 and PM10) [2, 3, 4]. Therefore, 1D Lighthill-Whitham-Richards traffic model and advection-diffusion-reaction pollution model for estimating the pollution emission rate due to traffic flow in big cities are proposed in [3]. The modeling of oxidation and hydrolysis of sulfur and nitrogen oxide used the convection-diffusion-reactions equation for higher order accurate solutions. The technique of Lax and Wendroff is introduced in [5]. A three-dimensional advection-diffusion equation of air pollutant is applied to a street tunnel configuration by using the FTCS finite difference method with air flow in x and y directions in [6]. In [7], a one-dimensional advection-diffusion equation with variable coefficients in semi-infinite media [8] is solved using explicit finite difference method for three dispersion problems: (i) solute dispersion along the steady flow, (ii) temporarily dependent solute dispersion along the uniform flow, and (iii) solute dispersion is temporarily dependent on the steady flow through inhomogeneous medium to find solutions. A study of vehicle exhaust dispersion within different street canyons models in urban ventilated by cross-wind by using the advanced computational and mathematical models. The pollutant concentrations are estimated for the street canyon models, which may include simplified photochemistry and particle deposition-suspension algorithms. After application of Box model to the street canyon. [9] can be calculated for CO, NOx, SPM, and PM10 by this box model. In [10], the effects of the variations of atmospheric stability classes and wind velocities on the three-dimensional air-quality models are observed. The fractional step method is used to solve the dispersion model for advection-diffusion equation.

In this research, we are interested in traffic density of the area the Bangkok sky train platform. We proposed the numerical modeling of air pollutant concentration in sky train platform with airflow obstacles on heavy traffic road. The estimated three-dimensional advection-diffusion equation is used by the finite difference method. In our research, we indicate that the air pollution modeling depends on air pollution flows and wind directions. In the second section, the governing equation corresponding to the model is the three-dimensional advection-diffusion equation including the initial condition and boundary conditions. The third section is numerical techniques. The finite difference technique introduced two methods for calculating the air pollutant concentration. The three-dimensional advection-diffusion equation is solved by using the Forward Time, Centered Space (FTCS) and Forward Time, Backward Space (FTBS) schemes. In order to illustrate the performance of the model, in section 4 the numerical experiments are presented. This is the comparisons between the two methods and the simulations of the proposed air pollution control. Finally, the discussion and conclusion are presented in section 5.

2. Governing equation

The street tunnel configuration is shown in Figure 1. That is, the street is flanked by buildings on both sides, including the top area is also closed. The bottom floor is the street floor, next up is the ticket floor and the top floor is the platform floor. For both sides of the street are the section of buildings. In this research, we assume that there are wind inflow in x- and y-directions and there are the obstacles as columns. The columns are on both sides of the street tunnel. The air pollutant concentrations are emitted from the entrance gate and the right side gap as Figure 2(a). We consider the wind inflow along x- and y-directions as Figure 2(b). Then the consider domain becomes : $\Omega = \{(x, y, z);$ $0 \le x \le L$, $0 \le y \le W$, $0 \le z \le H\}$, where W is the width (m), L is the length (m) and H is the height (m) of the street tunnel.

Figure 1: The street tunnel configuration.

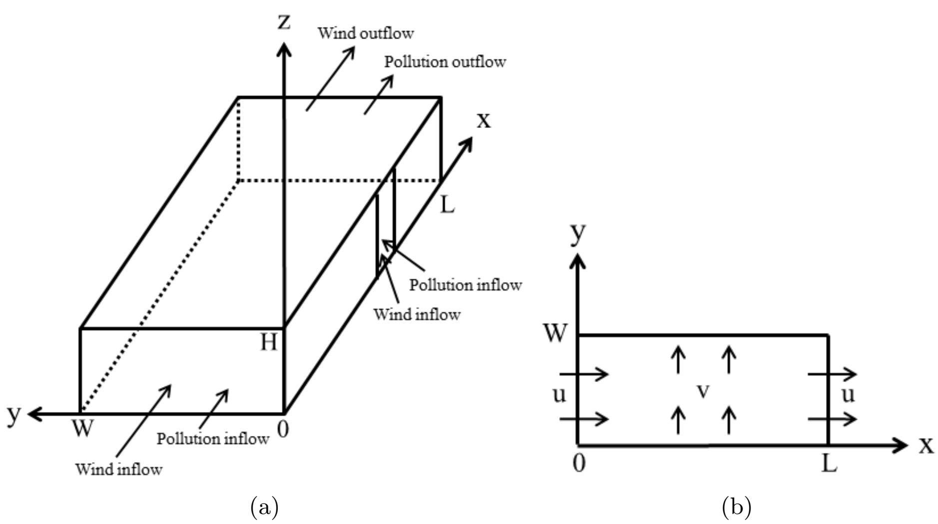


Figure 2: (a) The direction of pollution and wind flows. (b) The wind directions for street tunnel.

The air pollutant concentrations can be described by the three-dimensional advection-diffusion equation as follows:

$$\frac{\partial C}{\partial t} + V \cdot \nabla C = \nabla \cdot \left(\bar{K} \otimes \nabla C \right), \tag{1}$$

where $C = C(x, y, z, t)$ is the air pollutant concentration at point (x, y, z) in Cartesion coordinates at time t (kg/m^3), $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$, and $\otimes$ is matrix multiplication. The vector V is the wind velocity field (m/sec), $\bar{K}$ is the eddy-diffusivity or dispersion tensor (m^2/sec).

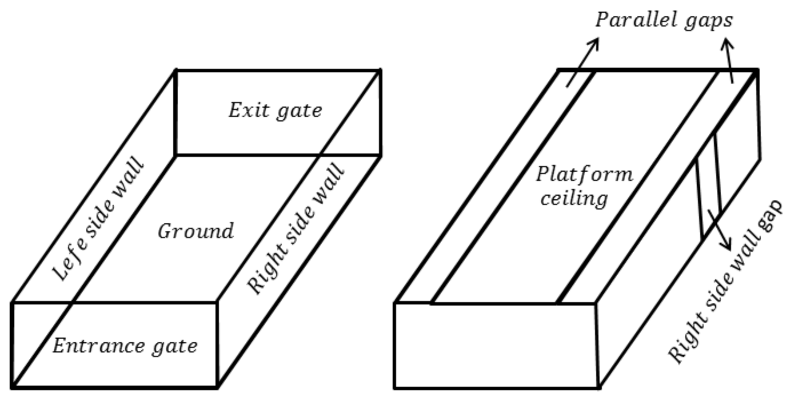


Figure 3: The components of the street tunnel.

