# SOFT ROUGH $BCI$-ALGEBRAS AND CORRESPONDING DECISION MAKING

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**Abstract.** In this paper, we study soft rough $BCI$-algebras with respect to $MS$-approximation spaces. Some new soft rough operations over $BCI$-algebras are explored. In particular, lower and upper soft rough $BCI$-algebras with another soft set are investigated. Finally, a kind of decision making method for soft rough $BCI$-algebras are originally investigated.
**Keywords:** Soft set, soft rough set, $MSR$-set, decision making method, $BCI$-algebra.

## 1. Introduction

The concept of rough sets was initiated by Pawlak [23] as an approach to copy with inexact and uncertain knowledge. As far as known that that an equivalence relation on a set partitions the set into disjoint classes and vice versa. We know that a subset can be written as union of these classes, which is called definable, otherwise it is not definable. In this case, it can be approximated by two definable subsets called lower and upper approximations of the set. Some general models can be found in [30–32]. Nowadays, this theory has been applied to many fields, such as patter recognition, intelligent systems, machines learning, cognitive science, image processing, signal analysis and so on. On the other hand, some researchers applied this theory to algebraic structures, for examples, see [5, 6, 11].

The concept of soft sets was initiated by Molodsov [22] as a new mathematical tool for dealing with uncertainties. It is free that soft set theory is free from the difficulties that have troubled the usual theoretical approaches. Nowadays, the research on soft sets is progressing rapidly. In 2003, Maji [20] proposed some basic operations. Further, Ali [1] revised some operations. Afterwards, a wide range of applications of soft sets have been studied in many different fields including game theory, operation researches, data analysis, mea-

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surement theory, decision making, forecasting, and so on. In recent years, there has been a rapid growth of interest in soft set theory and its applications, for examples, see [2–4, 15, 21, 25]. In particular, Zhan [29] reviewed on decision making methods based on (fuzzy) soft sets and rough soft sets. At the same time, many researchers applied this theory to algebraic structures, for examples, see [5, 6, 12, 13].

As far as known that the research of $t$-norm-based logical systems has become increasingly more important in the field of logic. It is well known that $BCK$ and $BCI$-algebras are two classes of algebras of logic which were introduced by Imai and Iseki [9, 10]. We know that these two classes of logical algebras have been investigated by many researchers, see [14, 16, 18, 19, 26, 27]. Most of the algebras related to the $t$-norm based logic, such as $MTL$-algebras, $BL$-algebras, $MV$-algebras and Boolean algebras et al, are extensions of $BCK$-algebras. This shows that $BCK/BCI$-algebras are considerably general structures, which means that it is an important topic on these two kinds of logical algebras.

Recently, Feng [7] proposed rough soft sets by combining Pawlak rough sets and soft sets, rough sets can be regarded as a collection of rough sets sharing a common Pawlak approximation space. In [28], Zhan initiated rough soft set theory to algebraic structures–hemirings. On the other hand, Ma [17] put forth rough soft $BCI$-algebras by means of an ideal of the $BCI$-algebra. In 2011, Feng [8] proposed soft rough sets by combining soft sets with rough sets, which can be regarded as a kind of new rough set as a soft set instead of an equivalence relation. However, the soft set must be a full soft set in order to resolve theoretical and practical aspects. Recently, Shabir [24] pointed out that there exist two problems on Feng's soft rough set as follows: (1) An upper approximation of a non-empty set may be empty; (2) The upper approximation of a subset $X$ may not contain the set $X$. To resolve this shortcoming, Shabir modified this concept and proposed a class of revised soft rough set, which is called an $MSR$-set. The $MSR$-sets are not only no restrictions on the soft sets but also the underlying concepts are very similar to classical rough sets.

Based on the above idea, in the present paper, we apply this novel soft rough set theory to $BCI$-algebras. In section 2, we recall some basic concepts on rough sets, soft sets and $BCI$-algebras. In section 3, we study some operations with respect to $MS$-approximation spaces and some new soft rough operations over $BCI$-algebras are explored. Further, some lower and upper $MSR$-$BCI$-algebras are investigated in section 4. In particular, in section 5 we discuss soft rough $BCI$-algebras based another soft set. Finally, we initiate to put forth a kind of decision making method for soft rough $BCI$-algebras in section 6.

## 2. Preliminaries

For any $BCI$-algebra $X$, the relation $\leq$ defined by $x \leq y$ if and only if $x * y = 0$ is a partial order on $X$. Throughout this paper, $X$ is always a $BCI$-algebra.

A non-empty subset $S$ of $X$ is called a *subalgebra* of $X$ if $x * y \in S$ whenever $x, y \in S$. A non-empty subset $I$ of $X$ is called an *ideal* of $X$, denoted by $I \lhd X$, if it satisfies: (1) $0 \in I$; (2) $x * y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$.

**Definition 2.1** ([22])**.** A pair $\mathfrak{S} = (F, A)$ is called a *soft set* over $U$, where $A \subseteq E$ and $F : A \to \mathscr{P}(U)$ is a set-valued mapping.

**Definition 2.2** ([7])**.** A soft set $\mathfrak{S} = (F, A)$ over $U$ is called a *full soft set* if $\bigcup\limits_{a \in A} F(a) = U$.

