A CERTAIN NEW FAMILIAR CLASS OF UNIVALENT ANALYTIC FUNCTIONS WITH VARYING ARGUMENT OF COEFFICIENTS INVOLVING CONVOLUTION

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Abstract. In this paper, we used the generalization of the modified-Hadamard products to obtain some interesting characterization theorems for certain general subclass of uniformly functions with positive coefficients.
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1. Introduction and preliminaries

Let $\mathcal{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic and univalent in the open unit disc $\mathbb{U} = \{z : |z| < 1\}$. This class of functions has been extensively exploited in some recent articles to study subclasses of functions satisfy certain conditions [16, 17, 18, 19, 20]. For functions $f, g \in \mathcal{A}$ of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} a_n z^n$, their Hadamard product or convolution $f(z) * g(z)$ is defined by

$$f(z) * g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \qquad z \in \mathbb{U}. \tag{1.2}$$

Definition 1.1 ([14])**.** Let $\kappa - ST(\alpha, \beta)$ denote the subclass of $\mathcal{A}$ consisting of functions $f(z)$ of the form (1.1) and satisfy the following inequality

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} - \alpha > \kappa \left|\frac{zf'(z)}{f(z)} - \beta\right| \tag{1.3}$$

$$(0 \leq \alpha < \beta \leq 1; \kappa(1-\beta) < 1-\alpha; z \in \mathbb{U}).$$

Also let $\kappa - UCV(\alpha,\beta)$ denote the subclass of $\mathcal{A}$ consisting of functions $f(z)$ of the form (1.1) and satisfy the following inequality

$$\text{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}-\alpha>\kappa\left|1+\frac{zf''(z)}{f'(z)}-\beta\right| \tag{1.4}$$

$$(0\le\alpha<\beta\le 1;\kappa(1-\beta)<1-\alpha;z\in\mathbb{U}).$$

The class $\kappa - ST(\alpha,\beta)$ denote the class of $\kappa-$uniformly starlike functions of order α and type β and the class $\kappa-UCV(\alpha,\beta)$ denote the class of $\kappa-$uniformly convex functions of order α and type β.
Specializing the parameters α,β and κ, we obtain many subclasses studied by various authors (see [1-4] and [7-13]).

We now introduce the familiar subclass $\kappa - \mathcal{SC}(\Phi,\Psi;\alpha,\beta)$ of the functions in class $\mathcal{A}$ as follows.

Definition 1.2. Given

$$\Phi(z)=z+\sum_{n=2}^{\infty}\lambda_n z^n \quad \textit{and} \qquad \Psi(z)=z+\sum_{n=2}^{\infty}\mu_n z^n$$

be analytic in $\mathbb{U}$, *such that* $\lambda_n\ge 0,\mu_n\ge 0$ *and* $\lambda_n\ge\mu_n$ *for* $n\ge 2$, *we say that* $f(z)\in\mathcal{A}$ *is in the class* $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$ *if* $f(z)*\Psi(z)\neq 0$ *and*

$$\text{Re}\left\{\frac{f(z)*\Phi(z)}{f(z)*\Psi(z)}\right\}-\alpha>\kappa\left|\frac{f(z)*\Phi(z)}{f(z)*\Psi(z)}-\beta\right|, \tag{1.5}$$

$$(0\le\alpha<\beta\le 1;\kappa(1-\beta)<1-\alpha;z\in\mathbb{U}).$$

It is easy to check that various subclasses of $\mathcal{A}$ referred to above can be represented as $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$ for suitable choices of Φ, Ψ. For example

(i) $\kappa-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};\alpha,\beta\right)=\kappa-ST(\alpha,\beta)$ and $\kappa-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};\alpha,\beta\right)=\kappa-UCV(\alpha,\beta)$ (see Sim et al. [14]);

(ii) $\kappa-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};\alpha,1\right)=SD(\kappa,\alpha)$ and $\kappa-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};\alpha,1\right)=KD(\kappa,\alpha)$ (see Shams et al. [12]);

(iii) $\kappa-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};0,1\right)=\kappa-ST$ and $\kappa-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};0,1\right)=\kappa-UCV$ (see Kanas and Wisniowska [9, 10]);

(iv) $1-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};\alpha,1\right)=S_p(\alpha)$ and $1-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};\alpha,1\right)=UCV(\alpha)$ (see Ronning [4]);

(v) $1-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};0,1\right)=S_p$ and $1-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};0,1\right)=UCV$ (see Goodman [1, 2], Ma and Minda [13] and Ronning [3, 4]).