The three-dimensional advection-diffusion equation in Eq. (1), can be written as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = k_x\frac{\partial^2 C}{\partial x^2} + k_y\frac{\partial^2 C}{\partial y^2} + k_z\frac{\partial^2 C}{\partial z^2}, \tag{2}$$

where u, v, and w are the constant wind velocity (m/sec) in x, y, and z-directions, respectively, k_x, k_y, and k_z are the constant diffusion coefficient (m^2/sec) in x, y, and z-directions, respectively.

The assumptions of Eq. (2) are defined that the wind inflow in x- and y-directions are the horizontal direction and in z-direction is the vertical direction. Consequently, the three-dimensional advection-diffusion equation in Eq. (2), can be written as:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = k_h\frac{\partial^2 C}{\partial x^2} + k_h\frac{\partial^2 C}{\partial y^2} + k_v\frac{\partial^2 C}{\partial z^2}, \tag{3}$$

where k_h is the constant dispersion coefficient in the horizontal direction (m^2/sec) and k_v is the constant dispersion coefficient in the vertical direction (m^2/sec).

We consider the components of the street tunnel shown in Figure 3. Figure 4 shows the model of the problem with A is the right parallel gap size along the ceiling, B is the left parallel gap size along the ceiling and $G - F$ is the right side wall gap of the beside consider domain. (D_{cn}, E_{rn}) is a center point of the columns for $cn = 1, 2, ..., ncl$ and $rn = 1, 2$, where ncl is the number of the columns. The initial condition, there is no initial pollutant $C(x, y, z, 0) = 0$, for all $(x, y, z) \in \Omega$. For the boundary conditions are assumed that

Entrance gate : $C(0, y, z, t) = c_1$.
Margin of entrance gate : $\frac{\partial C}{\partial x}(0, y, z, t) = c_2, y = 0, W, z = 0, H$.
Exit gate : $\frac{\partial C}{\partial x}(L, y, z, t) = c_3$.
Left side wall : $\frac{\partial C}{\partial y}(x, W, z, t) = c_4$.
Right side wall gap : $C(x, 0, z, t) = c_5, F \leq x \leq G$.

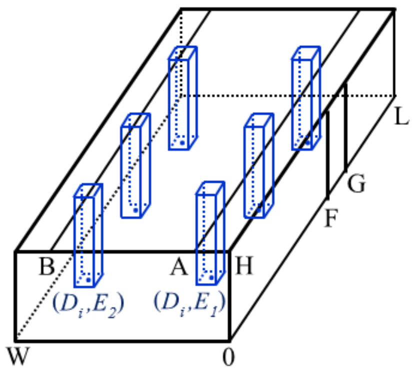


Figure 4: Model of the problem.

Right side wall : $\frac{\partial C}{\partial y}(x,0,z,t) = c_6$, otherwise.

Ground : $\frac{\partial C}{\partial z}(x,y,0,t) = c_7$.

Platform ceiling : $\frac{\partial C}{\partial z}(x,y,H,t) = c_8, A < y < B$.

Parallel gaps : $\frac{\partial C}{\partial z}(x,y,H,t) = c_9$, otherwise.

Center columns : $C(D_{cn}, E_{rn}, z, t) = 0, 0 \leq z \leq H$.

Front and back columns : $\frac{\partial C}{\partial x}(D_{cn} - 1, y, z, t) = \frac{\partial C}{\partial x}(D_{cn} + 1, y, z, t) = c_1 0, E_{rn} - 1 \leq y \leq E_{rn} + 1$, for all $t > 0$.

Left and right columns: $\frac{\partial C}{\partial y}(x, E_{rn} - 1, z, t) = \frac{\partial C}{\partial y}(x, E_{rn} + 1, z, t) = c_1 1, D_{cn} - 1 \leq x \leq D_{cn} + 1$, for all $t > 0$.
Where c_1 and c_5 are the air pollutant concentration inflow in $x-$ and $y-$ directions, respectively, $c_2, c_3, c_4, c_6, c_7, c_8, c_9, c_{10}$, and c_{11} are the rate of change of air pollutant concentration in each the boundary conditions.

3. Numerical techniques

We use the finite difference methods to compute a numerical approximation to the solutions of a three-dimensional advection-diffusion equation. The solution domain of the problem over time $0 \leq t \leq T$ is covered by a mesh of grid spacing: $x_i = i\Delta x, i = 0,1,2,...,M; y_j = j\Delta y, j = 0,1,2,...,N; z_k = k\Delta z, k = 0,1,2,...,P; t_n = n\Delta t, n = 0,1,2,...,Q$; parallel to the space and time coordinate axes, respectively. Approximation the solution of the air pollutant concentration $C^n_{i,j,k}$ to $C(i\Delta x, j\Delta y, k\Delta z, n\Delta t)$ are calculated at the point of intersection of these lines, namely, $(i\Delta x, j\Delta y, k\Delta z, n\Delta t)$ which is referred to as the (i,j,k,n) grid point. The constant spatial and temporal grid-spacing are $\Delta x = \frac{L}{M}, \Delta y = \frac{W}{N}, \Delta z = \frac{H}{P}, \Delta t = \frac{T}{Q}$, respectively. In this research, we distinguish two difference methods as following:

3.1 Forward time central space scheme

The first method, we use an explicit forward difference estimate for the time derivative (FT), and central difference approximations for the space derivative (CS), so the acronym FTCS. Consequently, the three-dimensional advection-diffusion equation in Eq. (3) becomes

$$\frac{C^{n+1}_{i,j,k}-C^{n}_{i,j,k}}{\Delta t}+u\left(\frac{C^{n}_{i+1,j,k}-C^{n}_{i-1,j,k}}{2\Delta x}\right)+v\left(\frac{C^{n}_{i,j+1,k}-C^{n}_{i,j-1,k}}{2\Delta y}\right)$$
$$=D_h\left(\frac{C^{n}_{i+1,j,k}-2C^{n}_{i,j,k}+C^{n}_{i-1,j,k}}{(\Delta x)^2}\right)+D_h\left(\frac{C^{n}_{i,j+1,k}-2C^{n}_{i,j,k}+C^{n}_{i,j-1,k}}{(\Delta y)^2}\right)$$
$$+D_v\left(\frac{C^{n}_{i,j,k+1}-2C^{n}_{i,j,k}+C^{n}_{i,j,k-1}}{(\Delta z)^2}\right). \tag{4}$$