**Definition 2.3** ([12, 13])**.** Let $(F, A)$ be a soft set over $X$. Then

(1) $(F, A)$ is called a *soft BCI-algebra* over $X$ if $F(x)$ is a subalgebra of $X$ for all $x \in \mathrm{Supp}(F, A)$,

(2) $(F, A)$ is called a *soft ideal* if $F(x)$ is an ideal of $X$ for all $x \in \mathrm{Supp}(F, A)$, where $\mathrm{Supp}(F, A) = \{x \in A | F(x) \neq \emptyset\}$ is called a soft support of the soft set $(F, A)$.

**Definition 2.4** ([23])**.** Let $R$ be an equivalence relation on the universe $U$, $(U, R)$ be a Pawlak approximation space. A subset $X \subseteq U$ is called *definable* if $R_* X = R^* X$; in the opposite case, i.e., if $R_* X - R^* X \neq \emptyset$, $X$ is said to be a *rough set*, where two operations are defined as:

$$R_* X = \{x \in U : [x]_R \subseteq X\},$$
$$R^* X = \{x \in U : [x]_R \cap X \neq \emptyset\}.$$

**Definition 2.5** ([8])**.** Let $\mathfrak{S} = (F, A)$ be a soft set over $U$. Then the pair $P = (U, \mathfrak{S})$ is called a *soft approximation space*. Based on $P$, we define the following two operations:

$$\underline{apr}_P(X) = \{u \in U | \exists a \in A[u \in F(a) \subseteq X]\},$$
$$\overline{apr}_P(X) = \{u \in U | \exists a \in A[u \in F(a), F(a) \cap X \neq \emptyset]\},$$

assigning to every subset $X \subseteq U$, two sets $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ are called the *lower* and *upper soft rough approximations* of $X$ in $P$, respectively. If $\underline{apr}_P(X) = \overline{apr}_P(X)$, $X$ is said to be soft definable; otherwise $X$ is called a *soft rough set*. In what follows, we call it *Feng-soft rough set*.

**Definition 2.6** ([24])**.** Let $(F, A)$ be a soft set over $U$ and $\varphi : U \to \mathscr{P}(A)$ be a map defined as $\varphi(x) = \{a | x \in F(a)\}$. Then the pair $(U, \varphi)$ is called an *$MS$-approximation space* and for any $X \subseteq U$, the lower $MSR$-approximation and upper $MSR$-approximation of $X$ are denoted by $\underline{X}_\varphi$ and $\overline{X}_\varphi$ respectively, which two operations are defined as

$$\underline{X}_\varphi = \{x \in X | \varphi(x) \neq \varphi(y) \text{ for all } y \in X^c\}$$

and

$$\overline{X}_\varphi = \{x \in U | \varphi(x) = \varphi(y) \text{ for some } y \in X\}$$

If $\underline{X}_\varphi = \overline{X}_\varphi$, then the $X$ is said to be *$MS$-definable*, otherwise $X$ is said to be *$MSR$-set*.

### 3. Soft rough approximations

In this section, we investigate some operations and fundamental properties of modified soft rough sets over $BCI$-algebras. In order to illustrate the roughness in $BCI$-algebra $X$ w.r.t. soft rough approximation spaces over $BCI$-algebras, we first introduce two special kinds of soft sets over $BCI$-algebras.

**Definition 3.1.** Let $\mathfrak{S} = (F, A)$ be a soft set over $X$ and $\varphi : X \to \mathscr{P}(A)$ be a map defined as $\varphi(x) = \{a|x \in F(a)\}$, then $\mathfrak{S}$ is called a *$C$-soft set* over $X$ if $\varphi(a) = \varphi(b)$ and $\varphi(c) = \varphi(d)$ imply $\varphi(a * c) = \varphi(b * d)$ for all $a, b, c, d \in X$.

**Example 3.2.** Let $X = \{0, a, b, c\}$ be a $BCI$-algebra with the following Cayley Table 1:

Table 1 table for $BCI$-algebra

| $*$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $0$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $0$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $0$ |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 2.

Table 2 table for soft set $\mathfrak{S}$

| | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 0 | 0 |
| $e_2$ | 1 | 1 | 1 | 1 |
| $e_3$ | 1 | 1 | 0 | 0 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of soft rough approximation space $(X, \varphi)$ is given by $\varphi(0) = \varphi(a) = \{e_2, e_3\}$, $\varphi(b) = \varphi(c) = \{e_2\}$.

By calculations, $\mathfrak{S}$ is called a $C$-soft set over $X$.

Let $Y$, $Z$ be any two non-empty subsets in any $BCI$-algebra $X$. Denote $Y * Z = \{y * z| \ \forall\ y \in Y, z \in Z\}$.

**Theorem 3.3.** Let $\mathfrak{S} = (F, A)$ be a $C$-soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space. For any two non-empty subsets $Y$, $Z$ in $X$. Then

$$\overline{Y}_{\varphi} * \overline{Z}_{\varphi} \subseteq \overline{Y * Z}_{\varphi}.$$

**Proof.** Let $c \in \overline{Y}_{\varphi} * \overline{Z}_{\varphi}$, then $c = a * b$, where $a \in \overline{Y}_{\varphi}$ and $b \in \overline{Z}_{\varphi}$, and so there exist $y \in Y$ and $z \in Z$ such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$. Since $\mathfrak{S}$ is a $C$-soft set, $\varphi(a * b) = \varphi(y * z)$ for $y * z \in Y * Z$. Hence $c = a * b \in \overline{Y * Z}_{\varphi}$. That is, $\overline{Y}_{\varphi} * \overline{Z}_{\varphi} \subseteq \overline{Y * Z}_{\varphi}$. □

The following example shows that the containment in Theorem 3.3 is proper.

