SOLVING A CLASS OF BOUNDARY VALUE PROBLEMS IN STRUCTURAL ENGINEERING AND FLUID MECHANICS USING HOMOTOPY PERTURBATION AND ADOMIAN DECOMPOSITION METHODS

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Abstract. In this article, the performance of two analytical methods known as the homotopy perturbation method (HPM) and Adomian decomposition method (ADM) on solving linear and nonlinear boundary value problems structural engineering and fluid mechanics are compared. In order to compare these mathematical models, various problems in inelastic and viscoelastic flows, deformation of beams, and plate deflection theory are chosen. In addition, the results of these two methods are compared with exact solutions to evaluate the precision and accuracy of these numerical methods.

Keywords: deformation of elastic beams, plate deflection theory, homotopy perturbation method, Adomian's decomposition method, boundary-value problems, exact solution.

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1. Introduction

Recently, several researches and works are devoted to develop analytical methods for solving nonlinear problems. In addition, many scientists try to compare the existed methods to evaluate the efficiency of them. This paper focuses on the analytical approximate solution of fourth-order equations when nonlinear boundary conditions contains third-order derivatives. Equation (1.1) presents The the general type of the equation when a integer n, $n \geq 2$, is fixed and positive. Thus, differential equation of order $2n$ is as follows:

$$y^{(2n)} + f(x, y) = 0 \tag{1.1}$$

The following equations define the boundary conditions

$$y^{(2j)}(a) = A_2 j, y^{(2j)}(b) = B_{2j}, j = 0, \cdots, n-1, \tag{1.2}$$

where $-\infty < a \leq x \leq b < \infty$, A_{2j}, B_{2j}, $j = 0, \cdots, n-1$ are finite constants.

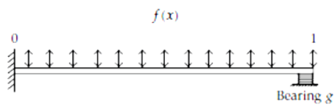

Figure 1: Beam on elastic bearing

In this modeling, it is assumed that the value of y is adequately different and a unique solution of (1.1) existed. One of main issue in this type of problem is plate-deflection theory. It is also crucial in fluid mechanics for modeling viscoelastic and inelastic flows [1], [2]. Usmani [1], [2] presented sixth order methods for solving linear differential equation $y^{(4)} + P(x)y = q(x)$ when the boundary conditions is as follows: $y(a) = A_0, y''(a) = A_2,\ y(b) = B_0,\ y''(b) = B_2$. According to the method described in [1], five diagonal linear systems are obtained and they contains p', p'', q', q'' at a and b, while the method described in [2] constrains nine diagonal linear systems.

In the other work, iterative solutions were proposed by Ma and Silva [3] for modeling beams (1.1) as elastic foundations. According to the classical beam theory, they found that if $u = u(x)$ represents the pattern of the deformed beam, the bending moment satisfies the following relation $M = -EIu''$, where E is the Young modulus of elasticity and I is the inertial moment. Since a load $f = f(x)$, induces the deformation, a free-body diagram, leads following $f = -v'$ and $v = M' = -EIu''$, where v represents the shear force and u is an elastic beam with length $L = 1$, with clamped at its left side ($x = 0$). The load f applied along its length and this causes deformations in the beam (Figure 1), Maand Silva [3] presented the following equation for this boundary value problem by assuming $EI = 1$:

$$u^{iv}(x) = f(x, u(x)),\ 0 < x < 1, \tag{1.3}$$

the boundary conditions are as below:

$$u(0) = u'(0) = 0, \tag{1.4}$$

$$u''(1) = 0,\ u'''(1) = g(u(1)), \tag{1.5}$$

where $f \in C([0,1]\times\mathbb{R})$ and $g \in C(\mathbb{R})$ are real functions. The boundary conditions are obtained through real modelling assumptions. $u'''(1)$ is obtained from the shear force at $x = 1$, and the second boundary condition equation (1.5) shows that the vertical force is equal to $g(u(1))$. Indeed, it presents a nonlinear relation between the vertical force and the displacement $u(1)$. In addition, this assumption $u''(1) = 0$ expresses that no bending moment occurs at $x = 1$, and the beam is assumed at rest on the bearing g.

Ma and Silva [3] presented solutions for this nonlinear equation by means of iterative procedures and explained that the accuracy of results is highly proportional to the integration method used in the iterative process.