Rearrangement and simplification of Eq. (4),

$$\begin{aligned} C^{n+1}_{i,j,k} &= \left(s_x+\frac{r_x}{2}\right)C^{n}_{i-1,j,k}+\left(s_y+\frac{r_y}{2}\right)C^{n}_{i,j-1,k}+(s_z)\,C^{n}_{i,j,k-1} \\ &+ \left(s_x-\frac{r_x}{2}\right)C^{n}_{i+1,j,k}+\left(s_y-\frac{r_y}{2}\right)C^{n}_{i,j+1,k}+(s_z)\,C^{n}_{i,j,k+1} \\ &+ (1-2s_x-2s_y-2s_z)\,C^{n}_{i,j,k}, \end{aligned} \tag{5}$$

in which $r_x=\frac{u\Delta t}{\Delta x}, r_y=\frac{v\Delta t}{\Delta y}, s_x=\frac{D_h\Delta t}{(\Delta x)^2}, s_y=\frac{D_h\Delta t}{(\Delta y)^2}$ and $s_z=\frac{D_v\Delta t}{(\Delta z)^2}$.

The finite difference scheme for the left end and the right end of the fictitious points are following:

$$C^{n}_{-1,j,k}=\frac{4C^{n}_{0,j,k}-C^{n}_{1,j,k}-2c_2\Delta x}{3}, \tag{6}$$

$$C^{n}_{i,-1,k}=\frac{4C^{n}_{i,0,k}-C^{n}_{i,1,k}-2c_6\Delta y}{3}, \tag{7}$$

$$C^{n}_{i,j,-1}=\frac{4C^{n}_{i,j,0}-C^{n}_{i,j,1}-2c_7\Delta z}{3}, \tag{8}$$

$$C^{n}_{M+1,j,k}=\frac{4C^{n}_{M,j,k}-C^{n}_{M-1,j,k}+2c_3\Delta x}{3}, \tag{9}$$

$$C^{n}_{i,N+1,k}=\frac{4C^{n}_{i,N,k}-C^{n}_{i,N-1,k}+2c_4\Delta y}{3}, \tag{10}$$

$$C^{n}_{i,j,P+1}=\frac{4C^{n}_{i,j,P}-C^{n}_{i,j,P-1}+2c_8\Delta z}{3}. \tag{11}$$

3.2 Forward time backward space scheme

The second method, we calculated by using an explicit forward difference estimate for the time derivative (FT), and backward difference approximations for the space derivative (BS), so the acronym FTBS. The approximate solution of a three-dimensional advection-diffusion equation in Eq. (3) use the FTCS scheme satisfies

$$\frac{C_{i,j,k}^{n+1}-C_{i,j,k}^{n}}{\Delta t}+u\left(\frac{C_{i,j,k}^{n}-C_{i-1,j,k}^{n}}{2\Delta x}\right)+v\left(\frac{C_{i,j,k}^{n}-C_{i,j-1,k}^{n}}{2\Delta y}\right)$$

$$=D_h\left(\frac{C_{i,j,k}^{n}-2C_{i-1,j,k}^{n}+C_{i-2,j,k}^{n}}{(\Delta x)^2}\right)+D_h\left(\frac{C_{i,j,k}^{n}-2C_{i,j-1,k}^{n}+C_{i,j-2,k}^{n}}{(\Delta y)^2}\right)$$

$$+D_v\left(\frac{C_{i,j,k}^{n}-2C_{i,j,k-1}^{n}+C_{i,j,k-2}^{n}}{(\Delta z)^2}\right). \tag{12}$$

Rearrangement and simplification of Eq. (12),

$$\begin{aligned} C_{i,j,k}^{n+1} &= (s_x)\,C_{i-2,j,k}^{n}+(s_y)\,C_{i,j-2,k}^{n}+(s_z)\,C_{i,j,k-2}^{n}\\ &+ (r_x-2s_x)\,C_{i-1,j,k}^{n}+(r_y-2s_y)\,C_{i,j-1,k}^{n}-(2s_z)\,C_{i,j,k-1}^{n}\\ &+ (1+s_x+s_y+s_z-r_x-r_y)\,C_{i,j,k}^{n}. \end{aligned} \tag{13}$$

The finite difference scheme for the left end of the fictitious points are following:

$$C_{-1,j,k}^{n}=\frac{4C_{0,j,k}^{n}-C_{1,j,k}^{n}-2c_2\Delta x}{3}, \tag{14}$$

$$C_{-2,j,k}^{n}=\frac{13C_{0,j,k}^{n}-4C_{1,j,k}^{n}-14c_2\Delta x}{9}, \tag{15}$$

$$C_{i,-1,k}^{n}=\frac{4C_{i,0,k}^{n}-C_{i,1,k}^{n}-2c_6\Delta y}{3}, \tag{16}$$

$$C_{i,-2,k}^{n}=\frac{13C_{i,0,k}^{n}-4C_{i,1,k}^{n}-14c_6\Delta y}{9}, \tag{17}$$

$$C_{i,j,-1}^{n}=\frac{4C_{i,j,0}^{n}-C_{i,j,1}^{n}-2c_7\Delta z}{3}, \tag{18}$$

$$C_{i,j,-2}^{n}=\frac{13C_{i,j,0}^{n}-4C_{i,j,1}^{n}-14c_7\Delta z}{9}. \tag{19}$$

4. Numerical experiments

4.1 Comparison between FTCS and FTBS solutions in sky train platform on a single

In this section, we describe about a comparision of some numerical methods for solving the three-dimensional advection-diffusion equation. There are two methods. These are forward time central space (FTCS) and forward time backward space (FTBS). We consider the domain as a single layer as shown in Figure 4 that the length(L), width(W) and height(H) of tunnel are 198, 21 and 28 meters, respectively. Then, the problem domain is $\Omega=\{(x,y,z);0\le x\le 198,0\le y\le 21,0\le z\le 28\}$. We assume that $\Delta x=\Delta y=\Delta z=2$ m, $\Delta t=0.06$ sec,