**Example 3.4.** Let $X = \{0, a, b, c\}$ be a $BCI$-algebra with the following Cayley Table 3:

Table 3 table for $BCI$-algebra

| $*$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $c$ |
| $a$ | $a$ | $0$ | $a$ | $c$ |
| $b$ | $b$ | $b$ | $0$ | $c$ |
| $c$ | $c$ | $c$ | $c$ | $0$ |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 4.

Table 4 table for soft set $\mathfrak{S}$

| | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 |
| $e_2$ | 0 | 0 | 0 | 0 |
| $e_3$ | 0 | 0 | 0 | 1 |
| $e_4$ | 1 | 1 | 1 | 0 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of soft rough approximation space $(X, \varphi)$ is given by $\varphi(0) = \varphi(a) = \varphi(b) = \{e_1, e_4\}$, $\varphi(c) = \{e_3\}$. Then we can check that $\mathfrak{S}$ is a $C$-soft set over $X$.

If we take $Y = \{0, c\}$ and $Z = \{c\}$, then $Y * Z = \{0, c\}$, and so, $\overline{Y}_\varphi = \{0, a, b, c\}$ and $\overline{Z}_\varphi = \{c\}$, so $\overline{Y}_\varphi * \overline{Z}_\varphi = \{0, c\}$. Also we have $\overline{Y * Z}_\varphi = \overline{\{0, c\}}_\varphi = \{0, a, b, c\}$. Thus $\overline{Y}_\varphi * \overline{Z}_\varphi \subsetneq \overline{Y * Z}_\varphi$.

**Definition 3.5.** Let $\mathfrak{S} = (F, A)$ be a $C$-soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space, then $\mathfrak{S}$ is called a *$CC$-soft set* over $X$ if for all $c \in X$, $\varphi(c) = \varphi(x * y)$, there exist $a, b \in X$, such that $\varphi(x) = \varphi(a)$ and $\varphi(y) = \varphi(b)$ satisfying $c = a * b$.

**Remark 3.6.** (1) $\mathfrak{S}$ in Example 3.4 is a $C$-soft set over $X$, but it is not a $CC$-soft set.

(2) $\mathfrak{S}$ in Example 3.2 is a $CC$-soft set over $X$.

If we strength the condition, we can obtain the following result:

**Theorem 3.7.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space. For any two non-empty subsets $Y$, $Z$ in $X$. Then*

$$\overline{Y}_\varphi \cdot \overline{Z}_\varphi = \overline{Y \cdot Z}_\varphi.$$

**Proof.** By Theorem 3.3, we have $\overline{Y}_\varphi * \overline{Z}_\varphi \subseteq \overline{Y * Z}_\varphi$. Now let $c \in \overline{Y * Z}_\varphi$, so $\varphi(c) = \varphi(y * z)$ for some $y \in Y$ and $z \in Z$. Then there exist $a, b \in X$, such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$ satisfying $c = a * b$ since $\mathfrak{S}$ is a $CC$-soft set over $X$. Thus $a \in \overline{Y}_\varphi$ and $b \in \overline{Z}_\varphi$. Hence $c \in \overline{Y}_\varphi * \overline{Z}_\varphi$. Summing up the above arguments, $\overline{Y}_\varphi * \overline{Z}_\varphi = \overline{Y * Z}_\varphi$. □

Next, we consider lower soft rough approximations over $BCI$-algebras.

**Theorem 3.8.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space. For any two non-empty subsets $Y$, $Z$ in $X$. Then*

$$\underline{Y}_\varphi * \underline{Z}_\varphi \subseteq \underline{Y * Z}_\varphi.$$

**Proof.** Suppose that $\underline{Y}_\varphi * \underline{Z}_\varphi \subseteq \underline{Y * Z}_\varphi$ does not hold, then there exists $c \in \underline{Y}_\varphi * \underline{Z}_\varphi$, but $c \notin \underline{Y * Z}_\varphi$. Then $c = a * b$, where $a \in \underline{Y}_\varphi$ and $b \in \underline{Z}_\varphi$. This means that $\varphi(a) \neq \varphi(y)$ and $\varphi(b) \neq \varphi(y)$ for all $y \in Y^c$ and $z \in Z^c$. $(\triangle)$

On the other hand, $c \notin \underline{Y * Z}_\varphi$, then we may have the following two conditions:

(i) $c \notin Y * Z$, which contradicts with $c \in \underline{Y}_\varphi * \underline{Z}_\varphi \subseteq Y * Z$;

(ii) $c \in Y * Z$ and $\varphi(c) = \varphi(y' * z')$ for some $y' * z' \in (Y * Z)^c$. Thus $y' \in Y^c$ or $z' \in Z^c$. In fact, if $y' \notin Y^c$ and $z' \notin Z^c$, we have $y' * z' \in Y * Z$, a contradiction. Since $\mathfrak{S} = (F, A)$ is a $CC$-soft set over $X$, then there exist $a', b' \in X$ such that $\varphi(a') = \varphi(y')$ and $\varphi(b') = \varphi(z')$ satisfying $a' * b' = c$, for some $y' \in Y^c$ or $z' \in Z^c$. This is contradiction with $(\triangle)$. Hence $\underline{Y}_\varphi * \underline{Z}_\varphi \subseteq \underline{Y \cdot Z}_\varphi$. $\square$

The following example shows that Theorem 3.8 is not true if $\mathfrak{S}$ is not a $CC$-soft set over $X$.

