# PYTHAGOREAN FUZZY HYBRID AVERAGING AGGREGATION OPERATOR AND ITS APPLICATION TO MULTIPLE ATTRIBUTE DECISION MAKING

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**Abstract.** In this paper, we introduce the notion of Pythagorean fuzzy hybrid averaging operator, which is the generalization of Pythagorean fuzzy weighted averaging operator and Pythagorean fuzzy ordered weighted averaging operator. We also study several properties of the propose operator. At the last we apply the the proposed operator to deal with MAGDM under Pythagorean fuzzy information.

**Keywords:** pythagorean fuzzy set, PFHA operator, some properties of PFHA operator, decision making.

## 1. Introducation

In [1] Zadeh introduced the notion of fuzzy set characterized by a membership function. In [2] Atanassov generalized the notion of fuzzy set and introduced the idea of intuitionistic fuzzy set characterized by a membership function and a non-membership function. In [3] Yager generalized the concept of intuitionistic fuzzy set and introduced the concept of Pythagorean fuzzy set. Pythagorean fuzzy set is more powerful tool to solve uncertain problems. Like intuitionistic fuzzy aggregation operators, Pythagorean fuzzy aggregation operators are also become an interesting and important area for research, after the advent of Pythagorean fuzzy set theory. In [4] Yager and Abbasov introduced the notion of PFWA operator, PFOWA operator. In [5] X. Zeng and Z. S. Xu introduced the notion

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of Topsis method using Pythagorean fuzzy numbers. In [6, 7] H. Garg used the Einstein sum and Einstein product and introduced the notion of several arithmetic and geometric aggregation operators and also applied them to group decision making. In [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] K. Rahman et al. introduced the notion of many aggregation operators based on PFNs and applied them to group decision making. Thus, in this paper, our aim to introduce the notion of Pythagorean fuzzy hybrid averaging operator.

The remainder of this paper is structured as follows. In section 2, we give some basic definitions and results. In section 3, we develop Pythagorean fuzzy hybrid averaging operator. In section 4, we apply the proposed operator to deal with MAGD based on PFNs. In section 5, we construct a numerical example. In section 6, we have conclusion.

## 2. Preliminaries

**Definition 1** ([3]). *Let $Z$ be a fixed set, then PFS can be defined as:*

$$P = \{(z, \mu_P(z), \nu_P(z)) \,|z \in Z\}, \tag{1}$$

*where $\mu_p(z)$ and $\nu_p(z)$ are mappings from $z$ to $[0,1]$, with the condition $0 \leq \mu_p^2(z) + \nu_p^2(z) \leq 1$, for all $z \in Z$. Let $\pi_p(z) = \sqrt{1 - \mu_p^2(z) - \nu_p^2(z)}$, then it is called the Pythagorean fuzzy index of element $z \in Z$ to set $P$, which shows the degree of hesitation or indeterminacy of $z$ to $p$. It is clear that $0 \leq \pi_p(z) \leq 1$, for every $z \in Z$.*

**Definition 2** ([5]). *Let $\beta = (\mu_\beta, \nu_\beta)\,, \beta_1 = (\mu_{\beta_1}, \nu_{\beta_1})$ and $\beta_2 = (\mu_{\beta_2}, \nu_{\beta_2})$ be the three PFVs, then*

$$\begin{aligned}
(1)\ \beta_1 \oplus \beta_2 &= \left(\sqrt{\mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1}^2\mu_{\beta_2}^2}, \nu_{\beta_1}\nu_{\beta_2}\right), \\
(2)\ \beta_1 \otimes \beta_2 &= \left(\mu_{\beta_1}\mu_{\beta_2}, \sqrt{\nu_{\beta_1}^2 + \nu_{\beta_2}^2 - \nu_{\beta_1}^2\nu_{\beta_2}^2}\right), \\
(3)\ \lambda\beta &= \left(\sqrt{1 - \left(1 - \mu_\beta^2\right)^\lambda}, \nu_\beta^\lambda\right), \lambda > 0, \\
(4)\ \beta^\lambda &= \left(\mu_{\bar{a}}^\lambda, \sqrt{1 - \left(1 - \nu_\beta^2\right)^\lambda}\right), \lambda > 0.
\end{aligned}$$