Recently, nonlinear sciences have extensively developed, and several methods were proposed to solve differential nonlinear equations such as boundary value problems (BVPS). Among various approaches, four methods are sophisticated and robust: the perturbation parameter method (PPM), homotopy perturbation method (HPM), Adomian decomposition method (ADM) and the variational iteration method (VIM) [4]–[15]. In this research, the abilities and accuracy of two perturbation methods, homotopy perturbation method and Adomian decomposition method, for solving different forms of equation (1.1) with various boundary conditions are compared.

2. The methods

2.1. Basic idea of homotopy perturbation method

Homotopy perturbation method (HPM) is summarized by the following function:

$$A(u) - f(r) = 0,\ r \in \Omega \tag{2.1}$$

with the following boundary conditions of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0,\ r \in \Omega, \tag{2.2}$$

where A, B, $f(r)$ and Ω are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain Ω, respectively. In this method, it is assumed that the operator A contains two parts: a linear part L and a non-linear part $N(u)$. Therefore, equation (2.1) rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \tag{2.3}$$

By applying the homotopy technique, we have the following relation:

$$H(\nu, p) = (1-p)[L(\nu) - L(u_0)] + p[A(\nu) - f(r)] = 0,\ p \in [0,1],\ r \in \Omega, \tag{2.4}$$

or

$$H(\nu, p) = L(\nu) - L(u_0) + pL(u_0) + p[A(\nu) - f(r)] = 0, \tag{2.5}$$

where $p \in [0, 1]$ is an embedding parameter, while u_0 is an initial estimation of equation (2.1), to satisfy the boundary conditions. Therefore, equations (2.4) and (2.5) are transferred to the followings relations:

$$H(\nu, 0) = L(\nu) - L(u_0) = 0, \tag{2.6}$$

$$H(\nu, 1) = A(\nu) - f(r) = 0, \tag{2.7}$$

As the p value is changed from zero to unity, $\nu(r, p)$ is transformed from u_0 to $u(r)$. In the topology, this is known as a deformation, and $L(\nu)L(u_0)$ and $A(\nu)f(r)$ are recognized as Homotopy. According to the HPM method, it is assumed that the parameter p is a small parameter in the initial steps, and the solutions of equations (2.4) and (2.5) is a power series in p:

$$\nu = \nu_0 + p^1\nu_1 + p^2\nu_2 + \cdots, \tag{2.8}$$

when $p = 1$, it changes in the approximate solution of equation (2.8) to:

$$u = \lim_{p\to 1} \nu = \nu_0 + \nu_1 + \nu_2 + \cdots \tag{2.9}$$

In recent decades, the perturbation method and the homotopy method are combined and called the HPM. This method removes the disadvantages of the conventional perturbation methods while it contains all of their advantages. The series (2.9) is convergent for most of physical problems. However, the convergent rate highly related to the nonlinear operator $A(\nu)$. Furthermore, He (He, 1999b) proposed the following suggestions:

- The second derivative term of $N(\nu)$ must be small in comparison with ν because the parameter may be relatively large, i.e., $p \to 1$.
- The norm of $L^{-1}\frac{\partial N}{\partial \nu}$ must be lesser than one to elevate the converges of series.

2.2. Basic idea of Adomian decomposition method

The Adomian decomposition method transfers a general nonlinear equations in the following form:

$$Lu + Ru + Nu = g. \tag{2.10}$$

In this equation, the highest order derivative term is assigned to L which is assumed to be easily invertible, R is the linear differential operator of terms with less order than L, Nu refers the nonlinear terms and g is known as the source term. In the next step, the inverse operator L^1 is applied to the both sides of equation (2.10), and we obtain following relation with the given conditions:

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu), \tag{2.11}$$

where the function $f(x)$ represents the terms obtained from the integration of the source term $g(x)$ with given boundary conditions.

In nonlinear differential equations, it is supposed that the nonlinear operator $N(u) = F(u)$ could be represented by an infinite series of the so-called Adomian polynomials

$$F(u) = \sum_{m=0}^{\infty} A_m. \tag{2.12}$$

The polynomials series (A_m) are produced for all types of nonlinearity terms so that A_0 is only proportional to u_0, A_1 depends on u_0 and u_1, and so on. According to the Adomian polynomials introduced above, it is found that the summation of subscripts of the component of u for each term of A_m is equal to n. The Adomian method presents a following series for the solution $u(x)$

$$u = \sum_{m=0}^{\infty} u_m. \tag{2.13}$$

In addition, the infinite series of a Taylor expansion about u_0 is proposed for $F(u)$ as follows:

$$F(u) = F(u_0) + F'(u_0)(u - u_0) + F''(u_0)\frac{(u - u_0)}{2!} + F'''(u_0)\frac{(u - u_0)^2}{3!} + \cdots \tag{2.14}$$