**Example 3.9.** Let $X = \{0, a, b, c, d\}$ be a $BCI$-algebra with the following Cayley Table 5:

Table 5 table for $BCI$-algebra

| $*$ | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $c$ | $c$ |
| $a$ | $a$ | 0 | $a$ | $c$ | $c$ |
| $b$ | $b$ | $b$ | 0 | $c$ | $c$ |
| $c$ | $c$ | $c$ | $c$ | 0 | 0 |
| $d$ | $d$ | $c$ | $d$ | $a$ | $c$ |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 6.

Table 6 table for soft set $\mathfrak{S}$

| | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $e_1$ | 1 | 0 | 1 | 1 | 0 |
| $e_2$ | 0 | 1 | 1 | 1 | 0 |
| $e_3$ | 0 | 0 | 0 | 0 | 1 |
| $e_4$ | 1 | 0 | 1 | 1 | 0 |

Clearly, $\mathfrak{S}$ is not a $CC$-soft set over $X$. If we take $Y = \{0, b\}$ and $Z = \{b, d\}$, then $\underline{Y}_\varphi = \{0\}$ and $\underline{Z}_\varphi = \{c\}$, so $\underline{Y}_\varphi * \underline{Z}_\varphi = \{c\}$. Also we have $\underline{Y * Z}_\varphi = \{0\}$. This means that $\underline{Y}_\varphi * \underline{Z}_\varphi \nsubseteq \underline{Y * Z}_\varphi$.

The following example shows that the containment in Theorem 3.8 is proper.

**Example 3.10.** Consider the $BCI$-algebra $X$ and the soft set $\mathfrak{S} = (F, A)$ in Example 3.2. If we take $Y = \{0, a, b\}$ and $Z = \{0, b, c\}$, then $\underline{Y}_\varphi = \{0, a\}$ and $\underline{Z}_\varphi = \{b, c\}$, so $\underline{Y}_\varphi * \underline{Z}_\varphi = \{b, c\}$. On the other hand, $\underline{Y * Z}_\varphi = \{0, a, b, c\}$. This means that $\underline{Y}_\varphi * \underline{Z}_\varphi \subsetneq \underline{Y * Z}_\varphi$.

## 4. Soft rough $BCI$-algebras

In this section, we study the operations of lower and upper $MSR$-approximations of soft rough $BCI$-algebras.

**Definition 4.1.** Let $\mathfrak{S} = (F, A)$ be a soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space. For any $Y \subseteq X$, the lower $MSR$-approximation and upper $MSR$-approximation of $Y$ are denoted by $\underline{Y}_\varphi$ and $\overline{Y}_\varphi$, respectively, which two operations are defined as

$$\underline{Y}_\varphi = \{x \in Y | \varphi(x) \neq \varphi(y) \text{ for all } y \in Y^c\}$$

and

$$\overline{Y}_\varphi = \{x \in X | \varphi(x) = \varphi(y) \text{ for some } y \in Y\}$$

If $\underline{Y}_\varphi \neq \overline{Y}_\varphi$, then
(i) $Y$ is called a *lower (upper) soft rough $BCI$-algebra (resp., ideal)* over $X$, if $\underline{Y}_\varphi$ $(\overline{Y}_\varphi)$ is a subalgebra (resp., ideal) of $X$;
(ii) $Y$ is called a *soft rough $BCI$-algebra (resp., ideal)* over $X$, if $\underline{Y}_\varphi$ and $\overline{Y}_\varphi$ are subalgebras (resp., ideals) of $X$.

**Example 4.2.** Let $X = \{0, a, b, c, d\}$ be a $BCI$-algebra with the following Cayley Table 7:

Table 7 table for $BCI$-algebra

| $*$ | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| $a$ | $a$ | 0 | $a$ | 0 | 0 |
| $b$ | $b$ | $b$ | 0 | 0 | $b$ |
| $c$ | $c$ | $b$ | $a$ | 0 | $b$ |
| $d$ | $d$ | $a$ | $d$ | $a$ | 0 |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 8.

Table 8 table for soft set $\mathfrak{S}$

| | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $e_1$ | 1 | 0 | 0 | 0 | 0 |
| $e_2$ | 0 | 1 | 0 | 1 | 0 |
| $e_3$ | 1 | 1 | 1 | 1 | 1 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of $MS$-approximation space $(S, \varphi)$ is given by $\varphi(0) = \{e_1, e_3\}$, $\varphi(a) = \varphi(c) = \{e_2, e_3\}$ and $\varphi(b) = \varphi(d) = \{e_1\}$.

Let $Y = \{0, b, c, d\}$, then $\underline{Y}_\varphi = \{0, b, d\}$ and $\overline{Y}_\varphi = \{0, a, b, c, d\}$. This shows that $\underline{Y}_\varphi$ and $\overline{Y}_\varphi$ are subalgebras of $X$. In other words, $Y$ is a soft rough $BCI$-algebra over $X$, but it is not a soft rough ideal over $X$ since $\underline{Y}_\varphi$ is a subalgebra of $X$.

**Example 4.3.** Let $X = \{0, a, b, c, d\}$ be a $BCI$-algebra with the following Cayley Table 9:

Table 9 table for $BCI$-algebra

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $c$ | $b$ |
| $a$ | $a$ | 0 | $c$ | $b$ |
| $b$ | $b$ | $b$ | 0 | $c$ |
| $c$ | $c$ | $c$ | $b$ | 0 |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 10.

