COMPARISON OF SURFACE FITTING METHODS FOR MODELLING LEAF SURFACE

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Abstract. A novel hybrid method for modelling leaf surface that combines Gaussian radial basis function (RBF) and Clough-Tocher (CT) methods to achieve a continuous surface is proposed by the author [20]. In this paper, we demonstrate the accuracy of the hybrid Gaussion RBF-CT approach by applying it to model the surface of frangipani and anthurium leaves. Furthermore, a comparison between the hybrid Gaussion RBF-CT method and the hybrid multiquadric RBF-CT method introduced by the author Oqielat et al. [17] is presented.

The development of the algorithm has been made to assist the understanding of leaf surface properties. It is found that the hybrid multiquadric RBF-CT surface fitting methodology produces more accurate and realistic leaf surface representation than using the hybrid Gaussion RBF-CT method.

Keywords: finite elements methods, interpolation, radial basis functions, Clough-Tocher method, virtual plants.

1. Introduction

The modelling of virtual plant has been researched extensively over the last decades [24] and models of leaf surfaces have generally been generated recently with accurate accuracy and level of detail by [5,9,10,11,19]. Leaves play an important role in the development of a plant, and therefore leaf model is required. Loch [13] proposed two methods to accurately model leaf surfaces.

Surface data can often be collected at a discrete set of points and a key problem is to reconstruct the surface, or perhaps capture important features of the surface from a discrete set of measurements. This representation may be used for visualization purposes only [13] or may be used to study biological processes such as photosynthesis and canopy light environments.

The minority of the leaf models that presented in the past were based on accurate measurements until 3D digitizers and faster computers with improved graphic capabilities became available. Virtual leaf models may be presented in an abstract way, where the leaf is represented by a disk [25] or more realistically, by a surface model that captures the surface shape and boundary [22]. Boundary algorithms were applied by [16] for modelling lobed leaves. Maddonni [15] used piecewise linear triangles to represent the leaf surface, where vertices along the boundary are estimated by allometric relationships.

Loch [13,14] presented two methods to model accurate leaf surfaces in three dimensions based on finite elements methods (piecewise linear triangular and piecewise cubic Clough-Tocher triangular). Loch modelled the leaf surfaces for Flame,Frangipani, Elephant's ear and Anthurium leaves.

Our research in this paper introduces a new surface fitting method that combine Clough-Tocher with Gaussian radial basis techniques for modelling the leaf surface. Then, a comparison of our method with a hybrid method introduced by the author [17] is presented. Finally, the proposed hybrid technique is applied to a large number of three-dimensional data points captured from an Frangipani and Anthurium leaf surface and we found that the hybrid method gives accurate representation of the leaf surface, see figure 1. This work form the bases for future research , for example, accurate leaf surface representation can be used in the context of modelling surface droplet movement.

The research is presented over four sections. In section 1, we overview of interpolation surface fitting methods based on the Clough-Tocher and the radial basis function method which are already outlined by the auother in [19,20]. In section 2 a new surface fitting method is proposed that combines the CT and Gaussian RBF methods for modelling leaf surfaces. In section 3, the application of the new method to a Frangipani leaf and Anthurium leaf is presented. Future work and further applications of the model are discussed in section 4.

2. Clough-Tocher finite element method

The Clough-Tocher method (CTM) is an interpolating finite element method that was introduced originally by Clough and Tocher [4]. This method is used to minimize the degree of the polynomial interpolant without losing the continuity of the gradient over the whole domain.

The CTM is a seamed element approach, whereby each triangle is treated as a macro-element that is split into subtriangles, which are called micro-elements. The CTM, has the advantage that it results in a smooth surface over the whole domain. It approximates the surface as an interpolating cubic polynomial constructed on each subtriangle which enables a bivariate piecewise cubic interpolant to be devised over the entire triangle that is continuously differentiable. The key result is that only twelve degrees of freedom are required for the CTM, namely the function values and the gradient at each vertex, as well as the normal derivative along the edges. The CT interpolant has the form:

$$\varphi(x,y) = \sum_{i=1}^{3} \left(f_i b_i + (c_i, d_i)^T \cdot \nabla f_i\right) + \sum_{j=1}^{3} \frac{\partial f}{\partial n_j} e_j. \tag{1}$$

In this representation the twelve functions $b_i(x,y), c_i(x,y), d_i(x,y)$ and $e_j(x,y)$, $i = 1,2,3$ are cardinal basis functions (see Lancaster [12]), having the property that just one of them is unity and the reminder zero at each of the node points.