$T = 2$ min, $u = 0.5$ m/sec, $v = 0$ m/sec, $D_h = D_v = 0.001$ m^2/sec, $c_1 = 0.5$, $c_3 = c_9 = -0.01$, $c_2 = c_4 = c_5 = c_6 = c_7 = c_8 = c_{10} = c_{11} = 0$, $A = 4$, $B = 17$, and $ncl = 9$. Figures 6 and 7 are solved by various methods. These are FTCS and FTBS methods, respectively. The solutions of air pollutant concentration by using the FTCS method in Eq. (5) are shown in Figure 6. This figure show about contour and surface plots of air pollutant concentration levels after 2 minutes passed. You can be seen that the maximum value of air pollutant concentration is 0.6 kg/m^3. Furthermore, the contour and surface plots of air pollutant concentration levels after 2 minutes passed for solutions of air pollutant concentration by using the FTBS method in Eq. (13) are shown in Figure 7. The maximum value of air pollutant concentration in Figure 7 is 0.5 kg/m^3.

The approximations of air pollutant concentration for FTCS and FTBS methods are compared in Figure 11. We choose $\Delta x = \Delta y = \Delta z = 2$ m, $\Delta t = 0.06$ sec, $T = 30$ sec, $u = 0.1$ m/sec, $v = 0$ m/sec, $D_h = D_v = 0.001$ m^2/sec, $c_1 = 0.5$, $c_3 = c_9 = -0.01$, $c_2 = c_4 = c_5 = c_6 = c_7 = c_8 = c_{10} = c_{11} = 0$, $A = 4$, $B = 17$ and $ncl = 9$. It can be seen that the trend of results from both methods in the same way. How do you know which one is better? The idea is to find out the method whether we will change the grid-spacing, the solution is stable. Table. 1 shows the stable of FTCS and FTBS approximate solutions. You can be seen that if we choose $\lambda = \gamma = 0.03$, and $\lambda = 0.06$ then the solution of the FTBS method are unstable but the FTCS method is stable. Consequently, the FTCS method gives better than the FTBS method.

4.2 Numerical simulations of air pollutant assessment in sky train platform on triple layers

In this section, explicit forward time and central space (FTCS) scheme have been presented. We distinguish three scenarios of released air pollutant phenomenons demonstrated by using the finite difference in Eq. (5). In all scenarios, the air pollutant concentration is flowing along the x and y-directions, these are constants or functions. In addition, there are two parallel gaps along the ceiling, the rate of change are decreased at the parallel gaps. Moreover, both sides were flanked by buildings. All of the building walls are non-absorbing air pollution materials, there is no rate of change.

For three scenarios, we consider the length, width and height of tunnel are 198, 21 and 28 meters, respectively. Then, the problem domain is $\Omega = \{(x, y, z); 0 \leq x \leq 198, 0 \leq y \leq 21, 0 \leq z \leq 28\}$, when $0 \leq z < 9, 9 \leq z < 22, 22 \leq z \leq 28$ are street floor, ticket floor and platform floor, respectively, see in Figure 5. We consider c_1 in the boundary condition in the entrance gate of each floor, we distinguish 3 scenarios in the consider domain as follows:

4.2.1 Scenario A : Air pollutant flowing into the street floor

If we consider BTS station area, we will see that the street floor has a lot of cars. This causes heavy traffic, as a result air pollution is higher than other

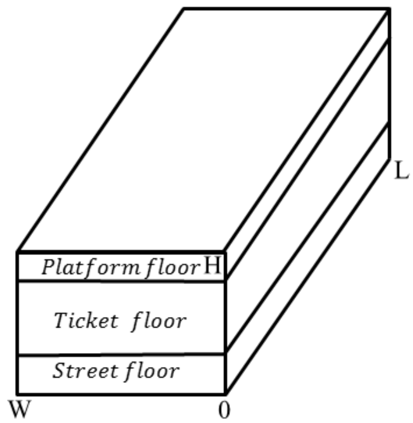


Figure 5: The problem domain.

floors. Therefore, we assumed the air pollutant concentration at the entrance gate of the street floor is constant. However, ticket floor and platform floor are assumed that the pollution cannot be reached. So, we assumed that there is no rate of change of air pollutant concentration at the entrance gate of the ticket and platform floors. Therefore, c_1 in the boundary condition of all 3 floors as follows:

Street floor : $c_1 = 0.5$.
Ticket floor : $c_1 = \frac{\partial C}{\partial x} = 0$.
Platform floor : $c_1 = \frac{\partial C}{\partial x} = 0$.

The problem domain is $\Omega = \{(x, y, z); 0 \leq x \leq 198, 0 \leq y \leq 21, 0 \leq z < 9\}$. The grid spacing: $\Delta x = \Delta y = \Delta z = 1$ m, $\Delta t = 0.06$ sec and for the time $T = 2$ min. We assume: $c_3 = c_9 = -0.01$, $c_5 = 0.2$, $c_2 = c_4 = c_6 = c_7 = c_8 = c_{10} = c_{11} = 0$, $A = 4$, $B = 17$, $F = 125$, $G = 135$ and $ncl = 9$. The wind velocity and diffusion coefficient are taken to be $u = 2.7778, v = u/20$ m/sec and $D_h = 0.1592, D_v = 0.05$ m^2/sec. Therefore, the results of ScenarioA for three different floors are shown in Figure 8. That is, in Figures 8(a), 8(c) and 8(e) show the contour plot of the air pollutant concentration levels for street, ticket and platform floors, respectively. Meanwhile, the surface plot of the air pollutant concentration levels for street, ticket and platform floors are shown in Figures 8(b), 8(d) and 8(f), respectively. It can be seen from Figure 8 that the air pollutant concentration in the platform floor is very low. It comes from only the pollution on the right side wall gap. So, the air pollution on platform floor is less than 0.2 kg/m^3. The air pollutant concentration of Scenario A with the different floors are shown in Figure 12(a).