Table 10 table for soft set $\mathfrak{S}$

| | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 1 | 0 | 0 |
| $e_2$ | 1 | 1 | 1 | 1 |
| $e_3$ | 0 | 0 | 1 | 1 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of $MS$-approximation space $(S, \varphi)$ is given by $\varphi(0) = \varphi(a) = \{e_1, e_2\}$, and $\varphi(b) = \varphi(c) = \{e_2, e_3\}$.

Let $Y = \{0, a, b\}$, then $\underline{Y}_\varphi = \{0, a\} \lhd X$ and $\overline{Y}_\varphi = \{0, a, b, c\} \lhd X$. This shows that $Y$ is a soft rough ideal over $X$.

**Proposition 4.4.** Let $(X, \varphi)$ be an $MS$-approximation space. If $Y$ and $Z$ are lower soft rough $BCI$-algebras (resp., ideals) over $X$, then so is $Y \cap Z$.

**Proof.** If $Y$ and $Z$ are lower soft rough $BCI$-algebras (resp., ideals) over $X$, then $\underline{Y}_\varphi$ and $\underline{Z}_\varphi$ are subalgebras (resp., ideals) of $X$, so $\underline{Y}_\varphi \cap \underline{Z}_\varphi$ is a subalgebra (resp., ideal) of $X$. By Theorem 3 in [24], we have $\underline{Y \cap Z}_\varphi = \underline{Y}_\varphi \cap \underline{Z}_\varphi$ is also a subalgebra (resp., ideal) of $X$. Hence $X \cap Y$ is a lower soft rough $BCI$-algebra (resp., ideal) over $X$. □

In general, $Y \cap Z$ is not an upper soft rough $BCI$-algebra (resp., ideal) over $X$, if $Y$ and $Z$ are upper soft rough $BCI$-algebras (resp., ideals) over $X$. Actually we have the following example.

**Example 4.5.** Consider the $BCI$-algebra $X$ and the soft set $\mathfrak{S} = (F, A)$ in Example 4.3. Let $Y = \{0, c\}$ and $Z = \{a, c\}$, then $\overline{Y}_\varphi = \overline{Z}_\varphi = \{0, a, b, c\}$ are subalgebras of $X$. That is, $Y$ and $Z$ are upper soft rough $BCI$-algebras over $X$. But $\overline{Y \cap Z}_\varphi = \overline{\{c\}}_\varphi = \{b, c\}$ is not subalgebra of $X$.

Finally, we study the upper and lower soft rough $BCI$-algebras.

**Theorem 4.6.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$. If $Y$ is a subalgebra of $X$, then $Y$ is a soft rough $BCI$-algebra over $X$ when $\underline{Y}_\varphi \neq \emptyset$.*

**Proof.** (1) By Theorem 3.8, $\underline{Y}_\varphi * \underline{Y}_\varphi \subseteq \underline{Y * Y}_\varphi$. Moreover, by Theorem 3 in [24], $\underline{Y * Y}_\varphi \subseteq \underline{Y}_\varphi$ since $Y * Y \subseteq Y$. Hence $\underline{Y}_\varphi * \underline{Y}_\varphi \subseteq \underline{Y}_\varphi$. This means that $Y$ is a lower soft rough $BCI$-algebra over $X$.

(2) Now, let $a, b \in \overline{Y}_\varphi$, then there exist $y, z \in Y$ such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$. Since $\mathfrak{S}$ is a $C$-soft set over $X$ , $\varphi(a * b) = \varphi(y * z)$. Also, $y * z \in Y$

since $Y$ is a subalgebra of $X$, and so, $Y$ is an upper soft rough $BCI$-algebra over $X$.

By (1) and (2), $Y$ is a soft rough $BCI$-algebra over $X$. $\square$

**Remark 4.7.** The above theorem shows that any soft rough $BCI$-algebra is a generalization of a subalgebra of $BCI$-algebras.

**Open Question:** Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$. If $Y$ is an ideal of $X$, is it a soft rough ideal over $X$ when $\underline{Y}_\varphi \neq \emptyset$?

## 5. Soft rough $BCI$-algebras w.r.t. another soft set

In this section, we investigate soft rough $BCI$-algebras based on another soft set.

**Definition 5.1.** Let $\mathfrak{S} = (F, A)$ be a soft set over $X$ and $(X, \varphi)$ an $MS$-approximation space. Let $\mathfrak{T} = (G, B)$ be another soft set defined over $X$. The lower $MSR$-approximation and upper $MSR$-approximation of $\mathfrak{T}$ w.r.t. $\mathfrak{S}$ are denoted by $\underline{(G,B)}_\varphi = (\underline{G}_\varphi, B)$ and $\overline{(G,B)}_\varphi = (\overline{G}_\varphi, B)$, respectively, which two operations are defined as

$$\underline{G(e)}_\varphi = \{x \in G(e) | \varphi(x) \neq \varphi(y) \text{ for all } y \in X - G(e)\}$$

and

$$\overline{G(e)}_\varphi = \{x \in X | \varphi(x) = \varphi(y) \text{ for some } y \in G(e)\},$$

for all $e \in B$.