In the modelling of leaf surface, the function value is assigned at the triangle vertices. However, the derivative information at the vertices and at the midpoints of each side is unavailable and needs to be estimated. The gradient at the triangle vertices are estimated from neighbouring data information and thereafter the edge normal derivatives are determined as the mean of the normal estimated at the two vertices associated with the edge. This approximation is based on the assumption that the normal slope along the sides of the triangle changes linearly. The author [1,26] presented analysis for the least square gradient estimation method as well as an error bounds for the method. A more detailed description of CTM including the list of cardinal basis functions for the standard triangular element can be found in (Lancaster [12]).

1.2 Radial basis functions

A Radial Basis Function (RBF) approximation to f is a function S of the form:

$$S(x) = \sum_{i=1}^{n} a_i \Phi_i(x), \quad x \in \mathrm{R}^2, \tag{2}$$

where $\Phi_i(x) = R(\|x - x_i\|)$, $R(r)$ is a non-negative real-valued function with non-negative argument r and $\|.\|$ denotes the Euclidean norm. The points $\{x_i\}$ belonging to R^2 are called the centres of the RBF approximation. The expansion coefficients $\{a_i\}$ are determined by satisfying some approximation criterion; in this application by interpolation (see equation 4).

Radial basis function method has found applications in areas such as geodesy and medical imaging, the theory of the RBF approximation is given by Powell [21]. A main problem of the radial basis function method concerns large sets of data points where the computational costs involved in fitting and evaluating the RBF can become time-consuming.

Well known examples of radial basis function methods include Gaussian RBF which is adopted in this paper:

$$R(\|x - x_i\|) = \exp -c^2 \|x - x_i\|^2 . \tag{3}$$

The parameter c is specified by the user, however, it is well known that the accuracy for interpolating scattered data with radial basis functions depends strongly on this parameter, see for example [3,6,23]. For some values of c the problem may become ill-conditioned. Franke [6] used $c = 1.25\frac{D}{\sqrt{n}}$ where D is the diameter of the minimal circle enclosing all data points.

Rippa [23] proposed an algorithm for selecting a good value for the parameter c based on minimizing a cost function that represents the error between the interpolating radial basis function and the unknown function (RMS), see equation 6. Rippa considered two sets of data points and nine different test functions defined on the unit square. A data vector $f = (f_1, f_2, ..., f_n)^T$ was constructed by evaluating each test function over the set of data points so that

$$S(x_j) = f_j, j = 1, 2, ..., n. \tag{4}$$

3. Hybrid Gaussian radial basis function Clough-Tocher method

The CT method requires derivative estimates at the vertices and midpoints of the elements for its evaluation. We propose a new hybrid approach [20] for surface fitting based on using Gaussian RBF (either local or global) to estimate the gradient at the vertices and mid-points of the Clough-Tocher triangle. The Gaussian RBF interpolant $Q(x)$ is given by equation 2. The gradient of Q is then given by

$$\nabla Q(x) = \sum_{i=1}^{n} a_i \nabla \Psi_i(x). \tag{5}$$

Where

$$\nabla \Psi_i(x) = \nabla R(\|x - x_i\|) = (dR/dx_k, dR/dy_k) =$$
$$(-2(x_k - x_i)c^2 \exp -r^2c^2, -2(y_k - y_i)c^2 \exp -r^2c^2).$$

The procedure that uses this hybrid approach for the purpose of surface fitting is summarised in the following algorithm:

Algorithm 1: Construction Surfaces using the Hybrid RBF-CT Method

INPUT:N data points $\{(x_i, f_i), i = 1, \ldots, N\}$

Step 1: *Choose a subset of* n *data points from the given* N *points to triangulate the surface.*

Step 2: *Using either a global multiquadric RBF interpolant constructed from the* n*triangulation points OR, a local multiquadric RBF interpolant constructed on each triangle using a local subset of* m *points, generate the RBF linear system.*