**Definition 3** ([5]). *Let $\beta = (\mu_\beta, \nu_\beta)$ be a $PFN$, then the score function and accuracy degree of $\beta$ can be denoted by: $s(\beta) = \mu_\beta^2 - \nu_\beta^2$ , $s(\beta) \in [-1, 1]$ and $h(\beta) = \mu_\beta^2 + \nu_\beta^2$ where $h(\beta) \in [0, 1]$.*

**Definition 4** ([4]). *Let $\beta_i = (\mu_{\beta_i}, \nu_{\beta_i})\,(i = 1, 2, ..., n)$ be a collection of PFVs, then a $PFWA$ operator of dimension $n$ is a mapping $PFWA : \Omega^n \to \Omega$, that has an associated vector $w = (w_1, w_2, w_3, ..., w_n)^T$ , such that $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Furthermore,*

$$PFWA_w\left(\beta_1, \beta_2, \beta_3, ..., \beta_n\right) = w_1\beta_1 \oplus w_2\beta_2 \oplus w_3\beta_3 \oplus ... \oplus w_n\beta_n. \tag{2}$$

**Definition 5** ([4]). *Let $\beta_i = (\mu_{\beta_i}, \nu_{\beta_i})\,(i = 1, 2, ..., n)$ be a collection of $PFVs$, then a $PFOWA$ operator of dimension $n$ is a mapping $PFOWA : \Omega^n \to \Omega$, that has an associated vector $w = (w_1, w_2, w_3, ..., w_n)^T$, such that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Furthermore*

(3) $PFOWA_w(\beta_1, \beta_2, \beta_3, ..., \beta_n) = w_1\beta_{\sigma(1)} \oplus w_2\beta_{\sigma(2)} \oplus w_3\beta_{\sigma(3)} \oplus ... \oplus w_n\beta_{\sigma(n)}$ ,

*where $(\sigma(1), \sigma(2), \sigma(3), ..., \sigma(n))$ is any permutation of $(1, 2, 3, ..., n)$, such that $\beta_{\sigma(i-1)} \geq \beta_{\sigma(i)}$ for all $i$.*

## 3. Pythagorean fuzzy hybrid averaging aggregation operator

**Definition 6.** *A Pythagorean fuzzy hybrid averaging operator of dimension $n$ is a mapping $PFHA : \Omega^n \to \Omega$, which has an associated vector $w = (w_1, w_2, ..., w_n)^T$, such that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Furthermore*

(4) $$PFHA_{\omega,w}(\beta_1, \beta_2, ..., \beta_n) = w_1\dot{\beta}_{\sigma(1)} \oplus w_2\dot{\beta}_{\sigma(2)} \oplus ... \oplus w_n\dot{\beta}_{\sigma(n)},$$

*where $\dot{\beta}_{\sigma(i)}$ is the ith largest of the weighted PFVs $\dot{\beta}_{\sigma(i)}\left(\dot{\beta}_{\sigma(i)} = n\omega_i\beta_i\right)$. $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weighted vector of $\beta_i\,(i = 1, 2, 3, ..., n)$ such that $\omega_i \in [0, 1]$, $\sum_{i=1\omega i}^{n} = 1$, and $n$ is the balancing coefficient, which plays a role of balance. If the vector $w = (w_1, w_2, ..., w_n)^T$ approaches $\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then the vector $(n\omega_1\beta_1, n\omega_2\beta_2, ..., n\omega_n\beta_n)^T$ approaches $(\beta_1, \beta_2, ..., \beta_n)^T$.*

**Theorem 1.** *Let $\beta_i = (\mu_{\beta_i}, \nu_{\beta_i})\,(i = 1, 2, 3, ..., n)$ be a collection of PFVs, then their aggregated value by using the PFHA operator is also a PFV, and*