Then, equation (2.13) is reformed as $uu_0 = u_1 + u_2 + u_3 + \cdots$, and replaced in equation (2.15). In the next step, two definitions of $F(u)$ presented in equation (2.14) and equation (2.12) become equals, results formulas for the Adomian polynomials in the form of

$$F(u) = A_1 + A_2 + \cdots = F(u_0) + F'(u_0)(u_1 + u_2 + \cdots) + F''(u_0)\frac{(u_1 + u_2 +)^2}{2!} + \cdots \tag{2.15}$$

Then, it is supposed that each terms in equation (2.15) is equivalent with Adomians polynomials A_0, A_1, A_2, A_3 and A_4 as follows:

$$A_0 = F(u_0) \tag{2.16}$$

$$A_1 = u_1 F'(u_0) \tag{2.17}$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) \tag{2.18}$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0) \tag{2.19}$$

$$A_4 = u_4 F'(u_0) + (\frac{1}{2!}u_2^2 + u_1 u_3)F''(u_0) + \frac{1}{2!}u_1^2 u_2 F'''(u_0) + \frac{1}{4!}u_1^4 F^{(iv)}(u_0). \tag{2.20}$$

3. Application of methods

In this section, various problems are solved with these two approaches and the result are compared with exact solution.

3.1. Application of HPM

Example 3.1. Equation (3.1) is chosen as our first linear boundary value problem:

$$u^{(4)}(x) = u(x) + 4e^x \tag{3.1}$$

with following boundary conditions

$$u(0) = 1,\ u'(0) = 2,\ u(1) = 2e,\ u'(1) = 3e. \tag{3.2}$$

It is clear that the exact solution of this problem is

$$u(x) = (1 + x)e^x. \tag{3.3}$$

In order to applied HPM for solving equation (3.1), the following processes are considered after the linear and nonlinear portions of the equation are separated. Thus, following equation is obtained by Homotopy method.

$$H(x,p) = (1-p)\left(\frac{d^4}{dx^4}u(x) - \frac{d^4}{dx^4}u_0(x)\right) + p\left(\frac{d^4}{dx^4}u(x) - u(x) - 4e^x\right). \tag{3.4}$$

Then, $\nu = \nu_0 + p\nu_1 + \cdots$, is replaced in to Equation (2.2) and the following equivalents are obtained:

$$p^1 : \frac{d^4}{dx^4}u_1(x) + \frac{d^4}{dx^4}u_0(x) - u_0(x) - 4e^x = 0, \tag{3.5}$$

$$p^2 : \frac{d^4}{dx^4}u_2(x) - u_1(x) = 0, \tag{3.6}$$

$$p^3 : \frac{d^4}{dx^4}u_3(x) - u_2(x) = 0, \tag{3.7}$$

$$p^4 : \frac{d^4}{dx^4}u_4(x) - u_3(x) = 0, \tag{3.8}$$

In order to solve this equation with HPM, we proposed an arbitrary initial approximation:

$$u_0(x) = ax^3 + bx^2 + cx + d. \tag{3.9}$$

By applying equation (3.9) in equations (3.5)-(3.8), we obtain the following:

$$\begin{aligned}
u_1(x) &= \frac{1}{840}ax^7 + \frac{1}{360}bx^6 + \frac{1}{120}cx^5 + 4e^x + \frac{1}{24}dx^4,\\
u_2(x) &= \frac{1}{6652800}ax^{11} + \frac{1}{1814400}bx^{10} + \frac{1}{362880}cx^9 + 4e^x + \frac{1}{40320}dx^8,\\
u_3(x) &= \frac{1}{217945728000}ax^{15} + \frac{1}{43589145600}bx^{14} + \frac{1}{6227020800}cx^{13}\\
&\quad +4e^x + \frac{1}{479001600}dx^{12},\\
u_4(x) &= \frac{1}{20274183401472000}ax^{19} + \frac{1}{3201186852864000}bx^{18}\\
&\quad +\frac{1}{355687428096000}cx^{17} + 4e^x + \frac{1}{20922789888000}dx^{16}.
\end{aligned}$$