4.2.2 Scenario B : Air pollutant flowing into every floors

In reality, we noticed that the air pollutant concentration depends on the height of the tunnel, so if the height increases, the air pollutant concentration will be less. Thus, the air pollutant concentration at the entrance gate for the street floor, ticket floor and platform floor can be described as different decreasing functions varied with the height of the tunnel. Therefore, c_1 in the boundary condition of all 3 floors as follows:

Street floor : $c_1 = 0.5 - 0.02z$.

Ticket floor : $c_1 = 0.32 - 0.015z$.

Platform floor : $c_1 = 0.01 - 0.005z$.

The problem domain is $\Omega = \{(x, y, z); 0 \leq x \leq 198, 0 \leq y \leq 21, 9 \leq z < 22\}$. The grid spacing: $\Delta x = \Delta y = \Delta z = 1$ m, $\Delta t = 0.06$ sec and for the time $T = 2$ min. We assume: $c_3 = c_9 = -0.01$, $c_5 = 0.2$, $c_2 = c_4 = c_6 = c_7 = c_8 = c_{10} = c_{11} = 0$, $A = 4$, $B = 17$, $F = 125$, $G = 135$ and $ncl = 9$. We choose $u = 2.7778, v = u/20$ m/sec, $D_h = 0.1592, D_v = 0.05$ m^2/sec. So, the results of Scenario B for three different floors are shown in Figure 9. That is, in Figures 9(a), 9(c) and 9(e) show the contour plot of the air pollutant concentration levels for street, ticket and platform floors, respectively. Meanwhile, the surface plot of the air pollutant concentration levels for street, ticket and platform floors are shown in Figures 9(b), 9(d) and 9(f), respectively. As the results, the air pollution continues to be released as the decreasing functions but gradually decreases. Therefore, the air pollution in three floors of this scenario is higher especially, the air pollutant concentration on the street floor is as high as 0.7 kg/m^3. The air pollutant concentration of Scenario B with the different floors are shown in Figure 12(b).

4.2.3 Scenario C : Air pollutant flowing through the street floor and their gaps

The air pollutant concentration on the street floor is assumed to be a constant. Furthermore, for more realism, we can see that the air pollutant concentration from the previous floor impact on the next floor. So we will define the pollution of the next floor by applying the principle of average. That is the air pollutant concentration at the entrance of the next floor is the average of air pollutant concentration of gaps on the previous floor. Therefore, c_1 in the boundary condition of all 3 floors as follows:

Street floor : $c_1 = 0.5$.

Ticket floor : $c_1 = c_{avS}$.

Platform floor : $c_1 = c_{avT}$,

where c_{avS} and c_{avT} are the average of air pollutant concentration of gaps on the street floor and ticket floor, respectively.

The problem domain is $\Omega = \{(x, y, z); 0 \leq x \leq 198, 0 \leq y \leq 21, 22 \leq z \leq 28\}$. The grid spacing: $\Delta x = \Delta y = \Delta z = 1$ m, $\Delta t = 0.06$ sec and for the time $T = 2$ min. We assume: $c_3 = c_9 = -0.01$, $c_5 = 0.2$, $c_2 = c_4 = c_6 = c_7 = c_8 = c_{10} = c_{11} = 0$, $A = 4$, $B = 17$, $F = 125$, $G = 135$ and $ncl = 9$. We choose $u = 2.7778, v = u/20$ m/sec, $D_h = 0.1592, D_v = 0.05$ m^2/sec. So, the results of Scenario C for three different floors are shown in Figure 10. That is, in Figures

10(a), 10(c) and 10(e) show the contour plot of the air pollutant concentration levels for street, ticket and platform floors, respectively. Meanwhile, the surface plot of the air pollutant concentration levels for street, ticket and platform floors are shown in Figures 10(b), 10(d) and 10(f), respectively. It can also be obtained by Figure 13 that the air pollutant concentration gradually decreases because we bring the value of the previous floor to the next floor. So, the pollution on the platform floor is least the pollution than other floors as 0.5 kg/m^3. The air pollutant concentration of Scenario C with the different floors are shown in Figure 12(c).

5. Discussion and conclusion

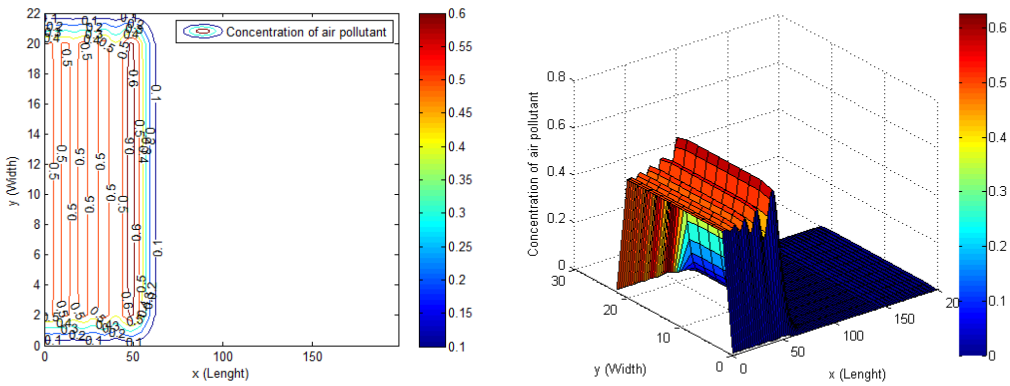
In this research, the air pollutant concentration model is presented. The finite difference methods such as FTCS and FTBS methods can be used to estimate the air pollutant concentration. Also, it is appealing that the grid spacing is different so FTCS method has been chosen because when making comparisons between FTCS and FTBS methods in some cases, the solution for FTBS method is unstable while the solution for FTCS method is stable. Hence, FTCS method provides a better result than FTBS method.

Furthermore, we proposed three scenarios for estimating the air pollutant concentration as follows; Scenario A: there is no rate of change of air pollutant concentration at the entrance gate of the ticket and platform floors. Based on the results, the air pollution in the platform floor is low because it only emits pollution at the wall gap at the right side. Scenario B: the air pollutant concentration at the entrance of three floors can be described by different decreasing functions depended on the height of the tunnel. As the results, the air pollution continues to be released as the decreasing functions but gradually decreases. Therefore, the air pollution in this scenario is high when compared to other scenarios. Scenario C: the air pollution at the entrance of the next floor is the average of air pollutant concentration of gaps on the previous floor. The results of this scenario show that the pollution gradually decreases because we used the value of the previous floor to the next floor which significantly reduces air pollution.