(5) $$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = \left(\sqrt{1 - \prod_{i=1}^{n}(1 - \mu^2_{\dot{\beta}_{\sigma(i)}})^{w_i}}, \prod_{i=1}^{n}(\nu^2_{\dot{\beta}_{\sigma(i)}})^{w_i}\right).$$

**Proof.** By mathematical induction we can prove that equation (5) holds for all $n$. First we show that equation (5) holds for $n = 2$. Since

$$w_1\dot{\beta}_{\sigma(1)} = \left(\sqrt{1 - (1 - \mu^2_{\dot{\beta}_{\sigma(1)}})^{w_1}}, \nu^{w_1}_{\dot{\beta}_{\sigma(1)}}\right), w_2\dot{\beta}_{\sigma(2)} = \left(\sqrt{1 - (1 - \mu^2_{\dot{\beta}_{\sigma(2)}})^{w_2}}, \nu^{w_2}_{\dot{\beta}_{\sigma(2)}}\right).$$

Thus

$$\begin{aligned} PFHA_{\omega,w}(\beta_1, \beta_2) &= w_1\dot{\beta}_{\sigma(1)} \oplus w_2\dot{\beta}_{\sigma(2)} \\ &= \left(\sqrt{1 - \prod_{i=1}^{2}\left(1 - \mu^2_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}}, \prod_{i=1}^{2}\nu^{w_i}_{\dot{\beta}_{\sigma(i)}}\right). \end{aligned}$$

Thus equation (5) holds for $n = 2$. Now we show that equation (5) holds for $n = k$. Thus

$$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_k) = \left(\sqrt{1 - \prod_{i=1}^{k}\left(1 - \mu^2_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}}, \prod_{i=1}^{k}\left(\nu_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}\right).$$

If equation (5) true for $n = k$, then we show that equation (5) holds for $n = k+1$. Thus

$$PFHA_{\omega,w}(\beta_1, \beta_2, ..., \beta_{k+1}) = \left( \sqrt{1 - \prod_{i=1}^{k} \left(1 - \mu^2_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}}, \prod_{i=1}^{k} \left(\nu_{\dot{\beta}_{\sigma(i)}}\right)^{w_i} \right)$$
$$+ \left( \sqrt{1 - \left(1 - \mu^2_{\dot{\beta}_{\sigma(k+1)}}\right)^{w_{k+1}}}, \left(\nu_{\dot{\beta}_{\sigma(k+1)}}\right)^{w_{k+1}} \right)$$
$$= \left( \sqrt{1 - \prod_{i=1}^{k+1} \left(1 - \mu^2_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}}, \prod_{i=1}^{k+1} \left(\nu_{\dot{\beta}_{\sigma(i)}}\right)^{w_i} \right).$$

Thus equation (5) holds for $n = k + 1$. Thus equation (5) holds for $n$. □

**Theorem 2.** *Let $\beta_i = (\mu_{\beta_i}, \nu_{\beta_i})\,(i = 1, 2, 3, ..., n)$ be a collection of PFVs, then the following conditions always true.*

(1) (Idempotency): If $\dot{\beta}_{\sigma(i)} = \dot{\beta}$, for all $i$, then

$$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = \dot{\beta}. \tag{6}$$

(2) (Boundary):

$$\dot{\beta}^-_{\sigma(i)} \preceq PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) \preceq \dot{\beta}^+_{\sigma(i)}, \tag{7}$$

where $\dot{\beta}^+_{\sigma(i)} = (\max_i(\mu_{\dot{\beta}_{\sigma(i)}}), \min_i(\nu_{\dot{\beta}_{\sigma(i)}}))$, $\dot{\beta}^-_{\sigma(i)} = (\min_i(\mu_{\dot{\beta}_{\sigma(i)}}), \max_i(\nu_{\dot{\beta}_{\sigma(i)}}))$.