Step 3: *Approximately solve this linear system using the TSVD method.*

Step 4: *Use the RBF coefficients to construct either the global or local gradient.*

Step 5: *Apply the hybrid CT-RBF method to construct the surface using either*$\nabla S_n(x)$ *(global) or* $\nabla S_m(x)$ *(local) to provide the necessary derivative information for the construction of the CT interpolant.*

The **local hybrid approach** applied here is based on choosing the set of 5 nearest neighbours to each vertex and to the center of the triangle. Next, a local radial basis function is built from the 20 points for each triangle, which is then used to estimate the directional derivative at the triangle vertices and midpoints. A **global hybrid approach** is also applied, which is based on building one single global RBF from the triangulation points and then using it to evaluate the gradients at the vertices and midpoints of all triangles. The parameter c in both cases was estimated globally using the triangulation points following the Rippa framework [23].

The selection of the local set of points to use for the construction of the RBF is important. For Frangipani leaf, we used the 5 points closest points to each vertex and to the centroid of the triangle for the construction of the local RBF. Using this point set produced the best results and we found that using more points, for example 10 or 20, did not improve the fit. For Anthurium leaf using closest 5 points struck problems because we did not produce enough points of the triangular element to enable a sufficient sampling of function values to produce reasonable gradient estimates. To avoid this problem we construct the local RBF by select the closest 30 points to each vertex and to the centroid of the triangle. This selection process ensured that the local RBF contains a sufficient set of data that enables a more accurate gradient to be produced.

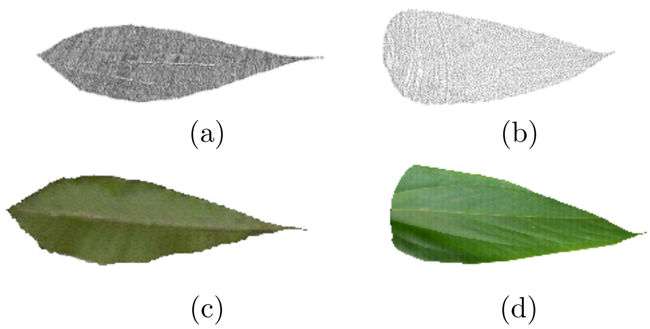
4. Application of hybrid method for the frangipani and anthurium leaves

A set of representative data points sampled from the surface are required to reconstruct the shape of a leaf using surface fitting techniques. The process of sampling data points from the leaf surface using a measuring device is called digitizing such that the visible exterior data points of the leaf are enough to capture the surface of the leaf. Loch [14] collected data points for different types of leaves (such as, Frangipani, Anthurium, Flame and Elephant's Ear) using a laser scanner. The boundary points were selected by hand from the complete set of points using the PointPicker, software written by Hanan [7].

To assess the accuracy of the hybrid Clough-Tocher Radial basis function interpolation method, we applied the method to the laser scanned Frangipani and Anthurium leaf data taken from (Loch [14]) to construct the surface of those two leaves. The Anthurium leaf data set consists of a set containing 4,688 points, which represent the entire leaf surface points and a second set containing 79 points representing the boundary points of the Anthurium leaf surface. The Frangipani leaf data set contains two subsets of data. The first set consists of 3,388 points, which represents the entire leaf surface scanned points; while the second set consists of 17 points representing the boundary points of the Frangipani leaf surface. These point sets are displayed in Figures 1 (a) and (b). Now, to apply the hybrid method to the leaf data, preprocessingsteps are essential which includes determination a new plane for the leaf data and then tringulation of the leaf surface. These two steps are discussed previously by the author [17,19,20] and will explain briefly in the following two subsections.

4.1 Leaf reference plane

The laser scanner returns the coordinate system of points on the leaf, These coordinates may not be suitable for interpolation due to the possibility of multivalued and vertical surfaces. To solve this problem we can use use a reference plane that is a least squares fit to these data points. We construct a reference plane by making a linear least squares fit to the data and rotating the coordinate system so that the reference plane becomes the $xy-$plane. This rotation can be achieved by rotating the normal vector of the reference plane about the $y-$axis into the $yz-$ plane and then about the $x-$axis into the $xz-$ plane (Oqielat [18]). If the vertical height of the data points is single valued in the transformed coordinate system, then the procedure is successful.