Summary of the three scenarios: Scenario A is a simple model that is an economical method to use. It requires a few of collected data. Scenario B is a realistic numerical simulation. The approximated air pollutant concentration depends on the height but requires a lot of field data. Scenario C is a fairly good model. The simulation needs to average the pollutant concentration level. It is used as the input air pollutant concentration to the above floor. However, the suitable model depends on the provided field data.

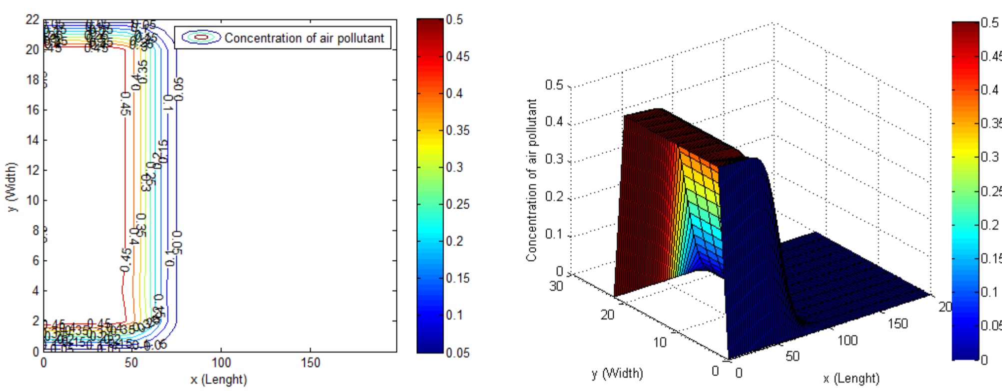
Table 1: The stable of FTCS and FTBS approximate solutions.

$\lambda = \frac{\Delta t}{\Delta x}$	$\gamma = \frac{\Delta t}{(\Delta x^2)}$	Δx	Δy	Δz	Δt	$FTCS$	$FTBS$
0.02	0.0067	3.0	3.0	3.0	0.06	stable	stable
	0.0267	1.5	1.5	1.5	0.03	stable	stable
0.03	0.015	2.0	2.0	2.0	0.06	stable	stable
	0.03	1.0	1.0	1.0	0.03	stable	unstable
0.06	0.06	1.0	1.0	1.0	0.06	stable	unstable
	0.12	0.5	0.5	0.5	0.03	stable	unstable



(a) (b)

Figure 6: Contour and surface plot of air pollutant concentration levels after the past 2 minutes computed by FTCS method.



(a) (b)

Figure 7: Contour and surface plot of air pollutant concentration levels after the past 2 minutes computed by FTBS method.

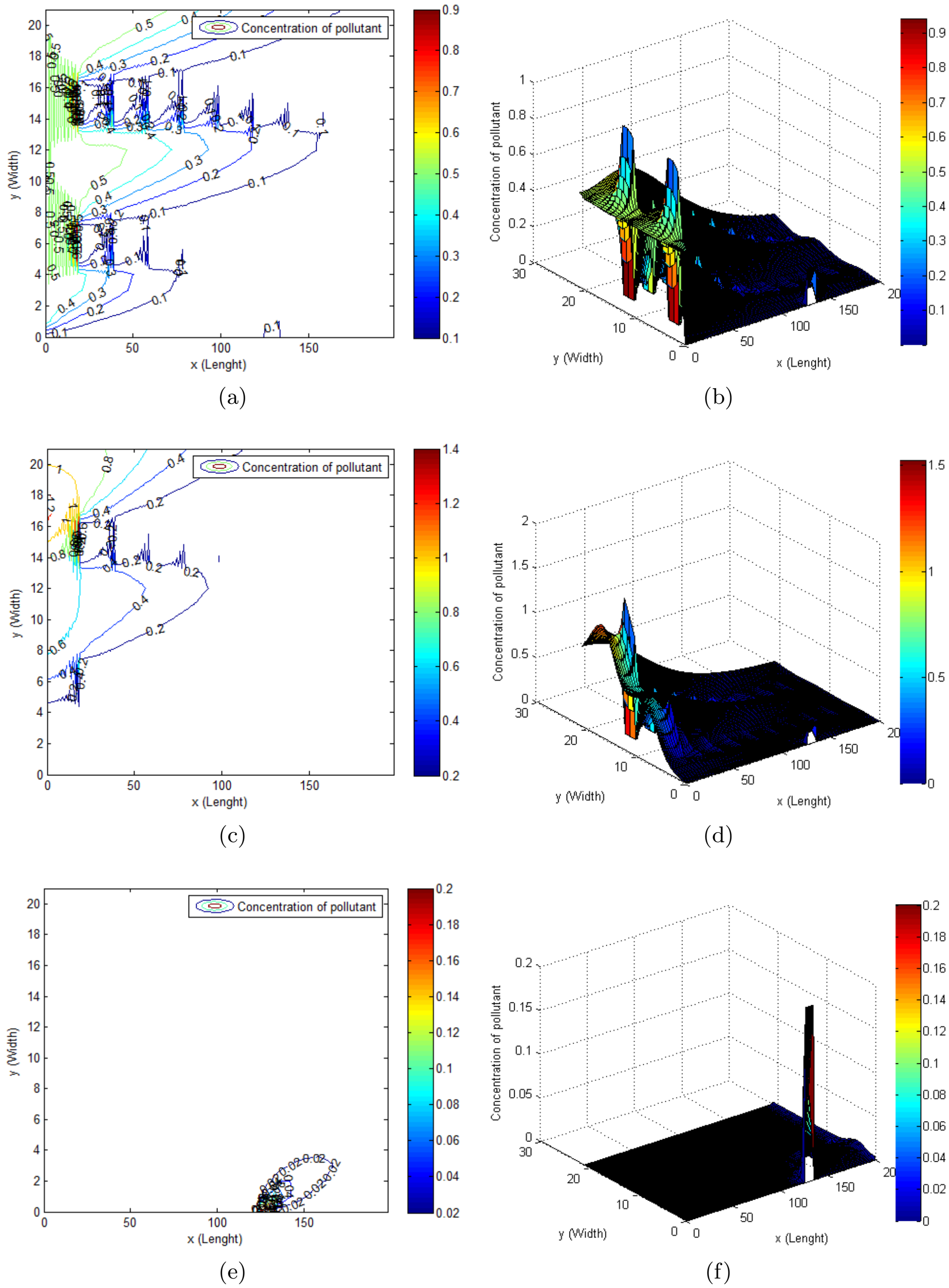


Figure 8: Contour and surface plot of air pollutant concentration levels after the past 2 minutes for the respective streets, tickets and platform floors. (Scenario A)