(3) (Monotonicity): Let $\dot{\beta}^*_{\sigma(i)} = (\mu_{\dot{\beta}^*_{\sigma(i)}}, \nu_{\dot{\beta}^*_{\sigma(i)}})(i = 1, 2, 3, ..., n)$ a collection of PFVs, if $\mu_{\dot{\beta}_{\sigma(i)}} \preceq \mu_{\dot{\beta}^*_{\sigma(i)}}$ and $\nu_{\dot{\beta}_{\sigma(i)}} \succeq \nu_{\dot{\beta}^*_{\sigma(i)}}$ for all $i$, then

$$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) \preceq PFHA_{\omega,w}(\beta^*_1, \beta^*_2, \beta^*_3, ..., \beta^*_n). \tag{8}$$

**Proof.** Idempotency: Since $PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = w_1\dot{\beta}_{\sigma(1)} \oplus w_2\dot{\beta}_{\sigma(2)} \oplus ... \oplus w_n\dot{\beta}_{\sigma(n)} = (w_1 \oplus w_2 \oplus ... \oplus w_n)\dot{\beta} = \dot{\beta}$.

Boundedness: Since $\dot{\beta}^-_{\sigma(j)} \preceq PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) \preceq \dot{\beta}^+_{\sigma(j)}$. Since

$$\Leftrightarrow \min_i \left(\mu_{\dot{\beta}_{\sigma(i)}}\right) \preceq \mu_{\dot{\beta}_{\sigma(i)}} \preceq \max_i \left(\mu_{\dot{\beta}_{\sigma(i)}}\right)$$
$$\Leftrightarrow \min_i \left(\mu_{\dot{\beta}_{\sigma(i)}}\right) \preceq \sqrt{1 - \prod_{i=1}^{n} \left(1 - \mu^2_{\dot{\beta}_{\sigma(i)}}\right)^{w_i}} \preceq \max_i \left(\mu_{\dot{\beta}_{\sigma(i)}}\right). \tag{9}$$

Again

$$\Leftrightarrow \min_i \left(\nu_{\dot{\beta}_{\sigma(i)}}\right) \preceq \nu_{\dot{\beta}_{\sigma(i)}} \preceq \max_i \left(\nu_{\dot{\beta}_{\sigma(i)}}\right)$$
$$\Leftrightarrow \min_i \left(\nu_{\dot{\beta}_{\sigma(i)}}\right) \preceq \prod_{i=1}^{n} \nu^{w_i}_{\dot{\beta}_{\sigma(i)}} \preceq \max_i \left(\nu_{\dot{\beta}_{\sigma(i)}}\right). \tag{10}$$

Let $PFHA_{\omega,w}(\beta_1, \beta_2, ..., \beta_n) = \dot{\beta}_{\sigma(i)} = (\mu_{\dot{\beta}_{\sigma(i)}}, \nu_{\dot{\beta}_{\sigma(i)}})$. Then $s(\dot{\beta}_{\sigma(i)}) \preceq s(\dot{\beta}^{+}_{\sigma(i)})$. Again $s(\dot{\beta}_{\sigma(i)}) \succeq s(\dot{\beta}^{-}_{\sigma(i)})$. Thus

$$\dot{\beta}^{-}_{\sigma(i)} \prec PFWA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) \prec \dot{\beta}^{+}_{\sigma(i)}. \quad (11)$$

If $s(\dot{\beta}_{\sigma(i)}) = s(\dot{\beta}^{+}_{\sigma(i)})$. Then $h(\dot{\beta}_{\sigma(i)}) = \mu^2_{\dot{\beta}_{\sigma(i)}} + \nu^2_{\dot{\beta}_{\sigma(i)}} = \max_i(\mu_{\dot{\beta}_{\sigma(i)}})^2 + \min_i(\nu_{\dot{\beta}_{\sigma(i)}})^2 = h(\dot{\beta}^{+}_{\sigma(i)})$. Thus

$$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = \dot{\beta}^{+}_{\sigma(i)}. \quad (12)$$