According to this approach, all of the components were determined. In addition, HPM results:

$$u(x) = \lim_{p\to 1} \nu(t) = \nu_0(x) + \nu_1(x) + \cdots \tag{3.10}$$

Then, the values of $u_0(x), u_1(x), u_2(x), u_3(x)$ and $u_4(x)$ are replaced to equation (3.10) and this yields:

$$\begin{aligned} u(x) = {} & ax^3 + bx^2 + cx + d + \frac{1}{840}ax^7 + \frac{1}{360}bx^6 + \frac{1}{120}cx^5 + 16e^x \\ & + \frac{1}{24}dx^4 + \frac{1}{6652800}ax^{11} + \frac{1}{1814400}bx^{10} + \frac{1}{362880}cx^9 + \frac{1}{40320}dx^8 \\ & + \frac{1}{217945728000}ax^{15} + \frac{1}{43589145600}bx^{14} + \frac{1}{6227020800}cx^{13} \\ & + \frac{1}{479001600}dx^{12} + \frac{1}{20274183401472000}ax^{19} + \frac{1}{3201186852864000}bx^{18} \\ & + \frac{1}{355687428096000}cx^{17} + \frac{1}{20922789888000}dx^{16}. \end{aligned} \tag{3.11}$$

As the boundary conditions (3.2) applied into $u(x)$, all of coefficients are determined:

$$a = -1.537,\ b = -6.756,\ c = -14.,\ d = -15. \tag{3.12}$$

Finally, the following approximate solution is obtained:

$$\begin{aligned} u(x) = {} & -1.537x^3 - 6.756x^2 - 14x - 15 - 0.0018298x^7 - 0.018767x^6 \\ & -0.11667x^5 + 16.e^x - 0.62500x^4 - 2.310310^{-7}x^11 - 0.0000037236x^{10} \\ & -0.000038580x^9 - 0.00037202x^8 - 7.052210^{-12}x^{15} - 1.5499x^210^{-10}x^{14} \\ & -2.248310^{-9}x^{13} - 3.131510^{-8}x^12 - 7.581110^{-17}x^{19} - 2.110410^{-15}x^{18} \\ & -3.936010^{-14}x^{17} - 7.169210^{-13}x^{16}, \end{aligned}$$

Example 3.2. Equation (3.13) is another example of linear boundary value problem:

$$u^{(4)}(x) = u(x) + u''(x) + (x-3)e^x, \tag{3.13}$$

with these boundary conditions:

$$u(0) = 1,\ u'(0) = 0,\ u(1) = 0,\ u'(1) = -e. \tag{3.14}$$

Equation (3.15) is the exact solution for this problem

$$u(x) = (1-x)e^x, \tag{3.15}$$

As mentioned in the previous example, the linear and nonlinear parts of the equation is initially separated to solve equation (3.13) by means of HPM. Hence, the following equation is formed:

$$H(x,p) = (1-p)(u^{(4)}(x) - u_0^{(4)}(x)) + p(u^{(4)}(x) - u(x) - u^{(2)}(x) - (x-3)e^x). \tag{3.16}$$

Then, this $\nu = \nu_0 + p\nu_1 + \cdots$ substituted into equation (3.16) and we obtain the following:

(3.17) $p^0 : 0,$

(3.18) $p^1 : \left(-e^x x + \frac{d^4}{dx^4}u_1(x) + \frac{d^4}{dx^4}u_0(x) - u_0(x) - \left(\frac{d^2}{dx^2}u_0(x)\right) + 3e^x\right) = 0,$

The following polynomial is chosen as an arbitrary initial approximation:

$$u_0(x) = ax^3 + bx^2 + cx + d. \tag{3.19}$$

By solving equations (3.18) and (3.19), we obtain the following:

$$u_1(x) = e^x x - 7e^x + \frac{1}{840}ax^7 + \frac{1}{360}bx^6 + \frac{1}{120}cx^5 + \frac{1}{20}ax^5 + \frac{1}{24}(d+2b)x^4. \tag{3.20}$$

In the same way, the rest of components were determined by using the Maple package. Hence, we found that:

$$u(x) = \lim_{p\to 1} \nu(t) = \nu_0(x) + \nu_1(x) + \cdots \tag{3.21}$$

In addition, the values of $u_0(x)$, $u_1(x)$ obtained from equations (3.19) and (3.20) is substituted in equation(3.21) and this yields:

$$\begin{aligned} u(x) &= ax^3 + bx^2 + cx + d + e^x x - 7e^x \\ &\quad + \frac{1}{840}ax^7 + \frac{1}{360}bx^6 + \frac{1}{12}cx^5 + \frac{1}{20}ax^5 + \frac{1}{24}(d+2b)x^4. \end{aligned} \tag{3.22}$$