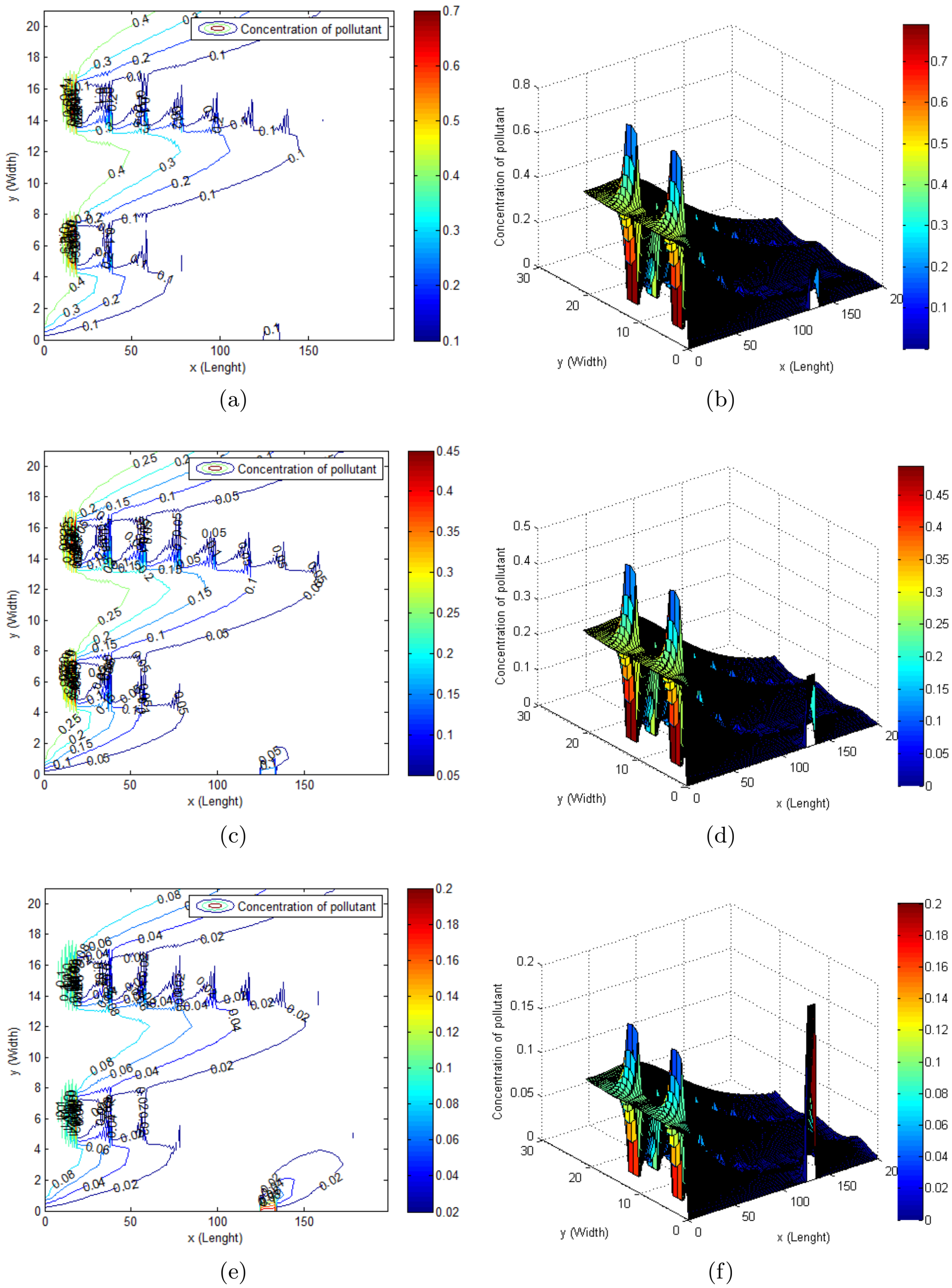


Figure 9: Contour and surface plot of air pollutant concentration levels after the past 2 minutes for the respective streets, tickets and platform floors. (Scenario B)