If $s(\dot{\beta}_{\sigma(i)}) = s(\dot{\beta}^{-}_{\sigma(i)})$. Then $h(\dot{\beta}_{\sigma(i)}) = \mu^2_{\dot{\beta}_{\sigma(i)}} + \nu^2_{\dot{\beta}_{\sigma(i)}} = \min_i(\nu_{\dot{\beta}_{\sigma(i)}})^2 + \max_i(\mu_{\dot{\beta}_{\sigma(i)}})^2 = h(\dot{\beta}^{-}_{\sigma(i)})$. Thus

$$PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = \dot{\beta}^{-}_{\sigma(i)}. \quad (13)$$

Thus from equation (11) to (13), we have $\dot{\beta}^{-}_{\sigma(i)} \preceq PFHA_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) \preceq \dot{\beta}^{+}_{\sigma(i)}$.

(3) (Monotonicity): Follows the proof of above. □

**Theorem 3.** *The PFWA operator is a special case of the PFHA operator.*

**Proof**. Let $w = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then
PFHA$_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = w_1\dot{\beta}_{\sigma(1)} \oplus w_2\dot{\beta}_{\sigma(2)} \oplus ... \oplus w_n\dot{\beta}_{\sigma(n)} = \frac{1}{n}(\dot{\beta}_{\sigma(1)} \oplus \dot{\beta}_{\sigma(2)} \oplus ... \oplus \dot{\beta}_{\sigma(n)}) = \omega_1\beta_1 \oplus \omega_2\beta_2 \oplus ... \oplus \omega_n\beta_n =$ PFWA$_\omega(\beta_1, \beta_2, \beta_3, ..., \beta_n)$.

**Theorem 4.** *The PFOWA operator is a special case of the PFHA operator.*

**Proof.** $\omega = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then $\dot{\beta}_{\sigma(i)} = \beta_{\sigma(i)}\,(i = 1, 2, 3, ..., n)$. Thus PFHA$_{\omega,w}(\beta_1, \beta_2, \beta_3, ..., \beta_n) = w_1\dot{\beta}_{\sigma(1)} \oplus w_2\dot{\beta}_{\sigma(2)} \oplus ... \oplus w_n\dot{\beta}_{\sigma(n)} = w_1\beta_{\sigma(1)} \oplus w_2\beta_{\sigma(2)} \oplus ... \oplus w_n\beta_{\sigma(n)} =$ PFOWA$_w(\beta_1, \beta_2, \beta_3, ..., \beta_n)$.

## 4. Approaches to multiple attribute decision making with pythagorean fuzzy information

**Algorithm 1.** *Let $A = (A_1, A_2, ..., A_m)$ be the set of alternatives and $C = (C_1, C_2, ..., C_n)$ be the set of attributes. Let $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weighted vector of attributes, $C_j\,(j = 1, 2, ..., n)$, such that $\omega_j \in [0, 1]$ and $\sum_{j=1\omega i}^{n} = 1$. Let us suppose that, $D = (d_{ij})_{m\times n} = (\mu_{ij}, \nu_{ij})_{m\times n}$ be the Pythagorean fuzzy decision matrix.*

Step 1: Construct decision matrix.
Step 2: Calculating $\dot{A}_{ij} = nw_jA_{ij}$.
Step 3: Apply PFHA operator to derive the overall preference values.
Step 4: Calculate the score function.
Step 5: Ranking to the given alternatives accoring to their scores function.

## 5. Numerical example

Suppose a man wants to invest money, for the investment the man has four possible options (1) $A_1$ : Mobile Company (2) $A_2$ : Car Company (3) $A_3$ : Fan Company (4) $A_4$ : Laptop Company. There are many factors that must be considered while selecting a suitable company for investment, but here, we consider the following four criteria. whose weighted vector is $w = (0.10, 0.20, 0.30, 0.40)^T$.

(1) $C_1$ : is the risk analysis,
(2) $C_2$ : is the growth analysis,
(3) $C_3$ : is the social political impact analysis,
(4) $C_4$ : is the environmental impact analysis.

Step 1. The decision makers give his decision in the following table.