(a) (b)

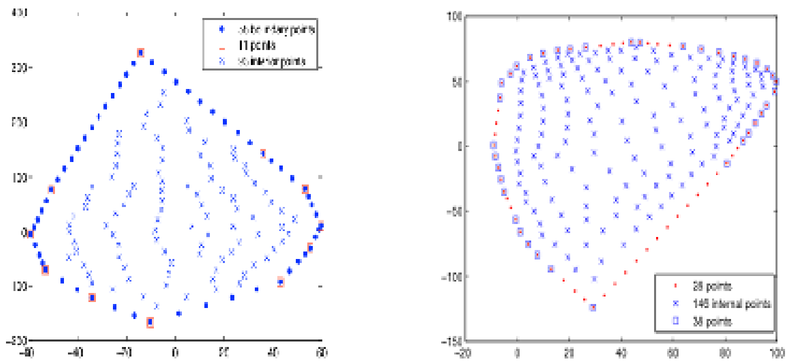
(c) (d)

Figure 1: Photos of the scanned (a) Frangipani and (b) Anthurium leaves and corresponding (c) Frangipani and (d) Anthurium leaf surface models for these point sets.

5. Triangulation of the leaf surface

To apply our method to the leaf data points, we need to construct a triangulation to the leaf surface. The data points that represent the leaf surface is large, so the computational expense for surface fitting can be reduced by selecting only a subset of these data to generate a triangulation of the leaf surface. In this research we constructed the triangulation of the leaf using the *EasyMesh* generator, which is software written in the C language by Bojan Niceno [2]. *EasyMesh* generates two-dimensional *Delaunay* and constrained *Delaunay* triangulations in general domains. This software return a good quality triangulation if the domain is convex. However, because the piecewise linear boundary defined by the boundary points do not enclose a convex set, *EasyMesh* was unable to produce the required triangulation. To solve this problem, an algorithm was used to generate a convex hull from the entire set of leaf data points.

Moreover, *EasyMesh* produce better shaped triangles if we define either a horizontal, or vertical, line Inside the convex hull (leaf surface). For the Frangipani and Anthurium leaves (Oqielat [17,18]) it appears that the vertical line produces a more suitable triangulation than the horizontal line. The final triangulation is given in figures 2 and 3. The hybrid Clough-Tocher- radial basis function method is then applied to construct the leaf surface after the triangulation of the leaf surface is constructed.

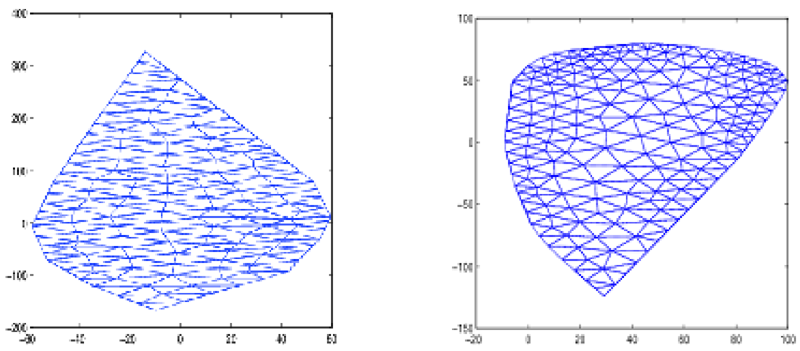


(a) (b)

Figure 2: (a) The vertices of the mesh structure generated using Easymesh. The square points represent the 11 boundary points that are given to Easymesh; the dot points represent the 58 extra points added by Easymesh, while the x points represent the 93 internal points. (b)The vertices of the mesh structure generated using Easymesh. The square points represent the 38 boundary points that are given to Easymesh; the dot points represent the 28 extra points added by Easymesh, while the x points represent the 146 internal points.

6. Numerical experiments

The results of comparisons and applying the hybrid methods to the Frangipani and Anthurium leaf data is present in this section. First, the triangulation points were selected, then the rest of the m data points (denoted by $z_k = z(x_k), k = 1, ..., m$) from the leaf data set were used to measure the quality of the approximation of the hybrid methods. Some of the m data points occurred outside of the virtual leaf mesh so these points were ignored in the quality analysis. The hybrid method then applied to estimate the surface values for the data points occurring inside the triangulation to construct the leaf surface, see Figure 1 (c) and (d).