Another subclasses are the subclasses

$$1-\mathcal{SC}\left(\frac{z}{(1-z)^2},\frac{z}{1-z};\alpha,\beta\right)=US(\alpha,\beta)\equiv\text{Re}\left\{\frac{zf'(z)}{f(z)}\right\}-\alpha>\left|\frac{zf'(z)}{f(z)}-\beta\right|$$

and

$$1-\mathcal{SC}\left(\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2};\alpha,\beta\right)$$
$$=UC(\alpha,\beta)\equiv \operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}-\alpha>\left|1+\frac{zf''(z)}{f'(z)}-\beta\right|.$$

For $p_i \geq 1$ and $\sum_{i=1}^{\infty}\frac{1}{p_i}=1$, the Hölder inequality is defined by (see [15]):

$$\sum_{i=2}^{\infty}\left(\prod_{j=1}^{m}a_{i,j}\right)\leq\prod_{j=1}^{m}\left(\sum_{i=2}^{\infty}a_{i,j}^{p_i}\right)^{\frac{1}{p_i}}. \tag{1.6}$$

Let $f_j\in\mathcal{A}\,(j=1,2)$ be given by

$$f_j\,(z)=z+\sum_{n=2}^{\infty}a_{n,j}z^n \quad (j=1,2)\,. \tag{1.7}$$

Then the modified Hadamard product or (convolution) $f_1 * f_2$ is defined by

$$(f_1 * f_2)\,(z)=z+\sum_{n=2}^{\infty}a_{n,1}a_{n,2}z^n. \tag{1.8}$$

For any real numbers p and q, the modified generalized Hadamard product $(f_1\Delta f_2)\,(p,q;z)$ defined by (see Choi et al. [6]):

$$(f_1\Delta f_2)\,(p,q;z)=z+\sum_{n=2}^{\infty}(a_{n,1})^p\,(a_{n,2})^q\,z^n. \tag{1.9}$$

In the special case, if we take $p=q=1$, then

$$(f_1\Delta f_2)\,(1,1;z)=(f_1 * f_2)\,(z) \quad (z\in\mathbb{U})\,. \tag{1.10}$$

In order to prove our results, we shall need the following lemma.

Lemma 1.3 ([5]). *Let the function $f(z)$ be given by* (1.1). *If*

$$\sum_{n=2}^{\infty}\left[(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n\right]|a_n|\leq 1-\alpha-\kappa(1-\beta), \tag{1.11}$$

where $\lambda_n\geq 0,\mu_n\geq 0$ and $\lambda_n\geq\mu_n$, then $f(z)\in\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$.

In the present paper, we will obtain several results for the generalized Hadamard product of functions in the class $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$.

2. Main results

Unless otherwise mentioned, we assume in the reminder of this paper that; $0 \le \alpha < \beta \le 1; \kappa(1-\beta) < 1-\alpha; z \in \mathbb{U}$.

Theorem 2.1. *If the function f_j $(j=1,2)$ defined by* (1.7) *belongs to the subclass $\kappa - \mathcal{SC}(\Phi, \Psi; \alpha_j, \beta)$ $(j=1,2)$, then*

$$(f_1 \Delta f_2)\left(\frac{1}{p}, \frac{1}{q}; z\right) \in \kappa - \mathcal{SC}(\Phi, \Psi; \sigma, \beta), \tag{2.1}$$

where p, $q > 1$ and σ is given by

$$\sigma = \min_{n \ge 2} \left\{ \frac{\begin{array}{c}(1-\kappa(1-\beta))\left(\frac{(1+\kappa)\lambda_n - (\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)}\right)^{\frac{1}{p}}\left(\frac{(1+\kappa)\lambda_n - (\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)}\right)^{\frac{1}{q}} \\ +\kappa\beta\mu_n - (1+\kappa)\lambda_n\end{array}}{\left(\frac{(1+\kappa)\lambda_n - (\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)}\right)^{\frac{1}{p}}\left(\frac{(1+\kappa)\lambda_n - (\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)}\right)^{\frac{1}{q}} - \mu_n} \right\}.$$

Proof. Let $f_j \in \kappa - \mathcal{SC}(\Phi, \Psi; \alpha_j, \beta)$. Then by using Lemma 1.3, we have

$$\sum_{n=2}^{\infty} \frac{(1+\kappa)\lambda_n - (\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)} |a_{n,j}| \le 1 \quad (j=1,2)\,. \tag{2.2}$$