Table 1 : Pythagorean Fuzzy Decision Making D

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $(0.60, 0.40)$ | $(0.70, 0.50)$ | $(0.80, 0.30)$ | $(0.70, 0.40)$ |
| $A_2$ | $(0.70, 0.60)$ | $(0.60, 0.40)$ | $(0.70, 0.40)$ | $(0.60, 0.50)$ |
| $A_3$ | $(0.60, 0.60)$ | $(0.70, 0.40)$ | $(0.80, 0.40)$ | $(0.70, 0.50)$ |
| $A_4$ | $(0.70, 0.40)$ | $(0.60, 0.50)$ | $(0.70, 0.30)$ | $(0.60, 0.40)$ |

Using $\dot{\beta}_{ij} = n\omega_j\beta_{ij}$, where $\omega = (0.10, 0.20, 0.30, 0.40)^T$ we have

$$\begin{aligned}
\dot{\beta}_{11} &= (0.40, 0.69), \dot{\beta}_{12} = (0.64, 0.57), \dot{\beta}_{13} = (0.84, 0.23), \dot{\beta}_{14} = (0.81, 0.23) \\
\dot{\beta}_{21} &= (0.48, 0.81), \dot{\beta}_{22} = (0.54, 0.48), \dot{\beta}_{23} = (0.74, 0.33), \dot{\beta}_{24} = (0.71, 0.32) \\
\dot{\beta}_{31} &= (0.40, 0.81), \dot{\beta}_{32} = (0.64, 0.48), \dot{\beta}_{33} = (0.84, 0.33), \dot{\beta}_{34} = (0.81, 0.32) \\
\dot{\beta}_{41} &= (0.48, 0.69), \dot{\beta}_{42} = (0.54, 0.57), \dot{\beta}_{43} = (0.74, 0.23), \dot{\beta}_{44} = (0.71, 0.23).
\end{aligned}$$

By the score function we have

$$\begin{aligned}
&\dot{\beta}_{\sigma(11)} = (0.84, 0.23), \dot{\beta}_{\sigma(12)} = (0.81, 0.23), \dot{\beta}_{\sigma(13)} = (0.64, 0.57), \dot{\beta}_{\sigma(14)} = (0.40, 0.69) \\
&\dot{\beta}_{\sigma(21)} = (0.74, 0.33), \dot{\beta}_{\sigma(22)} = (0.71, 0.32), \dot{\beta}_{\sigma(23)} = (0.54, 0.48), \dot{\beta}_{\sigma(24)} = (0.48, 0.81) \\
&\dot{\beta}_{\sigma(31)} = (0.84, 0.33), \dot{\beta}_{\sigma(32)} = (0.81, 0.32), \dot{\beta}_{\sigma(33)} = (0.64, 0.48), \dot{\beta}_{\sigma(34)} = (0.40, 0.81) \\
&\dot{\beta}_{\sigma(41)} = (0.74, 0.23), \dot{\beta}_{\sigma(42)} = (0.71, 0.23), \dot{\beta}_{\sigma(43)} = (0.54, 0.57), \dot{\beta}_{\sigma(44)} = (0.48, 0.69).
\end{aligned}$$

Step 2. Using Pythagorean fuzzy hybrid averaging operator, where $\omega = (0.10, 0.20, 0.30, 0.40)^T$ we have $d_1 = (0.65, 0.47), d_2 = (0.60, 0.53), d_3 = (0.60, 0.47), d_4 = (0.45, 0.41)$.
Step 3. Calculating scores function $s(d_i)(i = 1, 2, 3, 4)$, $s(d_1) = 0.14, s(d_2) = 0.08, s(d_3) = 0.15, s(d_4) = -0.01$.
Step 4. Thus $A_3$ : fan company is the best option for a man to invest his money.

## 6. Conclusion

In this paper, we have defined Pythagorean fuzzy hybrid averaging operator, which is the generalization of PFWA operator and PFOWA operator. At the last we applied the proposed operator to MCDM problem, based on Pythagorean fuzzy information.

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