(i) If $\underline{(G,B)}_\varphi = \overline{(G,B)}_\varphi$, then $\mathfrak{T}$ is called *definable*;

(ii) If $\underline{(G,B)}_\varphi \neq \overline{(G,B)}_\varphi$, then $\mathfrak{T}$ is called a *lower* (*upper*) *soft rough BCI-algebra (resp., ideal)* w.r.t. $\mathfrak{S}$ over $X$, if $\underline{G(e)}_\varphi$ ($\overline{G(e)}_\varphi$) is a subalgebra (resp., ideal) of $X$ for all $e \in \mathrm{Supp}(G, B)$;

Moreover, $\mathfrak{T}$ is called a *soft rough BCI-algebra (resp., ideal)* w.r.t. $\mathfrak{S}$ over $X$, if $\underline{G(e)}_\varphi$ and $\overline{G(e)}_\varphi$ are subalgebras (resp., ideals) of $X$ for all $e \in \mathrm{Supp}(G, B)$.

**Example 5.2.** Consider the $BCI$-algebra $X$ and the soft set $\mathfrak{S} = (F, A)$ as in Example 3.2. Define another soft set $\mathfrak{T} = (G, B)$ as the following Table 11:

Table 11 table for soft set $\mathfrak{T}$

| | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 0 | 0 | 0 |
| $e_2$ | 1 | 1 | 0 | 1 |
| $e_3$ | 1 | 0 | 1 | 0 |
| $e_4$ | 1 | 0 | 0 | 1 |

By calculations, $\underline{G(e_1)}_\varphi = \emptyset$, $\overline{G(e_1)}_\varphi = \{0, a\}$, $\underline{G(e_2)}_\varphi = \{0, a\}$, $\overline{G(e_2)}_\varphi = \{0, a\}$, $\underline{G(e_3)}_\varphi = \emptyset$, $\overline{G(e_3)}_\varphi = \{0, a, b, c\}$, $\underline{G(e_4)}_\varphi = \emptyset$ and $\overline{G(e_4)}_\varphi = \{0, a, b, c\}$.

Thus, $\mathfrak{T}$ is both a soft rough $BCI$-algebra and a soft rough ideal w.r.t. $\mathfrak{S}$ over $X$.

**Definition 5.3.** Let $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ be two soft sets over $X$ with $C = A \cap B \neq \emptyset$. The *product* * is defined as $\mathfrak{S} * \mathfrak{T} = (F, A) * (G, B) = (K, C)$, where $K(c) = F(c) * G(c)$, for all $c \in C$.

**Theorem 5.4.** *Let $\mathfrak{S} = (F, A)$ be a $C$-soft set over $X$ and $(X, \varphi)$ be an $MS$-approximation space. Let $\mathfrak{T}_1 = (G_1, B)$ and $\mathfrak{T}_2 = (G_2, C)$ be two soft sets over $X$ with $D = B \cap C \neq \emptyset$. Then*

$$\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi \subseteq \overline{(G_1 * G_2, D)}_\varphi.$$

**Proof.** For all $e \in \mathrm{Supp}(G_1, B) \cap \mathrm{Supp}(G_2, C)$, let $c \in \overline{G_1(e)}_\varphi * \overline{G_2(e)}_\varphi$, then $c = a * b$, where $a \in \overline{G_1(e)}_\varphi$ and $b \in \overline{G_2(e)}_\varphi$, and so there exist $y \in G_1(e)$ and $z \in G_2(e)$ such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$. Since $\mathfrak{S}$ is a $C$-soft set, $\varphi(a * b) = \varphi(y * z)$ for $y * z \in G_1(e) * G_2(e)$. Hence $c = a * b \in \overline{G_1(e) * G_2(e)}_\varphi$. That is, $\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi \subseteq \overline{(G_1 * G_2, D)}_\varphi$. □

The following example shows that the containment in Theorem 5.4 is proper.

**Example 5.5.** Let $X = \{0, a, b, c, d\}$ be a $BCI$-algebra with the following Cayley Table 12:

Table 12 table for $BCI$-algebra

| $*$ | $0$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $c$ | $c$ |
| $a$ | $a$ | $0$ | $0$ | $c$ | $c$ |
| $b$ | $b$ | $a$ | $0$ | $c$ | $c$ |
| $c$ | $c$ | $c$ | $c$ | $0$ | $0$ |
| $d$ | $d$ | $c$ | $c$ | $a$ | $0$ |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 13.

Table 13 table for soft set $\mathfrak{S}$

| | $0$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 | 0 |
| $e_2$ | 0 | 0 | 0 | 1 | 1 |
| $e_3$ | 1 | 1 | 1 | 1 | 1 |
| $e_4$ | 0 | 0 | 0 | 1 | 1 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of soft rough approximation space $(X, \varphi)$ is given by $\varphi(0) = \varphi(a) = \varphi(b) = \{e_1, e_3\}$, $\varphi(c) = \varphi(d) = \{e_2, e_3, e_4\}$. Then we can check that $\mathfrak{S}$ is a $C$-soft set over $X$, but it is not a $CC$-soft set over $X$.

Define two soft sets $\mathfrak{T}_1 = (G_1, B)$ and $\mathfrak{T}_2 = (G_2, C)$ over $X$, where $B = \{e_1, e_2\}$ and $C = \{e_2, e_3\}$ with $B \cap C = \{e_2\}$, by $G_1(e_2) = \{c\}$ and $G_2(e_2) = \{c\}$.