Moreover,

$$\left\{\sum_{n=2}^{\infty} \frac{(1+\kappa)\lambda_n - (\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)} |a_{n,1}|\right\}^{\frac{1}{p}} \le 1, \tag{2.3}$$

and

$$\left\{\sum_{n=2}^{\infty} \frac{(1+\kappa)\lambda_n - (\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)} |a_{n,2}|\right\}^{\frac{1}{q}} \le 1. \tag{2.4}$$

Applying the Hölder inequality (1.6) to (2.3) and (2.4), we obtain

$$\begin{aligned}&\sum_{n=2}^{\infty} \left[\frac{(1+\kappa)\lambda_n - (\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)}\right]^{\frac{1}{p}} \cdot \\ &\quad \cdot \left[\frac{(1+\kappa)\lambda_n - (\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)}\right]^{\frac{1}{q}} |a_{n,1}|^{\frac{1}{p}} \ |a_{n,2}|^{\frac{1}{q}} \le 1.\end{aligned} \tag{2.5}$$

Since

$$(f_1 \Delta f_2)\left(\frac{1}{p}, \frac{1}{q}; z\right) = z + \sum_{n=2}^{\infty} (a_{n,1})^{\frac{1}{p}} (a_{n,2})^{\frac{1}{q}} z^n, \tag{2.6}$$

we see that

$$\sum_{n=2}^{\infty}\left[\frac{(1+\kappa)\lambda_n-(\sigma+\kappa\beta)\mu_n}{1-\sigma-\kappa(1-\beta)}\right]|a_{n,1}|^{\frac{1}{p}}\quad|a_{n,2}|^{\frac{1}{q}}\leq 1, \tag{2.7}$$

with

$$\sigma\leq\min_{n\geq 2}\left[\frac{\begin{array}{c}(1-\kappa(1-\beta))\left(\frac{(1+\kappa)\lambda_n-(\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)}\right)^{\frac{1}{p}}\left(\frac{(1+\kappa)\lambda_n-(\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)}\right)^{\frac{1}{q}}\\ +\kappa\beta\mu_n-(1+\kappa)\lambda_n\end{array}}{\left(\frac{(1+\kappa)\lambda_n-(\alpha_1+\kappa\beta)\mu_n}{1-\alpha_1-\kappa(1-\beta)}\right)^{\frac{1}{p}}\left(\frac{(1+\kappa)\lambda_n-(\alpha_2+\kappa\beta)\mu_n}{1-\alpha_2-\kappa(1-\beta)}\right)^{\frac{1}{q}}-\mu_n}\right].$$

Thus, by using Lemma 1.3, the proof of Theorem 2.1 is completed . □

Putting $\alpha_j=\alpha\,(j=1,2)$ in Theorem 2.1, we obtain the following corollary.

Corollary 2.2. *If the functions $f_j\,(j=1,2)$ defined by (1.7) are in the subclass $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$. Then*

$$(f_1\ \Delta f_2\ )\left(\frac{1}{p},\frac{1}{q};z\right)\in\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)\quad(p,q>1)\,. \tag{2.8}$$

Theorem 2.3. *If the function $f_j\ \ (j=1,2,\cdots,m)$ defined by (1.7) belongs to the subclass $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha_j,\beta)$ $(j=1,2,\cdots,m)$, and $\mathcal{G}_m\,(z)$ defined by*

$$\mathcal{G}_m\,(z)=z+\sum_{n=2}^{\infty}\left(\sum_{j=1}^{m}(a_{n,j})^p\right)z^n, \tag{2.9}$$

then

$$\mathcal{G}_m\,(z)\in\kappa-\mathcal{SC}(\Phi,\Psi;\sigma_m,\beta), \tag{2.10}$$

where

$$\sigma_m=\min_{n\geq 2}\left\{1-\kappa(1-\beta)-\frac{(\kappa(1-2\beta)-1)m\mu_n+(1-\kappa)m\lambda_n}{\left[\frac{(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n}{1-\alpha-\kappa(1-\beta)}\right]^p-m\mu_n}\right\},\alpha=\min_{1\leq j\leq m}\{\alpha_j\}.$$