As the boundary conditions (3.14) are applied into $u(x)$, all of the coefficients are defined:

$$a = \frac{7730702}{323149} - \frac{2950080}{323149}e,\ b = -\frac{11761596}{323149} + \frac{4640400}{323149}e,\ c = 6,\ d = 8 \tag{3.23}$$

Thus, the final results are obtained as follows:

$$\begin{aligned} u(x) = &\left(\frac{7730702}{323149} - \frac{2950080}{323149}e\right)x^3 + \left(-\frac{11761596}{323149} + \frac{4640400}{323149}e\right)x^2 \\ &+6x + 8 + e^x x - 7e^x + \frac{1}{840}\left(\frac{7730702}{323149} - \frac{2950080}{323149}e\right)x^7 \\ &+\frac{1}{360}\left(-\frac{11761596}{323149} + \frac{4640400}{323149}e\right)x^6 + \frac{1}{20}x^5 \\ &+\frac{1}{20}\left(\frac{7730702}{323149} - \frac{2950080}{323149}e\right)x^5 + \frac{1}{24}\left(-\frac{20938000}{323149} + \frac{9280800}{323149}e\right)x^4, \end{aligned}$$

3.2. Application of ADM

Example 3.3. In this example, equation (3.1) are solved by means of ADM. Hence, the equation is reformed as

$$u(x) = L_{4x}u - 4e^x, \tag{3.24}$$

where $L_t = \dfrac{\partial}{\partial t},\ L_{2t} = \dfrac{\partial^2}{\partial t^2},\ L_{3t} = \dfrac{\partial^3}{\partial t^3}, \cdots$

According to the decomposition method, the approximate solution has the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{3.25}$$

Thus,

$$u(x) = L_{4x}\left(\sum_{n=0}^{\infty} u_n(x)\right) - 4e^x, \tag{3.26}$$

so, we find:

$$u_1(x) = L_{4x}(u_0) - 4e^x = -4e^x, \tag{3.27}$$

As the values of $u_0(x)$, $u_1(x)$ obtained from equations (3.9) and (3.27) substituted into equation (3.25), it yields the following:

$$u(x) = ax^3 + bx^2 + cx + d - 4e^x. \tag{3.28}$$

Then, the boundary conditions (2.1) applied into $u(x)$, and the following coefficients are obtained:

$$a = 16 - 5e,\ b = -27 + 11e,\ c = 6,\ d = 5 \tag{3.29}$$

Thus, the solution $u(x)$ is determined:

$$u(x) = (16 - 5e)x^3 + (-27 + 11e)x^2 + 6x + 5 - 4e^x, \tag{3.30}$$

Example 3.4. Here, equation (3.13) is solved by means of ADM. Hence, the equation is presented as

$$u(x) = L_{4x}u - L_{2x}u - (x - 3)e^x, \tag{3.31}$$

where $L_t = \dfrac{\partial}{\partial t},\ L_{2t} = \dfrac{\partial^2}{\partial t^2},\ L_{3t} = \dfrac{\partial^3}{\partial t^3}, \cdots$

In the decomposition method, the approximate result is as follows:

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{3.32}$$

Thus,

$$u(x) = L_{4x}\left(\sum_{n=0}^{\infty} u_n(x)\right) - L_{2x}\left(\sum_{n=0}^{\infty} u_n(x)\right) - (x - 3)e^x. \tag{3.33}$$

So, we find the following:

$$u_1(x) = L_{4x}(u_0) - L_{2x}(u_0) - (x-3)e^x = -6ax - 2b - (x-3)e^x, \tag{3.34}$$

$$u_2(x) = L_{4x}(u_1) - L_{2x}(u_1) - (x-3)e^x = -2e^x - (x-3)e^x. \tag{3.35}$$

Furthermore, the values of $u_0(x), u_1(x), u_2(x)$ obtained from equations (3.19), (3.34) and (3.35) applied into equation (3.32) lead to the following:

$$u(x) = ax^3 + bx^2 + cx + d - 6ax - 2b - 2(x-3)e^x - 2e^x, \tag{3.36}$$

As the boundary conditions (3.14) applied into $u(x)$ and the coefficients are obtained; the following solution $u(x)$ is resulted:

$$u(x) = (-8+3e)x^3 + (13-5e)x^2 + (-50+18e)x - 3 - 6(-8+3e)x - 2(x-3)e^x - 2e^x.$$

Tables (1) and (2) compare the approximate solutions with exact solution.