By calculations, $\overline{G_1(e_2)}_\varphi = \{c, d\}$ and $\overline{G_2(e_2)}_\varphi = \{c, d\}$, so $\overline{G_1(e_2)}_\varphi * \overline{G_2(e_2)}_\varphi = \{0, a\}$. But $G_1(e_2) * G_2(e_2) = \{0\}$, $\overline{G_1(e_2) * G_2(e_2)}_\varphi = \overline{\{0\}}_\varphi = \{0, a, b\}$. Thus $\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi \subsetneq \overline{(G_1 * G_2, D)}_\varphi$.

If we strength the condition, we can obtain the following result:

**Theorem 5.6.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$ and $(X, \varphi)$ be an $MS$-approximation space. Let $\mathfrak{T}_1 = (G_1, B)$ and $\mathfrak{T}_2 = (G_2, C)$ be two soft sets over $X$ with $D = B \cap C \neq \emptyset$. Then*

$$\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi = \overline{(G_1 * G_2, D)}_\varphi.$$

**Proof.** By Theorem 5.4, we have $\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi \subseteq \overline{(G_1 * G_2, D)}_\varphi$.

For all $e \in \mathrm{Supp}(G_1, B) \cap \mathrm{Supp}(G_2, C)$, let $x \in \overline{G_1(e) * G_2(e)}_\varphi$, so $\varphi(x) = \varphi(y * z)$ for some $y \in G_1(e)$ and $z \in G_2(e)$. Then there exist $a, b \in X$, such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$ satisfying $x = a * b$ since $\mathfrak{S}$ is a $CC$-soft set over $X$. Thus $a \in \overline{G_1(e)}_\varphi$ and $b \in \overline{G_2(e)}_\varphi$. Hence $x \in \overline{G_1(e)}_\varphi * \overline{G_2(e)}_\varphi$. This shows that $\overline{(G_1 * G_2, D)}_\varphi \subseteq \overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi$.

Summing up the above arguments, $\overline{(G_1, B)}_\varphi * \overline{(G_2, C)}_\varphi = \overline{(G_1 * G_2, D)}_\varphi$. $\square$

Combining Theorems 3.8 and 5.6, we can obtain the following result:

**Theorem 5.7.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$ and $(X, \varphi)$ be an $MS$-approximation space. Let $\mathfrak{T}_1 = (G_1, B)$ and $\mathfrak{T}_2 = (G_2, C)$ be two soft sets over $X$ with $D = B \cap C \neq \emptyset$. Then*

$$\underline{(G_1, B)}_\varphi * \underline{(G_2, C)}_\varphi \subseteq \underline{(G_1 * G_2, D)}_\varphi.$$

Finally, we investigate the upper and lower soft rough $BCI$-algebras with respect to another soft set.

**Theorem 5.8.** *Let $\mathfrak{S} = (F, A)$ be a $CC$-soft set over $X$ and $(X, \varphi)$ be an $MS$-approximation space. If $\mathfrak{T} = (G, B)$ is a soft $BCI$-algebra over $X$, then $\mathfrak{T}$ is a soft rough $BCI$-algebra over $X$ w.r.t. $\mathfrak{S}$ when $\underline{\mathfrak{T}}_\varphi \neq \emptyset$.*

**Proof.** (1) By Theorem 5.7, $\underline{(G, B)}_\varphi * \underline{(G, B)}_\varphi \subseteq \underline{(G * G, B)}_\varphi$. Since $\mathfrak{T} = (G, B)$ is a soft $BCI$-algebra over $X$, for any $e \in \mathrm{Supp}(G, B)$, $G(e)$ is a subalgebra of $X$, then $G(e) * G(e) \subseteq G(e)$. By Theorem 9(14) in [24], $\underline{G(e) * G(e)}_\varphi \subseteq \underline{G(e)}_\varphi$, that is, $\underline{(G * G, B)}_\varphi \subseteq \underline{(G, B)}_\varphi$. Thus, $\underline{(G, B)}_\varphi * \underline{(G, B)}_\varphi \subseteq \underline{(G, B)}_\varphi$. This means that $\underline{(G, B)}_\varphi$ is a soft $BCI$-algebra over $X$, that is, for any $e \in \mathrm{Supp}(G, B)$, $\underline{G(e)}_\varphi$ is a subalgebra of $X$. Hence $\mathfrak{T}$ is a lower soft rough $BCI$-algebra w.r.t. $\mathfrak{S}$ over $X$.

(2) For any $e \in \mathrm{Supp}(G, B)$, let $a, b \in \overline{G(e)}_\varphi$, then there exist $y, z \in G(e)$ such that $\varphi(a) = \varphi(y)$ and $\varphi(b) = \varphi(z)$. Since $\mathfrak{S}$ is a $C$-soft set over $X$, $\varphi(a * b) = \varphi(y * z)$. Also, $y * z \in G(e) * G(e) \subseteq G(e)$ since $\mathfrak{T} = (G, B)$ is a soft $BCI$-algebra over $X$, and so, $a * b \in \overline{G(e)}_\varphi$, that is, $\overline{G(e)}_\varphi$ is a subalgebra of $X$. Hence, $\mathfrak{T}$ is an upper soft rough $BCI$-algebra w.r.t. $\mathfrak{S}$ over $X$.

By (1) and (2), $\mathfrak{T}$ is a soft rough $BCI$-algebra w.r.t. $\mathfrak{S}$ over $X$. $\square$

## 6. Soft rough $BCI$-algebras in decision making methods

In this section, we illustrate a kind of new decision making method for Shabir's soft rough sets to $BCI$-algebras.