Proof. Since $f_j\in\kappa-\mathcal{SC}(\Phi,\Psi;\alpha_j,\beta)$ by using Lemma 1.3, we have

$$\sum_{n=2}^{\infty}\frac{(1+\kappa)\lambda_n-(\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)}\,|a_{n,j}|\leq 1\quad(j=1,2,\cdots,m;n\geq 2)\,. \tag{2.11}$$

and

$$\sum_{n=2}^{\infty}\left\{\frac{(1+\kappa)\lambda_n-(\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)}\right\}^p|a_{n,j}|^p$$

$$\leq\left\{\sum_{n=2}^{\infty}\frac{(1+\kappa)\lambda_n-(\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)}|a_{n,j}|\right\}^p\leq 1, \tag{2.12}$$

it follows from (2.12) , that

$$\sum_{n=2}^{\infty}\left(\frac{1}{m}\sum_{j=1}^{m}\left\{\frac{(1+\kappa)\lambda_n-(\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)}\right\}^p|a_{n,j}|^p\right)\leq 1. \tag{2.13}$$

Putting

$$\alpha=\min_{1\leq j\leq m}\{\alpha_j\}, \tag{2.14}$$

and by virtue of Lemma 1.3, we find that

$$\sum_{n=2}^{\infty}\frac{(1+\kappa)\lambda_n-(\sigma_m+\kappa\beta)\mu_n}{1-\sigma_m-\kappa(1-\beta)}\sum_{j=1}^{m}|a_{n,j}|^p$$

$$\leq\sum_{n=2}^{\infty}\left\{\frac{1}{m}\left[\frac{(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n}{1-\alpha-\kappa(1-\beta)}\right]^p\sum_{j=1}^{m}|a_{n,j}|^p\right\}$$

$$\leq\sum_{n=2}^{\infty}\left\{\frac{1}{m}\sum_{j=1}^{m}\left[\frac{(1+\kappa)\lambda_n-(\alpha_j+\kappa\beta)\mu_n}{1-\alpha_j-\kappa(1-\beta)}\right]^p|a_{n,j}|^p\right\}\leq 1, \tag{2.15}$$

if

$$\sigma_m\leq\min_{n\geq 2}\left\{1-\kappa(1-\beta)-\frac{(\kappa(1-2\beta)-1)\,m\mu_n+(1-\kappa)m\lambda_n}{\left[\frac{(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n}{1-\alpha-\kappa(1-\beta)}\right]^p-m\mu_n}\right\}. \tag{2.16}$$

Thus the proof of Theorem 2.3 is completed. □

Taking $p=2$ and $\alpha_j=\alpha(j=1,2,\cdots,m)$ in Theorem 2.3, we obtain the following corollary:

Corollary 2.4. *Let the functions $f_j(z)$ $(j=1,2,\cdots,m)$ defined by* (1.7) *be in the class $\kappa-\mathcal{SC}(\Phi,\Psi;\alpha,\beta)$ and let the function $\mathcal{G}_m(z)$ be defined by*

$$\mathcal{G}_m(z)=z+\sum_{n=2}^{\infty}\left(\sum_{j=1}^{m}(a_{n,j})^2\right)z^n, \qquad z\in\mathbb{U}. \tag{2.17}$$

Then $\mathcal{G}_m(z) \in \kappa - \mathcal{SC}(\Phi, \Psi; \zeta_m, \beta)(z \in \mathbb{U})$*, where*

$$\zeta_m = \min_{n\geq 2}\left\{1-\kappa(1-\beta)-\frac{(\kappa(1-2\beta)-1)\, m\mu_n+(1-\kappa)m\lambda_n}{\left[\frac{(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n}{1-\alpha-\kappa(1-\beta)}\right]^2-m\mu_n}\right\}. \tag{2.18}$$

Taking $m = 2$ in Corollary 2.4, we obtain

Corollary 2.5. *Let the functions* $f_j(z)$ $(j = 1, 2)$ *defined by* (1.7) *be in the class* $\kappa - \mathcal{SC}(\Phi, \Psi; \alpha, \beta)$ *and let the function* $\mathcal{G}_2(z)$ *defined by*

$$\mathcal{G}_2(z) = z + \sum_{n=2}^{\infty}(a_{n,1}^2 + a_{n,2}^2)z^n\,, \qquad z \in \mathbb{U}. \tag{2.19}$$

Then $\mathcal{G}_2(z) \in \kappa - \mathcal{SC}(\Phi, \Psi; \zeta_2, \beta)$ $(z \in \mathbb{U})$*, where*

$$\zeta_2 = \min_{n\geq 2}\left\{1-\kappa(1-\beta)-\frac{(\kappa(1-2\beta)-1)\, \mu_n+(1-\kappa)\lambda_n}{\left[\frac{(1+\kappa)\lambda_n-(\alpha+\kappa\beta)\mu_n}{\sqrt{2}(1-\alpha-\kappa(1-\beta))}\right]^2-\mu_n}\right\}. \tag{2.20}$$

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