Table 1: Comparison between HPM & ADM with exact solution for equation (3.1)

x	HPM	ADM	Exact solution	Error(HPM)	Error(ADM)
0	1.0000000000	1.0000000000	1.0000000000	0.0000000000	0.0000000000
0.1	1.2135895620	1.2107359200	1.2156880100	0.0020984480	0.0049520900
0.2	1.4589237340	1.4497016990	1.4656833100	0.0067595760	0.0159816110
0.3	1.7429453950	1.7266957310	1.7548164500	0.0118710550	0.0281207190
0.4	2.0727440300	2.0510270400	2.0885545770	0.0158105470	0.0375275370
0.5	2.4555841900	2.4314638000	2.4730819060	0.0174977160	0.0416181060
0.6	2.8989385100	2.8761764650	2.9153900800	0.0164515700	0.0392136150
0.7	3.4105262700	3.3926748900	3.4233796020	0.0128533320	0.0307047120
0.8	3.9983578000	3.9877388780	4.0059736700	0.0076158700	0.0182347920
0.9	4.6707856300	4.6673413760	4.6732459110	0.0024602810	0.0059045350
1	5.4365636600	5.4365636569	5.4365636569	−0.0000000031	0.0000000000

Table 2: Comparison between HPM & ADM with exact solution for equation (3.13)

x	HPM	ADM	Exact solution	Error(HPM)	Error(ADM)
0	1.0000000000	1.0000000000	1.0000000000	0.0000000000	0.0000000000
0.1	0.9998588457	0.9938902421	0.9946538262	−0.0052050195	0.0007635841
0.2	0.9937091960	0.9746323273	0.9771222064	−0.0165869896	0.0024898791
0.3	0.9737227060	0.9404739520	0.9449011656	−0.0288215404	0.0044272136
0.4	0.9330798160	0.8891236830	0.8950948188	−0.0379849972	0.0059711358
0.5	0.8659651150	0.8176672130	0.8243606355	−0.0416044795	0.0066934225
0.6	0.7675654620	0.7224719740	0.7288475200	−0.0387179420	0.0063755460
0.7	0.6340703500	0.5990785600	0.6041258121	−0.0299445379	0.0050472521
0.8	0.4626739700	0.4420772650	0.4451081856	−0.0175657844	0.0030309206
0.9	0.2515788300	0.2449677980	0.2459603111	−0.0056185189	0.0009925131
1	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000

Moreover, Figures 1 and 2 illustrate these values and show a remarkable agreement between these methods. Of course, the accuracy of solutions could be significantly enhanced.

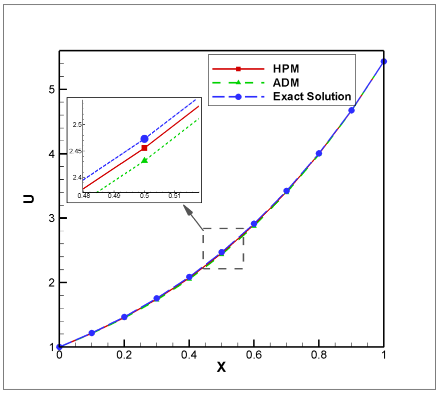

Fig. 1 Comparison between different solutions for equation (3.1)

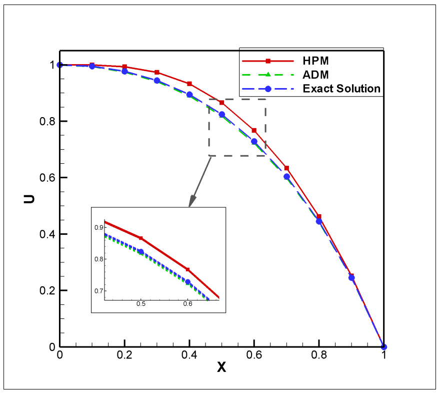

Fig. 2 Comparison between different solutions for equation (3.13)

4. Conclusion

In this study, the results of homotopy perturbation method (HPM) and Adomian decomposition method (ADM) are compared and it is found that both approaches are remarkably effective for solving boundary value problems. In these methods, a fourth-order differential equation was solved for specific engineering applications to prove their effectiveness. In this research, boundary conditions are chosen base on real physical condition of the problem. Also, the results and error of each method are compared with exact solutions to evaluate the accuracy of both approaches. Our findings show that the HPM and ADM are recommended for solving partial differential equations with minimum calculation process.

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