We will put forth a new method to find which is the best parameter $e$ of a given soft set $\mathfrak{S} = (F, A)$. In other words, $F(e)$ is the nearest accurate $BCI$-algebra on $\mathfrak{S}$ based on another soft set $\mathfrak{T}$ over $BCI$-algebras.

**Decision making method:**

Let $X$ be a $BCI$-algebra and $E$ a set of related parameters. Let $A = \{e_1, e_2, \cdots, e_m\} \subseteq E$, $\mathfrak{S} = (F, A)$ be an original description soft set over $X$ and $(X, \varphi)$ be an $MS$-approximation space. Let $\mathfrak{S} = (G, B)$ be another soft set over $X$. Then we present the decision algorithm for soft rough $BCI$-algebras as follows:

**Step 1** Input the original description $BCI$-algebra $X$, soft set $\mathfrak{S}$ and $(X, \varphi)$ be an $MSR$-approximation space. Consider be another soft set $\mathfrak{S} = (G, B)$ over $X$.

**Step 2** Compute the lower and upper rough soft approximation operators $\underline{(G,B)}_{\varphi}$ and $\overline{(G,B)}_{\varphi}$ w.r.t. $\mathfrak{S}$, respectively.

**Step 3** Compute the different values of $\|G(e_i)\|$, where $\|G(e_i)\| = \frac{|\overline{G(e_i)}_{\varphi}| - |\underline{G(e_i)}_{\varphi}|}{|G(e_i)|}$.

**Step 4** Find the minimum value $\|G(e_k)\|$ of $\|G(e_i)\|$, where $\|G(e_k)\| = \min\limits_{i} \|G(e_i)\|$.

**Step 5** The decision is $G(e_k)$.

**Example 6.1.** Assume that we want to find the nearest accurate $BCI$-algebra. Let $X = \{0, a, b, c, d\}$ be a $BCI$-algebra with the following Cayley Table 14:

Table 14 table for $BCI$-algebra

| $*$ | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $c$ | $c$ |
| $a$ | $a$ | 0 | 0 | $c$ | $c$ |
| $b$ | $b$ | $b$ | 0 | $c$ | $c$ |
| $c$ | $c$ | $c$ | $c$ | 0 | 0 |
| $d$ | $d$ | $c$ | $c$ | $a$ | 0 |

Define a soft set $\mathfrak{S} = (F, A)$ over $X$ which is given by Table 15.

Table 15 table for soft set $\mathfrak{S}$

| | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 | 0 |
| $e_2$ | 1 | 1 | 1 | 1 | 1 |
| $e_3$ | 1 | 1 | 1 | 1 | 1 |
| $e_4$ | 0 | 0 | 0 | 0 | 0 |

Then the mapping $\varphi : X \to \mathscr{P}(A)$ of soft rough approximation space $(X, \varphi)$ is given by $\varphi(0) = \varphi(a) = \varphi(b) = \{e_1, e_2, e_3\}$, $\varphi(c) = \varphi(d) = \{e_2, e_3\}$.

Define another soft set $\mathfrak{T} = (G, B)$ over $X$ which is given by Table 16.

Table 16 table for soft set $\mathfrak{S}$

| | 0 | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 0 | 0 | 0 |
| $e_2$ | 1 | 1 | 0 | 1 | 1 |
| $e_3$ | 0 | 0 | 1 | 1 | 1 |
| $e_4$ | 1 | 0 | 1 | 1 | 0 |
| $e_5$ | 1 | 1 | 1 | 1 | 0 |

That is, $G(e_1) = \{0, a\}$, $G(e_2) = \{0, a, c, d\}$, $G(e_3) = \{b, c, d\}$, $G(e_4) = \{0, b, c\}$ and $G(e_5) = \{0, a, b, c\}$.

By calculations, $\underline{G(e_1)}_\varphi = \emptyset$, $\overline{G(e_1)}_\varphi = \{0, a, b\}$, $\underline{G(e_2)}_\varphi = \{c, d\}$, $\overline{G(e_2)}_\varphi = \{0, a, b, c, d\}$, $\underline{G(e_3)}_\varphi = \{c, d\}$, $\overline{G(e_3)}_\varphi = \{c, d\}$, $\underline{G(e_4)}_\varphi = \emptyset$, $\overline{G(e_4)}_\varphi = \{0, a, b, c, d\}$, $\underline{G(e_5)}_\varphi = \{0, a, b\}$ and $\overline{G(e_5)}_\varphi = \{0, a, b, c, d\}$.

Then, we can calculate $\|G(e_1)\| = 1.5$, $\|G(e_2)\| = 0.75$, $\|G(e_3)\| = 1$, $\|G(e_4)\| = 1.67$ and $\|G(e_5)\| = 0.5$. This means the minimum value for $\|G(e_i)\|$ is $\|G(e_5)\| = 0.5$. That is, $G(e_5) = \{0, a, b, c\}$ is the closest accurate $BCI$-algebra.

**Remark 6.2.** (1) In [17], Ma applied rough soft set theory to $BCI$-algebra in order to find the nearest accurate $BCI$-algebra, but in the present paper, we try to find the nearest accurate $BCI$-algebra based on Shabir's soft rough set theory by means of two soft sets $\mathfrak{S}$ and $\mathfrak{T}$.

(2) Given a soft set $\mathfrak{S} = (F, A)$ over $X$, the decision maker can obtain different object by adjusting another soft set $\mathfrak{T} = (G, B)$. This means that by adjust different $\mathfrak{T} = (G, B)$, the decision maker can obtain the optimal one.

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