(a) (b)

Figure 3: (a) Triangulation of 151 points of Frangipani leaf surface generated using EasyMesh. (b) Triangulation of 212 points of Anthurium leaf surface generated using EasyMesh.

The root mean square error RMS, is used in this paper as an error metric to assess the accuracy of our methods which is given by:

$$RMS = \sqrt{\frac{\sum_{k=1}^{k=m}[Q(x_k) - z_k]^2}{m}}. \tag{6}$$

$Q(x_k)$ represents the CT estimated value at the m data points and z_k represents the given function values at the same data points. The second error metric measured the quality in terms of the maximum error associated with the surface fit in relation to the maximum variation in z.

$$\text{Scaled Max. Error} = \frac{\max\left(|Q(x_k) - z_k|\right)}{\max(z_k) - \min(z_k)}, k = 1, 2, ..., m.$$

Tables 1 and 2 show a comparison between the scaled maximum errors and the scaled RMS $= \frac{RMS}{\max(z_k) - \min(z_k)}$ for the Frangipani and the Anthurium leaf data sets respectively using the local and global hybrid multiquadric method and hybrid Gaussian method. For the Anthurium leaf there were a total of 4,460 data points used to assess the accuracy of the surface. Note the *EasyMesh* triangulation comprised 212 vertices. There were about 59 points ignored in the analysis because these points were deemed to lie outside the leaf mesh structure.

One observes for the Frangipani leaf that using the local hybrid multiquadric RBF method produced slightly more accurate RMS value than using the global hybrid multiquadric RBF method while it is the converse for the maximum error. The trends depicted in Table 1 for the Frangipani leaf appear consistent with observations from Table 1 for the Athurium leaf. Moreover, the hybrid (local and global) multiquadric RBF method produced more accurate RMS and maximum error than using the hybrid (local and global) Gaussian RBF method.

Table 1: A comparison of RMS computed using hybrid local and global RBF for the Frangipani leaf data points as well as the maximum error associated with the surface fit.

	Hybrid multiquadric local	Hybrid Gaussian local	Hybrid multiquadratic global	Hybrid Gaussian global
	RBF	RBF	RBF	RBF
Scaled RMS	0.0086	0.0218	0.0139	0.1163
Scaled maximum error	0.0700	0.3603	0.0655	0.4749
Boundary points	48	48	48	48
Points tested	3155	3155	3155	3155
Triangulation points	141	141	141	141
Outside points	104	104	104	104
No. of triangles	257	257	257	257

Table 2: A comparison of RMS computed using hybrid local and global Gaussian RBF for the Anthurium leaf data points as well as the maximum error associated with the surface fit.

	Hybrid multiquadric local	Hybrid Gaussian local	Hybrid multiquadratic global	Hybrid Gaussian global
	RBF	RBF	RBF	RBF
Scaled RMS	0.0043	0.02313	0.0068	0.0100
Scaled maximum error	0.0537	0.8209	0.0435	0.0561
Boundary points	66	66	66	66
Points tested	4460	4460	4460	4460
Triangulation points	212	212	212	212
Outside points	59	59	59	59
No. of triangles	387	387	387	387

6.1 Conclusion and future research

The research described here proposed a new mathematical surface fitting technique for modelling the leaf surface based on a hybrid CT-Gaussion-RBF methodology. The hybrid method has been successfully applied and compared with another interpolation method introduced by the author (Oqielat [17]) and shown to produce a good accuracy for the leaf surface representation compared with the other method.

Our method allows the user to construct an accurate leaf surface based on three-dimensional data points. Moreover, the research provides a basis on which future research can be built. Surface representations can be extended to models determining a water droplet, or pesticide paths along a leaf surface before it falls from or comes to a standstill on the surface; for example, the simulation of a pesticide application to plant surfaces presented by Hanan [5,8,11]. An advantage of the leaf models described in this paper is that they may be used in different plant modelling environments such as AMAP, xfrog or L-Studio.

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