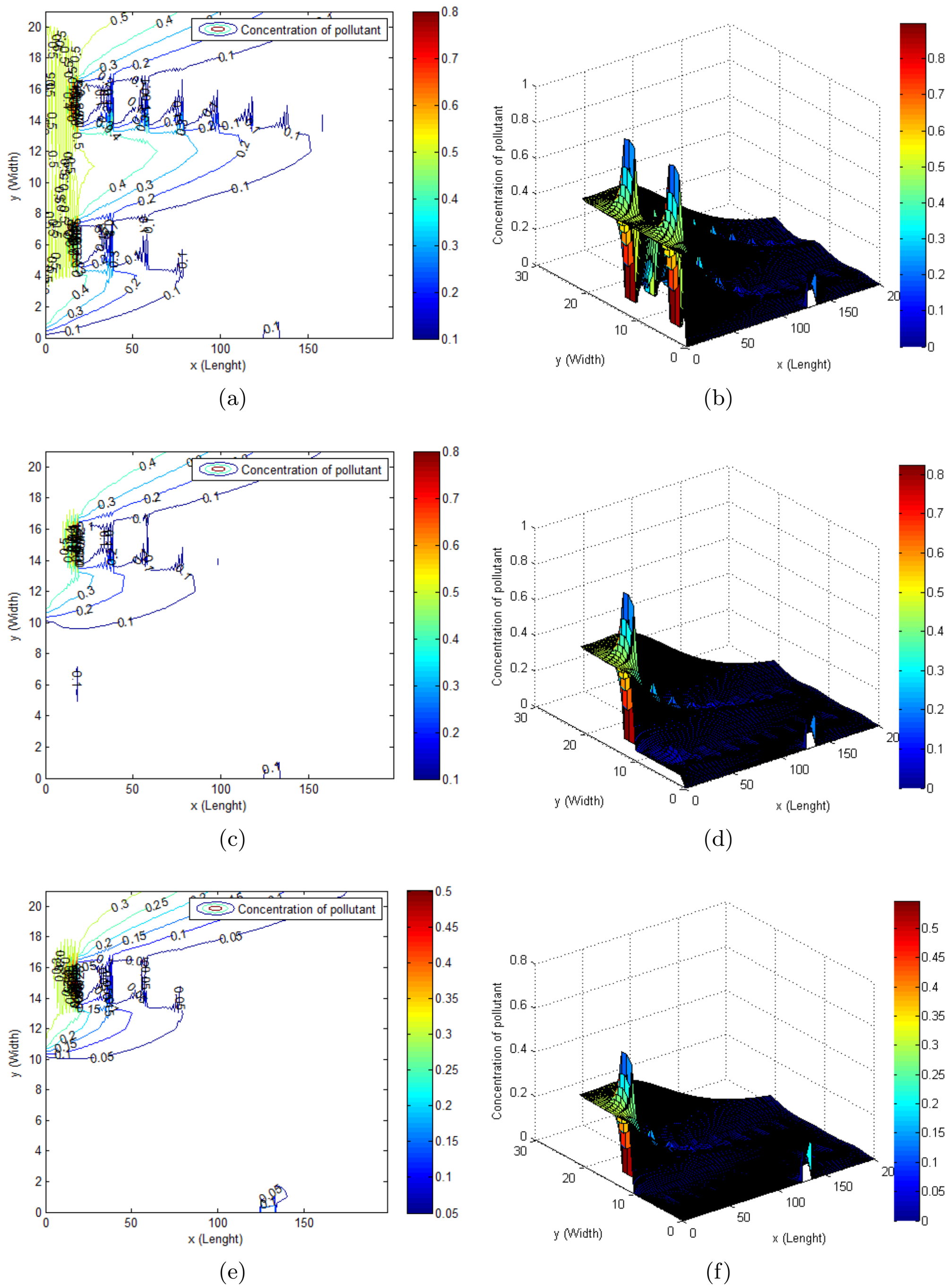


Figure 10: Contour and surface plot of air pollutant concentration levels after the past 2 minutes for the respective streets, tickets and platform floors. (Scenario C)

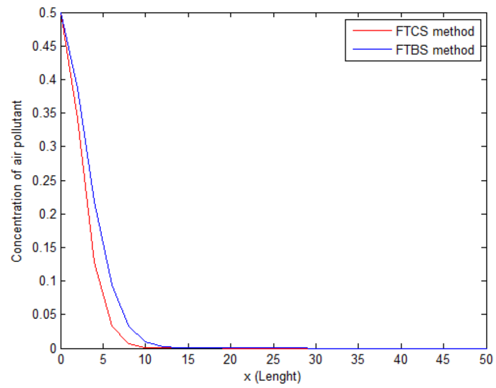


Figure 11: Comparison of air pollutant concentration between FTCS and FTBS methods after the past 30 seconds.

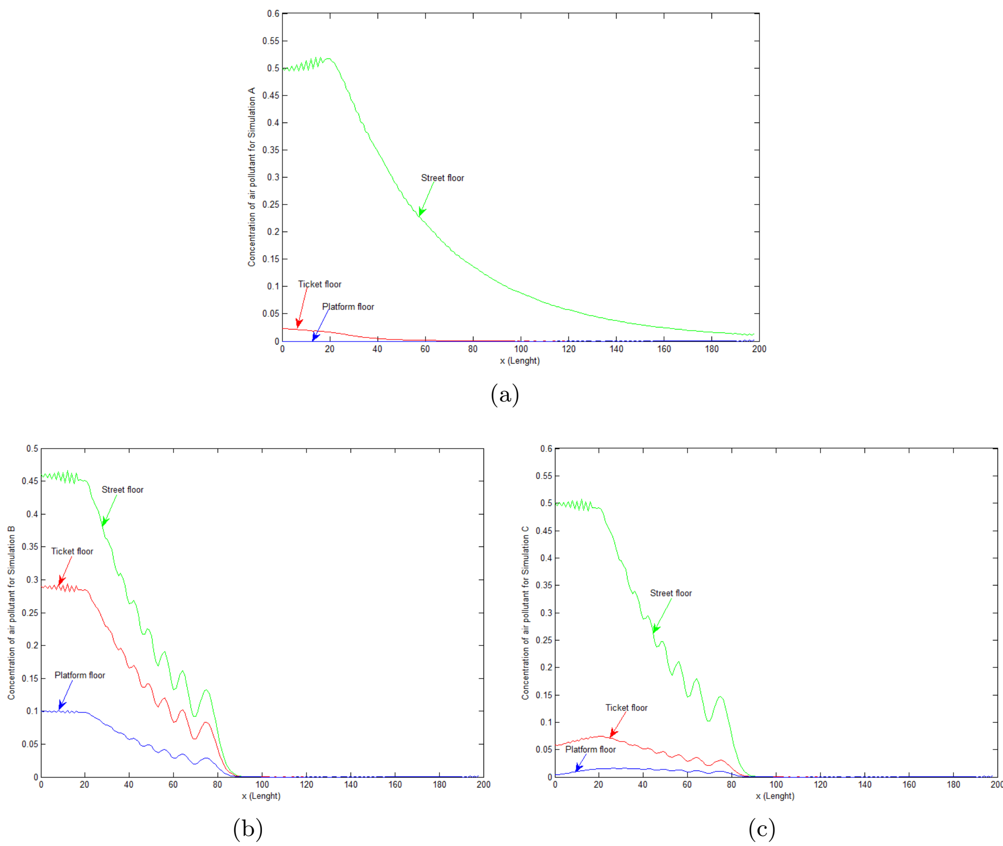


Figure 12: The air pollutant concentration with the different floors of (a) Scenario A. (b) Scenario B. (c) Scenario C.

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